Amplitude Modulation in Traveling Wave Tube Phase Modulator

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Abstract

Several problems of the amplitude modulation caused by the traveling wave tube used as a microwave phase modulator are discussed.

Using Pierce's linear theory, some expressions for phase modulation and the amplitude modulation which occurs simultaneously are derived. From the derived composite modulation formula, various effects of the amplitude modulation on amplitude of carrier and each side band are calculated. To decrease amplitude modulation, experimental studies are performed by using the 1st anode to compensate it and a satisfactory improvement has been obtained.

1. Introduction

The traveling wave tube is a microwave electronic tube possessing a special quality of tremendous wide band amplification which is now widely used in microwave relay system. Many other applications such as of oscillator and modulator are being studied. As a phase modulator, signal voltage is usually applied by adding it to the helix potential and this method has a good sensitivity and a simple circuit composition. However, because of the curved characteristic between the helix voltage and output power, this phase modulator is usually accompanied by an undesirable amplitude modulation.

In a microwave relay system, the amplitude modulation is eliminated by using a heterodyne detector and an amplitude limiter in i-f circuit plays that role. But for a special purpose, such as an experiment of microwave spectroscopy, a FM signal sometimes is applied immediately to the frequency discriminator operated in a microwave region. In such cases, this undesirable amplitude modulation gives a serious effect on its observation.

When we wish to observe the absorption spectrum in microwave region, as shown in Fig. 1(a), by the source frequency modulation, a microwave, which is slightly fre-

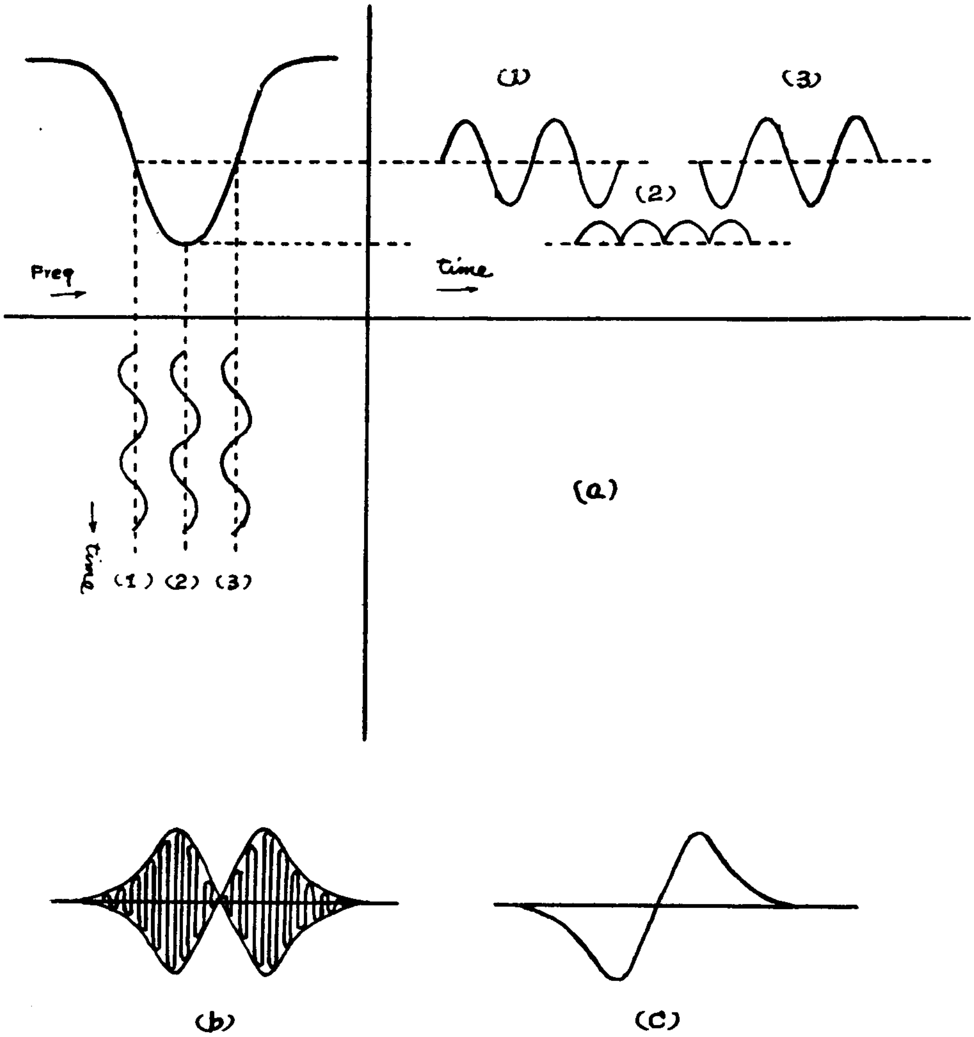

Fig. 1. Observation of absorption spectrum by the source frequency modulation

quency modulated, is transmitted into an absorption cell in which measuring gas is sealed off. Therefore, the absorption line acts as a discriminator and converts a frequency modulated signal into an amplitude modulated signal as illustrated in Fig. 1(a) and (b). When the degree of modulation is sufficiently small and a narrow band amplifier and a phase sensitive detector are used, the receiver detects the first derivative of the absorption line contour as shown in Fig. 1(c). From this figure, we can decide its center frequency.

However, if the amplitude modulation with the same modulation frequency occurs simultaneously in the modulator, the two detected signals having same frequency are applied to the phase sensitive detector. Of course, there are phase differences of 90 degrees between the detected signals of frequency modulation and those of amplitude modulation and they can be separated from one another. But, if there is a missetting, however small it may be, in the phase sensitive detector, a considerable error will occur in deciding the center frequency. In order to prevent this error, the amplitude modulation must be suppressed to the smallest possible extent.

In this paper, using Pierce's linear theory, the special quality of phase modulation in the traveling wave tube and the amplitude modulation which occurs simultaneously are discussed from its amplifying function and for the purpose of decreasing the amplitude modulation, some experimental studies of compensation are performed on a 7 kMC band tube by using the 1st anode.

2. Phase Modulation

In this section we will consider the phase modulated wave produced when a signal voltage is added to the helix.

The output of the traveling wave tube is mainly due to an increasing wave excited by an electron beam. The increasing wave at the output end of a tube ($z=L$), E_L, is expressed from Pierce's linear theory, as

$$\begin{aligned} E_L &= E_0 \exp\{-j\beta_e(1+jC\delta)L\} \\ &= E_0 \exp\{-j\beta_e(1-Cy)L\} \exp(\beta_e CxL) \end{aligned} \tag{1}$$

where E_0 is the amplitude of the increasing wave at the input terminal. δ is the increment of propagation constant of increasing wave against the beam phase constant $\beta_e(=\frac{\omega}{v}$: v is a beam velocity) and, x and y represent its real- and imaginary parts respectively.

L is the circuit length of the tube.

C is the gain parameter defined as the ratio of circuit-to-beam impedance.

$$C^3 = Z_0 I_0 / 4V_0 . \tag{2}$$

For a typical tube C is about 0.02.

From the energy relation, $\frac{1}{2}mv^2 = eV_0$, the following equation can be obtained at once for the small change of helix voltage, ΔV_0.

$$\frac{\Delta\beta_e}{\beta_e} = -\frac{1}{2}\frac{\Delta V_0}{V_0}. \tag{3}$$

The circuit wave phase constant in the absence of an electron beam, β_1, is related to the beam phase constant, β_e, as

$$\beta_1 = \beta_e(1+Cb) \tag{4}$$

where b is a velocity parameter.

Pierce gives many curves relating b to x and y as a function of the circuit loss parameter, d, and the space charge parameter, QC.

As β_1 is independent of beam current, we can obtain the following relation from eq. (4) for the small change of helix voltage.

$$\Delta\beta_1 = 0 = \Delta\beta_e(1+Cb)+\beta_e(C\cdot\Delta b+b\cdot\Delta C)$$

$$\frac{\Delta\beta_e}{\beta_e} = -\frac{(C\cdot\Delta b+b\cdot\Delta C)}{1+Cb}. \tag{5}$$

In the normal operation, $Cb \ll 1$ and, moreover, $\frac{\Delta b}{b} \gg \frac{\Delta C}{C}$ is valid for $\frac{\Delta V_0}{V_0} \ll 1$. Using these conditions and eq. (3), we obtain

$$\frac{1}{2}\frac{\Delta V_0}{V_0} = C\cdot\Delta b. \tag{6}$$

Now, let us consider the phase angle variation of an increasing wave. The phase constant of an increasing wave, β, is given from eq. (1).

$$\beta = \beta_e(1-Cy). \tag{7}$$

For the small change of helix voltage,

$$\Delta\beta = \Delta\beta_e(1-Cy)+\beta_e(-C\cdot\Delta y-y\cdot\Delta C).$$

By the same assumption used in eq. (6), $\frac{\Delta y}{y} \gg \frac{\Delta C}{C}$ is valid. Therefore,

$$\Delta\beta = \Delta\beta_e(1-Cy)+\beta_e(-C\cdot\Delta y). \tag{8}$$

A linear approximation to Pierce's curves for an increasing wave relating y and b is

$$\Delta y = -(0.4+0.09QC)\cdot\Delta b. \tag{9}$$

Then, from eq. (5), (8) and (9)

$$\Delta\beta = \frac{\Delta V_0}{2V_0}\beta_e[(1+Cb)(0.4+0.09QC)+Cy-1]. \tag{10}$$

The total phase shift is given by:

$$\Delta\phi = \Delta\beta\cdot L = \frac{\Delta V_0}{2V_0}\beta_e[(1+Cb)(0.4+0.09QC)+Cy-1]L. \tag{11}$$

Hence, bC, QC and Cy are very small compared with unity, eq. (11) reduces to the approximate formula of

$$\Delta\phi \fallingdotseq -0.3\beta_e\frac{\Delta V_0}{V_0}L \quad \text{radians.} \tag{12}$$

If the sinusoidal wave voltage with amplitude ΔV_0 and angular frequency p_m is supplied to the helix voltage, we can obtain the following phase modulated wave:

$$E_L = |E_L| \sin\{\omega_0 t + \Delta\phi \sin p_m t\}$$
$$= |E_L| \sin\{\omega_0 t + (0.3\beta_e \frac{\Delta V_0}{V_0} L) \sin p_m t\}\,. \tag{13}$$

As the non-linearity of modulation characteristic in FM relay link causes a phase distortion in telephony signal, a high degree of linearity is always required. Many experiments have been carried out on the traveling wave tube and its modulation characteristic, i.e. linearity and sensitivity are measured.

Some results of experiments carried out in our laboratory are shown in Fig. 2 and Fig. 3. Fig. 2 shows the modulation characteristic obtained by the experiments on a 7 kMC band tube and the linearity is retained until about 9 radians of maximum phase deviation. The modulation sensitivity is 0.17 radians/volt. It shows a good agreement with the calculated value derived from eq. (12), 0.165 radians/volt. The

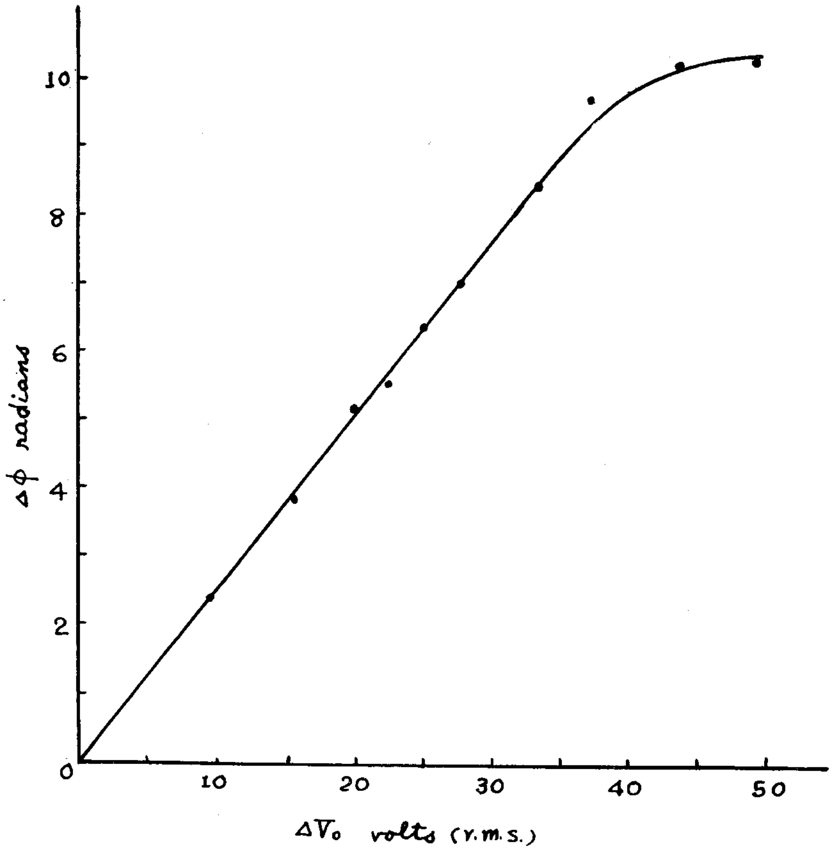

Fig. 2. Measurement of phase modulation sensitivity for 7 kMC band tube.

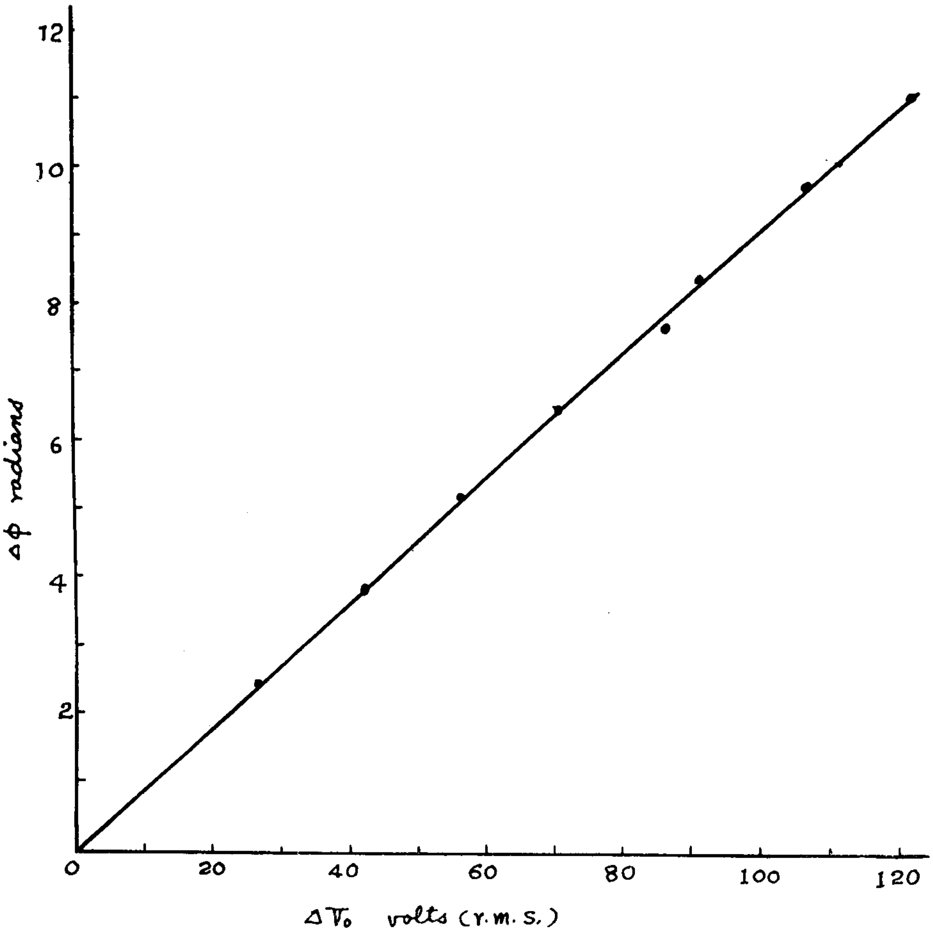

Fig. 3. Measurement of phase modulation sensitivity for 4 kMC band tube.

operating condition of this tube is $f_0=8$ kMC, $V_0=940$ volts and $L=0.18$ m.

The results of measurement carried on a 4 kMC band tube are shown in Fig. 3. In this case the tube is operated as a frequency multiplier and phase modulator at the same time. The operating conditions are: input frequency, 2430 MC; output frequency, 4860 MC; $V_0=1900$ volts and $L=0.3$ m. Although the theoretical study is very difficult in this case, the experimental results show that the linearity is secured at least until 11 radians and its sensitivity is 0.066 radians/volt.

3. Phase Modulated Wave affected by Amplitude Modulation

The amplitude modulation which occurs in the traveling wave tube phase modulator has already been partially explained in section 1. In this section its magnitude and effect on the phase modulated wave are discussed by using Pierce's linear theory.

Taking the part of amplitude from eq. (1)

$$|E_L| = |E_0| \exp(\beta_e CxL) . \tag{14}$$

Now, pertaining to the region where the maximum output can be obtained for the changes of helix voltage, we approximate the $b-x$ curve, given by Pierce, to the following quadratic equation

$$x = x_0 - k(b - b_0)^2 \tag{15}$$

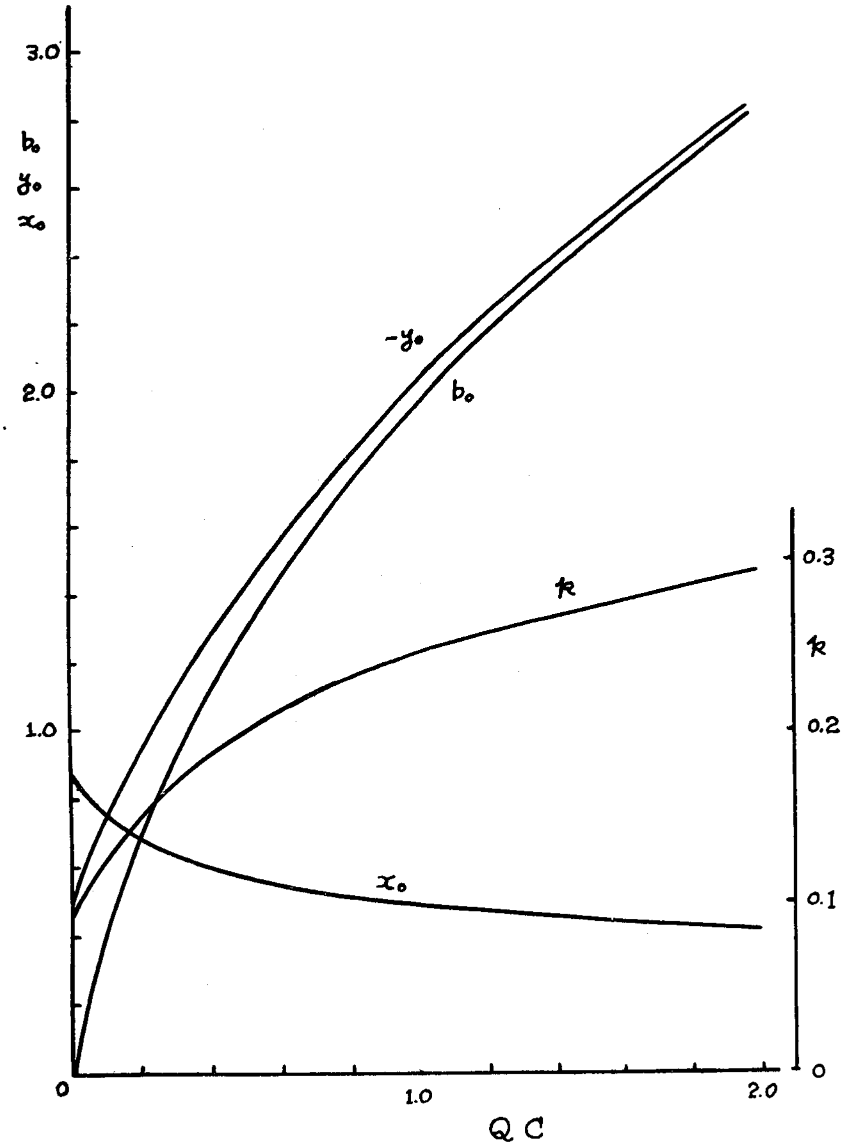

Fig. 4. Changes of b_0, x_0, y_0 and k with QC.

where b_0 and x_0 are constants and they are the values of b and x corresponding to the helix voltage V_0 which gives a maximum output. k is a proportional constant and is determined by the shape of the quadratic curve, i.e. a function of circuit loss parameter, d, and space charge parameter, QC.

In the case of $d=0$, the value obtained by calculation, in which b_0, x_0, y_0 and k are taken as functions of QC, is given in Fig. 4, where k is a value determined from the curvature at the point $b=b_0$.

Substituting eq. (15) into eq. (14)

$$|E_L| = |E_0| \exp[\beta_e\{x-k(b-b_0)^2\}L]. \tag{16}$$

Therefore, when the sinusoidal wave with amplitude ΔV_0 and angular frequency p_m is added to the helix voltage, the amplitude modulation will occur in the phase modulated wave derived in the preceeding section

$$|E_L| = |E_0| \exp[(\beta_e+\Delta\beta_e \sin p_m t)C\{x_0-k(b-b_0+\Delta b \sin p_m t)^2\}L]$$

and if the modulating voltage is small,

$$\begin{aligned} |E_L| &\fallingdotseq |E_0| \exp[\beta_e C\{x_0-k(b-b_0+\Delta b \sin p_m t)^2\}L] \\ &= |E_0| \exp[\beta_e C x_0 L] \exp\left[-\beta_e Ck\left\{(b-b_0)^2+\frac{(\Delta b)^2}{2}\right\}L\right] \\ &\quad \times \exp\left[-\beta_e Ck\{2(b-b_0)\Delta b \sin p_m t-\frac{(\Delta b)^2}{2}\cos 2p_m t\}L\right]. \end{aligned}$$

Using eq. (6), we can obtain for operating helix voltage V,

$$\begin{aligned} |E_L| &= |E_0| \exp(\beta_e C x_0 L) \exp\left[-\frac{\beta_e}{4C}k\left\{\left(\frac{V-V_0}{V_0}\right)^2+\frac{1}{2}\left(\frac{\Delta V_0}{V_0}\right)^2\right\}L\right] \\ &\quad \times \exp\left[-\frac{\beta_e}{4C}k\left\{2\left(\frac{V-V_0}{V_0}\right)\left(\frac{\Delta V_0}{V_0}\right)\sin p_m t-\frac{1}{2}\left(\frac{\Delta V_0}{V_0}\right)^2\cos 2p_m t\right\}L\right]. \end{aligned} \tag{17}$$

Moreover, assuming $\dfrac{V-V_0}{V_0} \ll 1$

$$\begin{aligned} |E_L| = |E_0| \exp(\beta_e C x_0 L) \Bigg[1 &- \frac{\beta_e}{4C}k\left\{\left(\frac{V-V_0}{V_0}\right)^2+\frac{1}{2}\left(\frac{\Delta V_0}{V_0}\right)^2\right\}L \\ &- \frac{\beta_e}{4C}k\left\{2\left(\frac{V-V_0}{V_0}\right)\left(\frac{\Delta V_0}{V_0}\right)\sin p_m t-\frac{1}{2}\left(\frac{\Delta V_0}{V_0}\right)^2\cos 2p_m t\right\}L\Bigg]. \end{aligned} \tag{18}$$

Changing the notations for the sake of simplicity as:

$$\left.\begin{aligned} g_0 &= \exp(\beta_e C x_0 L) \\ a^2 &= \frac{\beta_e}{4C}k\left(\frac{V-V_0}{V_0}\right)^2 L \\ b^2 &= \frac{\beta_e}{4C}k\left(\frac{\Delta V_0}{V_0}\right)^2 L \end{aligned}\right\} \tag{19}$$

we obtain:

$$|E_L| = g_0|E_0|\left[\left\{1-\left(a^2+\frac{b^2}{2}\right)\right\}-2ab \sin p_m t+\frac{b^2}{2}\cos 2p_m t\right]. \tag{20}$$

Therefore, substituting eq. (20) into eq. (13), the following composite modulation wave can be obtained.

$$E_L=g_0|E_0|\left[\left\{1-\left(a^2+\frac{b^2}{2}\right)\right\}-2ab\sin p_m t+\frac{b^2}{2}\cos 2p_m t\right]\sin\{\omega_0 t+\Delta\phi\sin p_m t\}\,. \quad (21)$$

Expanding eq. (21) with Bessel functions

$$\left.\begin{aligned}\frac{E_L}{g_0|E_0|}=&\left[\left\{1-\left(a^2+\frac{b^2}{2}\right)\right\}J_0+\frac{b^2}{2}J_2\right]\sin\omega_0 t-2abJ_1\cos\omega_0 t\\&+\frac{1}{2}\left[2\left\{1-\left(a^2+\frac{b^2}{2}\right)\right\}J_1+\frac{b^2}{2}(J_{-1}+J_3)\right]\{\sin(\omega_0+p_m)t-\sin(\omega_0-p_m)t\}\\&+ab(J_0-J_2)\{\cos(\omega_0+p_m)t-\cos(\omega_0-p_m)t\}\\&+\frac{1}{2}\left[2\left\{1-\left(a^2+\frac{b^2}{2}\right)\right\}J_2+\frac{b^2}{2}(J_0+J_4)\right]\{\sin(\omega_0+2p_m)t+\sin(\omega_0-2p_m)t\}\\&+ab(J_1-J_3)\{\cos(\omega_0+2p_m)t+\cos(\omega_0-2p_m)t\}\\&+\cdots\cdots.\end{aligned}\right\} \quad (22)$$

The arguments of the Bessel functions are $\Delta\phi$. Then, the amplitude of carrier and each side band are given by:

$$\left.\begin{aligned}&\text{carrier} && \sqrt{\left[\left\{1-\left(a^2+\frac{b^2}{2}\right)\right\}J_0+\frac{b^2}{2}J_2\right]^2+4a^2b^2J_1^2}\\&\text{the first side band} && \frac{1}{2}\sqrt{\left[2\left\{1-\left(a^2+\frac{b^2}{2}\right)\right\}J_1+\frac{b^2}{2}(J_{-1}+J_3)\right]^2+4a^2b^2(J_0-J_2)^2}\\&\text{the second side band} && \frac{1}{2}\sqrt{\left[2\left\{1-\left(a^2+\frac{b^2}{2}\right)\right\}J_2+\frac{b^2}{2}(J_0+J_4)\right]^2+4a^2b^2(J_1-J_3)^2}\\&\text{etc.}\end{aligned}\right\} \quad (23)$$

The amplitude of the carrier and of each side band in the pure phase modulated wave are given by the Bessel functions $J_0(\Delta\phi)$, $J_1(\Delta\phi)$, $J_2(\Delta\phi)$, ······ respectively. However, the amplitude modulation will change these amplitude as shown in eq. (23).

For instance, the calculated results by eq. (23) performed on a 7 kMC band tube are shown in Fig. 5. The operating helix voltage is $V=V_0$ (namely, $a=0$) and the assumptions of $k=0.1$ and $C=0.02$ are used. These results show good qualitative agreements with the experimental results on 4 kMC band tube measured by W. J. Bray.

Measurement of modulation sensitivity is usually performed by separating each side band and measuring the modulating voltage which makes their amplitude zero. In the composite modulation, although the amplitude of each side band differs considerably from the Bessel function, the arguments which make each amplitude zero, do not vary so much. In other word, it goes to show that, even though the phase modulated wave is accompanied with amplitude modulation, measurement of sensitivity is scarecely affected.

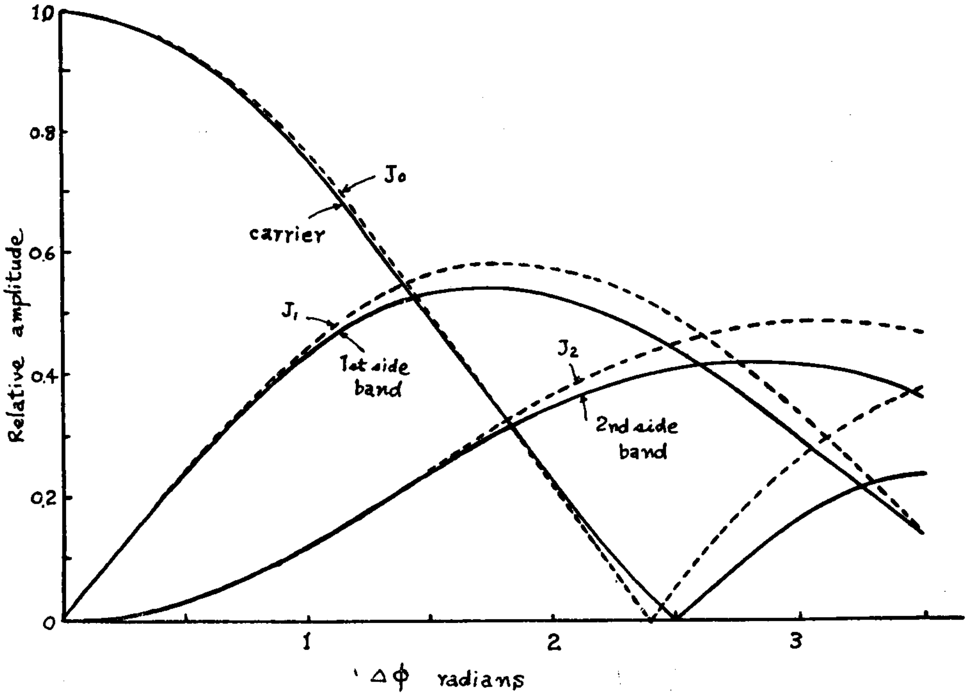

Fig. 5. Effects of amplitude modulation on carrier and each side band.

4. Compensation of the Amplitude Modulation Using the 1st Anode

As shown in the preceeding section, when the signal voltage is applied to the helix voltage, the composite wave is obtained as in eq. (21). And this undesirable amplitude modulation sometimes gives a serious effect as explained in section 1.

For the purpose of decreasing amplitude modulation, a compensating circuit may be contrived in which other electrode, which affects only on its amplitude, is used. Therefore, we performed an experiment of compensation, using the 1st anode as compensating electrode. Before describing details of the experiment, let us explain the method of its operation.

Assuming that, for the change of the 1st anode voltage, a beam current is varied in accordance with the Langmuir law,

$$I_0 = KV_1^{\frac{3}{2}}. \tag{24}$$

Hence

$$\frac{\Delta I_0}{I_0} = \frac{3}{2}\frac{\Delta V_1}{V_1}. \tag{25}$$

Using eq. (2)

$$\frac{\Delta C}{C} = \frac{1}{2}\frac{\Delta V_1}{V_1} \quad \text{or} \quad \Delta C = \frac{C}{2}\frac{\Delta V_1}{V_1}. \tag{26}$$

Therefore, when the sinusoidal voltage with amplitude ΔV_1 and angular frequency

p_m is imposed to the 1st anode voltage, a gain parameter, C, changes

$$C+\Delta C=C\left(1+\frac{1}{2}\frac{\Delta V_1}{V_1}\sin p_m t\right). \tag{27}$$

Substituting into eq. (17)

$$|E_L|=|E_0|\exp\left\{\beta_e C\left(1+\frac{1}{2}\frac{\Delta V_1}{V_1}\sin p_m t\right)x_0 L\right\}\exp\left[-\frac{\beta_e}{4C}k\left\{\left(\frac{V-V_0}{V_0}\right)^2+\frac{1}{2}\left(\frac{\Delta V_0}{V_0}\right)^2\right\}L\right]$$
$$\times\exp\left[-\frac{\beta_e}{4C}k\left\{2\left(\frac{V-V_0}{V_0}\right)\left(\frac{\Delta V_0}{V_0}\right)\sin p_m t-\frac{1}{2}\left(\frac{\Delta V_0}{V_0}\right)^2\cos 2p_m t\right\}L\right]. \tag{28}$$

Here, the higher terms are neglected by assuming $\frac{V-V_0}{V_0}\ll 1$ and $\frac{\Delta V_0}{V_0}\ll 1$.

Therefore, to compensate the frequency component of p_m alone in the amplitude modulation, all we need is to impose the voltage on the 1st anode given by

$$\frac{\Delta V_1}{V_1}=\frac{k}{x_0}\frac{1}{C^2}\frac{V-V_0}{V_0}\cdot\frac{\Delta V_0}{V_0}. \tag{29}$$

Similarly, to compensate the $2p_m$ frequency component we can simply impose the following voltage

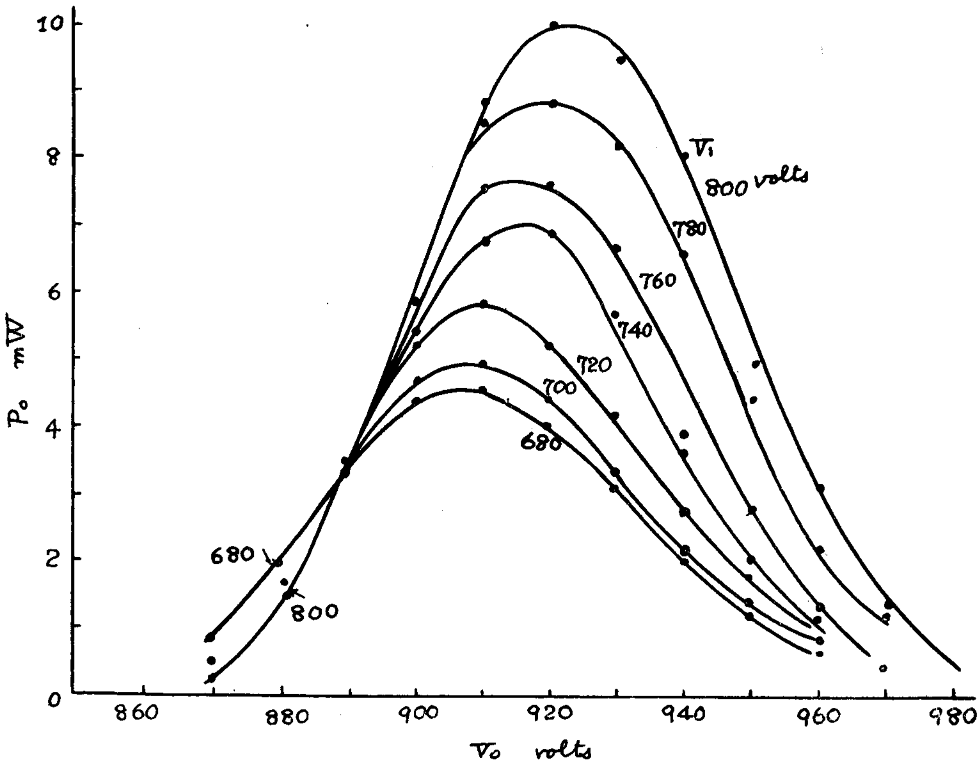

Fig. 6. Changes of output power with helix voltage.

$$\frac{\Delta V_1'}{V_1}=\frac{k}{4x_0}\cdot\frac{1}{C^2}\left(\frac{\Delta V_0}{V_0}\right)^2 . \qquad (30)$$

The experiments of compensation are performed on a 7 kMC band tube. The input signal is sufficiently low so that the tube can be regarded as having operated according to the small signal theory. At first, in order to determine the effect of the 1st anode voltage on the output signal, measurements of output/helix voltage characteristic are made for several 1st anode voltages as a parameter. These results are

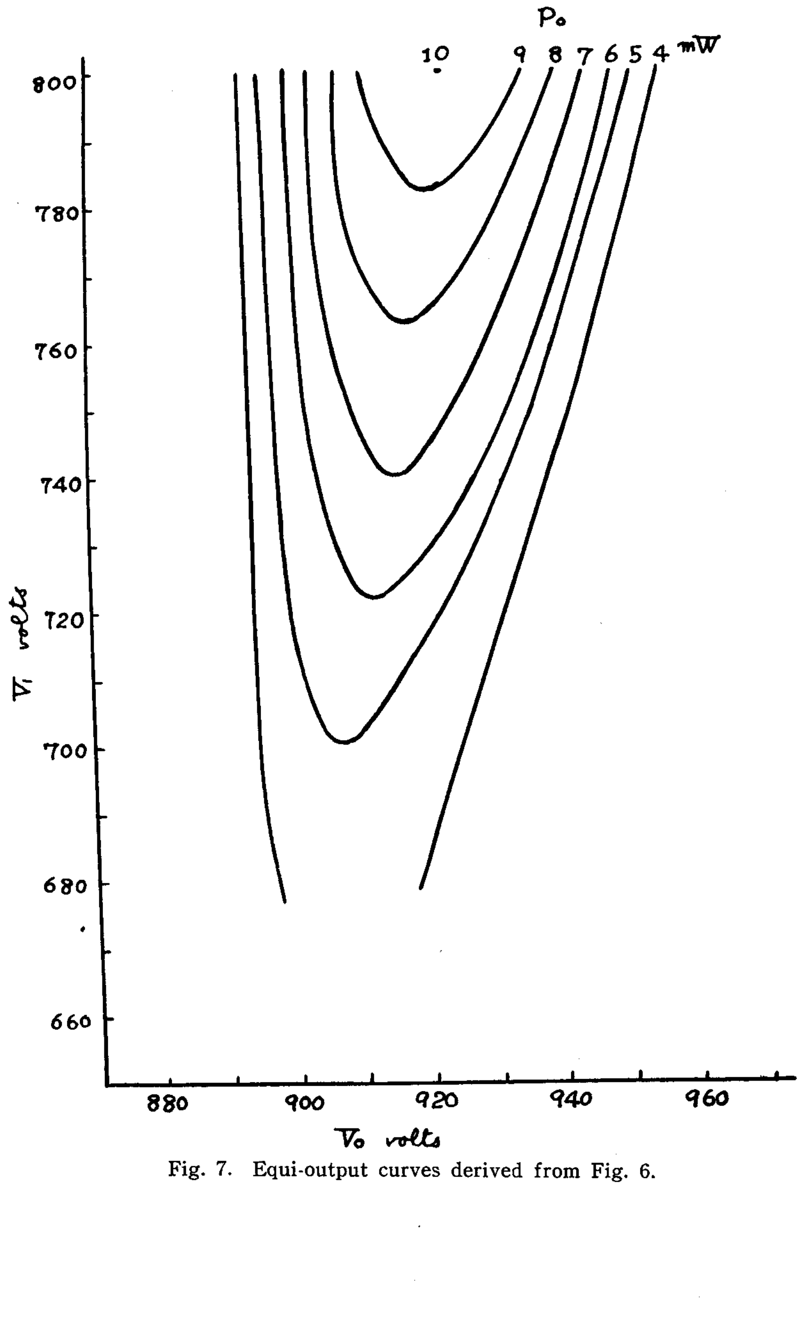

Fig. 7. Equi-output curves derived from Fig. 6.

shown in Fig. 6. The phenomenon that the helix voltage, giving maximum output, gradually increases as the first anode voltage is raised, can be explained as the space charge effect due to the increased beam current. From Fig. 6, the relation of the helix to the 1st anode voltage characteristics, holding a constant signal level, i.e. equi-output curves, will be obtained and are shown in Fig. 7.

Taking the region where the equi-output curves may be assumed to be linear and imposing the modulating- and compensating voltages, ΔV_0 and ΔV_1, of same frequency to the helix and the 1st anode respectively, and making adjustments so that the instantaneous operating point always remains in the equi-output line, the amplitude modulation can be reduced.

Fig. 8 shows the circuit arrangement used for the experiment of compensation. The modulating frequency is 60 c/s.

In Fig. 9, the variation of degrees of amplitude modulation due to the compensating voltage are shown for two modulating voltages of $\Delta V_0=2.5$ and 5 volts (rms). The operating conditions are $V_0=940$ volts and $V_1=770$ volts. The dotted line drawn in Fig. 9 shows the degrees of amplitude modulation at the helix voltage, $V_0=920$ volts, which gives the maximum output when the 1st anode voltage is $V_1=770$ volts.

For the degree of amplitude modulation, the carrier amplitude has been calculated by using DC voltages which appears in the crystal load resistance, 20 kΩ.

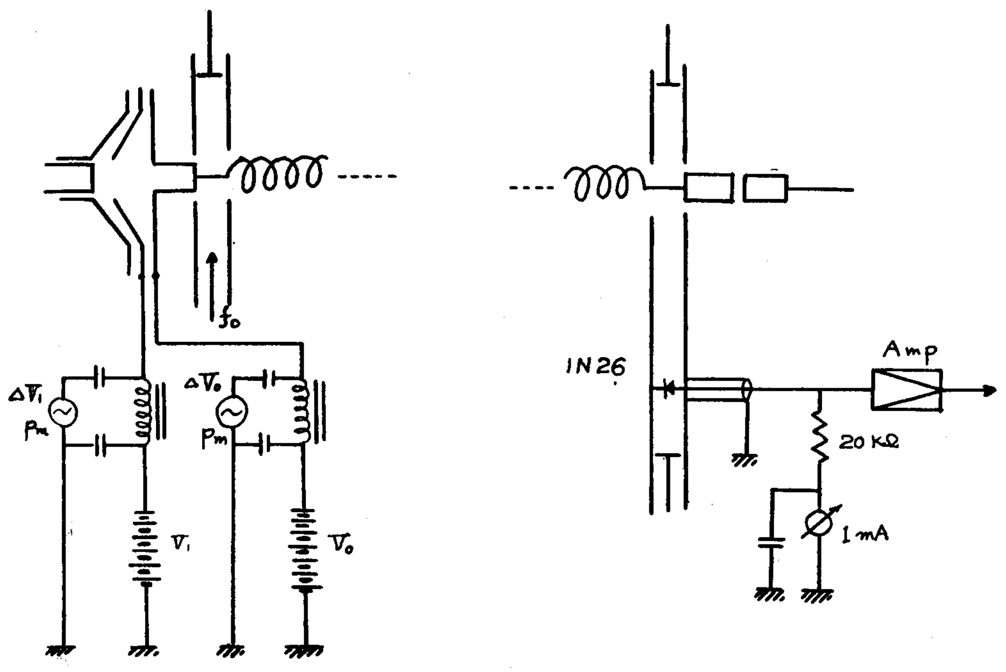

Fig. 8. Circuit arrangement used for experiment of compensation.

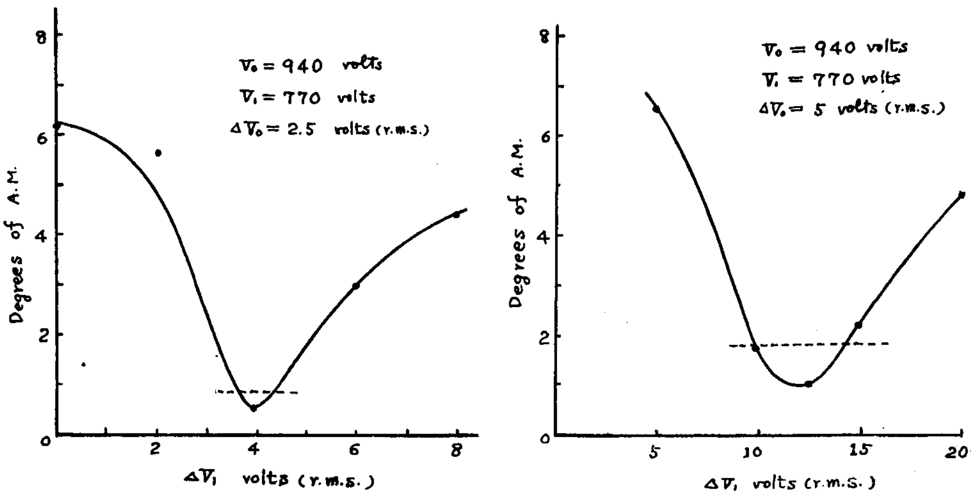

Fig. 9. Changes of degrees of amplitude modulation with compensating voltages.

From these results it has been made clear that the amplitude modulation can be reduced considerably by applying the compensating voltage having the same frequency as the modulating signal, if we choose a proper region where the equi-output curve can be assumed to be linear.

The phase modulation sensitivity of the 1st anode voltage is very low compared with that of the helix voltage. Therefore, the compensation by the 1st ande voltage is considered to be a very useful method to decrease the unwanted amplitude modulation without giving any effect upon the modulation sensitivity of the tube.

5. Conclusion

Nowadays, the klystron modulators are being widely used in microwave relay system. However, the traveling wave modulator also has many characteristics, such as a high degree of linearity and simplicity of its circuit composition. The simultaneous amplitude modulation, which also occurs in the klystron modulator, can be eliminated by using a heterodyne detector. But in special cases there is a possibility that it may cause a serious effect. However, by using the compensation explained in this paper, the amplitude modulation can be reduced and its application seems to be a very convenient method to eliminate the source of errors.

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