Concentration Gradient in a Continuous Countercurrent Extraction Column with Longitudinal Back-Mixing

By

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The following relations have been derived by the authors for the variation of concentration in a continuous countercurrent extraction column in which there is longitudinal back-mixing of the continuous phase, on the assumption that the diffusivity of the back-mixing and liquid velocity are constant and the solutions are dilute :

When $1+\beta>0$

$$\frac{C_c-\alpha}{C_{co}-\alpha}=\frac{(1+\sqrt{1+\beta})e^{2\gamma\sqrt{1+\beta}(1-x/L)}-(1-\sqrt{1+\beta})}{2\sqrt{1+\beta}e^{\gamma(1+\sqrt{1+\beta})(1-x/L)}}$$

and when $1+\beta<0$

$$\frac{C_c-\alpha}{C_{co}-\alpha}=\frac{i\sqrt{1+\beta}\cos[\gamma\,i\sqrt{1+\beta}(1-x/L)]+\sin[\gamma\,i\sqrt{1+\beta}(1-x/L)]}{i\sqrt{1+\beta}\,e^{\gamma(1-x/L)}}$$

where :

$$\alpha=\frac{C_{co}^*-(mF_c/F_d)C_{co}}{1-(mF_c/F_d)},\quad \beta=\frac{2MN}{(M+NQ)^2},\quad \gamma=M+NQ$$

$$M=\frac{Lu_c}{2E_cH_c},\quad N=K_c a'\Big(1-\frac{mF_c}{F_d}\Big)\Big(\frac{LH_c}{u_c}\Big),\quad Q=\frac{1}{2}\frac{1}{(F_d/mF_c)-1}.$$

The applicability of these equations is discussed, and the performance of an extraction column is analyzed.

Introduction

It has generally been thought that both the continuous and the dispersed phases flow countercurrently in a manner closely resembling the unidirectional movement of a piston as in such classical extraction columns as the packed and baffle-tower types etc. Recently, improved extractors have come into use in which some form of mechanical agitation is employed, e. g. stirrers, centrifugal force, pulsation, etc. In some of these cases, it is probable that back-mixing is not negligible in the column,

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and a study of the performance data of a number of investigators tends to confirm this view.

However, there are very few studies in which the effect of back-mixing has been quantitatively determined; furthermore, there are practically no studies in which the effect of back-mixing has been considered in the design of countercurrent equipment. This paper therefore suggests a new approach to the problem of back-mixing in countercurrent extraction.

Concentration Gradient in a Column for an Ideal System.

Consider the operation of a differential continuous countercurrent extractor as shown in Fig. 1. In actual operation the flow of the dispersed phase closely resembles piston-type flow, and it will be assumed, therefore, that back-mixing exists only in the flow of the continuous phase. For the flow of the continuous phase in the column, the following assumptions may be made:

1. Since the diffusivity in the direction normal to the flow is very large, each phase has an uniform concentration over any given cross-section, C_c and C_d, and the concentration change in the vertical direction is expressed by a continuous function of a single independet variable, x.
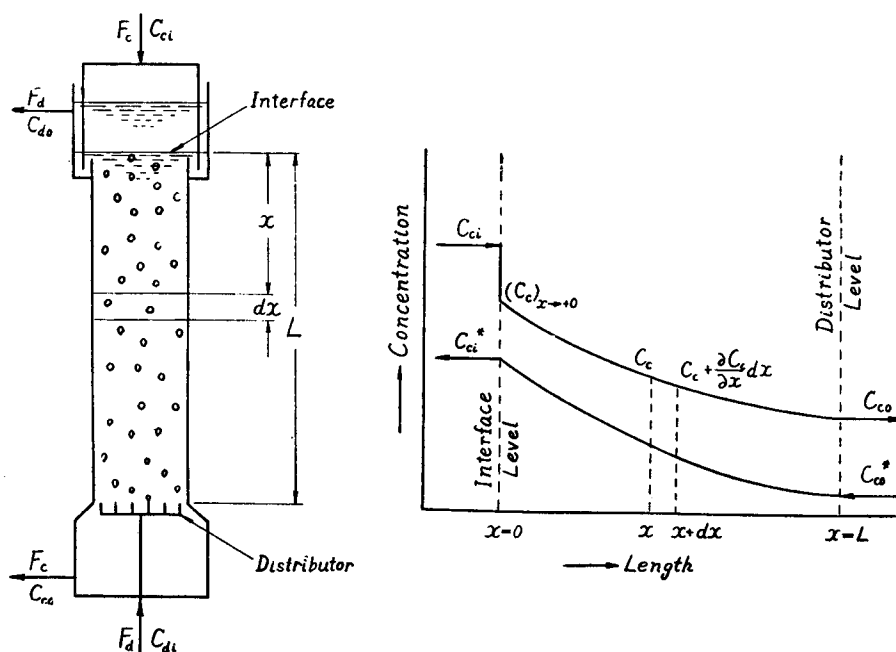

Fig. 1. Extraction column model and concentration gradient model.

2. The holdup of both the continuous and the dispersed phases (the volume of each phase per unit volume of the column), H_c and H_d, is constant throughout the column.

3. Assuming the solutions to be dilute, the volumetric flow rates of the two phase, F_c and F_d, are constant throughout the column regardless of the mass transfer which may occur. Therefore, the mean flow velocity of the continuous phase in the column, $u_c'(=u_c/H_c$, which is different from the mean velocity based on the empty column, $u_c=F_c/S$), is assumed constant.

4. For the flow of the continuous phase, a mean back-mixing diffusivity, E_c, is assumed which is constant throughout the column.

5. The overall mass transfer coefficient in the continuous phase, K_c, and the contacting surface area per unit volume of the continuous phase, a', are both constant throughout the column.

6. The overall mass transfer rate per unit volume of the continuous phase is given by $K_c a'(C_c - C_c^*)$.

On the basis of the above assumptions, from a material balance over the differential element dx at a distance x from the interface, as shown in Fig. 1, the following basic equation is derived for steady-state operation:

$$E_c \frac{d^2 C_c}{dx^2} - u_c' \frac{dC_c}{dx} - K_c a'(C_c - C_c^*) = 0 \qquad \cdots(1)$$

Expressing the effective height to the extraction column by L, and assuming that the mass transfer takes place only between $0 < x < L$ and that the back-mixing outside the effective section, i. e. when $x < 0$ and $x > L$, is negligible in comparison with the back-mixing in the column itself, the boundary conditions are obtained as follows:

At $x=0$

$$E_c S H_c \left(\frac{dC_c}{dx} \right)_{x=0} = u_c' S H_c (C_c)_{x \to +0} - F_c C_{ci} \qquad \cdots(2)$$

and at $x=L$

$$E_c S H_c \left(\frac{dC_c}{dx} \right)_{x=L} = 0 \qquad \cdots(3)$$

where C_{ci} is the concentration of the continuous phase at the inlet, i. e. at $x < 0$, and $(C_c)_{x \to +0}$ is the limiting value of C_c when $x \to +0$.

For dilute solutions in which it may be assumed that both the distribution coefficient, m, and the mutual solubility do not change, the equilibrium relationship is given by the following equation:

$$m = C_c^*/C_d = C_c/C_d^* = \text{constant} \qquad \cdots(4)$$

The operating line equation is given as follows:

$$C_d = C_{do} + \frac{F_c}{F_d}(C_c - C_{ci}) - \frac{E_c H_c}{u_d}\frac{dC_c}{dx} \qquad \cdots(5)$$

and the total solute balance is:

$$C_{do} = C_{di} - (F_c/F_d)(C_{co} - C_{ci}) \qquad \cdots(6)$$

where subscripts i and o refer to the inlet and outlet flows respectively.

Substituting equations (4), (5) and (6) into equation (1), the following equation is obtained:

$$E_c\frac{d^2C_c}{dx^2} - \left(\frac{u_c}{H_c} + \frac{K_c a' m E_c H_c}{u_d}\right)\frac{dC_c}{dx} - K_c a'\left(1 - \frac{mF_c}{F_d}\right)\left(C_c - \frac{C_{co}^* - \dfrac{mF_c}{F_d}C_{co}}{1 - \dfrac{mF_c}{F_d}}\right) = 0 \cdots(7)$$

In the case where the continuous phase is the extractant phase and $C_{ci}=0$, the solution of equation (7) is as follows:
When $1+\beta > 0$

$$\frac{C_c - \alpha}{C_{co} - \alpha} = \frac{(1+\sqrt{1+\beta})\,e^{2\gamma\sqrt{1+\beta}(1-x/L)} - (1-\sqrt{1+\beta})}{2\sqrt{1+\beta}\,e^{\gamma(1+\sqrt{1+\beta})(1-x/L)}} \qquad \cdots(8a)$$

and when $1+\beta < 0$

$$\frac{C_c - \alpha}{C_{co} - \alpha} = \frac{i\sqrt{1+\beta}\cos[\gamma i\sqrt{1+\beta}(1-x/L)] + \sin[\gamma i\sqrt{1+\beta}(1-x/L)]}{i\sqrt{1+\beta}\,e^{\gamma(1-x/L)}} \qquad \cdots(8b)$$

where:

$$\alpha = \frac{C_{co}^* - (mF_c/F_d)C_{co}}{1-(mF_c/F_d)}, \quad \beta = \frac{2MN}{(M+NQ)^2}, \quad \gamma = M+NQ \qquad \cdots(9)$$

and where:

$$M = \frac{Lu_c}{2E_c H_c}, \quad N = K_c a'\left(1 - \frac{mF_c}{F_d}\right)\left(\frac{LH_c}{u_c}\right), \quad Q = \frac{1}{2}\frac{1}{(F_d/mF_c) - 1} \qquad \cdots(10)$$

By the use of either equation (8a) or (8b), the correlation between C_c and x is obtained in terms of the three dimensionless groups M, N and Q. These groups express the effects of back-mixing diffusivity, mass transfer coefficient, and extraction factor, respectively.

Generally, the concentration gradient can be calculated from equation (8a). However, when $mF_c/F_d > 1$ and E_c is large, $(1+\beta)$ becomes negative and equation (8a) is inapplicable so that equation (8b) must be used.

If E_c is small, i.e. $M \gg NQ$, since then $(C_c)_{x \to +0}$ approaches closely to C_{ci}, the boundary condition at $x=0$ can be approximately expressed by the following equation instead of equation (2):

$$(C_c)_{x \to +0} = C_{ci} \qquad \qquad \cdots(2')$$

In conditions where equation (2') can be used, the third term on the right hand side of equation (5) is usually negligible in comparison with the sum of the other two terms. In this case the solution to equation (1) is as fallows:

$$\frac{C_c - C_c^*}{C_{co} - C_{co}^*} = \frac{(1 + \sqrt{1 + 2N/M})\, e^{2M\sqrt{1 + 2N/M}(1 - x/L)} - (1 - \sqrt{1 + 2N/M})}{2\sqrt{1 + 2N/M}\, e^{M(1 + \sqrt{1 + 2N/M})(1 - x/L)}} \qquad \cdots(11)$$

Equation (11) is applicable when either the continuous or the dispersed phase is the extractant phase.

For the cases when $C_{ci} \neq 0$, solving again equation (7) under the boundary conditions as expressed by equations (2) and (3), the following relation is obtained for an ideal system.

$$\frac{C_c - \alpha}{C_{ci} - \alpha} = \frac{2M(1 - \sqrt{1 + \beta})\, e^{\gamma(1 + \sqrt{1 + \beta})(x/L)} - 2M(1 + \sqrt{1 + \beta})\, e^{2\gamma\sqrt{1 + \beta}} e^{\gamma(1 - \sqrt{1 + \beta})(x/L)}}{[\gamma(1 - \sqrt{1 + \beta}) - 2M](1 + \sqrt{1 + \beta})\, e^{2\gamma\sqrt{1 + \beta}} - [\gamma(1 + \sqrt{1 + \beta}) - 2M](1 - \sqrt{1 + \beta})}$$
$$\cdots(12)$$

Equation (12) is also applicable when either the continuous or the dispersed phase is the extractant phase.

Application to the Design of Extraction Columns

Substituting $x = 0$, together with the corresponding boundary conditions, i. e. equation (2'), into equation (11), the relation between the inlet and outlet concentrations of the continuous phase is obtained:

$$\frac{C_{ci} - C_{ci}^*}{C_{co} - C_{co}^*} = \frac{(1 + \sqrt{1 + 2N/M})\, e^{2M\sqrt{1 + 2N/M}} - (1 - \sqrt{1 + 2N/M})}{2\sqrt{1 + 2N/M}\, e^{M(1 + \sqrt{1 + 2N/M})}} \qquad \cdots(13)$$

On the other hand, the relation between C_{ci} and C_{co} when the back-mixing diffusivity has a relatively large value cannot be obtained by applying equation (8a) or (8b) at $x = 0$. However, it can be obtained by substituting that result into equation (2), as follows:

Differentiating equation (8a) with respect to x, the following relation is obtained

$$\frac{dC_c}{dx} = \frac{\beta\gamma}{2L\sqrt{1 + \beta}} \Big[e^{-\gamma(1 + \sqrt{1 + \beta})(1 - x/L)} - e^{-\gamma(1 - \sqrt{1 + \beta})(1 - x/L)} \Big] \qquad \cdots(14a)$$

Substituting equation (14a) into equation (2), $(C_c)_{x \to +0}$ is expressed as follows, when $C_{ci} = 0$

$$(C_c)_{x \to +0} = \frac{N}{2\gamma\sqrt{1 + \beta}} \Big[e^{-\gamma(1 + \sqrt{1 + \beta})} - e^{-\gamma(1 - \sqrt{1 + \beta})} \Big] \qquad \cdots(15a)$$

Applying equation (8a) at $x = 0$, it becomes as follows:

$$\frac{(C_c)_{x \to +0} - \alpha}{C_{co} - \alpha} = \frac{(1+\sqrt{1+\beta})\,e^{2\gamma\sqrt{1+\beta}} - (1-\sqrt{1+\beta})}{2\sqrt{1+\beta}\,e^{\gamma(1+\sqrt{1+\beta})}} \qquad \cdots(16a)$$

Knowing m, C_{co}^*, F_c/F_d, $K_c a'$, E_c and H_c and assuming a value of L, and hence of M and N, $(C_c)_{x \to +0}$ is evaluated from equation (15a) and substituted into equation (16a) to give C_{co}. Hence the latter is obtained as a function of L and the required height of extraction column can be calculated by trial and error for the given fraction of solute extracted.

Furthermore, using the value of C_{co} thus calculated, the values of C_c at any point are calculated from equation (8a) and C_{do} is estimated from equation (6). Substituting these values together with equation (14a) into equation (5), C_d is determined at any point from the following equation:

$$C_d = C_{do} + \frac{F_c}{F_d}\left\{ C_c + \frac{N}{2\gamma\sqrt{1+\beta}}\left[e^{-\gamma(1-\sqrt{1+\beta})(1-x/L)} - e^{-\gamma(1+\sqrt{1+\beta})(1-x/L)} \right] \right\} \qquad \cdots(17a)$$

When equation (8b) is used instead of equation (8a), corresponding relations are obtained as follows:

$$(C_c)_{x \to +0} = \frac{-N}{\gamma i\sqrt{1+\beta}} \cdot \frac{\sin{(\gamma i\sqrt{1+\beta})}}{e^\gamma} \qquad \cdots(15b)$$

$$\frac{(C_c)_{x \to +0} - \alpha}{C_{co} - \alpha} = \frac{i\sqrt{1+\beta}\cos{(\gamma i\sqrt{1+\beta})} + \sin{(\gamma i\sqrt{1+\beta})}}{i\sqrt{1+\beta}\,e^\gamma} \qquad \cdots(16b)$$

$$C_d = C_{do} + \frac{F_c}{F_d}\left\{ C_c + \frac{N}{\gamma i\sqrt{1+\beta}}\frac{\sin{[\gamma i\sqrt{1+\beta}\,(1-x/L)]}}{e^{\gamma(1-x/L)}} \right\} \qquad \cdots(17b)$$

When $C_{ci} \neq 0$, applying equation (12) at $x/L = 1$, the following equation is derived.

$$\frac{C_{ci}}{C_{co}} - \alpha' =$$
$$\frac{[\gamma(1-\sqrt{1+\beta}) - 2M](1+\sqrt{1+\beta})\,e^{2\gamma\sqrt{1+\beta}} - [\gamma(1+\sqrt{1+\beta}) - 2M](1-\sqrt{1+\beta})}{4M\sqrt{1+\beta}\,e^{\gamma(1+\sqrt{1+\beta})}}$$
$$\times (\alpha' - 1) \qquad \cdots(18)$$

where

$$\alpha' = \frac{\alpha}{C_{co}} = \frac{(mC_{di}/C_{co}) - (mF_c/F_d)}{1 - (mF_c/F_d)} \qquad \cdots(19)$$

Knowing C_{ci}, C_{di}, M, N and Q, the value of C_{co} is evaluated from equation (18). In particular, if $C_{di} = 0$ the fraction of solute extracted is easily calculated because the following equation is derived for this case.

$$\alpha' = -2Q$$

Accordingly, equation (18) is applicable to the determination of the required height of the extraction column in which the dispersed phase is the extractant phase.

Discussion

Equations (8a), (8b) and (12) are more rigorous, while equation (11) is an approximate expression; however, the latter is convenient when applicable since the left hand side is expressed in terms of the driving forces.

In the case of piston-like flow, i.e. the limiting case where E_c tends to zero, equation (11) becomes:

$$\lim_{E_c \to 0} \frac{C_c - C_c^*}{C_{co} - C_{co}^*} = e^{N(1 - x/L)} \qquad \cdots (20)$$

This agrees with the classical expression for ideal systems.

In the limiting cases where either θ_c $(=LH_c/u_c)$ or $K_c a'$ tends to zero, the following relation is customarily obtained and is independent of the flow ratio:

$$\lim_{\theta_c \to 0} \frac{C_c - C_c^*}{C_{co} - C_{co}^*} = \lim_{K_c a' \to 0} \frac{C_c - C_c^*}{C_{co} - C_{co}^*} = 1 \qquad \cdots (21)$$

This means that the operating line becomes a point, and there is no mass transfer.

Furthermore in the limiting case where mF_c/F_d tends to unity, the following equation is obtained:

$$\lim_{mF_c/F_d \to 1} \frac{C_c - C_c^*}{C_{co} - C_{co}^*} = 1 \qquad \cdots (22)$$

This shows that the operating line is parallel to the equilibrium line.

Fig. 2 illustrates typical plots of equation (11) for a few values of N, using M as the parameter. Similar plots of equations (8a) or (8b) are obtained for each value of mF_c/F_d (or of Q), as shown in Fig. 3. For a given pair of values of M and N, these equations express the change in concentration and not the absolute values of the latter.

In Fig. 4, the effect of M on the operating curves is illustrated for the case where the operating conditions are as follows: $C_{ci}=0$, $C_{di}=5.0$, $m=2$, $F_c/F_d=1$ and $N=-2$. In this figure, the solid curves are the operating curves calculated from the expressions given in the previous section and the dotted curves show the relation between the concentrations and the position in the column.

The operating curve departs further from the equilibrium curve as M decreases, corresponding to an increase in E_c, and the difference between $(C_c)_{x \to +0}$ and C_{ci} increases at the same time. Furthermore, the slopes of the operating curves are greater at the continuous phase outlet than at the inlet end, and this effect increases with a decrease in M. It is clear from Fig. 4 that the fraction of solute extracted decreases with an increase in back-mixing even when the capacity coefficient, flow ratio, and holding time are constant.
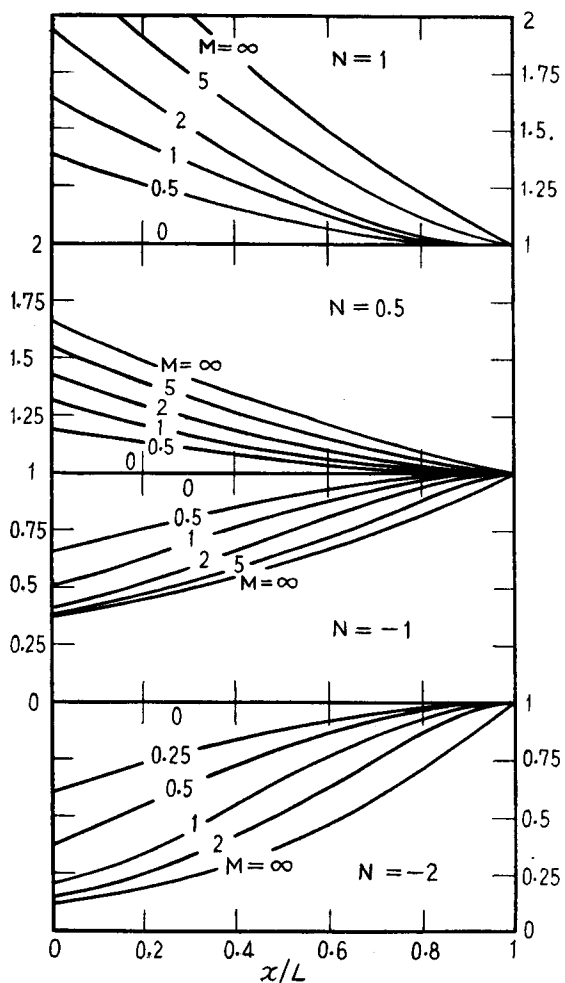
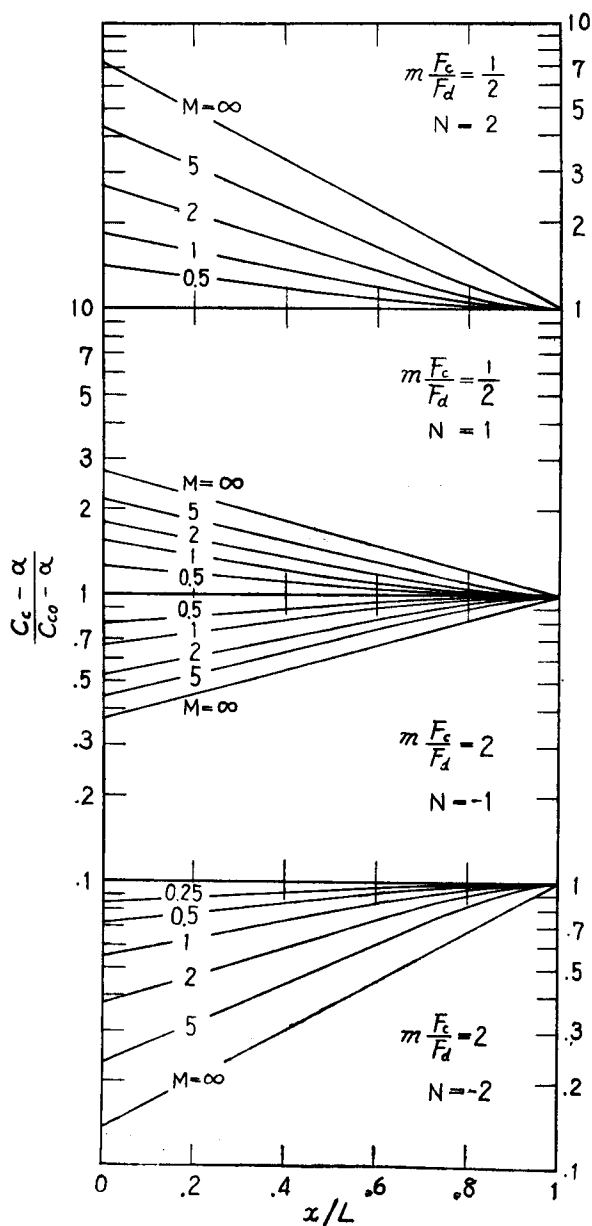Fig. 2. Driving force ratio expressed by equation (11).

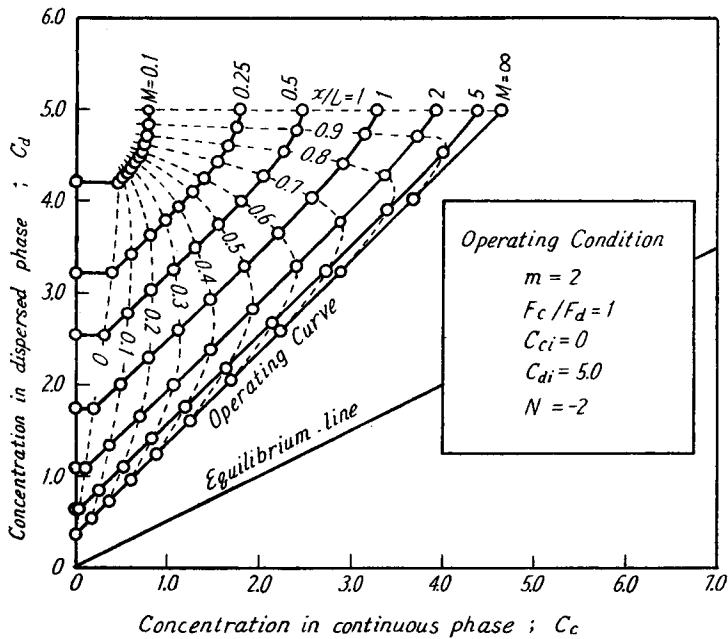Fig. 3. Concentration change expressed by equation (8a) or (8b).

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Fig. 4. Effect of back-mixing on the operating curves.

Fig. 5 shows the operating curves for some pairs of values of M and N when the fraction of solute extracted is 0.652. This shows that the concentration curves change with change in M in the case where inlet and outlet concentrations are unchanged and flow rates, flow ratio, holdup and holding time (but not capacity coefficient) are constant.

It is further apparent from the above results that the H.T.U. or Murphree efficiency changes with the position in a column. The Murphree efficiencies at each cross section when the column is sub-divided into ten equal sections are shown in Fig. 6, for the case where the operating curves are given by Fig. 4. In the case of piston-like flow, i. e. $M=\infty$ $(E_c=0)$, the efficiencies are constant throughout the column. On the other hand, the change of efficiency with change in position becomes considerable with large values of E_c; in particular, it decreases remarkably at a position adjacent to the continuous phase outlet.

Therefore, if the concentration change is measured in an experimental column, and β and γ (or M and N) are estimated by trial and error calculation from either equation (8a) or (8b), the values of $K_c a'$ and E_c can be determined from the resulting values of N and M. Fig. 7 shows a few examples in which measured and calculated concentrations are compared, the points being the measured values and the curves the calculated values.
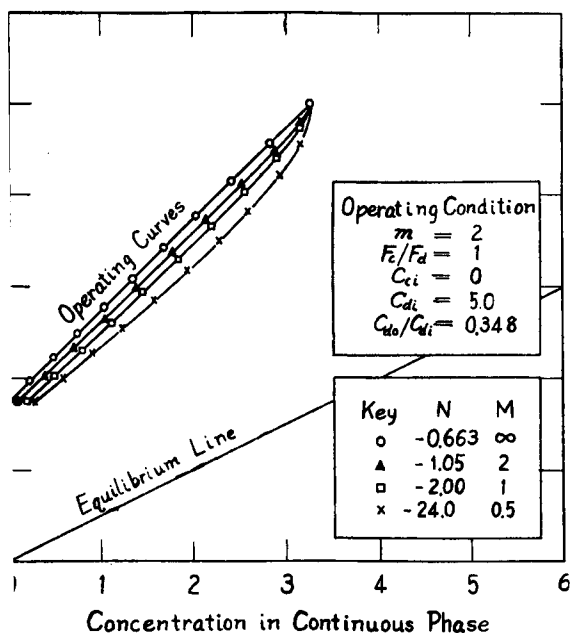
Operating Condition

$m = 2$
$F_c/F_d = 1$
$C_{ci} = 0$
$C_{di} = 5.0$
$C_{do}/C_{di} = 0.348$

Key	N	M
o	−0.663	∞
▲	−1.05	2
□	−2.00	1
×	−24.0	0.5

Concentration in Continuous Phase

Fig. 5. Effect of back-mixing on the concentration change in column.
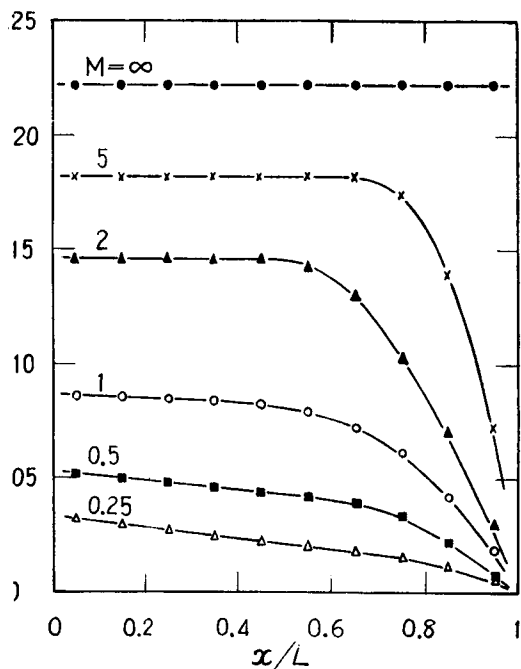
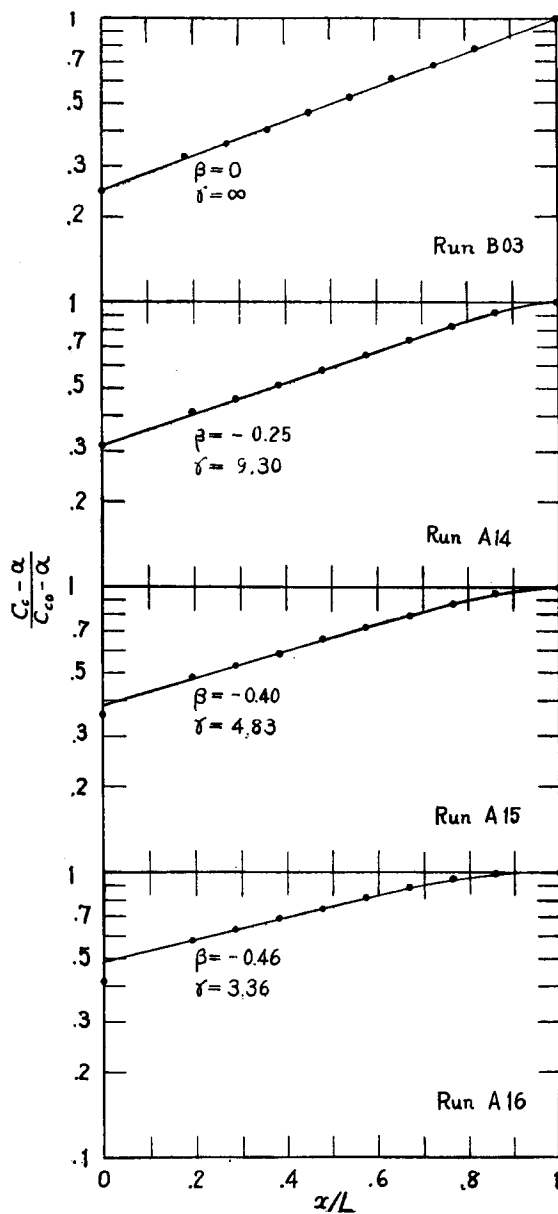Fig. 6. Change of Murphree efficiency with the position in a column.

$\beta = 0$
$\gamma = \infty$

Run B 03

$\beta = -0.25$
$\gamma = 9.30$

Run A 14

$\beta = -0.40$
$\gamma = 4.83$

Run A 15

$\beta = -0.46$
$\gamma = 3.36$

Run A 16

$\dfrac{C_i - \alpha}{C_{co} - \alpha}$

x/L

Fig. 7. Comparison of the observed and calculated concentration change in the extraction column.

In order to determine the values of β and γ from the experimental data, it is convenient to perform the first approximations as follows:

From the concentrations observed at several points in a column, the logarithm of $(C_c - \alpha)/(C_{co} - \alpha)$ is plotted against x/L as shown in Fig. 7, and extrapolated to $x/L = 0$. Since this curve approximates to a straight line in the region adjacent to $x/L = 0$, both the intercept and the slope of the curve at $x/L = 0$ can easily be determined. On the other hand, the intercept of the curve is given by equation (16a) or (16b), and the slope is derived from equation (8a) or (8b) as follows:

when $1 + \beta > 0$

$$\left[\frac{d\left(\ln \frac{C_c - \alpha}{C_{co} - \alpha}\right)}{d(x/L)}\right]_{x/L=0} = \frac{-\beta\gamma}{1 + \sqrt{1+\beta}\left(\frac{1 + e^{-2\gamma\sqrt{1+\beta}}}{1 - e^{-2\gamma\sqrt{1+\beta}}}\right)} \qquad \cdots(23a)$$

and when $1 + \beta < 0$

$$\left[\frac{d\left(\ln \frac{C_c - \alpha}{C_{co} - \alpha}\right)}{d(x/L)}\right]_{x/L=0} = \frac{-\beta\gamma}{1 + i\sqrt{1+\beta}\cot\left[\gamma i\sqrt{1+\beta}\right]} \qquad \cdots(23b)$$

The correlation between the intercepts and slopes at $x/L = 0$, as given by the these equations, is shown in Fig. 8 using β and $\beta\gamma$ as parameters. If the intercept and slope at $x/L = 0$ are estimated from observed data as described above, the first approximations to β and γ are obtained from Fig. 8. It is then checked whether or not the curve calculated from equation (8a) or (8b) using the first approximations passes through the measured points. If the points do not lie on the calculated curve, the trial and error calculations are repeated in the neighbourhood of the first approximations until satisfactory agreement is obtained.

Nonideal Systems

All the above relations were obtained for ideal systems, and the case will now be considered in which the distribution coefficient is not constant but the equilibrium curve may be approximated by a straight line over the range of operation.

In the range of operation, if the equilibrium curve can be approximated by a straight line with the slope $1/m'$, the equilibrium concentrations are given by the following equation, as shown in Fig. 9:

$$C_c^* = m' C_d + (m - m') C_d' \qquad \cdots(4')$$

In equation (4'), $1/m$ is the slope of the line joining the origin 0 with the point A (i. e. C_c', C_d'), on the straight portion of the equilibrium curve.
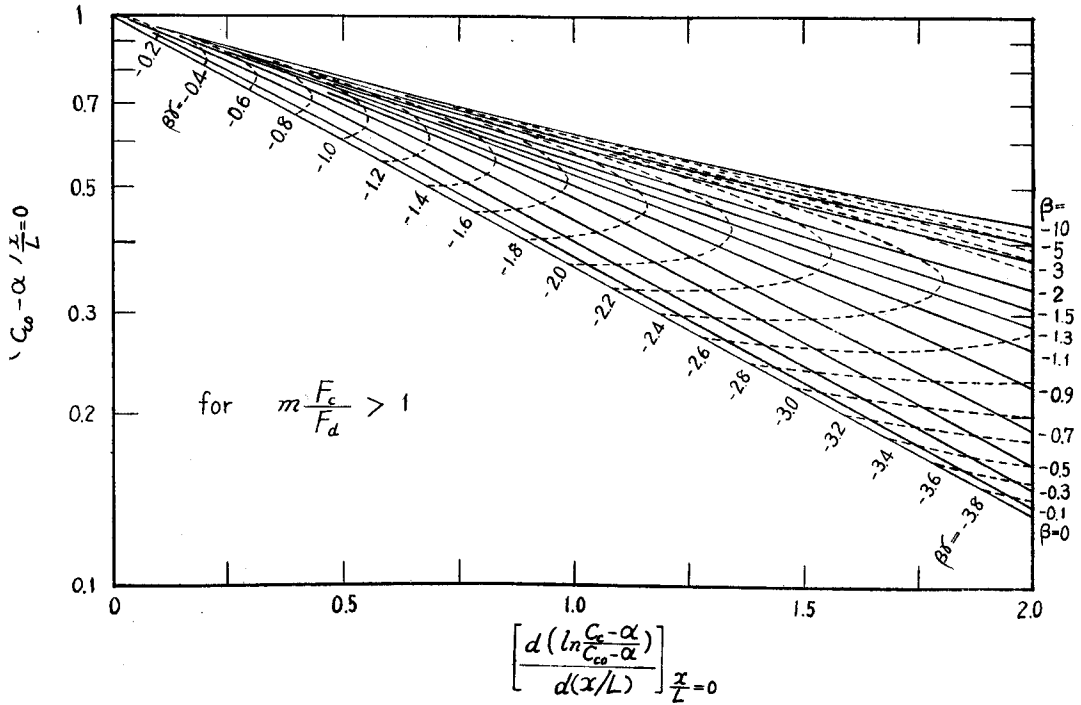
Fig. 8.   Correlation between the intercepts and slopes of equations
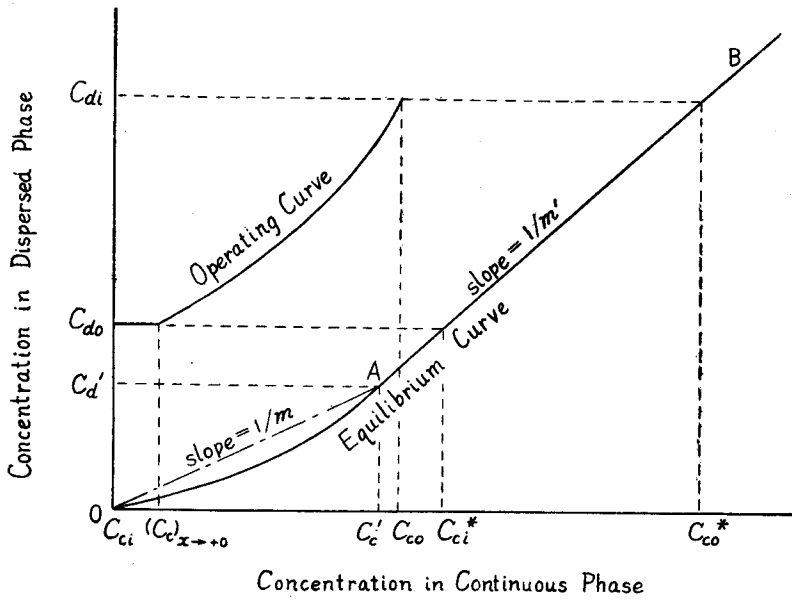(8a) and (8b) at $x/L=0$.

Fig. 9. Operating curve in a nonideal system.

In a case such as that shown in Fig. 9, the operating equation and the total solute balance are again given by equations (5) and (6) respectively. Substituting equations (4′), (5) and (6) into equation (1), the following equation is obtained:

$$E_c \frac{d^2 C_c}{dx^2} - \left(\frac{u_c}{H_c} + \frac{K_c a' m' E_c H_c}{u_d} \right) \frac{dC_c}{dx} - K_c a' \left(1 - \frac{m' F_c}{F_d} \right) \left(C_c - \frac{C_{co}^* - \dfrac{m' F_c}{F_d}}{1 - \dfrac{m' F_c}{F_d}} \right) = 0 \quad \cdots (7')$$

Equation (7′) is the same as equation (7) with the exception that m is replaced by m'. Accordingly, substituting m' for m, the relations obtained for the ideal system can be applied to such a nonideal system as shown in Fig. 9.

Conclusions

Theoretical equations are given for the concentration gradient in an extraction column in which back-mixing of the continuous phase takes place. The equations provide a reasonable explanation of the performance of an extraction column.

By the application of these equations, both the capacity coefficient and the back-mixing diffusivity can be estimated from the measured concentration gradient in an extractor. Alternatively, the outlet concentrations can be determined from the capacity coefficient, back-mixing diffusivity and operating conditions.

It is further apparent that the H.T.U. or average stage efficiency, etc., as estimated from inlet and outlet concentrations alone, cannot be applied to the design of an extraction column in which back-mixing occurs.

Equations (8a), (8b), (11) and (12) are also applicable to other countercurrent contacting operations such as absorption, distillation, etc.

Acknowledgment

Nomenclature

$a' = a/H_c$: contacting surface area per unit volume of the continuous phase $[m^2/m^3]$

C_c : concentration of solute in the continuous phase $\qquad$ $[kg\text{–moles}/m^3]$

C_c^* : equilibrium concentration for C_d $\qquad$ $[kg\text{–moles}/m^3]$

C_d : concentration of solute in the dispersed phase $\qquad$ $[kg\text{–moles}/m^3]$

E : back-mixing diffusivity or dispersion coefficient $\qquad$ $[m^2/hr]$

F_c : volumetric flow rate of the continuous phase $\qquad$ $[m^3/hr]$

F_d : volumetric flow rate of the dispersed phase $\qquad$ $[m^3/hr]$

H_c : holdup of the continuous phase in the column $\qquad$ $[m^3/m^3]$

K_c : overall mass transfer coefficient without the effect of
    back-mixing in the continuous phase $\qquad$ $[kg\text{–moles}/m^2 \cdot hr \cdot (kg\text{–moles}/m^3)]$

L : effective length of the extraction column $\qquad$ $[m]$

$M = L u_c/2E_cH_c$: $\qquad$ $[-]$

m : distribution coefficient $\qquad$ $[(kg\text{–moles}/m^3)/(kg\text{–moles}/m^3)]$

$N = K_c a'[1-(mF_c/F_d)](LH_c/u_c)$: $\qquad$ $[-]$

$Q = (1/2)[1/(F_d/mF_c - 1)]$: $\qquad$ $[-]$

S : cross sectional area of the column $\qquad$ $[m^2]$

u_c : superficial velocity of the continuous phase based on the empty column $[m/hr]$

$u_c' = u_c/H_c$: mean velocity of the continuous phase $\qquad$ $[m/hr]$

x : distance from interface at the inlet side of the continuous phase $\qquad$ $[m]$

$\alpha = \dfrac{C_{co}^* - (mF_c/F_d)C_{co}}{1-(mF_c/F_d)}$ $\qquad$ $[-]$

$\beta = 2MN/(M+NQ)^2$: $\qquad$ $[-]$

$\gamma = M + NQ$ $\qquad$ $[-]$

subscript

i : inlet condition

o : outlet condition