

# Effect of Liquid- and Gas-Mixing on the Rate of Mass Transfer between Two Phases in Crosscurrent Flow

By

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The theoretical correlations have been derived by the authors for the concentration gradients in the froth layer on a sieve or bubble-cap plate, based on the assumption that the flow velocities of the gas and liquid in each direction are constant throughout the froth layer. The degree of liquid- and gas-mixing was expressed by the backmixing diffusion.

Point efficiency varies with change in the number of transfer units and with the degree of gas-mixing. The liquid-mixing has no effect on the point efficiency.

Plate efficiency is affected by both the liquid- and gas-mixing. The effect of the liquid-mixing is more marked than that of gas-mixing.

## 1. Introduction

In the mass transfer operation between gas and liquid or two immiscible liquids on bubble-cap or sieve plates, the plate efficiency is affected by the diameter of the column. It is considered to be the concentration gradient on a plate.

Since the first attempt to account for the variation of the concentration gradient on the plates was made by Kirschbaum<sup>1)</sup>, many theoretical or experimental studies have been performed by several investigators. Sherwood and Walter<sup>2)</sup> gave the correlation between the plate efficiency and diameter of the column on the assumption that the liquid flows in a piston type flow on a plate and the vapor passing through the froth is perfectly mixed. On this assumption, however, the fact that the plate efficiency exceeds 100% under certain conditions could not be explained. Lewis<sup>3)</sup> derived the expressions for the point and plate efficiencies on the assumption that the liquid flow is of a piston type in the direction of its flow and is of perfect mixing in the direction of the gas flow. Kamei and Takamatsu<sup>4)</sup> have studied the concentration gradients in both phases of the froth layer on a plate on the assumption that the liquid and gas flow are in a perfect piston type state.

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All the works mentioned above are based on the assumption that the fluid flows are either in a piston or perfect mixing type, but the actual flows on a plate are really intermediate between the above two extreme types. To find the plate efficiency, O'Connell and Gautreaux<sup>5)</sup> assumed that a plate consists of the number of pools in series in the direction of liquid flow which have the equal volume and are perfectly mixed in each pool, and found the correlation between the length of travel of the liquid and the number of pools, empirically. Oliver and Watson<sup>6)</sup> assumed a imaginary recirculation for the liquid-mixing on a tray, and the plate efficiency was connected with the point efficiency and the fraction of mixing. Recently, the liquid-mixing and its effects on the plate efficiency have been studied in terms of liquid residence time distribution function by Foss, Gerster and Pigford.<sup>7)</sup> Marangozis and Johnson<sup>8)</sup> suggested a mechanism for the liquid-mixing, i.e. the mixing of liquid phase in the direction of flow results from the splashing of the liquid.

Authors<sup>9,10,11)</sup> have already shown that the mixing in continuous homogeneous flow or countercurrent two phase flow could be expressed on the assumption that the backmixing diffusivity is constant throughout the equipment. This paper is based on the above assumption for the crosscurrent operation on a tray.

## 2. Assumptions and Basic Equations

Let the case be considered where the two phases contact crosscurrently on a plate as shown in Fig. 1.  $\xi$ -axis is taken in the direction of liquid flow on a tray,  $\zeta$ -axis in vertical direction to the tray and  $\eta$ -axis in vertical direction to  $\xi$ - $\zeta$  plane. For the flow of both phases on a tray, the following assumptions are made:

1. The tray consists of a rectangular plate with constant width,  $Y$ . The height of the froth layer is  $Z$ . The holdup volume of both phases per unit volume of the froth layer,  $h_L$  and  $h_G$ , and the contacting surface area per unit volume of the froth layer,  $a$ , are constant throughout the froth layer on a tray.

2. Both phases are perfectly mixed before entering into the froth section, i.e. fed at  $C_{Li}$  and  $C_{Gi}$  respectively, where  $C_L$  is the molar concentration of volatile component in liquid phase,  $C_G$  is that in gas phase and subscript  $i$  means the entering stream. Also, both phases are again perfectly mixed after leaving the

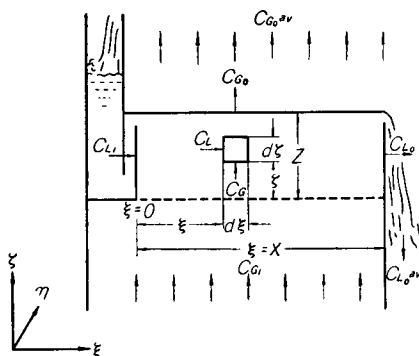


Fig. 1. Schematic diagram of the flows on a sieve plate.

froth layer, and so the concentration of each phase may be taken as  $C_{L,av}$  and  $C_{G,av}$ , respectively.

3. The volumetric flow rates of the two phases,  $F_L$  and  $F_G$ , are constant throughout the froth layer regardless of the mass transfer which may occur. In the froth layer, the velocity gradient over any vertical cross section to the direction of each flow may be considered negligible. By the above assumptions, the superficial mean velocities of both phases,  $u_L$  and  $u_G$ , are constant in the direction of flow.

4. For the flow of the liquid phase, the concentration gradient in the  $\eta$ -direction is negligible, and the mean backmixing diffusivities in the  $\xi$ - and  $\zeta$ -direction,  $E_{L\xi}$  and  $E_{L\zeta}$ , are assumed constant throughout the froth layer. Similarly, the average backmixing diffusivities for the gas-mixing in the froth layer,  $E_{G\xi}$  and  $E_{G\zeta}$ , are assumed constant.

5. The overall mass transfer coefficients in both phases,  $K_L$  and  $K_G$ , are constant throughout the froth layer, and the mass transfer rate per unit volume of the froth layer is given by  $K_L a(C_L - C_L^*)$  or  $K_G a(C_G^* - C_G)$ .

On the basis of the above assumptions, let the case be considered where the volume element in the froth layer,  $d\xi d\zeta$ , has unit width in the  $\eta$ -direction. Fig. 2 shows the material balance in this element referring to the flow, backmixing diffusion and mass transfer. From the material balances for both phases in the volume element, the following equations are derived at steady operation.

For the liquid phase

$$E_{L\xi} h_L \frac{\partial^2 C_L}{\partial \xi^2} + E_{L\zeta} h_L \frac{\partial^2 C_L}{\partial \zeta^2} - u_L \frac{\partial C_L}{\partial \xi} - K_L a(C_L - C_L^*) = 0 \tag{1}$$

For the gas phase

$$E_{G\xi} h_G \frac{\partial^2 C_G}{\partial \xi^2} + E_{G\zeta} h_G \frac{\partial^2 C_G}{\partial \zeta^2} - u_G \frac{\partial C_G}{\partial \xi} + K_G a(C_G^* - C_G) = 0 \tag{2}$$

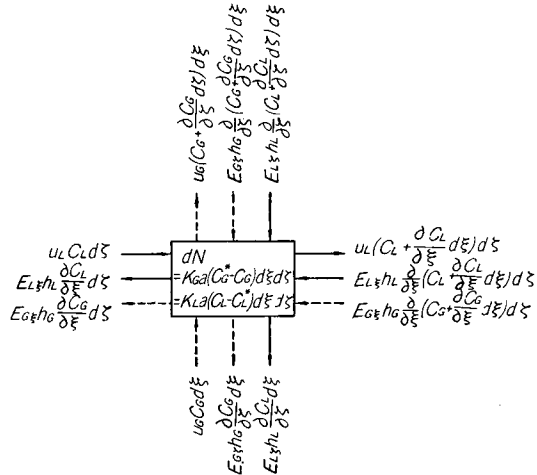


Fig. 2. Material balance in a volume element.

The equilibrium relationship is given by the following equations in the range of the concentrations on a tray.

$$C_G = m' C_L^* + b' \quad (3a)$$

$$C_G^* = m' C_L + b' \quad (3b)$$

$m'$  and  $b'$  in equation (3a) may not be strictly equal to those in equation (3b), but they are considered as equal in this case.

Solving the above equations under the proper boundary conditions, the concentrations of both phases at any position in the froth layer may be obtained; however, it is rather difficult to obtain rigorous general solution of the above simultaneous equations. Therefore, let the special case be considered as follows:

6. Since the liquid mixing in the  $\zeta$ -direction on a tray is very intense, the concentration gradient in the  $\zeta$ -direction of the liquid phase may be considered negligible.

$$E_{L\zeta} = \infty, \quad \partial C_L / \partial \zeta = 0 \quad (4)$$

7. Since the gas-mixing in the  $\xi$ -direction in the froth layer is very weak, backmixing diffusivity of the gas phase in this direction may be negligible.

$$E_{G\xi} = 0 \quad (5)$$

### 3. Concentration Gradients on a Tray

Substituting equation (5) in equation (2), the following equation is obtained.

$$E_{G\xi} h_G \frac{\partial^2 C_G}{\partial \zeta^2} - u_G \frac{\partial C_G}{\partial \zeta} + K_G a (C_G^* - C_G) = 0 \quad (6)$$

$C_G^*$  is the equilibrium concentration for  $C_L$ . According to Assumption 6, or equation (4),  $C_L$  is independent of  $\zeta$  and constant over any vertical cross section  $\xi$  to the  $\xi$ -direction. Equation (6) is rearranged as follows by substituting the value for  $C_G^*$  of equation (3b)

$$\frac{\partial^2 C_G}{\partial (\zeta/Z)^2} - 2M_G \frac{\partial C_G}{\partial (\zeta/Z)} - 2M_G N_G [C_G - (m' C_L + b')] = 0 \quad (7)$$

where,

$$M_G = \frac{u_G Z}{2E_{G\xi} h_G} \quad (8a)$$

$$N_G = \frac{K_G a Z}{u_G} = \frac{K_L a Z}{m' u_G} \quad (8b)$$

The boundary condition at the entry to the froth layer,  $\zeta/Z=0$ , where the backmixing diffusion across the plate does not exist, is obtained by taking material balance as follows:

$$\left[ \frac{\partial C_G}{\partial (\zeta/Z)} \right]_{\zeta/Z \rightarrow 0^+} = 2M_G[(C_G)_{\zeta/Z \rightarrow 0^+} - C_{Gi}] \quad (9)$$

where,  $C_{Gi}$  is the concentration of the gas stream entering into the froth layer, and  $(C_G)_{\zeta/Z \rightarrow 0^+}$  is the limiting value of  $C_G$  when  $\zeta/Z \rightarrow 0^+$  in the froth layer.

A similar relation is obtained for  $C_G$  at the exit from the froth layer,  $\zeta/Z=1$ , but it is replaced by the following equation as shown by Danckwerts<sup>12)</sup> and others.

$$\left[ \frac{\partial C_G}{\partial (\zeta/Z)} \right]_{\zeta/Z \rightarrow 1^-} = 0 \quad (10)$$

Solving equation (7) under the conditions of equations (9) and (10), the change of  $C_G$  in the  $\zeta$ -direction is obtained as follows.

$$\frac{C_G - C_G^*}{C_{Gi} - C_G^*} = \frac{2(1+p)e^{M_G(1+p)}e^{M_G(1-p)(\zeta/Z)} - 2(1-p)e^{M_G(1-p)}e^{M_G(1+p)(\zeta/Z)}}{(1+p)^2e^{M_G(1+p)} - (1-p)^2e^{M_G(1-p)}} \quad (11)$$

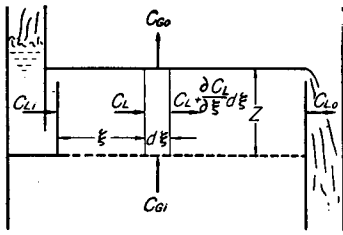


Fig. 3. Approximate flow on a plate.

where,

$$p = [1 + (2N_G/M_G)]^{1/2} \quad (12)$$

Since the second term in equation (1) becomes infinite by the condition of equation (4), equation (1) for the material balance on the liquid in the element  $d\xi d\zeta$  can not be used in this case; however, the following equation is derived from the material balance for the liquid phase in the element  $Zd\xi$  as shown in Fig. 3.

$$E_{L\xi}Zh_L \frac{d^2C_L}{d\xi^2} - u_L Z \frac{dC_L}{d\xi} - \int_0^Z K_L a (C_L - C_L^*) d\zeta = 0 \quad (13)$$

The third term in the above equation is the mass transferred from liquid to gas phase in the element  $Zd\xi$ . By the Assumption 6, or equation (4),  $C_L$  is independent of  $\zeta$  over any cross section  $\xi$ , but  $C_L^*$  which is the equilibrium concentration for  $C_G$  is dependent on  $\zeta$ . Applying equation (11) to  $C_L^*$ , the integration term in equation (13) is expressed as follows.

$$\begin{aligned} & \int_0^Z K_L a (C_L - C_L^*) d\zeta \\ &= 2K_L a (C_L - C_{Li}^*) \int_0^Z \frac{(1+p)e^{M_G(1+p)}e^{M_G(1-p)(\zeta/Z)} - (1-p)e^{M_G(1-p)}e^{M_G(1+p)(\zeta/Z)}}{(1+p)^2e^{M_G(1+p)} - (1-p)^2e^{M_G(1-p)}} d\zeta \\ &= \frac{K_L a Z}{N_G} \left[ 1 - \frac{4pe^{2M_G}}{(1+p)^2e^{M_G(1+p)} - (1-p)^2e^{M_G(1-p)}} \right] (C_L - C_{Li}^*) \end{aligned} \quad (14)$$

where,

$$C_{Li}^* = (C_{Gi} + b')/m'$$

Substituting equation (14) in equation (13), the following equation is obtained.

$$\frac{d^2 C_L}{d(\xi/X)^2} - 2M_L \frac{dC_L}{d(\xi/X)} - 2M_L N_L \alpha (C_L - C_{Li}^*) = 0 \quad (15)$$

where,

$$M_L = \frac{u_L X}{2E_L \xi h_L} \quad (16a)$$

$$N_L = \frac{K_L a X}{u_L} = \frac{m' K_G a X}{u_L} = \left( \frac{m' X Y u_G}{Y Z u_L} \right) \left( \frac{K_G a Z}{u_G} \right) = \lambda N_G \quad (16b)$$

$$\alpha = \frac{1}{N_G} \left[ 1 - \frac{4p e^{2M_G}}{(1+p)^2 e^{M_G(1+p)} - (1-p)^2 e^{M_G(1-p)}} \right] \quad (16c)$$

$$\lambda = m' F_G / F_L \quad (17)$$

The boundary conditions for equation (15) are derived from the material balance in liquid phase at  $\xi/X=0$  and  $\xi/X=1$  in the same manner as in equations (9) and (10).

$$\left[ \frac{dC_L}{d(\xi/X)} \right]_{\xi/X \rightarrow 0^+} = 2M_L [(C_L)_{\xi/X \rightarrow 0^+} - C_{Li}] \quad (18)$$

$$\left[ \frac{dC_L}{d(\xi/X)} \right]_{\xi/X \rightarrow 1^-} = 0 \quad (19)$$

Solving equation (15) under the above conditions, the concentration in the liquid at any cross section  $\xi$  on a tray,  $C_L$ , is given as follows.

$$\frac{C_L - C_{Li}^*}{C_{Li} - C_{Li}^*} = \frac{2(1+q) e^{M_L(1+q)} e^{M_L(1-q)(\xi/X)} - 2(1-q) e^{M_L(1-q)} e^{M_L(1+q)(\xi/X)}}{(1+q)^2 e^{M_L(1+q)} - (1-q)^2 e^{M_L(1-q)}} \quad (20)$$

where,

$$q = [1 + (2N_L \alpha / M_L)]^{1/2} = [1 + (2N_G \alpha \lambda / M_L)]^{1/2} \quad (21)$$

Substituting the relation of equation (20) in equation (11) with an equilibrium relationship, the concentration in gas at any position ( $\xi, \zeta$ ),  $C_G$ , is shown by the following equation.

$$\frac{C_G - C_{Gi}}{C_{Gi}^* - C_{Gi}} = \left\{ \frac{2(1+q) e^{M_L(1+q)} e^{M_L(1-q)(\xi/X)} - 2(1-q) e^{M_L(1-q)} e^{M_L(1+q)(\xi/X)}}{(1+q)^2 e^{M_L(1+q)} - (1-q)^2 e^{M_L(1-q)}} \right\} \\ \times \left\{ 1 - \frac{2(1+p) e^{M_G(1+p)} e^{M_G(1-p)(\zeta/Z)} - 2(1-p) e^{M_G(1-p)} e^{M_G(1+p)(\zeta/Z)}}{(1+p)^2 e^{M_G(1+p)} - (1-p)^2 e^{M_G(1-p)}} \right\} \quad (22)$$

Equations (20) and (22) are evaluated on the basis of the volumetric flow rates and molar concentrations. If the molar flow rates and molar volumes of both phases are constant throughout the froth layer, the following relations are derived from the compositions expressed in mole fractions of both phases,  $x$  and  $y$ , instead of molar concentrations,  $C_L$  and  $C_G$ .

$$\frac{x - x_i^*}{x_i - x_i^*} = \frac{C_L - C_{Li}^*}{C_{Li} - C_{Li}^*} \quad (20')$$

$$\frac{y-y_i}{y_i^*-y_i} = \frac{C_G-C_{Gi}}{C_{Gi}^*-C_{Gi}} \quad (22')$$

where,  $x$  is the mole fraction of volatile component in liquid phase at any cross section  $\xi$ , and  $y$  is that in gas phase at any point  $(\xi, \zeta)$  in the froth layer. Subscript  $i$  means the entering stream to the froth layer.  $x_i^*$  and  $y_i^*$  are equilibrium compositions for  $y_i$  and  $x_i$  respectively, and the equilibrium relationship is given by the following equation.

$$y = mx^* + b \quad (3a')$$

$$y^* = mx + b \quad (3b')$$

where,  $m$  is the slope of the equilibrium curve when the compositions of both phases are expressed in mole fraction. Expressing the molar flow rates of the gas and liquid as  $G$  and  $L$ , respectively, equation (17) is expressed as follows.

$$\lambda = m'F_G/F_L = mG/L \quad (17')$$

Applying equation (20) at  $\xi/X=1$ , the concentration of the liquid exit stream from froth layer,  $C_{Lo}$ , is given by equation (23).

$$\begin{aligned} \frac{C_{Lo}-C_{Li}^*}{C_{Li}-C_{Li}^*} &= \frac{x_o-x_i^*}{x_i-x_i^*} \\ &= \frac{4qe^{2M_L}}{(1+q)^2e^{M_L(1+q)} - (1-q)^2e^{M_L(1-q)}} \end{aligned} \quad (23)$$

Since the concentration gradient of liquid in the  $\zeta$ -direction is negligible by Assumption 6,  $C_{Lo}$  is equal to the concentration of down flow to the lower tray,  $C_{Loav}$ .

$$C_{Lo} = C_{Loav}, \quad x_o = x_{oav}. \quad (24)$$

Applying equation (22) at  $\zeta/Z=1$ , the concentration of the gas exit stream from the froth layer at any  $\xi$  is obtained.

$$\begin{aligned} \frac{C_{Go}-C_{Gi}}{C_{Gi}^*-C_{Gi}} = \frac{y_o-y_i}{y_i^*-y_i} &= \left[ 1 - \frac{4pe^{2M_G}}{(1+p)^2e^{M_G(1+p)} - (1-p)^2e^{M_G(1-p)}} \right] \\ &\times \left[ \frac{2(1+q)e^{M_L(1+q)}e^{M_L(1-q)(\xi/X)} - 2(1-q)e^{M_L(1-q)}e^{M_L(1+q)(\xi/X)}}{(1+q)^2e^{M_L(1+q)} - (1-q)^2e^{M_L(1-q)}} \right] \end{aligned} \quad (25)$$

Then, the concentration of up flow to the upper tray,  $C_{Goav}$ , is evaluated by the integral mean of equation (25) in the range from  $\xi/X=0$  to  $\xi/X=1$ ; however,  $C_{Goav}$  is also calculated by the following equation from the material balance using  $C_{Loav}$  evaluated from equation (23).

$$(C_{Goav}-C_{Gi}) = (F_L/F_G)(C_{Li}-C_{Loav}) \quad (26)$$

$$(y_{oav}-y_i) = (L/G)(X_i-x_{oav}) \quad (26')$$

#### § 4. Point Efficiency

Point efficiency,  $E_p$ , is defined by the following equation.

$$E_p = \frac{C_{G_o} - C_{G_i}}{C_{G^*} - C_{G_i}} = \frac{y_o - y_i}{y^* - y_i} \quad (27)$$

The concentration of the gas phase,  $y_o$ , leaving the froth at any  $\xi$  on a tray is given by equation (25) and that of the liquid phase,  $x$ , at any  $\xi$ , is given by equation (20); therefore, substituting those relations in equation (27) with equilibrium relation, the following relation is derived.

$$E_p = 1 - \frac{4pe^{2M_G}}{(1+p)^2 e^{M_G(1+p)} - (1-p)^2 e^{M_G(1-p)}} = N_G \alpha = \frac{N_L}{\lambda} \alpha \quad (28)$$

According to equation (28), point efficiency is affected by the holding time, mass transfer coefficient and gas-mixing, but is independent of the position on the tray, and constant throughout the froth layer. For scale up of the equipment, however, it is rather difficult to establish the condition where the values of  $E_p$  are equal.

Now, assuming that the gas flow is perfectly mixed in the  $\zeta$ -direction, the following relation is obtained.

$$\lim_{M_G \rightarrow 0} E_p = \frac{N_G}{1 + N_G} = \frac{N_L/\lambda}{1 + (N_L/\lambda)} \quad (29)$$

If the gas flow has no mixing throughout the froth layer, the following relation is obtained.

$$\lim_{M_G \rightarrow \infty} E_p = 1 - e^{-N_G} = 1 - e^{-N_L/\lambda} \quad (30)$$

Equation (30) coincides with the relation derived by Gerster, et al.<sup>13)</sup> on the assumption that there is no gas-mixing.

Fig. 4 shows the correlation between  $E_p$  and  $N_G$  or  $N_L$  using  $M_G$  as the parameter.

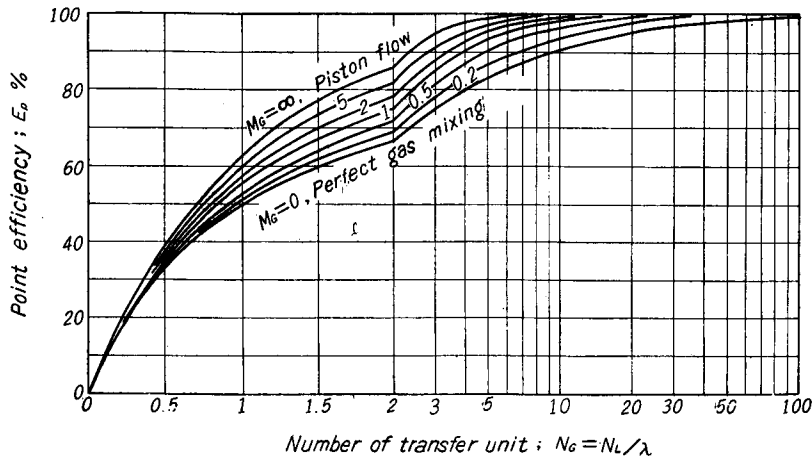


Fig. 4. Effects of  $M_G$  and  $N_G$  on  $E_p$ .



## § 5. Plate Efficiency

Murphree plate efficiency in the gas phase,  $E_{MG}$ , is defined by the following equation.

$$E_{MG} = \frac{C_{Goav} - C_{Gi}}{C_{Goav}^* - C_{Gi}} = \frac{y_{oav} - y_i}{y_{oav}^* - y_i} \quad (31)$$

where,  $C_{Goav}^*$  and  $y_{oav}^*$  are equilibrium composition for the liquid flowing down to the lower tray.

Substituting equations (23) and (26) in the above equation, the following relation is obtained.

$$E_{MG} = \frac{1 - \frac{4qe^{2M_L}}{(1+q)^2 e^{M_L(1-q)} - (1-q)^2 e^{M_L(1-q)}}}{\lambda \frac{4qe^{2M_L}}{(1+q)^2 e^{M_L(1+q)} - (1-q)^2 e^{M_L(1-q)}}} \quad (32)$$

Similarly, Murphree plate efficiency in the liquid phase,  $E_{ML}$ , is given by the following equation.

$$\begin{aligned} E_{ML} &= \frac{C_{Li} - C_{Loav}}{C_{Li} - C_{Loav}^*} = \frac{x_i - x_{oav}}{x_i - x_{oav}^*} \\ &= \frac{1 - \frac{4qe^{2M_L}}{(1+q)^2 e^{M_L(1+q)} - (1-q)^2 e^{M_L(1-q)}}}{1 - \frac{1}{\lambda} \left[ 1 - \frac{4qe^{2M_L}}{(1+q)^2 e^{M_L(1+q)} - (1-q)^2 e^{M_L(1-q)}} \right]} \end{aligned} \quad (33)$$

If the liquid phase is perfectly mixed on a tray, i.e.  $M_L \rightarrow 0$ , the following equations are derived from equations (32) and (33).

$$\lim_{M_L \rightarrow 0} E_{MG} = 1 - \frac{4pe^{2M_G}}{(1+p)^2 e^{M_G(1+p)} - (1-p)^2 e^{M_G(1-p)}} \quad (34)$$

$$\lim_{M_L \rightarrow 0} E_{ML} = \frac{\lambda \left[ 1 - \frac{4pe^{2M_G}}{(1+p)^2 e^{M_G(1+p)} - (1-p)^2 e^{M_G(1-p)}} \right]}{1 + (\lambda - 1) \left[ 1 - \frac{4pe^{2M_G}}{(1+p)^2 e^{M_G(1+p)} - (1-p)^2 e^{M_G(1-p)}} \right]} \quad (35)$$

Substituting equation (28) which was previously obtained for point efficiency into equations (34) and (35), the following relations are obtained.

$$\lim_{M_L \rightarrow 0} E_{MG} = E_p \quad (36)$$

$$\lim_{M_L \rightarrow 0} E_{ML} = \frac{\lambda E_p}{1 + (\lambda - 1) E_p} \quad (37)$$

Similarly, the following equations are derived on the assumption that the liquid phase is not mixed on the tray.

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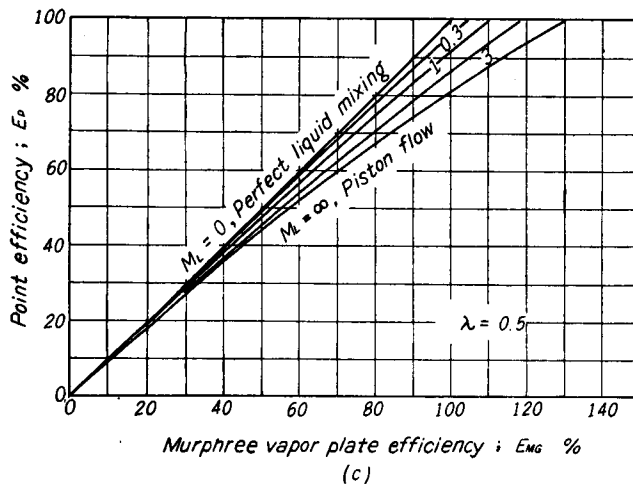
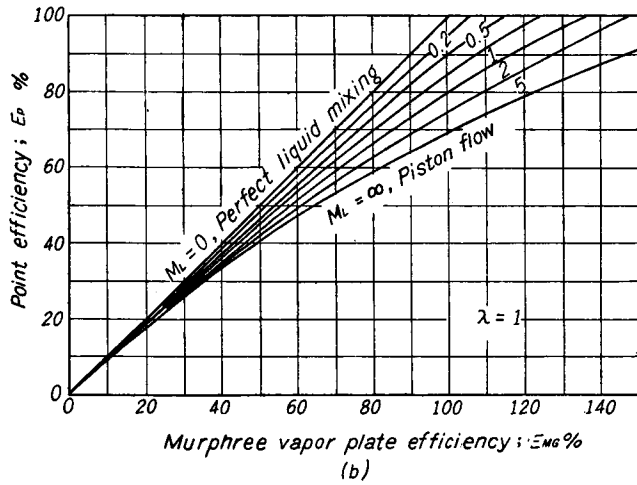
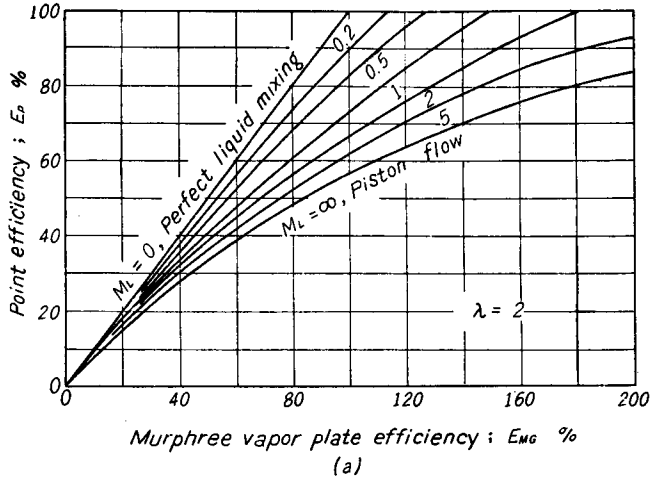


Fig. 5 Effects of  $M_L$  and  $\lambda$  on  $E_{MG}$ .

$$\lim_{M_L \rightarrow \infty} E_{MG} = (1/\lambda)[e^{\lambda E_p} - 1] \quad (38)$$

$$\lim_{M_L \rightarrow \infty} E_{ML} = \frac{1 - e^{-\lambda E_p}}{1 - (1/\lambda)[1 - e^{-\lambda E_p}]} \quad (39)$$

Equation (38) agrees with the correlation derived by Lewis<sup>3)</sup> on the assumption that the liquid is not mixed on a tray.

According to equation (32),  $E_{MG}$  is a function which has three variables,  $\lambda$ ,  $M_L$  and  $q$ ; however,  $q$  is the function which has three variables,  $\lambda$ ,  $M_L$  and  $E_p$ , as shown by equations (21) and (28). Accordingly,  $E_{MG}$  is also expressed as function which has three independent variables,  $\lambda$ ,  $M_L$  and  $E_p$ . Fig. 5 shows the correlation between  $E_p$  and  $E_{MG}$  for a few values of  $\lambda$ , using  $M_L$  as the parameter. Knowing  $\lambda$ ,  $M_G$ ,  $M_L$  and  $N_G$  or  $N_L$ , Murphree plate efficiency,  $E_{MG}$ , is evaluated using Figs. 4 and 5.

Fig. 6 illustrates an example of the calculation of plate efficiency. Let the case be considered where a sieve tray having the liquid path length  $X=100$  cm, is operated at the following conditions :

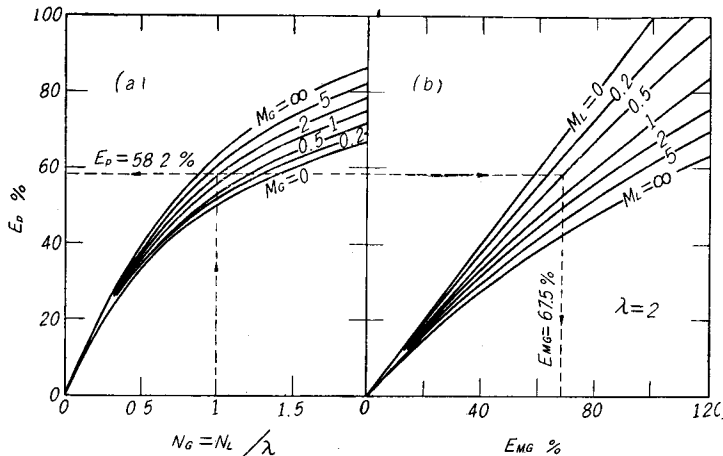


Fig. 6. Illustration of the Calculation of  $E_{MG}$ .

$$u_G = 15.0 \text{ cm/sec}, \quad u_L = 0.20 \text{ cm/sec}$$

$$G/L = 1, \quad m = 2$$

Under these operating conditions, the froth height, capacity coefficient, holdup and backmixing diffusivities are as follows :

$$Z = 20 \text{ cm}, \quad K_{Ga} = 0.75 \text{ sec}^{-1}$$

$$h_L = 0.40 \text{ cm}^3/\text{cm}^3, \quad h_G = 0.60 \text{ cm}^3/\text{cm}^3,$$

$$E_{L\xi} = 50 \text{ cm}^2/\text{sec}, \quad E_{G\xi} = 100 \text{ cm}^2/\text{sec}$$

By equations (8a) and (8b),  $M_G$  and  $N_G$  are evaluated as follows :

$$M_G = \frac{u_G Z}{2E_{G\zeta} h_G} = \frac{(15)(20)}{2(100)(0.60)} = 2.50$$

$$N_G = K_G a \frac{Z}{u_G} = 0.75 \frac{20}{15} = 1.00$$

With these conditions,  $E_p$  is given by the above values of  $M_G$  and  $N_G$  from Fig. 4 or equation (28) as shown in Fig. 6(a).

$$E_p = 0.582$$

From equation (17'),  $\lambda$  is evaluated.

$$\lambda = m(G/L) = (2)(1) = 2$$

And  $M_L$  is calculated by equation (16a).

$$M_L = \frac{u_L X}{2E_{L\xi} h_L} = \frac{(0.20)(100)}{2(50)(0.40)} = 0.50$$

Since  $\lambda$  is equal to 2,  $E_{MG}$  is obtained for  $E_p=0.582$  and  $M_L=0.50$  by using Fig. 5(a) as shown in Fig. 6(b).

$$E_{MG} = 0.675$$

## 6. Discussion

Fig. 7 illustrates the correlation calculated by equation (32) between  $E_{MG}$  and  $N_G$  or  $N_L$  using  $M_G$  and  $M_L$  as the parameters. Plate efficiency is largely affected by the increase in the values of  $N_G$  or  $N_L$ , and is decreased with the increase in both the liquid- and gas-mixing. The effect of the liquid-mixing on plate efficiency

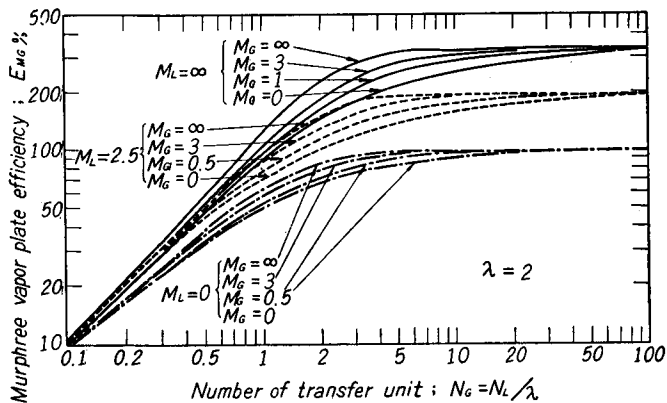


Fig. 7. Effects of  $M_G$ ,  $M_L$  and  $N_G$  on  $E_{MG}$ .

is more marked than that of gas-mixing, and is also more apparent with the increase in the values of  $N_G$  or  $N_L$ . On the other hand, the effect of gas-mixing is only striking in the range  $N_G$  gives the values from 0.5 to 20. In Fig. 8, the ratio of  $E_{MG}$  at any gas-mixing to value at maximum gas-mixing,  $(E_{MG})_{M_G=0}$ , is plotted against  $N_G$ . According to this figure, the difference between  $E_{MG}$  at maximum gas-mixing and that at minimum mixing is lower than 10% in the range where  $N_G$  is less than 0.5; but this difference increases with the increase in  $N_G$ , reaches a maximum value at  $N_G=2.5$  to 3, and then decreases with the increase in  $N_G$ , and is again lower than 10% at  $N_G$  over 20. Although the result of Fig. 8 is obtained at the condition of  $\lambda=2$  and  $M_L=2.5$ , similar relations are also obtained at the other values of  $\lambda$  and  $M_L$ , and the effect of  $M_G$  on  $E_{MG}$  is more conspicuous with the increase in  $M_L$ . In general, the effect of the gas-mixing may be negligible when  $N_G$  is less than 0.5 or greater than 20.

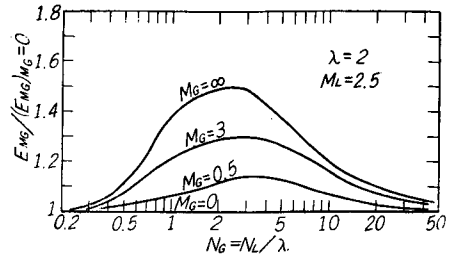


Fig. 8. Effect of the gas-mixing on the plate efficiency.

Now, let the case be considered where the diameter of the plate column is enlarged. For the scale up of the sieve plate column,  $m$ ,  $G/L$  and  $u_G$  are generally kept constant. If the plate spacing is kept constant, the froth height,  $Z$ , may also be kept at about the same height. For the sake of convenience, assume the values of those factors as follows.

$$m(G/L) = 2, \quad u_G = 15.0 \text{ cm/sec}, \quad Z = 20 \text{ cm}$$

In the case where  $G/L$ ,  $u_G$  and  $Z$  are kept constant and the liquid path width  $Y$  is enlarged in proportion to liquid path length  $X$ , the flow velocity of liquid  $u_L$  increases in proportion to  $X$ . Since the holdups of both the phases  $h_G$  and  $h_L$ , capacity coefficient  $K_{Ga}$ , and backmixing diffusivities  $E_{G\xi}$  and  $E_{L\xi}$  are the functions of  $u_G$  and  $u_L$ , the values of these factors change with the value of  $X$ ; however, the effect of  $u_L$  on  $h_G$ ,  $h_L$ ,  $E_{G\xi}$  and  $E_{L\xi}$  are relatively insignificant at the condition which  $Z$  and  $u_G$  are kept at constant value. Then, assume the values of the above factors as follows:

$$h_G = 0.60 \text{ cm}^3/\text{cm}^3, \quad h_L = 0.40 \text{ cm}^3/\text{cm}^3, \quad E_{G\xi} = 100 \text{ cm}^2/\text{sec}$$

Although  $K_{Ga}$  changes with the change in  $u_L$ , let the case be considered where  $K_{Ga}$  is kept constant as follows, regardless of the change in  $u_L$ ,

$$K_{Ga} = 0.75 \text{ sec}^{-1}$$

Under the above assumptions, the correlation between  $E_{MG}$  and  $X$  is shown for some values of  $E_{L\xi}$  in Fig. 9. According to this figure,  $E_{MG}$  at any constant  $E_{L\xi}$  gradually shifts from the maximum mixing state to the minimum mixing state with the increase in liquid path length.

Equations (28), (32) and (33) are derived on the assumptions stated in §2. Of these assumption, the one which is most doubtful may be Assumption 6, i.e.  $\partial C_L / \partial \zeta = 0$ . Fig. 10 shows an example of experimental results to verify the validity of Assumption 6.

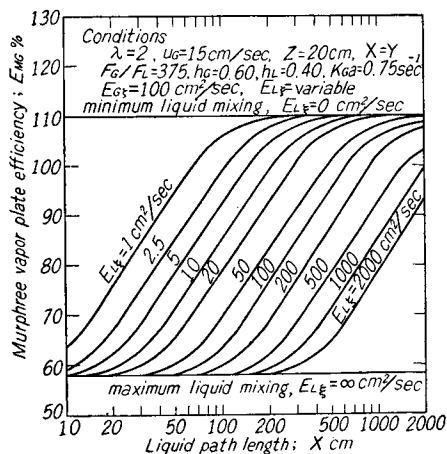


Fig. 9 Effect of the liquid path length on the plat efficiency

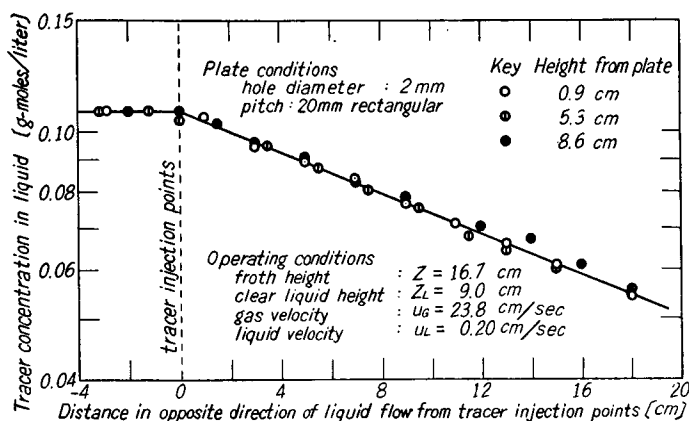


Fig. 10. Concentration gradients for the backward mixing of tracer.

A schematic diagram of the experimental apparatus is shown in Fig. 11. The apparatus consisted essentially of a single rectangular perforated plate, A, 10.5 cm width and 65.0 cm long, and the effective liquid path length was varied from 41.0 cm to 60.0 cm by changing the location of the weirs, B and C. The plate has 1.0, 1.5 or 2.0 mm perforations spaced on a 5, 10 and 20 mm triangular or rectangular pitch. In order to prevent the break down of the froth layer over the weir and to form a stable froth layer, the gauzes, D and D', were set up at the upper parts of the weirs. Water introduced into one end of plate, E, flowed over the inlet weir, B, and reached the outlet weir, C, and then discharged. Air was introduced into the air chamber under the plate, F, flowed up through a plate

and contacted crosscurrently with the water. Tracer solution (aqueous solution saturated with sodium chloride) was continuously fed with a constant small flow rate to the injection points, G. These injection points were arranged on a line at the distances of 10.0 cm from the exit weir and of 1.1 cm from the plate floor.

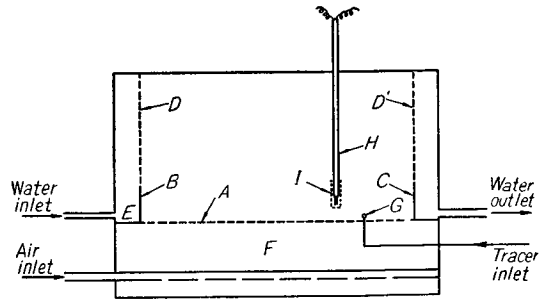


Fig. 11. Experimental apparatus.

Electrode, H, used for the measurement of the concentration of salt, was covered with a brass gauze, I. Then, the concentration gradient of salt in the liquid in the horizontal direction was measured at several heights in the froth layer by the electric conductivity method.

Fig. 10 shows an example of the results in such an experiment. According to this figure, the salt concentration is nearly equal in different froth levels either upstream or downstream from the injection points. Accordingly, it may be verified that Assumption 6 is approximately correct. Moreover, from the fact that the concentration gradient of the salt upstream from the injection points gives a linear relation on a semi-logarithmic paper, it may also be concluded that the backmixing may be expressed by constant backmixing diffusivity.

#### Nomenclature

$a$	: contacting surface area per unit volume of froth layer	$[\text{cm}^2/\text{cm}^3 \text{ of froth}]$
$C_G$	: concentration of volatile component in gas phase	$[\text{g-moles}/\text{cm}^3]$
$C_G^*$	: concentration of volatile component in gas phase in equilibrium with $C_L$	$[\text{g-moles}/\text{cm}^3]$
$C_L$	: concentration of volatile component in liquid phase	$[\text{g-moles}/\text{cm}^3]$
$C_L^*$	: concentration of volatile component in liquid phase in equilibrium with $C_G$	$[\text{g-moles}/\text{cm}^3]$
$E_{G\xi}, E_{G\zeta}$	: back-mixing diffusivity in gas phase in froth, in $\xi$ - and $\zeta$ -direction, respectively.	$[\text{cm}^2/\text{sec}]$
$E_{L\xi}, E_{L\zeta}$	: back-mixing diffusivity in liquid phase in froth, in $\xi$ - and $\zeta$ -directions, respectively	$[\text{cm}^2/\text{sec}]$
$E_{MG}$	: Murphree plate efficiency in the gas phase	$[\text{—}]$
$E_{ML}$	: Murphree plate efficiency in the liquid phase	$[\text{—}]$
$E_p$	: point efficiency	$[\text{—}]$
$F_G, F_L$	: volumetric flow rate of gas and liquid, respectively	$[\text{cm}^3/\text{sec}]$

$G$	: molar gas flow rate	[g-moles/sec]
$h_G = (Z - Z_L)/Z$	: holdup of the gas phase in the froth layer	[cm <sup>3</sup> /cm <sup>3</sup> of froth]
$h_L = Z_L/Z$	: holdup of the liquid phase in the froth layer	[cm <sup>3</sup> /cm <sup>3</sup> of froth]
$K_G$	: overall mass transfer coefficient in gas phase	[g-moles/cm <sup>2</sup> ·sec·(g-mole/cm <sup>3</sup> )]
$K_L$	: overall mass transfer coefficient in liquid phase	[g-moles/cm <sup>2</sup> ·sec·(g-mole/cm <sup>3</sup> )]
$L$	: molar liquid flow rate	[g-moles/sec]
$M_G = Zu_G/2E_G\zeta h_G$	: gas-mixing parameter	[—]
$M_L = Xu_L/2E_L\xi h_L$	: liquid-mixing parameter	[—]
$m$	: slope of equilibrium curve based on mole fraction	[—]
$m'$	: slope of the equilibrium curve based on molar concentration	[—]
$N_G = K_G a(Z/u_G)$	: number of transfer unit in the gas phase	[—]
$N_L = K_L a(X/u_L)$	: number of transfer unit in the liquid phase	[—]
$p = [1 + (2N_G/M_G)]^{1/2}$		[—]
$q = [1 + (2N_L\alpha/M_L)]^{1/2} = [1 + (2N_G\alpha\lambda/M_L)]^{1/2}$		[—]
$u_G$	: superficial linear gas velocity based on effective tray area	[cm/sec]
$u_L$	: superficial linear liquid velocity based on froth area	[cm/sec]
$X$	: liquid path length	[cm]
$x$	: mole fraction of volatile component in liquid phase	[—]
$x^*$	: mole fraction of volatile component in liquid phase in equilibrium with $y$	[—]
$Y$	: tray width	[cm]
$y$	: mole fraction of volatile component in gas phase	[—]
$y^*$	: mole fraction of volatile component in gas phase in equilibrium with $x$	[—]
$Z$	: height of aerated liquid above tray floor	[cm]
$Z_L$	: clear liquid height	[cm]
$\alpha = E_p/N_G$		[—]
$\zeta$	: height from plate floor	[cm]
$\xi$	: distance from the weir in the direction of liquid flow	[cm]
$\lambda = m'F_G/F_L = mG/L$		[—]

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