The Selection of the Representative Year of Stream Flow for Electric Power Generation

By

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An analysis of the actual state of the combination of the load and the hydro- and steam-power in a large combined hydro-steam power system is necessary for any economic comparison between various types of hydro- and steam-power generation; and for the power generation by a plant of the run-off-river type as well as of the pondage and storage types, it sometimes involves an inspection of the daily stream flow in connection with the daily load curve throughout a year.

In such cases, it being difficult to draw a daily stream flow diagram for the future, it sometimes becomes necessary to select the representative year representing a typical stream flow from the data in the past. This paper describes the method for its selection.

I. Introduction

An analysis of the actual state of the combination of the load and the hydro- and steam-power in a large combined hydro-steam power system is necessary for any economic comparison between various types of hydro- and steam-power generation in their development and management, but since it is extremely difficult to assume a daily stream flow in connection with the load curve in the future, we have proposed a method considering the project value of the annual average stream flow (i.e. available power)[1].

The method of estimating the annual average stream flow expected to hold on an average for a long term is shown in Ref. 2).

It is sometimes necessary, however, not only in the case of pondage and storage types, but also in the run-off-river type, to calculate the characteristics of the power generation by combining the daily stream flow diagram with the data for the daily load curve throughout a year for a newly developed plant[1]. In this case, as it is difficult to make up a new daily stream flow diagram expected to hold for a long term, there seems to be no better method than that of selecting a typical year from the past, since there is little data available, at least in Japan.

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It is not always reasonable, however, merely to take as typical that year when the percentage of stream flow is nearest to its mean value for all past years. We should first find the value of the annual average stream flow which holds on an average for a long term taking into consideration the fact that the annual average stream flow includes a prominent fluctuation having a hidden period of 10~11 years, and then select the year which presents a typical flowing condition from the years whose annual average stream flows are approximately that of the long-term average.

II. The Annual Average Stream Flow

Since it is almost impossible to find any particular year whose annual average stream flow equals that which hold on an average for a long term, we are required to select a particular year with a typical annual average stream flow by postulating an adequate interval as a basis of selection.

Here, the following points are to be borne in mind:

(i) The purpose here is to find the particular year, not the average of the stream flows.

(ii) The year chosen as representative should be preferably a recent year.

The interval, therefore, which is taken as a basis is to be determined from N-year average stream flow (where N indicates the longest fundamental fluctuation period).

The method for determining the variation interval of the N-year average stream flow is as follows:

(1) We calculate an unbiased estimate of variance given by

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\{z(i)-\bar{z}\{^2 \qquad (1)$$

from the n data of the residual series or random components $z(i)$ obtained by subtracting the mean and the periodic component from the annual average stream flow $x(i)$. Then, as $z(i)$ may be regarded as samples obtained from the population $N(0, \sigma^2)$[2], the annual average stream flow $x(i)$ will be expected to fall, with a probability of about 68%*, within the variation interval

$$I \equiv [m(i)+\xi(i)+S,\ m(i)+\xi(i)-S], \qquad (2)$$

where $m(i)$ is the mean of $x(i)$,

$\xi(i)$ is the periodic component.

In Figs. 1 (a), (b) and 2 (a), (b) the dotted lines show the variation intervals

* In the following pages the qualification "about" is omitted.

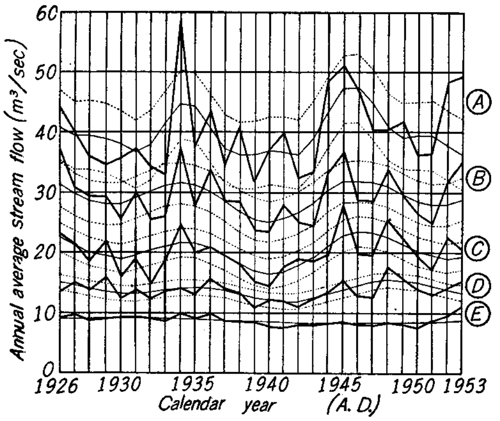

Fig. 1(a). (the River A)

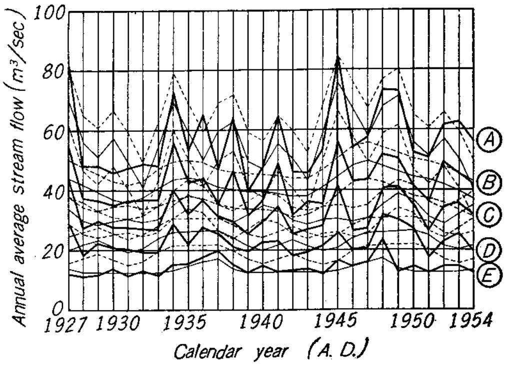

Fig. 1(b). (the River B)

The fluctuation curves of the annual average stream flows and their respective periodic components obtained by harmonic analysis.

Note: ━━ Data
―― Periodic components
┈┈┈ 68% variation intervals
Ⓐ··· The total annual average stream flow
Ⓑ··· where the 3-month-flow for each year is taken as the maximum available
Ⓒ··· 〃 6-month-flow 〃 〃
Ⓓ··· 〃 9-month-flow 〃 〃
Ⓔ··· 〃 drought-flow 〃 〃

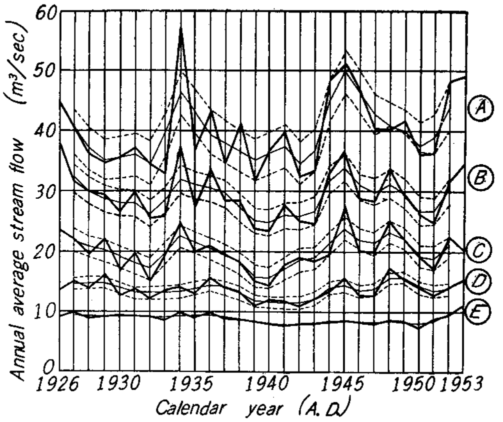

Fig. 2(a). (the River A)

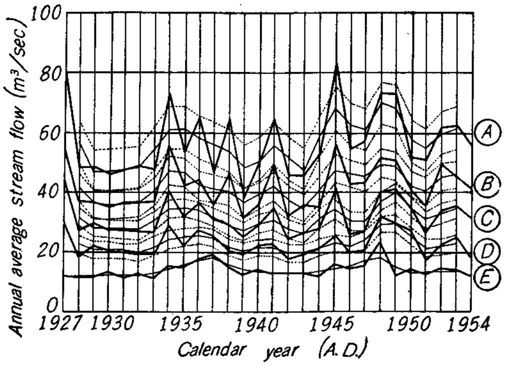

Fig. 2(b). (the River B)

The fluctuation curves of the annual average stream flows and their respective periodic components obtained by the moving average of Eq. (3).

Note: ━━ Data
―― Periodic components
┈┈┈ 68% variation intervals
Ⓐ··· The total annual average stream flow
Ⓑ··· where the 3-month-flow for each year is taken as the maximum available
Ⓒ··· 〃 6-month-flow 〃 〃
Ⓓ··· 〃 9-month-flow 〃 〃
Ⓔ··· 〃 drought-flow 〃 〃

drawn by this method, taking the total annual average stream flow, 3-, 6-, 9-month-flow and the drought-flow for each year as the maximum available stream flows for the River A and B respectively[2]. In Fig. 1 the periodic component was detected by harmonic analysis and in Fig. 2 by the weighted 3-year moving average

$$y_2(i) = \frac{1}{4}\{x(i-1)+2x(i)+x(i+1)\}. \qquad (3)$$

In the following calculations, the results shown in Fig. 1 will be used for the sake of convenience.

(**2**) The N-year average of $z(i)$ can be regarded as a sample obtained from $N(0, \sigma^2/N)$. Generally speaking, the fluctuation in the N-year moving average of the functional component is so small that it can be approximately looked upon as a constant value $\overline{m}_N$, provided that any trend detected be subtracted beforehand. It will be admissible, therefore, in the case of N-year average stream flow to adopt the 68% variation interval, which takes into account such trends.

$$I_N \equiv [\overline{m}_N + S/\sqrt{N},\ \overline{m}_N - S/\sqrt{N}]. \qquad (4)$$

The dotted lines in Figs. 3 (a) and (b) represent the 68% variation intervals

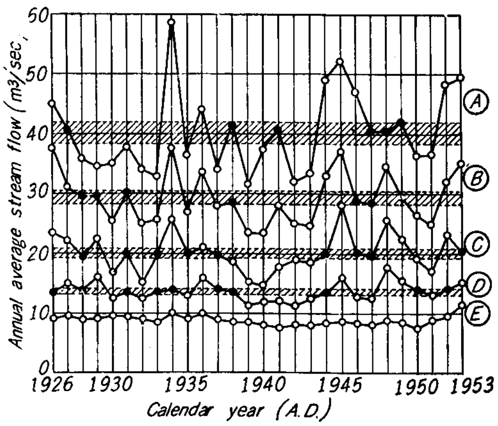

Fig. 3(a). (the River A)

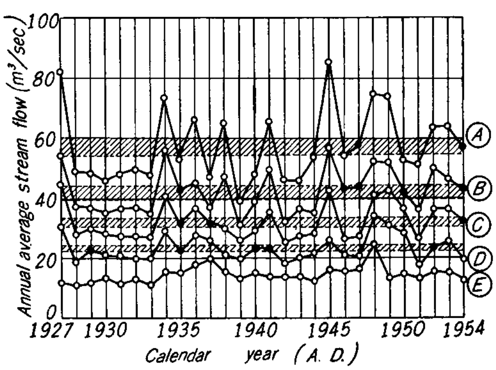

Fig. 3(b). (the River B)

68% variation intervals of the respective 11-year moving average stream flows.

Note: ○…… The annual average stream flow
●…… Those falling within the respective variation intervals
////// 68% variation intervals of the respective 11-year moving average stream flows

for various kinds of N-year average stream flows of the Rivers A and B (for $N=11$), where the years falling within these intervals are marked with "●".

Table 1. S and $S/\sqrt{N}$.

Kind of annual average stream flows \ Name of rivers \ Interval	S (m³/sec)		$S/\sqrt{N}$ (m³/sec)	
	A	B	A	B
Total annual average stream flow	6.07	8.10	1.83	2.70
where the 3-month-flow for each year is taken as the maximum available	3.62	7.60	1.10	2.30
" 6-month-flow " " "	2.56	5.10	0.77	1.50
" 9-month-flow " " "	1.56	3.60	0.47	1.00

In addition, the values of S and $S/\sqrt{N}$ (for $N=11$) for the Rivers A and B are shown in Table 1.

(3) The decision as to the suitability of the annual average stream flow for any year should depend upon whether the average stream flow for that year falls within this interval or not. The years which pass the test by means of the variation intervals in Figs. 3 (a) and (b) are shown in Table 2 for the Rivers A and B respectively, where the years enclosed in parentheses indicate those which have passed the test by means of the 95% variation intervals* as well.

Table 2. The years passing the test of the annual average stream flow.

Kind of annual average stream flows \ Name of rivers	A	B
Total annual average stream flow	1927, (1931), 1938, (1940), 1941, 1947, 1948, 1949	(1935), (1944), (1946), 1947, (1950), 1954
where the 3-month-flow for each year is taken as the maximum available	(1927), 1928, 1929, 1931, (1937), 1938, (1941), 1946, 1947, 1949	1935, (1936), (1940), 1946, 1947, 1950, 1953, 1954
" 6-month-flow " " "	1928, 1931, 1933, 1935, (1936), 1937, (1938), (1942), (1943), 1944, 1946, 1947, (1950), 1953	(1929), 1935, 1937, (1938), (1940), (1952), 1954
" 9-month-flow " " "	1926, 1928, 1931, 1933, 1934, (1935), 1937, 1938, 1944, (1946), 1950, (1951), 1952	1929, 1935, (1938), 1940, 1941, (1944), 1952

III. The Flowing Condition

When we have obtained the fundamental fluctuation period N (years) of the

* We have only to substitute 2S for S in Eqs. (2) and (4) to find the 95% variation interval.

annual average stream flow, we have to proceed to find the annual flowing condition holding on an average during those N years.

Though the flowing condition is represented by the daily stream flow diagram throughout a year, it includes, as is evident from Figs. 4 (a) and (b), such a

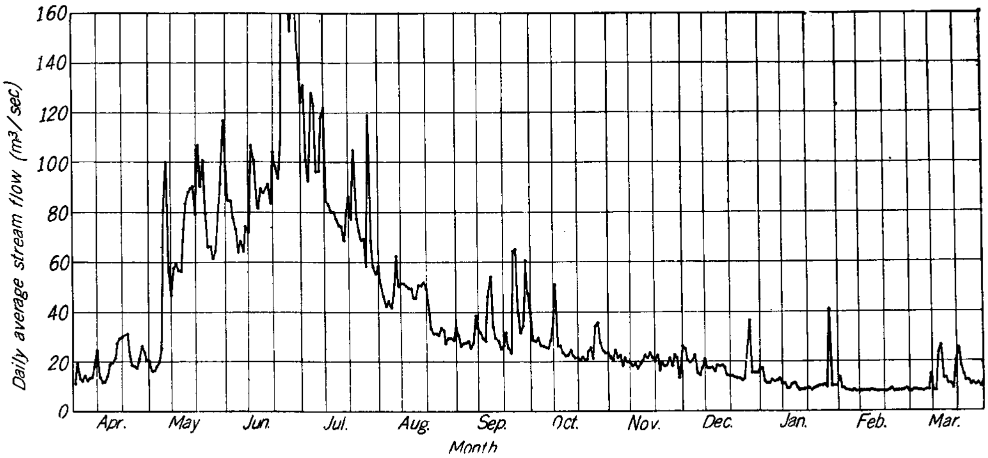

Fig. 4(a). (the River A)

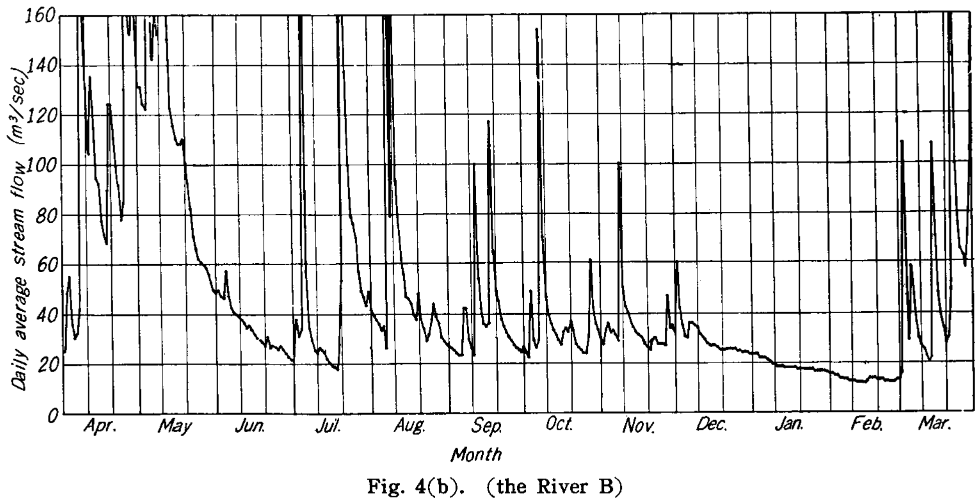

Fig. 4(b). (the River B)
The daily stream flow diagram.

complicated fluctuation from year to year that it is difficult to analyse and examine that fluctuation directly. It will be investigated, therefore, by the following two methods.

(1) The Duration Curve of Stream Flow

By finding out and plotting the duration days corresponding to various values of stream flow from the daily stream flow diagram for each year, we obtain the variation curve of the annual duration days for each stream flow. This curve usually abounds in fluctuations and by representing its variation range in the form of the duration curve we can determine the variation range of the abscissa-mean duration curve of stream flow.

The thick lines in Figs. 5 (a) and (b) show the variation range of the annual duration curves of stream flows of the Rivers A and B respectively.

As may be seen from these Figures, the variation in the duration days are extremely violent and irregular, so that it is difficult to detect its period by means of the correlogram as is done with the annual average stream flow.

After we have compared, for respective values of k, the errors of estimates which occur if we adopt the average duration days of stream flow for an arbitrary number of k successive years as extrapolatory estimates to hold for the next k years (the results of those calculations are omitted here), we shall have to conclude that there will be no optimum value of k if we are to use the k-year average duration days of stream flow as the projected value of duration days for k years to come. This may readily be understood from the fact that the correlograms of the 3-year moving average stream flows for the different parts, obtained by dividing horizontally the duration curve of the stream flow for each year as shown in Figs. 6 (a) and (b), present different periods.

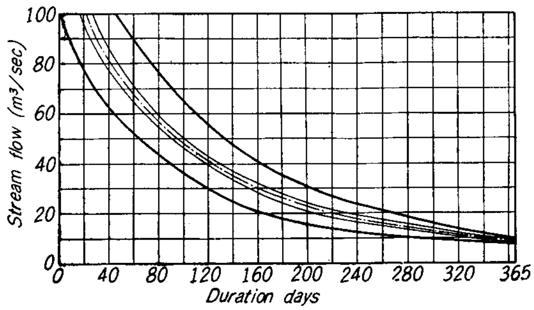

Fig. 5(a). (the River A)

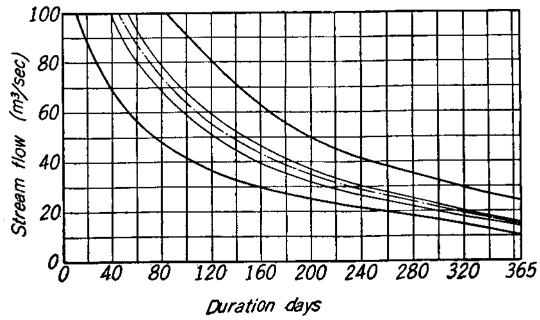

Fig. 5(b). (the River B)

The variation range of the annual duration curve of stream flows and the 11-year abscissa-mean duration curves.

Note: ━━ The annual duration curve
—— The 11-year abscissa-mean duration curve
—·— The 28-year abscissa-mean duration curve

But, as it seems pertinent to take the fundamental fluctuation period (N years) of the annual average stream flow as a basis for considering the annual

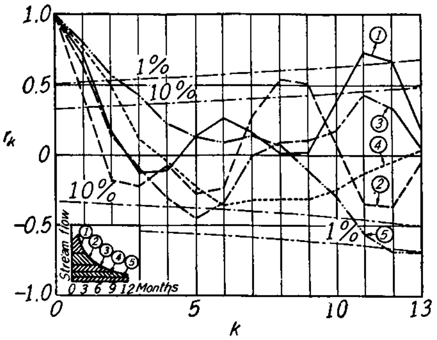

Fig. 6(a). (the River A).

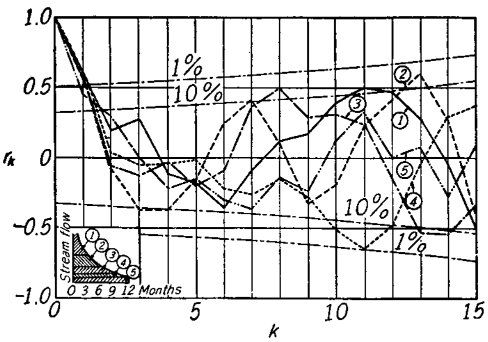

Fig. 6(b). (the River B)

The correlograms of the 3-year moving average stream flows for $K=2$ of 5 respective parts.

duration curve of stream flow in connection with the annual average stream flow, we propose to examine the N-year synthesized duration curve of stream flow. There have been two methods for obtaining the N-year synthesized duration curve of stream flow; namely, the so-called abscissa-mean and ordinate-mean methods. While the flowing condition for the i-th year of the N years in the past is given by the relation of the daily stream flow q_i and duration days d_i, the abscissa-mean method represents the relation between the mean d of the duration days for every year corresponding to the stream flow q, i.e.

$$d \equiv \bar{d}_i = \frac{1}{N}\left(\sum_{i=1}^{N} d_i\right)$$

and q. This is evidently none other than the reduction to $1/N$ of the time scale of the duration days in the synthesized duration curve of stream flow obtained by arranging all the data for N years in order of magnitude.

The ordinate-mean, on the other hand, represents the relation between the mean q of the duration stream flows for every year corresponding to the duration days d, i.e.

$$q \equiv \bar{q}_i = \frac{1}{N}\left(\sum_{i=1}^{N} q_i\right)$$

and d.

Both these curves furnish the same result as to the calculation of the average stream flow or the generated energy. But we think the method of abscissa-mean should be adopted in the development of hydro-power, because to estimate the N-year synthesized duration curve obtained by all data of the stream flow for N years is none other than to fit a curve to all these data arranged and plotted in order of magnitude.

In our case, the curves obtained by these two methods are practically identical except for the regions close to flood- and drought-flow due to the large number of years considered. It is also possible, in estimating the abscissa-mean duration curve of stream flow, to regard the curve obtained by connecting in order the plotted points as the estimated curve of the synthesized flowing condition for N years.

If it is necessary to fit a curve to these plotted points, we can do so either by the method of least squares or a method for estimating a probability distribution function such as the logarithmico-normal distribution. It has, however, little practical utility as far as the study of combined hydro-steam power is concerned, as there is no simultaneity between the duration curves of stream flow and load.

It follows, therefore, that in order to find the abscissa-mean duration curve of stream flow, we have to adopt the variation range of the abscissa-mean duration curve of stream flow for every N years as the variation range of the project duration curve of stream flow, and then choose the year whose duration curve of stream flow falls within this range. The thin lines in Figs. 5 (a) and (b) represent the variation range for the River A and B respectively for $N=11$.

As is evident from these figures, the range of variation in the 11-year abscissa-mean duration curve of stream flow is very small as compared with that of the annual duration curve of stream flow, though both have for their central axis (the chain line) the 28-year abscissa-mean duration curve of stream flow.

(2) The Correlation between the Annual Average Stream Flow and the Wet-, Moderate- and Drought-flow

The monthly flowing condition fluctutates much more conspicuously than the annual flow for each year, making it harder to deal with, and the monthly plan for power generation based upon the monthly flowing condition is in the last resort summed up for the whole year. Therefore we should adopt, as the project value of the monthly average stream flow or the monthly flowing condition upon which the monthly plan is to be based, the flows of the year determined for the annual flowing condition. Consequently, we shall study the suitability of the monthly flowing condition as a typical year by the following method.

In our analysis, we divide a year into three "seasons" for convenience' sake

The wet season: May, Jun. and Jul.

The moderate season: Apr., Aug., Sep. and Oct.

The drought season: Nov., Dec., Jan., Feb. and Mar.

However, such a division should be decided reasonably according to the flowing condition of the river. By applying the same method to each seasonal average stream flow as we have done to the annual average stream flow, we can obtain the variation interval for the seasonal average stream flow. Then we draw the correlation diagrams for the seasonal average stream flows for each year with various annual average stream flows of the Rivers A and B, then draw the 68% and 95% variation intervals for the *N*-year average of the annual and seasonal average stream flows in each correlation diagram, and thereby test the suitability of regarding the flowing condition of a year as typical according as whether or not the data for that year falls within the two rectangular regions obtained by these two sets of variation intervals.

In this case the year falling within the 68% variation intervals is to be regarded as the best, and the year falling within the 95% variation intervals as the second best.

The results obtained are shown in Figs. 7 (a), (b)-1, -2 and -3. (Due to considerations of space, only those for the total annual average stream flow are

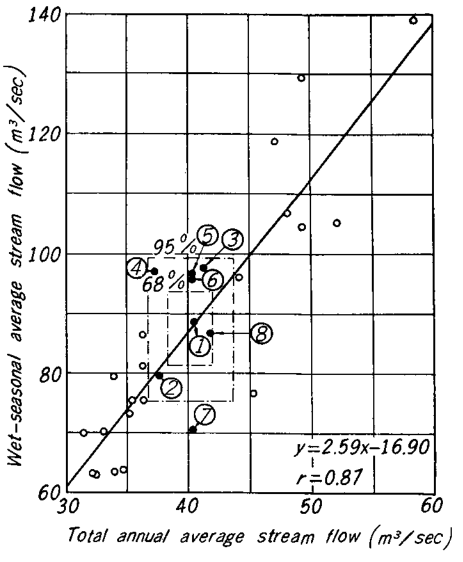

Fig. 7 (a)-1. (the River A)

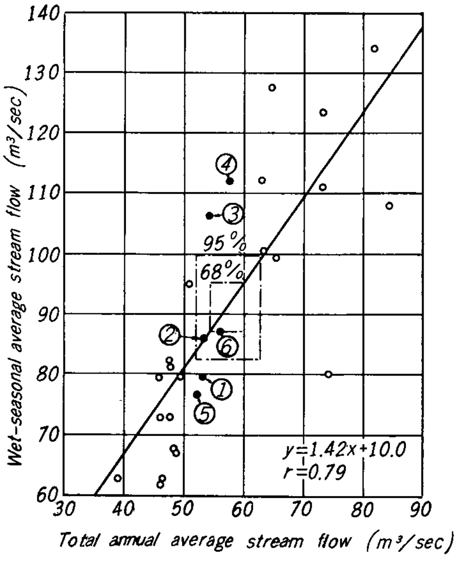

Fig. 7 (b)-1. (the River B)

The correlation diagrams between the wet-seasonal average stream flow and the annual average stream flow for each year.

Note: Inner rectangle 68% variation interval
Outer " 95% " "
○…… Data for each year
●…… That passing the test of the annual average stream flow
The River A…① 1927, ② 1931, ③ 1938, ④ 1940, ⑤ 1941, ⑥ 1947, ⑦ 1948, ⑧ 1949
The River B…① 1935, ② 1944, ③ 1946, ④ 1947, ⑤ 1950, ⑥ 1954

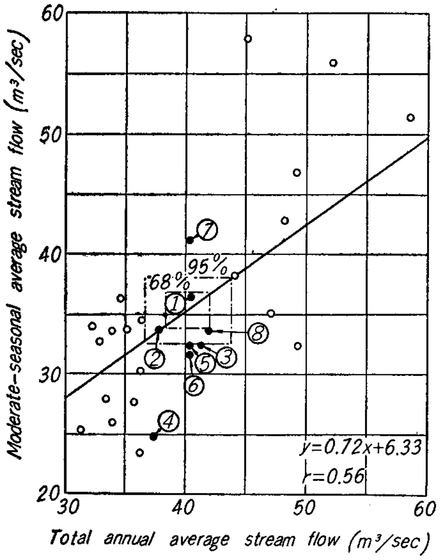

Fig. 7 (a)-2. (the River A)

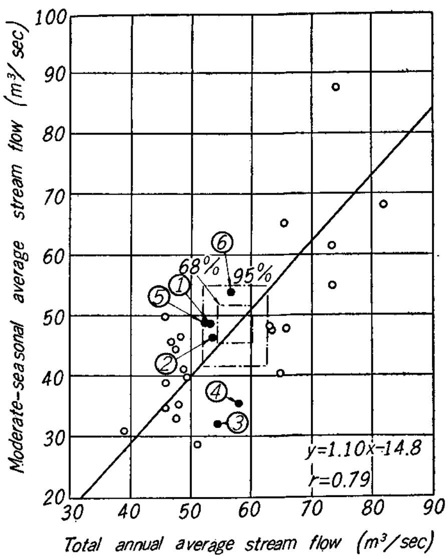

Fig. 7 (b)-2. (the River B)

The correlation diagrams between the moderate-seasonal average stream flow and the annual average stream flow for each year.

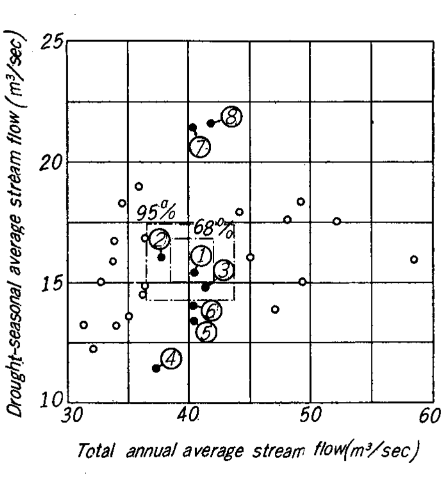

Fig. 7 (a)-3. (the River A)

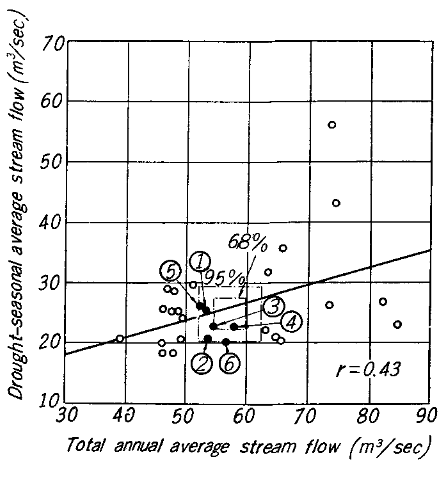

Fig. 7 (b)-3. (the River B)

The correlation diagrams between the drought-seasonal average stream flow and the annual average stream flow for each year.

Note: Inner rectangle 68% variation interval
Outer " 95% " "
○…… Data for each year
●…… That passing the test of the annual average stream flow
The River A…① 1927, ② 1931, ③ 1938, ④ 1940, ⑤ 1941, ⑥ 1947, ⑦ 1948, ⑧ 1949
The River B…① 1935, ② 1944, ③ 1946, ④ 1947, ⑤ 1950, ⑥ 1954

shown.) According to these Figures, the average stream flows for the wet and moderate seasons generally show a significant correlation with the annual average stream flow. (The regression line is shown for reference). On the other hand, the average stream flow for the drought season may be regarded as almost uncorrelated to the annual average stream flow.

These Figures also serve to estimate the respective seasonal average stream flows from the annual average stream flow.

IV. The Determination of the Representative Year

We may choose as representative the latest year which has a suitable annual average stream flow and which has passed the test of the flowing condition by the above two methods.

In the following, we examine the flowing conditions of the Rivers A and B by the above methods in order to select the representative year.

(1) Of the year shown in Table 2, those falling within the variation range of the 11-year abscissa-mean duration curve of stream flow represented by the thin lines in Fig. 5, are shown in Table 3.

(2) Of the years shown in Table 2, those of which every seasonal stream flow passes the test by means of the respective variation interval (cf. Fig. 7), are shown in Table 4.

The meaning of the parentheses in Tables 3 and 4 is the same as in Table 2.

Now that we have obtained by the above two methods, the results shown in Tables 3 and 4 which satisfy every requirement for the flowing condition suitable as a typical year, we can select the years common to both Tables and get Table 5, from which we select the latest year as representative.

Table 3. The years possing the test of the flowing condition by means of the duration curve of stream flow.

Kind of annual average stream flows \ Name of rivers	A	B
Total annual average stream flow	1938, 1947	(1935)
where the 3-month-flow for each year is taken as the maximum available	1931, (1937), 1938, 1946, 1947	1935, 1946, 1947
" 6-month-flow " " "	1928, 1931, 1933, 1935, 1937, 1944, 1946, 1947	1935
" 9-month-flow " " "	1928, 1933, 1937, 1938, 1944	1929, 1935, 1940, 1941

Table 4. The years passing the test of the flowing condition by means of the correlation of the annual and the seasonal average stream flows.

Kind of annual average stream flows \ Name of rivers	A	B
Total annual average stream flow	1927, (1931), (1938), (1947)	(1935), (1944), (1947), (1950), (1954)
where the 3-month-flow for each year is taken as the maximum available	1931, (1938), (1947)	1935, (1950)
" 6-month-flow " " "	(1928), 1931, (1935), 1937, (1838), (1943), 1944, 1947, (1950)	1935, (1937)
" 9-month-flow " " "	1926, 1928, 1931, 1933, (1934), (1935), (1937), (1938), 1944, (1950), (1952)	(1929), 1940, (1952)

Table 5. The years passing the test both of the annual average stream flow and the flowing condition.

Kind of annual average stream flows \ Name of rivers	A	B
Total annual average stream flow	1938, 1947	1935
where the 3-month-flow for each year is taken as the maximum available	1931, 1938, 1947	1935
" 6-month-flow " " "	1928, 1931, 1935, 1937, 1944, 1947	1935
" 9-month-flow " " "	1928, 1933, 1937, 1938, 1944	1929, 1940

V. Conclusion

If it sometimes becomes necessary in a large combined hydro-steam power system to analyse the actual state of the combination of the daily load, and the hydro- and steam-power throughout a year, we have to assume a diagram of the daily stream flow throughout a year which is expected to hold on an average for a long term to come, corresponding to the load curve. But as it is difficult to draw at first hand the diagram of the daily stream flow anew, we must be content to substitute the data for a year with a typical stream flow. It will do well, then, to determine the suitable variation region of the annual average stream flow beforehand, taking into account the fundamental fluctuation period of the stream flow, and then, as to the past years whose average stream flow throughout a year falls within this region, to inspect the flowing condition by means of (i) the variation range of the annual abscissa-mean duration curve of stream flow during a proper period of years (the fundamental fluctuation period of the stream flow or its multiple), and (ii) the respective correlation of the annual average stream flow with the wet-, moderate- and drought-flow. Thus we can select as representative the latest year which satisfys all these requirements.

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