Analysis and Design of a Valve-Controlled Hydraulic Device

By

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When the load of an actuator consists of a mass and friction for a valve-controlled hydraulic device, the exact responses are obtained analytically for rectilinear inputs. As any type of input can be approximated by a connection of straight lines, the responses to various types of inputs can be calculated.

An important application of the analysis is to obtain the frequency characteristics. As the sinusoidal input can be approximated by a proper trapezoid, the rectilinear inputs are applied to each straight range of the trapezoid. By this approach, the frequency characteristics for the case of considerably high frequency or heavy load for common hydraulic servos are made clear. The analytical results are shown in normalized forms and are approximated by the responses which are calculated by the equivalent transfer functions. The analytical results were verified by experiments.

Another important application of the analysis is to make clear the effects of the dither applied to the spool. The wave shape of the dither is also approximated by a proper trapezoid, and the responses to the rectilinear inputs are applied to each straight range of the trapezoid. The results of calculations are represented by an equivalent transfer function, and it was made clear how the dither affects the gain and the time constant.

The equivalent transfer functions are very useful to design the hydraulic servomechanism. In this paper, the design criterion which is based on the performance of the hydraulic device, particularly that of the frequency characteristics, is given.

1. Introduction

A spool-type 4-way valve is an important element in a valve-controlled hydraulic device, but it has inherently nonlinear characteristics because the flow rate is proportional to the square root of the pressure drop through a control orifice. The flow rate from the spool valve is approximately proportional to the spool displacement for a small load, but the analysis of the characteristics is very difficult for heavy loads[1),2)].

However, when the load of an actuator consists of a mass and Coulomb friction as is commonly the case, the correct responses are obtained analytically

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for rectilinear inputs. As any type of input can be approximated by a connection of straight lines, the responses to various types of inputs are analyzed even for considerably heavy loads.

In this paper, the characterstics of a 4-way valve are analyzed for rectilinear inputs, and then the responses are obtained respectively for either a step or ramp or sinusoidal input by straight-line approximations. The effects of the dither applied to the spool are also made clear. The design criterion is given for certain specifications, using the equivalent transfer functions which are obtained from the above approximations. The validity of the analysis is verified by experiments.

2. Responses of a 4-Way Valve for Rectilinear Inputs

When a load of an actuator consists of a mass and Coulomb friction and the compressibility of the oil is negligible, the characteristics of a 4-way valve are shown by the following equation[3]:

$$v=\frac{K_q y}{\sqrt{2}A_a}\sqrt{p_s-\frac{F_l}{A_a}-\frac{y}{|y|}\frac{M_l}{A_a}\frac{dv}{dt}} \tag{1}$$

where

y =displacement of a spool
v =velocity of an actuator piston
K_q =coefficient of a control orifice
A_a =effective area of an actuator piston
p_s =supply pressure
F_l =Coulomb friction of an actuator
M_l =mass of a load
t =time

The following normalized variables are used to normalize Eq. (1), where y_0 is a constant displacement of the spool:

$$\left.\begin{aligned} q&=\frac{A_a v}{K_q y_0\sqrt{p_s-(F_l/A_a)}}, & \eta&=\frac{y}{y_0},\\ \tau&=\frac{K_q\sqrt{p_s-(F_l/A_a)}}{A_a}t, & m&=\frac{K_q^2 y_0 M_l}{A_a^3}. \end{aligned}\right\} \tag{2}$$

Eq. (1) is transformed by substituting Eq. (2) as follows for $y>0$:

$$q=\frac{\eta}{\sqrt{2}}\sqrt{1-m\frac{dq}{d\tau}} \tag{3}$$

Eq. (3) is a normalized equation and is solved for rectilinear η.

(1) The case of $d\eta/d\tau > 0$

In this case, the normalized input is given by $\eta = \eta_0 + \nu\tau$, where η_0 and ν are positive constants. By transforming $\tau_1 = \tau + (\eta_0/\nu)$, Eq. (3) is reduced to a homogeneous differential equation, and two types of solutions are obtained:

$$q = \nu\tau_1\left(\frac{m\nu}{4}\right)\left\{-1 \pm \psi(m\nu)\right\}, \tag{4}$$

and

$$q = \nu\tau_1\left(\frac{m\nu}{4}\right)\frac{\{\psi(m\nu)-1\}C_1\tau_1^{\psi(m\nu)} - \{\psi(m\nu)+1\}}{C_1\tau_1^{\psi(m\nu)}+1} \tag{5}$$

where

$$\psi(m\nu) = \sqrt{1+\frac{1}{2}\left(\frac{4}{m\nu}\right)^2}. \tag{6}$$

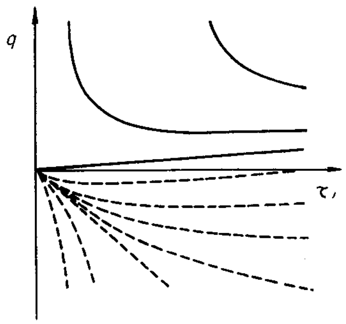

Fig. 1. Normalized velocity of an actuator piston in the case of opening control orifices.

In Eq. (5), C_1 is an integration constant and is determined as follows for the initial condition of $q = q_A$ at $\tau = 0$:

$$C_1 = \left(\frac{\eta_0}{\nu}\right)^{-\psi(m\nu)}\frac{\eta_0\left(\frac{m\nu}{4}\right)\{\psi(m\nu)+1\} + q_A}{\eta_0\left(\frac{m\nu}{4}\right)\{\psi(m\nu)-1\} - q_A} \tag{7}$$

Fig. 1 shows the relations between q and τ_1 given by Eqs. (4) and (5). As q is positive for $\eta > 0$ by Eq. (3), the solid curves in Fig. 1 are valid.

(2) The case of $d\eta/d\tau = 0$

In this case, the normalized input is given by $\eta = \eta_0$. By substituting η_0 for η in Eq. (3), the equation is reduced to a differential equation which can be directly integrated, and the solution is obtained as follows:

$$q = \frac{\eta_0}{\sqrt{2}}\tanh\left\{\frac{\sqrt{2}}{m\eta_0}(\tau + C_2)\right\} \tag{8}$$

where C_2 is an integration constant and is determined as follows by the initial condition of $q = q_B$ at $\tau = 0$:

$$C_2 = \frac{m\eta_0}{\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2}\,q_B}{\eta_0}\right). \tag{9}$$

(3) The case of $d\eta/d\tau < 0$

In this case, the normalized input is given by $\eta = \eta_0 - \nu\tau$. By transforming $\tau_2 = (\eta_0/\nu) - \tau$, Eq. (3) is reduced to a homogeneous equation which is obtained by substituting $-m$ for m in the equation for $d\eta/d\tau > 0$. From this equation, two types of solutions are obtained;

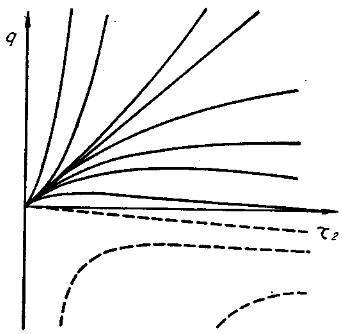

Fig. 2. Normalized velocity of an actuator piston in the case of closing control orifices.

$$q = \nu\tau_2\left(\frac{m\nu}{4}\right)\{1\pm\psi(m\nu)\} \tag{10}$$

and

$$q = -\nu\tau_2\left(\frac{m\nu}{4}\right)\frac{\{\psi(m\nu)-1\}C_3\tau_2{}^{\psi(m\nu)}-\{\psi(m\nu)+1\}}{1+C_3\tau_2{}^{\psi(m\nu)}} \tag{11}$$

where C_3 is an integration constant and is determined by the initial condition of $q=q_C$ at $\tau=0$:

$$C_3 = \left(\frac{\eta_0}{\nu}\right)^{-\psi(m\nu)}\frac{\eta_0\left(\frac{m\nu}{4}\right)\{\psi(m\nu)+1\}-q_C}{\eta_0\left(\frac{m\nu}{4}\right)\{\psi(m\nu)-1\}+q_C} \tag{12}$$

Fig. 2 shows the relations between q and τ_2 given by Eqs. (10) and (11), and the solid curves are valid.

3. Responses to Various Types of Inputs

To obtain the responses to various types of inputs, the input curves are approximated by connections of straight lines to which the above mentioned solutions are applied. The approximations are determined by means of the principle that the integration of squared errors between the actual and the approximated inputs should be a minimum. The following are analytical results of responses to typical types of inputs, and then the equivalent transfer functions for these responses are given.

(1) Response to a step input

When a step input, $y=y_0$, is given to the spool at the initial condition of $v=0$ at $t=0$, the following response is obtained from Eq. (8):

$$q = \frac{1}{\sqrt{2}}\tanh\left(\frac{\sqrt{2}\,\tau}{m}\right). \tag{13}$$

This response is obtained approximately by an equivalent transfer function, $G_v(s)$, of a first-order system. The time constant is approximated by the time for q to reach 63.2% of the stationary value.

$$G_v(s) = \frac{K_v}{1+T_v s} \tag{14}$$

where

$$\left.\begin{aligned} K_v &= \frac{K_q\sqrt{p_s-(F_l/A_a)}}{\sqrt{2}\,A_a} \\ T_v &= \frac{0.527\,K_q y_0 M_l}{A_a^2\sqrt{p_s-(F_l/A_a)}} \end{aligned}\right\}. \tag{15}$$

(2) Response to a ramp input

When a ramp input, $y=wt$, is given to the spool at the initial condition of $v=0$ at $t=0$, the following response is obtained from Eq. (4):

$$q = \nu_r \tau \left(\frac{m\nu_r}{4}\right)\{\psi(m\nu_r)-1\} \tag{16}$$

where

$$\nu_r = \frac{wA_a}{K_q y_0 \sqrt{p_s-(F_l/A_a)}}\,. \tag{17}$$

This response is obtained by an equivalent transfer function $G_v(s)$:

$$G_v(s) = K_v = \frac{K_q M_l w}{4A_a^3}\left\{\sqrt{1+\frac{p_s-(F_l/A_a)}{2}\left(\frac{4A_a^2}{wM_l}\right)^2}-1\right\}. \tag{18}$$

(3) Response to a sinusoidal input

The sinusoidal input is represented by

$$y = y_0 \sin \omega t = y_0 \sin \theta\,. \tag{19}$$

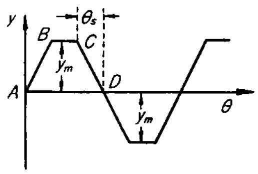

Fig. 3. Trapezoid approximating a sinusoidal input.

This is approximated by a trapezoidal wave of Fig. 3, where y_m is the height of the trapezoid, θ_s being a projection of the trapezoid slope on the θ axis. When the fundamental component of the Fourier expansion of the trapezoidal wave is assumed to be the same as the actual input, the trapezoidal wave is represented as

$$y = \sum_{n=1,3,5,\cdots} \frac{y_0}{n^2}\frac{\sin n\theta_s}{\sin\theta_s}\sin\frac{n\pi}{2}\sin n\theta\,. \tag{20}$$

The integration of the squared error during one period, $I(\theta_s)$, is obtained by Parseval's theorem:

$$I(\theta_s) = y_0\pi \sum_{n=3,5,\cdots}\left(\frac{1}{n^2}\frac{\sin n\theta_s}{\sin\theta_s}\right)^2. \tag{21}$$

As $dI(\theta_s)/d\theta_s=0$, so

$$\sum_{n=3,5,\cdots}\frac{2}{n^4}\frac{\sin n\theta_s}{\sin^2\theta_s}(n\cos n\theta_s-\sin n\theta_s\cot\theta_s) = 0\,. \tag{22}$$

Solving Eq. (22) numerically, the optimum condition to minimize $I(\theta_s)$ is:

$$\left.\begin{aligned}\theta_s &= 62°0'20''\\ y_m &= 0.96254\,y_0\end{aligned}\right\}. \tag{23}$$

When Eq. (23) holds true, the responses are obtained as follows to each straight range of Fig. 3:

(a) $0 \leqq \theta \leqq \theta_s$

In this case, the spool displacement is represented as $y = y_m \omega t/\theta_s$, and the normalized input is obtained by $\eta = \nu_s \tau$, where

$$\nu_s = \frac{y_m \omega A_a}{y_0 \theta_s K_q \sqrt{p_s - (F_l/A_a)}} . \tag{24}$$

By substituting ν_s in Eq. (4), the following response is obtained:

$$q = 0.21405\, m\Omega\{-1 + \psi(m\Omega)\}(\theta/\theta_s) \tag{25}$$

where

$$\left.\begin{aligned} \Omega &= \frac{A_a \omega}{K_q \sqrt{p_s - (F_l/A_a)}} \\ m\Omega &= \frac{K_q y_0 M_l \omega}{A_a^2 \sqrt{p_s - (F_l/A_a)}} \\ \psi(m\Omega) &= \sqrt{1 + \{10.111/(m\Omega)^2\}} \end{aligned}\right\} . \tag{26}$$

(b) $\theta_s \leqq \theta \leqq (\pi - \theta_s)$

In this case, the spool displacement is constant, and the following response is obtained from Eq. (8):

$$q = 0.68062 \tanh\left\{\frac{1.5899}{m\Omega}\left(\frac{\theta}{\theta_s} - 1\right) + \tanh^{-1}(1.4693\, q_B)\right\} \tag{27}$$

where

$$q_B = 0.21405\, m\Omega\{-1 + \psi(m\Omega)\} . \tag{28}$$

(c) $(\pi - \theta_s) \leqq \theta \leqq \pi$

In this case, the response is obtained as follows by substituting Eq. (24) in Eq. (11):

$$q = 0.21405\, m\Omega\left(1 - \frac{\theta}{\theta_s}\right) \frac{\{\psi(m\Omega) + 1\} - \{\psi(m\Omega) - 1\} C_3 \left(1 - \frac{\theta}{\theta_s}\right)^{\psi(m\Omega)}}{1 + C_3\left(1 - \frac{\theta}{\theta_s}\right)^{\psi(m\Omega)}} \tag{29}$$

where

$$C_3 = \frac{0.21405\, m\Omega\{\psi(m\Omega) + 1\} - q_C}{0.21405\, m\Omega\{\psi(m\Omega) - 1\} + q_C} \tag{30}$$

$$q_C = 0.68062 \tanh\left\{\frac{1.4361}{m\Omega} + \tanh^{-1}(1.4693\, q_B)\right\} . \tag{31}$$

The velocity of an actuator piston during a half period is represented by the connection of curves obtained by Eqs. (25), (27) and (29). The normalized velocity, q, depends only on the normalized parameter, $m\Omega$. Fig. 4 shows both the input trapezoid and the output q during a half period. The fundamental component of the output, $q_1 \sin(\omega t + \varphi_1)$, is obtained by Fourier expansion of q

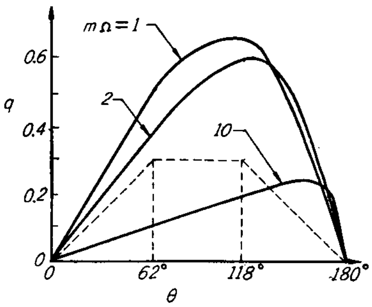

Fig. 4. Normalized velocity responses to trapezoidal inputs.

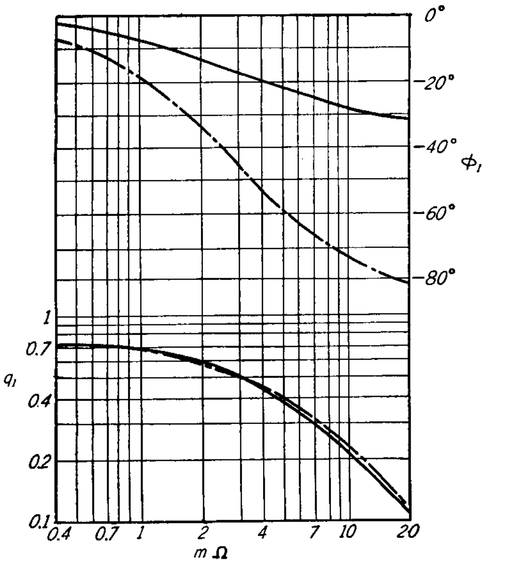

Fig. 5. Normalized gain and phase angle in frequency responses.

during one period, where q_1 is the fundamental amplitude and represents the gain of a 4-way valve, and φ_1 is the phase angle. The solid curves in Fig. 5 show q_1 and φ_1. The characteristics are approximated by an equivalent transfer function, $G_v(j\omega)$, of a first-order system:

$$G_v(j\omega) = \frac{K_v}{1+T_v j\omega}. \tag{32}$$

If the gain of $G_v(j\omega)$ is assumed to become $-3\,db$ at $m\Omega=3$, then

$$\left.\begin{aligned} K_v &= \frac{K_q\sqrt{p_s-(F_l/A_a)}}{\sqrt{2}\,A_a} \\ T_v &= \frac{K_q y_0 M_l}{3A_a^2\sqrt{p_s-(F_l/A_a)}} \end{aligned}\right\}. \tag{33}$$

The chain lines in Fig. 5 show the gain and phase of the equivalent transfer function.

4. Effect of a dither applied to a spool

When a high-performance servo is operated by high pressure oil, a dither should be superimposed on the spool motion to prevent the spool from sticking and to improve its reliability. In this case, the spool movement corresponds to the input signal, accompanied by a small sinusoidal oscillation. The oscillation is represented by $y_0 \sin \omega t$ and is approximated by a trapezoid. Fig. 6 shows the trapezoid accompanied by a constant deviation y_d. The DC gain is determined by the average velocity of an actuator piston during one sinusoidal period, that is, the time from A to G in Fig. 6. The normalized displacement of the actuator piston is obtained by integrating q as follows:

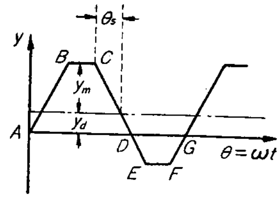

Fig. 6. Trapezoid approximating a spool displacement with a dither.

$$\xi = \frac{x}{y_0} = \int_0^t \frac{v}{y_0} dt = \int_0^T q d\tau \tag{34}$$

where

ξ = normalized displacement of an actuator piston

x = displacement of an actuator piston.

The average velocity of the actuator piston during one period, q_T, is obtained by the displacement during one period divided by one normalized period, $\tau_T = 2\pi/\Omega$. The displacement of the actuator piston is obtained for each straight range of Fig. 6 as follows:

(a) From A to B

The displacement during this period is obtained by integrating Eq. (4). The contribution of this displacement to the average velocity is represented by ζ_A which is obtained from the displacement during this period divided by τ_T. ζ_A is obtained as

$$\zeta_A = 0.018433\, m\Omega \{\psi(m\Omega) - 1\} \left(1 + \frac{y_d}{y_m}\right)^2. \tag{35}$$

(b) From B to C

By integrating Eq. (8), the contribution of the displacement during this period, ζ_B, is obtained:

$$\zeta_B = 0.073727\, m\Omega \left(1 + \frac{y_d}{y_m}\right)^2 \log\cosh \left\{ \frac{1.4361}{m\Omega\left(1 + \frac{y_d}{y_m}\right)} + \tanh^{-1}\left(\frac{1.4693\, q_B}{1 + \frac{y_d}{y_m}} \right) \right\}$$
$$+ 0.036864\, m\Omega \left(1 + \frac{y_d}{y_m}\right)^2 \log \left\{ 1 - 2.1587 \left(\frac{q_B}{1 + \frac{y_d}{y_m}} \right)^2 \right\} \tag{36}$$

where

$$q_B = 0.21405\, m\Omega \{\psi(m\Omega) - 1\} \left(1 + \frac{y_d}{y_m}\right). \tag{37}$$

(c) From C to D

The displacement of the actuator piston during this period is obtained by integrating numerically the velocity obtained by Eq. (11) as follows:

$$q = 0.21405\left(1+\frac{y_d}{y_m}-\frac{\theta}{\theta_s}\right)m\Omega\,\frac{\{\psi(m\Omega)+1\}-\{\psi(m\Omega)-1\}C_3\left(1+\frac{y_d}{y_m}-\frac{\theta}{\theta_s}\right)^{\psi(m\Omega)}}{1+C_3\left(1+\frac{y_d}{y_m}-\frac{\theta}{\theta_s}\right)^{\psi(m\Omega)}} \tag{38}$$

where

$$C_3 = \frac{0.21405\, m\Omega\left(1+\frac{y_d}{y_m}\right)\{\psi\,(m\Omega)+1\}-q_C}{0.21405\, m\Omega\left(1+\frac{y_d}{y_m}\right)\{\psi\,(m\Omega)-1\}+q_C}\cdot\left(1+\frac{y_d}{y_m}\right)^{-\psi(m\Omega)} \tag{38}$$

$$q_C = 0.68062\left(1+\frac{y_d}{y_m}\right)\tanh\left\{\frac{1.4361}{m\Omega\left(1+\frac{y_d}{y_m}\right)}+\tanh^{-1}\left(\frac{1.4693\,q_B}{1+\frac{y_d}{y_m}}\right)\right\}. \tag{40}$$

The contribution of the displacement during this period, ζ_C, is obtained by a numerical integration of q divided by τ_T.

The contribution of the displacement during A and D to the average velocity, ζ, is obtained as follows:

$$\zeta = \zeta_A+\zeta_B+\zeta_C \tag{41}$$

Fig. 7 shows the relation between ζ and $m\Omega$ with a parameter of y_d/y_m. During D and G of Fig. 6, the actuator piston moves inversely. The contribution of the displacement during this period to the average velocity is obtained by substituting $-(y_d/y_m)$ for y_d/y_m in the above calculations, and is obtained from Fig. 7 easily. The average velocity, q_T, is obtained by the difference between ζ during A and D and ζ during D and G. The result shows that the average velocity is proportional to y_d/y_m and depends only on $m\Omega$ as shown in Fig. 8. In Fig. 8, α is the DC gain of the 4-way valve with the dither to the spool, and reduces to

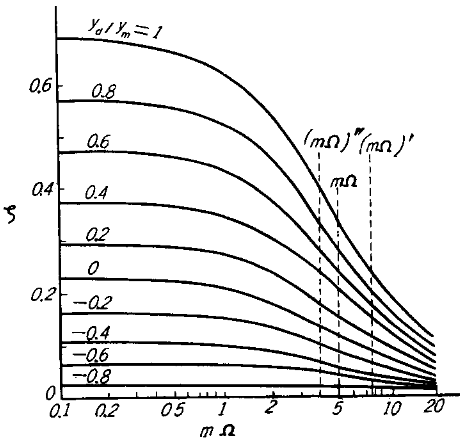

Fig. 7. Relation between ζ and $m\Omega$ with a parameter of y_d/y_m.

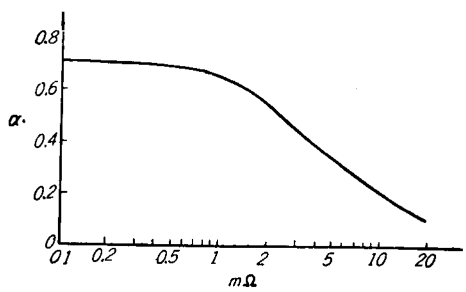

Fig. 8. Relation between α and $m\Omega$.

$1/\sqrt{2}$ for small $m\Omega$. The actual average velocity of the actuator piston is obtained as

$$v_T = \frac{\alpha K_q y_d \sqrt{p_s - (F_l/A_a)}}{A_a}. \tag{42}$$

When a static load is given, the basic equation will be:

$$v = \frac{K_q y}{\sqrt{2} A_a} \sqrt{p_s - \frac{F_l}{A_a} - \frac{y}{|y|} \frac{M_l}{A_a} \frac{dv}{dt} - \frac{y}{|y|} \frac{P_l}{A_a}} \tag{43}$$

Eq. (43) differs from Eq. (1) in the following points: $\{p_s - (F_l/A_a)\}$ in Eq. (1) is replaced by $\{p_s - (F_l/A_a) - (P_l/A_a)\}$ when $y > 0$, by $\{p_s - (F_l/A_a) + (P_l/A_a)\}$ when $y < 0$. Therefore, if Eq. (43) is valid, the average velocity q_T is obtained by substituting the following $(m\Omega)'$ and $(m\Omega)''$ for $m\Omega$:

$$\left.\begin{aligned} (m\Omega)' &= \frac{m\Omega}{\sqrt{1-\lambda}} \qquad \text{when } y > 0 \\ (m\Omega)'' &= \frac{m\Omega}{\sqrt{1+\lambda}} \qquad \text{when } y < 0 \end{aligned}\right\} \tag{44}$$

where

$$\lambda = \frac{(P_l/A_a)}{p_s - (F_l/A_a)}. \tag{45}$$

In Fig. 7, broken lines show $m\Omega$, $(m\Omega)'$ and $(m\Omega)''$ for $m\Omega = 5$ and $\lambda = 0.6$. q_T is obtained graphically by using the ordinates of the intersections of the solid curves and broken lines. Fig. 9 shows the relation between q_T and λ with a

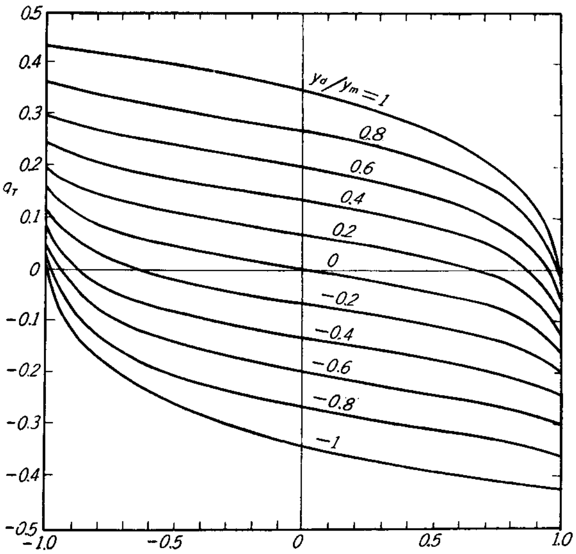

Fig. 9. Static characteristics of a 4-way valve with a dither to a spool.

parameter of y_d/y_m for $m\Omega=5$. The relation as shown in Fig. 9 indicates the static characteristics of the 4-way valve with the dither to a spool, and it changes by the normalized parameter $m\Omega$.

The curves near the q_T axis in Fig. 9 are approximated as

$$q_T = \alpha \frac{y_d}{y_0} - \beta\lambda \tag{46}$$

where α and β are constants and depend on $m\Omega$. α is given by Fig. 8, and β is calculated for various values of $m\Omega$ as shown in Fig. 10.

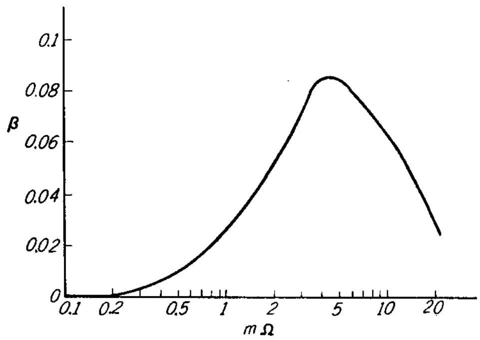

Fig. 10. Relation between β and $m\Omega$.

Though P_l has been considered to be a static load until now, Eq. (43) holds on the average during one period of the dither when P_l changes very slowly in comparison with the dither period. When the load of the actuator consists of a mass and Coulomb friction and the average displacement of the spool changes very slowly, the following relation is valid on the average during one period of the dither:

$$P_l = M_l \frac{dv_T}{dt} \tag{47}$$

where v_T is the average velocity of the actuator piston during the period. Using Eqs. (46) and (47), the equivalent transfer function is represented as follows:

$$G_v(s) = \frac{K_v}{1 + T_v s} \tag{48}$$

where

$$\left.\begin{aligned} K_v &= \frac{\alpha K_q \sqrt{p_s - (F_l/A_a)}}{A_a} \\ T_v &= \frac{\beta K_q y_0 M_l}{A_a^2 \sqrt{p_s - (F_l/A_a)}} \end{aligned}\right\} . \tag{49}$$

The gain and time constant are independent of the input signal, but depend on the normalized parameter $m\Omega$ which is a function of the amplitude and frequency of the dither.

5. Design of a Valve-Controlled Hydraulic Device

In designing a valve-controlled hydraulic device, the dimensions of the spool valve must be determined by various factors[4]. The following is a design criterion which depends on the performance, particularly the frequency response.

(1) The case where a dither is not given to the spool

The zero-lapped spool valve is used in the high-speed or high-accuracy servo, because the steady error and the phase lag are small in the steady state and the overshoot does not occur in the transient response. The dimensions of the valve are determined by the following specifications:

(a) The maximum speed of the actuator piston is v_m.

(b) The maximum displacement of the spool is smaller than y_a.

(c) The angular frequency where the gain becomes $-3\,db$ in the frequency response is ω_m.

The maximum speed of the actuator piston is obtained from Eq. (1) by substituting $M_l=P_l=0$ or from the gain constant K_v of Eq. (33). If the maximum speed v_m corresponds to the maximum spool displacement y_a, the relation between v_m and y_a is as follows:

$$v_m=\frac{K_q y_a \sqrt{p_s-(F_l/A_a)}}{\sqrt{2}\,A_a}. \tag{50}$$

The angular frequency ω_m is the reciprocal of T_v in Eq. (33) and the following equation is obtained:

$$\omega_m=\frac{3A_a^2\sqrt{p_s-(F_l/A_a)}}{K_q y_a M_l}. \tag{51}$$

From Eq. (50), the following is obtained:

$$K_q=\frac{\sqrt{2}\,A_a v_m}{y_a\sqrt{p_s-(F_l/A_a)}}. \tag{52}$$

Substituting Eq. (52) in Eq. (51), the following equation is obtained.

$$A_a p_s=\frac{\sqrt{2}}{3}M_l v_m \omega_m+F_l\,. \tag{53}$$

Using Eq. (53), the effective area of the actuator piston, A_a, is determined by the supply pressure, p_s, or inversely p_s is determined by A_a. Substituting A_a and p_s in Eq. (52), K_q is determined and the width of the control orifice is determined by the following equation.

$$b=\frac{K_q}{C_q}\sqrt{\frac{\rho}{2}} \tag{54}$$

where

b = width of a control orifice.

C_q = discharge coefficient of a control orifice,

ρ = density of oil.

In the ideal zero-lapped valve, the steady error does not occur theoretically for any load acting on the actuator piston. In actual practice, however, not only is it very difficult to manufacture the ideal zero-lapped valve, but also the discharge coefficient becomes very small for openings the control orifice less than 0.01 mm. Therefore, it is not possible to expect that the position error will become less than 0.01 mm by using the hydraulic servomechanism which consits of an ordinary spool valve, an actuator and a mechanical unity-feedback mechanism.

(2) The case where a dither is given to the spool

When a dither is given to the spool, the characteristics of a valve-controlled hydraulic device approach the linear as shown in Fig. 9, and the control system is synthesized by the method for the linear system. If the specifications are given by the maximum piston speed v_m, the maximum spool displacement y_a and the angular frequency ω_m at the breaking point, the following relations are obtained by the equivalent transfer function of Eq. (48).

$$K_q = \frac{A_a v_m}{\alpha y_a \sqrt{p_s - (F_l/A_a)}} \tag{55}$$

$$A_a p_s = \frac{\beta}{\alpha} M_l v_m \omega_m + F_l \,. \tag{56}$$

Using Eq. (56), A_a is determined by p_s, or inversely p_s is determined by A_a. Substituting A_a and p_s in Eq. (55), K_q is calculated, the width of the control orifice being determined by Eq. (54). In the above calculations, it is noted that α and β in Eqs. (55) and (56) are the functions of both the amplitude and the frequency of the dither applied to the spool.

When a dither is applied to the spool, the curves of the static characteristics shown in Fig. 9 are similar to those for an underlapped valve. However, the actuator piston moves surely in the direction correspoding to the control orifice opening in this case, and the steady error does not occur. As the spool opens the two control orifices alternately, the actuator piston oscillates in the dither frequency, and moves in the direction that corresponds to the average displacement of the spool. The small oscillation in the actuator piston is useful to reduce the friction, but must be small enough not to disturb the output displacement. To reduce the amplitude of the actuator piston, the dither amplitude of the spool, y_0, must be small, and the dither frequency, ω, must be high. It is also effective to make the normalized parameter $m\Omega$ larger than 10.

6. Experiment

Experiments for frequency characteristics were carried out to verify the validity of the analysis. A two-stage servo valve as shown in Fig. 11 was used. In this valve, the displacement of the spool is restricted by stoppers which face the two ends of the spool. Principal dimensions are: $b=2$ mm, $A_a=1.374\ \mathrm{cm}^2$, $p_s=20\ \mathrm{kg/cm}^2$, $F_t=6$ kg and $y_m=0.6$ mm. When a rectangular pulse current is applied to the torquemotor, the spool moves in the shape of a trapezoidal pulse train. The inclinations of the sides of the trapezoids change with the supply pressure to the flapper-nozzle mechanism, which is adjusted so that the spool motion approaches the trapezoid in Fig. 4. The spool motion was measured with a strain gauge which was attached to a beam pushed in to the notch of the spool. The spool velocity was measured by this method, and it was verified that the spool velocity was almost constant in spite of such various forces affecting the spool motion as hydraulic axial forces and friction between the spool land and the sleeve. However, the detailed movement of the spool could not be measured due to the backlash of the measuring apparatus.

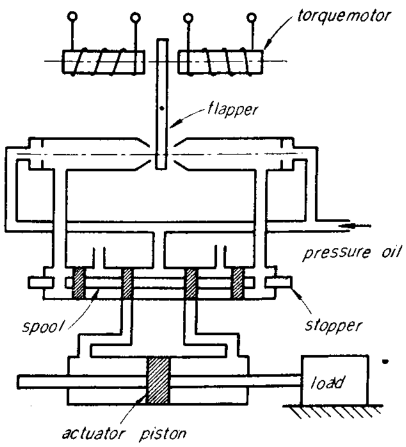

Fig. 11. Schematic diagram of an experimental apparatus.

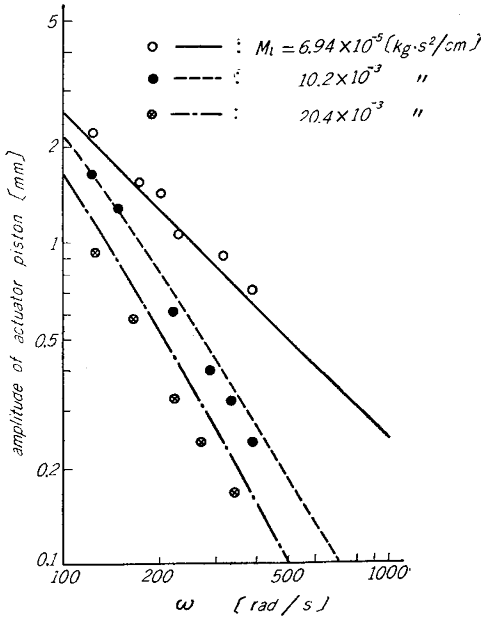

Fig. 12. Experimental and analytical results.

Fig. 12 shows the experimental results, and the curves there show the analytical results. The amplitudes of the experimental results are in good agreement with the analytical. The phase is not shown because the spool displacement can not be measured. In this experiment, the normalized parameter, $m\Omega$, covers the range from 0.1 to 20, and it corresponds to a very high input frequency or very heavy load for common hydraulic servos.

7. Conclusion

When the load of an actuator consists of a mass and Coulomb friction for a valve-controlled hydraulic device, the exact responses are obtained analytically for rectilinear inputs. As an application of the analytical results, the responses for either a step, or ramp, or sinusoidal input are respectively obtained, and approximated by the responses calculated from the equivalent transfer functions. The effects of the dither applied to the spool are also clarified by this approach. Using the equivalent transfer functions, the design criterion which depends on the performance of the hydraulic device, particularly on the frequency characteristics, is given. The validity of the analysis is verified by experiment.

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