Estimation of the Average Stream Flow in the Immediate Future for Hydro-electric Power Generation

By

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(Received January 31, 1961)

In making an economical study of a large combined hydro-steam power system, it is necessary to estimate the annual average available power of the rivers for any given year in the immediate future, and, especially for the control of a plant of pondage or storage type, to estimate the drought-seasonal average stream flow during the coming winter.

In this paper we shall study the method for estimating the annual average stream flow by means of the theoretically based method of statistical extrapolation, and the method for estimating the drought-seasonal average stream flow during the coming winter by means of the method of statistical extrapolation for a time series with two members, by analysing the existing data of the seasonal average stream flows during winter and autumn over a number of recent years, including that for autumn of the estimated year, taking the data of certain rivers as examples.

I. Introduction

In making an economical study of a large combined hydro-steam power system, we must estimate the annual average available power of the rivers for any given year in the immediate future. In the case of a plant of the pondage or storage type, it is especially necessary to estimate the drought-seasonal available power for winter considering the heavy load demand for electric power supply during this season, in order to develop a hydro-power control plan in relation to the capacity of the reservoir, the estimated load and the steam-power.

Since we will estimate the stream flow for a particular year in this case, it is incorrect to use the stream flow of a typical year, for instance that of the representative year[1].

It is very difficult to estimate the monthly average available power for any given year in the immediate future, because of random fluctuations in the stream flow[2]. Therefore, we decided to take as a basis for our estimation in this paper

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the annual average stream flow and the drought-seasonal average stream flow. Moreover, in order to consider the external factors such as meteorological conditions, we decided to use a statistical method.

In the following study, we will analyse the fluctuations contained in the data of the annual average stream flow over a number of recent years. Then we will present the method for estimating the annual average stream flow for any given year in the immediate future, by means of the so-called "method of statistical extrapolation"; and we will also present the method for estimating the drought-seasonal average stream flow during the coming winter, by means of the method of statistical extrapolation for a time series with two members, by making use of the existing data on the average stream flows during winter and autumn over a number of recent years including that for autumn of the estimated year, taking the data of the Rivers A and B as examples, following the calculation stepwise.

II. The Estimation of the Annual Average Stream Flow for Any given Year in the Immediate Future

(1) The Determination of the Fundamental Fluctuation Period

In order to detect the fundamental fluctuation period in the series $x(i)$ of the annual average stream flow, we first obtain correlogram, i.e. the graph of the serial auto-correlation coefficient r_k of the 3-year moving average series $y(i)$

$$r_k = \frac{1}{n-k}\sum_{i=1}^{n-k}\{y(i)-\bar{y}_1\}\{y(i+k)-\bar{y}_2\}/S_1S_2\,, \qquad k=1,2,\cdots$$

where

$$\bar{y}_1 = \frac{1}{n-k}\sum_{i=1}^{n-k}y(i)\,,\quad \bar{y}_2 = \frac{1}{n-k}\sum_{i=k+1}^{n}y(i)\,, \tag{1}$$

$$S_1^2 = \frac{1}{n-k}\sum_{i=1}^{n-k}\{y(i)-\bar{y}_1\}^2\,,\quad S_2^2 = \frac{1}{n-k}\sum_{i=k+1}^{n}\{y(i)-\bar{y}_2\}^2\,,$$

obtained by applying to $x(i)$ the 3-year moving average for $K=2$

$$y(i) = \frac{1}{4}\{x(i-1)+2x(i)+x(i+1)\}\,. \tag{2}$$

Then we test whether or not there is any correlation in the graph for some values of k, comparing the correlogram which is taken as a sample correlation coefficient out of a non-correlated normal population, with the 1 or 10% level of significance[2].

In general statistical hypothesis tests, the 5% level of significance is used. However, in this paper, we consider that the most remarkable periodic components only exist in case the correlogram exceeds the 1% level of significance for some values of k, and in cases where the correlogram does not reach the 10% level, we consider that there are no periodic components.

However, if it is found that the correlogram of $y(i)$ does not seem to reach the above-mentioned 10% level effected by the component like a trend, in such a case, we will determine it together with the correlogram of the original series $x(i)$.

The total annual average stream flow and the annual average stream flows in the case where we take the 3-, 6-, 9-month-flow for a typical year as the

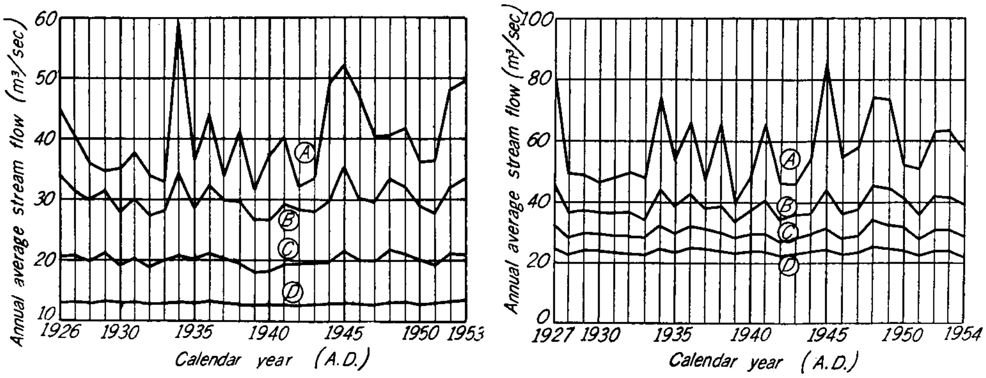

Fig. 1 (a) (the River A) Fig. 1 (b) (the River B)

The fluctuation curves of the annual average stream flows.

Note: Ⓐ ··· The total annual average stream flow
Ⓑ ··· where the 3-month-flow for a typical year is taken as the maximum available
Ⓒ ··· 〃 6-month-flow 〃 〃
Ⓓ ··· 〃 9-month-flow 〃 〃

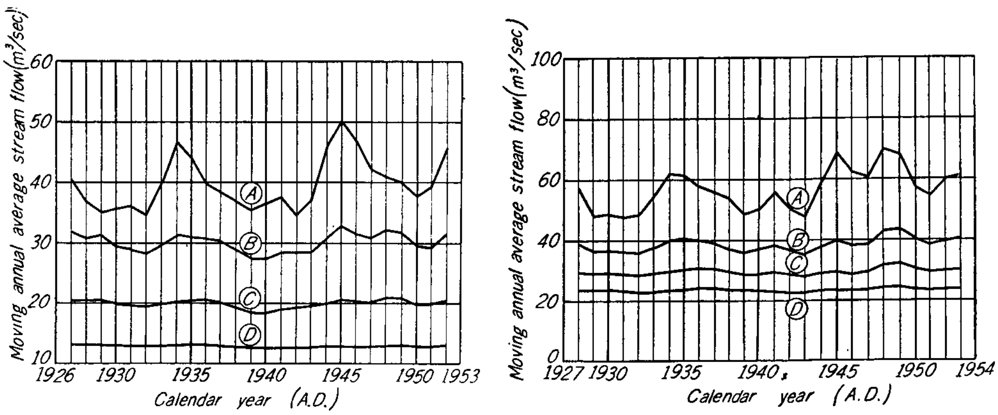

Fig. 2 (a) (the River A) Fig. 2 (b) (the River B)

The fluctuation curves after applying the moving average of Eq. (2) to the annual average stream flows.

Note: Ⓐ ··· The total annual average stream flow
Ⓑ ··· where the 3-month-flow for a typical year is taken as the maximum available
Ⓒ ··· 〃 6-month-flow 〃 〃
Ⓓ ··· 〃 9-month-flow 〃 〃

maximum available, for the River A, are shown in their calendar year order as in Fig. 1(a). Moreover the result which was obtained by applying to $x(i)$ the 3-year moving average represented by Eq. (2) is shown in Fig. 2(a).

When we compare these two Figures, it is easily seen that the fluctuation of the latter is less than that of the former, and the periodic fluctuation of the stream flow becomes very evident. The correlograms of these series are shown in Figs. 3(a) and 4(a). The correlogram of the latter sometimes exceeds the levels of significance, when k has certain values (approaching $k=5$ or $k=10\sim11$), while, the former do not have such a marked periodic component[2].

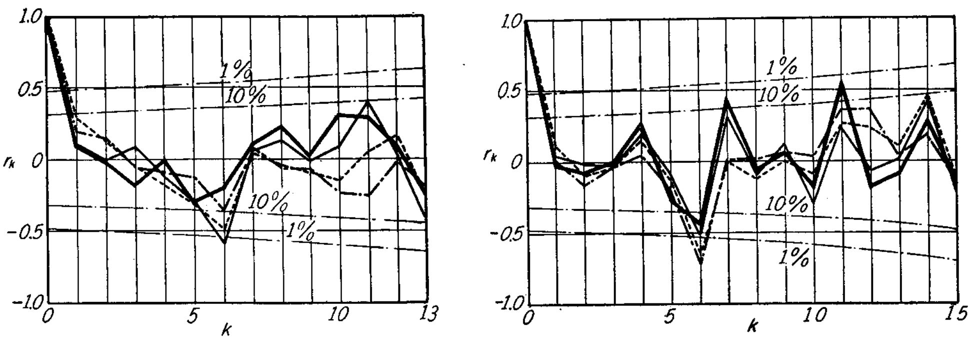

Fig. 3 (a) (the River A) Fig. 3 (b) (the River B)

The correlogram of the annual average stream flows.

Note: ——— The total annual average stream flow
——— where tho 3-month-flow for a typical year is taken as the maximum available
- - - - " 6-month-flow " "
— - — " 9-month-flow " "

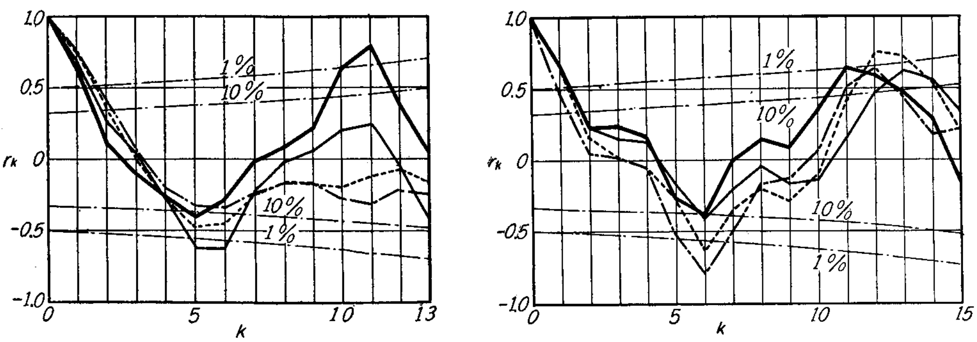

Fig. 4 (a) (the River A) Fig. 4 (b) (the River B)

The correlogram after applying the moving average of Eq. (2) to the annual average stream flows.

Note: ——— The total annual average stream flow
——— where the 3-month-flow for a typical year is taken as the maximum available
- - - - " 6-month-flow " "
— - — " 9-month-flow " "

The results for River B are shown in Figs. 1 (b)$\sim$4(b), and are nearly the same as those for River A[2].

(2) The Determination of the Mean $m(i)$ of $x(i)$ which contains the Trend

In order to determine the mean, it is generally correct to fit the curve to the moving average series of $x(i)$ over the years of the fundamental fluctuation period (for example, 11 years) by the method of least squares. However, in this case, since the sample size is small, in order to obtain the periodic components by means of harmonic analysis (cf. following (3) (i)), for the sake of convenience, we shall appropriate the 1/2 harmonic components of the fundamental fluctuation period to the trend (for example, the 22-year periodic components)*.

In order to obtain the periodic components of the annual average stream flow $x(i)$, by means of the moving average of Eq. (2) (cf. following (3) (ii)), it is correct to fit a straight line or a simple curve to the 3-year moving average stream flow $y(i)$, by the method of least squares.

(3) The Determination of the Hidden Periodic Components $\xi(i)$ with Period L Contained in the Series $\{x(i)-m(i)\}$

(i) Harmonic Analysis

When we take L years as the fundamental fluctuation period of the annual average stream flow, in order to obtain the hidden periodic components $\xi(i)$, we first consider the equation

$$\xi_1(i) = A_1 \cos\frac{2\pi}{L}i + B_1 \sin\frac{2\pi}{L}i + A_2 \cos\frac{4\pi}{L}i + B_2 \sin\frac{4\pi}{L}i + \cdots, \qquad (3)$$

where the coefficients A_1, B_1, A_2, B_2 $\cdots$ can be found from the following Eq. (4):

* If the fundamental fluctuation period is taken as L, at first we put

$$m(i) = A_0 + A_{1/2}\cos\frac{2\pi}{2L}i + B_{1/2}\sin\frac{2\pi}{2L}i,$$

then the coefficients may be determined by the next equation;

$$\begin{pmatrix} n & \sum\cos\frac{2\pi}{2L}i & \sum\sin\frac{2\pi}{2L}i \\ \sum\cos\frac{2\pi}{2L}i & \sum\cos^2\frac{2\pi}{2L}i & \sum\cos\frac{2\pi}{2L}i\cdot\sin\frac{2\pi}{2L}i \\ \sum\sin\frac{2\pi}{2L}i & \sum\cos\frac{2\pi}{2L}i\cdot\sin\frac{2\pi}{2L}i & \sum\sin^2\frac{2\pi}{2L}i \end{pmatrix} \begin{pmatrix} A_0 \\ A_{1/2} \\ B_{1/2} \end{pmatrix} = \begin{pmatrix} \sum x(i) \\ \sum x(i)\cdot\cos\frac{2\pi}{2L}i \\ \sum x(i)\cdot\sin\frac{2\pi}{2L}i \end{pmatrix}.$$

But, it is permissible to combine with the next calculation for finding the periodic component.

$$\begin{pmatrix} \sum\cos^2\frac{2\pi}{L}i & \sum\cos\frac{2\pi}{L}i\sin\frac{2\pi}{L}i & \sum\cos\frac{2\pi}{L}i\cos\frac{4\pi}{L}i & \sum\cos\frac{2\pi}{L}i\sin\frac{4\pi}{L}i & \cdots \\ \sum\cos\frac{2\pi}{L}i\sin\frac{2\pi}{L}i & \sum\sin^2\frac{2\pi}{L}i & \sum\sin\frac{2\pi}{L}i\cos\frac{4\pi}{L}i & \sum\sin\frac{2\pi}{L}i\sin\frac{4\pi}{L}i & \cdots \\ \sum\cos\frac{2\pi}{L}i\cos\frac{4\pi}{L}i & \sum\sin\frac{2\pi}{L}i\cos\frac{4\pi}{L}i & \sum\cos^2\frac{4\pi}{L}i & \sum\cos\frac{4\pi}{L}i\sin\frac{4\pi}{L}i & \cdots \\ \sum\cos\frac{2\pi}{L}i\sin\frac{4\pi}{L}i & \sum\sin\frac{2\pi}{L}i\sin\frac{4\pi}{L}i & \sum\cos\frac{4\pi}{L}i\sin\frac{4\pi}{L}i & \sum\sin^2\frac{4\pi}{L}i & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ \vdots \end{pmatrix}$$

$$= \begin{pmatrix} \sum\{x(i)-m(i)\}\cos\frac{2\pi}{L}i \\ \sum\{x(i)-m(i)\}\sin\frac{2\pi}{L}i \\ \sum\{x(i)-m(i)\}\cos\frac{4\pi}{L}i \\ \sum\{x(i)-m(i)\}\sin\frac{4\pi}{L}i \\ \cdots\cdots \end{pmatrix}. \tag{4}$$

Here $\sum$ represents $\sum_{i=1}^{n}$ (n represents the sample size).

When testing the significance of the correlation coefficient concerning the residual series, which was substracted $\xi_1(i)$ from $\{x(i)-m(i)\}$ as in the above-mentioned case (**1**), if no significant periodic components remain, we may take $\xi_1(i)$ as equal to $\xi(i)$.

But in the case where some significant periodic components remain, we must repeat our calculations in the same manner, until no significant periodic components do remain. Then, we may take the total sum of all the periodic components as $\xi(i)$.

(ii) The Method of the 3-year Moving Average of Eq. (2)

To find the periodic components of the series $r(i)$ which were obtained by subtracting the mean containing the trend (cf. above-mentioned **II.** (**2**)) from the 3-year moving average stream flow $y(i)$, we will consider the following two methods.

a) First, we take $r(i)$ as the total of the periodic component and the random component of the annual average stream flow $x(i)$, then we divide $r(i)$ into L parts successively (L is the length of the fundamental fluctuation period which we have already obtained), and then we can find the mean of the j-th terms of these parts ($j=1, 2, \cdots, L$) in order, after that, we take these mean values thus obtained as the periodic component of $x(i)$.

b) Having found the periodic component of $r(i)$ with the period L by means of harmonic analysis, we can find the periodic component of the annual average stream flow, considering also the results obtained by the extrapolatory estimation.

However, since method **b**) is considered to make use of some parts of the above-mentioned harmonic analysis, in this paper, we will describe only method **a**), but we are not going to dismiss method **b**) summarily, because it possesses greater precision in the estimation of the moving average stream flow than that of the original series.

(4) **The Inspection and the Extrapolation of the Residual Series**

The residual series $z(i)$ is obtained by subtracting the mean $m(i)$ (containing the trend) and all sorts of significant periodic components $\xi(i)$ from the original series $x(i)$. $z(i)$ is considered as the sample obtained from the process with a continuous spectrum which has no singular process, and therefore called the random component. We are going to inspect this component and pick up the effective information contained in this component, i.e. to apply the method of so-called extrapolatory estimation, taking $z(i)$ for the total annual average stream flow for the River A as an example. (We omit the explanation of the other examples for the manifold average stream flows.)

It is evident that the correlogram of $z(i)$ no longer contains any significant periodic components, as is shown in Fig. 5. The chain lines in Fig. 5 show the 90% confidence limit of the correlation coefficient. The confidence limit (ρ_k', ρ_k'') of the confidence coefficient $\alpha\%$ of the correlation coefficient r_k, is obtained by means of the following well-known equations:

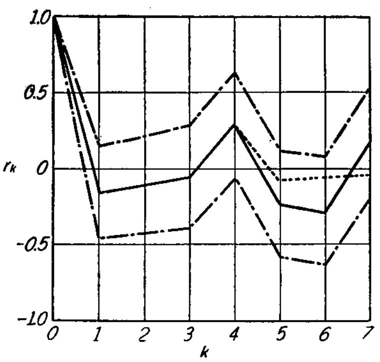

Fig. 5. (the River A)

The correlogram of the random component $z(i)$ of the total annual average stream flow with the 90% confidence limit.

Note: ——— Correlogram
- - - - The estimate of ρ_k when we put $h=4$
— - — 90% confidence limit

$$\zeta_k' = Z_k - \frac{\lambda_\alpha}{\sqrt{n-k-3}}, \qquad \zeta_k'' = Z_k + \frac{\lambda_\alpha}{\sqrt{n-k-3}}, \tag{5}$$

where

$$\zeta_k = \frac{1}{2}\log_e\frac{1+\rho_k}{1-\rho_k}, \qquad Z_k = \frac{1}{2}\log_e\frac{1+r_k}{1-r_k}, \tag{5$'$}$$

and λ_α is shown in Table 1. The transformations $\zeta \leftrightarrow \rho$ and $Z \leftrightarrow r$ are easily found in the table of Z-transformation.

Table 1. λ_α for various values of α.

α (%)	50	90	95	99
λ_α	0.67	1.64	1.96	2.58

The series without any singular components thus obtained, is considered as the realized value of the discrete stationary stochastic process with a continuous spectrum. Consequently, in this case, the statistical extrapolation formula are applicable[3),4)].

Our experience suggests that the correlogram of $z(i)$ for the annual average stream flow possesses the first kind of persistency* or cyclic periodicity[3)] as is shown clearly in Fig. 5.

In such case, ρ_k $(k=1,2,\cdots)$, the correlation coefficient of the stochastic process (in population) $\{Z(i)\}$, will satisfy the difference equation, as follows:

$$\rho_k+a_1\rho_{k-1}+\cdots+a_h\rho_{k-h}=0\,,\qquad k=1,2,\cdots, \tag{6}$$

then the stochastic process $Y(i)$, which is given by the following Eq.

$$Y(i)=Z(i)+a_1Z(i-1)+\cdots+a_hZ(i-h) \tag{7}$$

is non-autocorrelative, nor is it correlative with $Z(i-1)$, $Z(i-2)$, $\cdots$, moreover its mean is zero.

Putting the correlation coefficients $r_1, r_2, \cdots, r_h$ (see Fig. 5) of $z(i)$ into ρ's of the first h equations of Eq. (6), the following simultaneous equations are obtained:

$$\begin{aligned} &r_1+a_1+a_2r_1+\cdots+a_hr_{h-1}=0\\ &r_2+a_1r_1+a_2+\cdots+a_hr_{h-2}=0\\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ &r_h+a_1r_{h-1}+a_2r_{h-2}+\cdots+a_h=0\,. \end{aligned} \tag{8}$$

Thus the coefficients $a_1, a_2, \cdots, a_h$ are determined by solving these equations.

If the value of the above-mentioned h is proper, the solution of Eq. (6) where $k=h+1$, $h+2$, $\cdots$, ρ_{h+1}, ρ_{h+2}, $\cdots$, and r_{h+1}, r_{h+2}, $\cdots$ are in agreement with each other. (The degree of agreement is decided by the well-known confidence limit of the correlation coefficients, which may usually be taken as $h=2\sim3$.) If the above-mentioned two terms are not in sufficient agreement with each other, the determination must be repeated with a different value of h.

If h and $a_1, a_2, \cdots, a_h$ pass this test, we estimate the extrapolated values of the $\{z(i)\}$ series by means of the extrapolation formula.

* This means the process whose auto-correlation coefficient converges monotonously to zero as k increases to infinity[3)].

$$\begin{aligned} \zeta(n+1) &= -a_1 z(n) - a_2 z(n-1) - \cdots - a_h z(n-h+1), \\ \zeta(n+2) &= -a_1 \zeta(n+1) - a_2 z(n) - \cdots - a_h z(n-h+2), \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned} \quad (9)$$

In this case $z(n)$ is the data for the most recent year. The extrapolated value of $x(i)$ is obtained by adding the estimated value thus obtained to the extended value of the periodic component and the mean containing the trend which we have already subtracted.

In the case of the River A, regarding its total annual average stream flow, the values of the a coefficients, when $h=2$, 4 or 6, are shown in Table 2.

Table 2. The a coefficients for various values of h.

h	a_1	a_2	a_3	a_4	a_5	a_6
2	0.198	0.163	—	—	—	—
4	0.180	0.120	0.070	-0.280	—	—
6	0.098	0.052	0.122	-0.189	0.223	0.345

The dotted lines in Fig. 5 show the estimated values of $\rho_5, \rho_6, \cdots$ in that order, with regard to the River A, when we put $h=4$. All terms in this calculation, being within the 90% confidence limit, pass the test. It is the same as in the case when $h=2$ or $h=6$. Therfore, in this case it is sufficient to put $h=2$.

The dotted lines and the chain lines in Fig. 6 (a) show the estimated results of the annual average stream flows in 1954 and 1955, comparing with the actual data, in which we found the periodic components by means of the above-mentioned two methods with the data of the total annual average stream flow and the annual average stream flows where the 3-, 6-, 9-month-flow for a typical year for the River A were taken as the maximum available. Fig. 6 (b) shows the same kind of estimated results with regard to the River B.

As these figures clearly show, the estimated result proves to have nearly the same tendency as the actual fluctuation, and we expect that we could make our estimation more precise, by considering such external factors as meteorological conditions from the statistical view point. When we compare the above-mentioned two methods, the annual fluctuation is a little larger in the method where we estimate the periodic components by means of the moving average of Eq. (2), than in the other method. In any case, however, it may be considered that the results found by the two methods are similar to each other.

Of course, because a hydrological quantity such as the flowing conditions of rivers have some random fluctuating components which cannot be perfectly

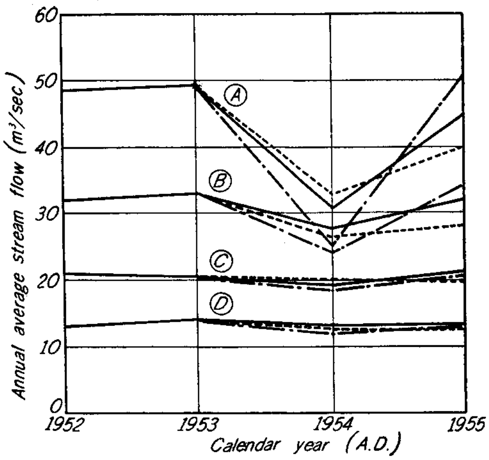

Fig. 6 (a) (the River A)

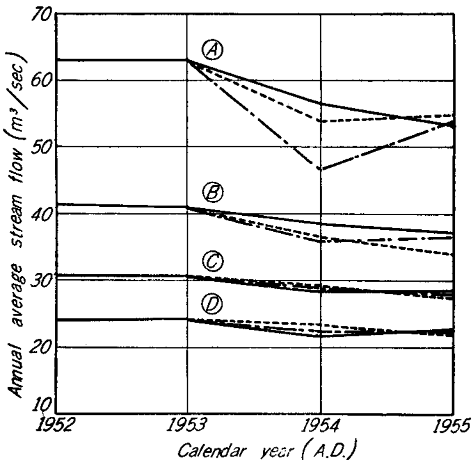

Fig. 6 (b) (the River B)

The comparison between the estimate and data of the annual average stream flows.

Note: Ⓐ ··· The total annual average stream flow
Ⓑ ··· where the 3-month-flow for a typical year is taken as the maximum available
Ⓒ ··· " 6-month-flow " "
Ⓓ ··· " 9-month-flow " "
——— Data
- - - - The estimate when the periodic component is obtained by harmonic analysis
— - — " " by the method of moving average of Eq. (2)

estimated, it is very difficult to expect a high degree of precision in the estimation of the stream flow. Some degree of error is inevitable. However, in order to make an appropriate plan, we must develop the analysis which serves as the basis of the plan by means of a theoretically sound method, and the above-mentioned methods are adopted for this purpose.

Furthermore, it is possible for us to estimate the conditional estimate of the annual average stream flow for each year by means of the estimated result of the 3-year moving average stream flow. Then we believe that the precision in this method will be the same as that of the above-mentioned method. In the following paragraphs, we will describe example calculations of this method.

III. The Estimation of the Drought-seasonal Average Stream Flow during the Coming Winter

We are going to describe the two methods for estimating the drought-seasonal average stream flow during the coming winter by means of the method of statistical extrapolation for a time series with two members, by making use of the existing data of the average stream flows during winter and autumn over a number of recent years including that for autumn of the estimated year, taking the data of the Rivers A and B as examples, following the calculation stepwise. The method for determining the range of these seasons depends on the flowing conditions.

(1) In the Case of the Direct Estimation of the Drought-seasonal Average Stream Flow during the Coming Winter

We take the seasonal average stream flows during autumn and winter of the i-th year as $x_a(i)$ and $x_w(i)$ respectively.

(i) First we are going to find both the mean containing the trend and the significant periodic (singular) component, employing the method described in the above-mentioned **II** or in Ref. 2) of the series $x_a(i)$ and $x_w(i)$. Then we find the random components, $z_a(i)$ and $z_w(i)$, with the mean of zero by subtracting the above-mentioned means and periodic components from the original series.

When we take $x_a(i)$ average stream flow for September and October, and $x_w(i)$ as that for December, January and February, for the River A, we will find the following results about $x_a(i)$: (the unit is m^3/sec)

$$\begin{aligned} \bar{x}_a = 34.5;\quad & A_{22} = 0.82\ , && B_{22} = -1.45; \\ & A_{11} = -2.26, && B_{11} = -1.62; \\ & A_{11/2} = 0.91\ , && B_{11/2} = 0.40\ ; \\ & A_{11/3} = -0.76, && B_{11/3} = -0.45. \end{aligned}$$

And about $x_w(i)$ as follows:

$$\begin{aligned} \bar{x}_w = 14.0;\quad & A_{26} = 0.77\ , && B_{26} = -0.15; \\ & A_{13} = -0.20, && B_{13} = -0.86. \end{aligned}$$

(ii) We represent the auto-correlation coefficients of $z_w(i)$ and $z_a(i)$ as $r_w(k)$ and $r_a(k)$ respectively and the mutual correlation coefficient of $z_w(i+k)$ with $z_a(i)$ as $r_{wa}(k)$. $r_w(k)$, $r_a(k)$ and $r_{wa}(k)$ are shown in Figs. 7 (a), (b) and (c). The chain lines in these Figures show the 90% confidence limit.

(iii) When we represent the stochastic processes of the realized values $z_w(i)$ and $z_a(i)$ as $Z_w(i)$ and $Z_a(i)$, and extensively apply and modify the method cited in Refs. 3) and 4), we find

$$\begin{aligned} V(i) = & Z_w(i) + c_1 Z_w(i-1) + \cdots + c_f Z_w(i-f) \\ & + d_0 Z_a(i) + d_1 Z_a(i-1) + \cdots + d_g Z_a(i-g), \end{aligned} \tag{10}$$

where $V(i)$ thus obtained is the non-autocorrelative process whose mean is zero, and it is presumed that $V(i)$ is non-correlative with $Z_w(i-1)$, $Z_w(i-2)$, $\cdots$ and with $Z_a(i)$, $Z_a(i-1)$, $\cdots$. This is necessary in the formation of the extrapolation formula (13). Then the difference equations

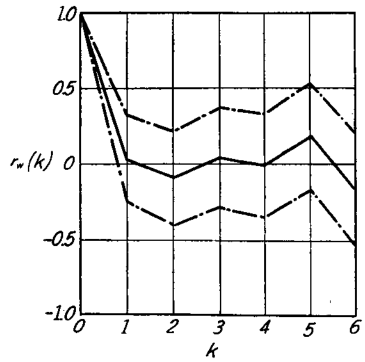

Fig. 7 (a) (winter)

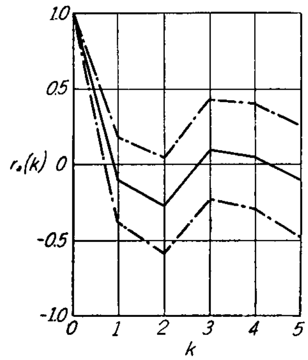

Fig. 7 (b) (autumn)

The correlograms of the random components of the seasonal average stream flows during winter and autumn with the 90% confidence limits (the River A).

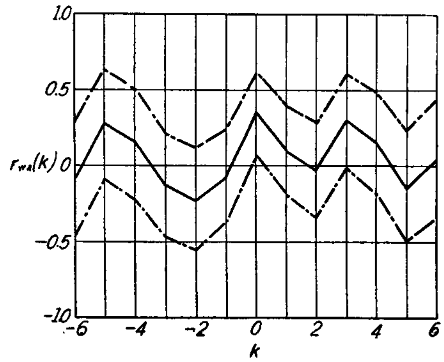

Fig. 7 (c). The correlogram of the mutual correlation of $z_w(i+k)$ with $z_a(i)$ with the 90% confidence limit (the River A).

Note: ——— Correlogram
— - — 90% confidence limit

$$\begin{aligned}&\rho_w(k)+c_1\rho_w(k-1)+\cdots+c_f\rho_w(k-f)\\&\quad+d_0'\rho_{wa}(-k)+d_1'\rho_{wa}(1-k)+\cdots+d_g'\rho_{wa}(g-k)=0\,,\\&\rho_{wa}(k-1)+c_1\rho_{wa}(k-2)+\cdots+c_f\rho_{wa}(k-1-f)\\&\quad+d_0'\rho_a(k-1)+d_1'\rho_a(k-2)+\cdots+d_g'\rho_a(k-1-g)=0\end{aligned}\tag{11}$$

are obtained, where the ρ's are the population correlation coefficients corresponding to the r's, and

$$d_j'=\frac{D(Z_a)}{D(Z_w)}\cdot d_j\,,\qquad j=0,1,\cdots,g\,.\tag{12}$$

where D represents the square root of the population variance.

(iv) If these conditions are satisfied, the conditional estimate $\zeta_w(i)$ of $Z_w(i)$ in the case where the sample values of Z_w and Z_a, $z_w(i-1)$, $z_w(i-2)$, $\cdots$ and $z_a(i)$, $z_a(i-1)$ $\cdots$ respectively are all given, is obtained as follows:

$$\zeta_w(i) = -c_1 z_w(i-1) - \cdots - c_f z_w(i-f) \\ -d_0 z_a(i) - \cdots - d_g z_a(i-g) \quad . \tag{13}$$

The estimated values for the c and d' coefficients are obtained by putting the sample correlation coefficients $r's$ into the $\rho's$ of Eq. (11), and by putting the square root of the sample variance into $D(Z_a)$ or $D(Z_w)$ of Eq. (12). Eq. (13) is the extrapolation formula to be used.

The results obtained by assuming $f=2$ and $g=5$ for the River A,

$c_1 = 0.240$, $c_2 = 0.195$,

$d'_0 = -0.071$, $d'_1 = -0.066$, $d'_2 = -0.067$, $d'_3 = -0.092$,

$d'_4 = -0.063$, $d'_5 = -0.021$.

The comparison between the data and the estimated results calculated by means of these coefficients of the seasonal average stream flow for the River A during winter is shown in Fig. 8 (a). The same kind of estimated result with regard to the River B is shown in Fig. 8(b).

Furthermore, since $z_w(i)$ is almost non-autocorrelative, as is shown in Fig. 7 (a), even if we omit all of the c coefficients and assume that ζ_w is determined only by z_a, the result thus obtained is as precise as that of Fig. 8 (a).

(2) In the Case of Estimation of the Drought-seasonal Average Stream Flow during the Coming Winter from the Estimated Value of the Moving Drought-seasonal Average Stream Flow during Winter

We are going to obtain the estimated value of the moving

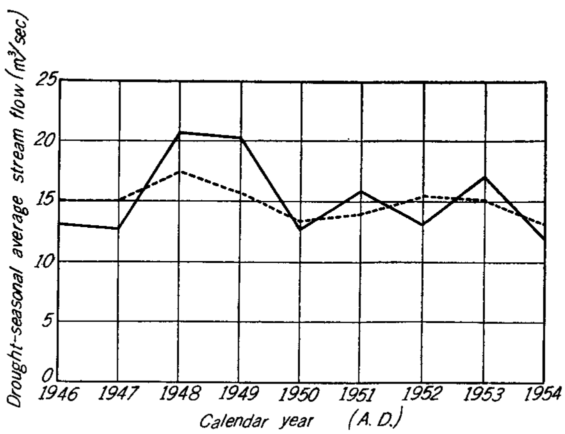

Fig. 8 (a) (the River A)

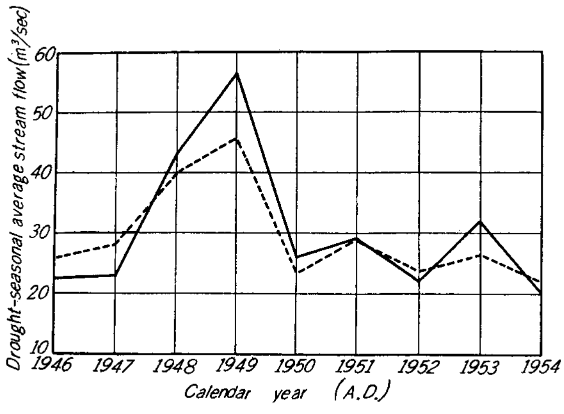

Fig. 8 (b) (the River B)

The comparison between the estimate and date of the seasonal average stream flow during winter.

Note: By means of the direct estimation of the seasonal average stream flow during the coming winter

——— Data

- - - - Estimate

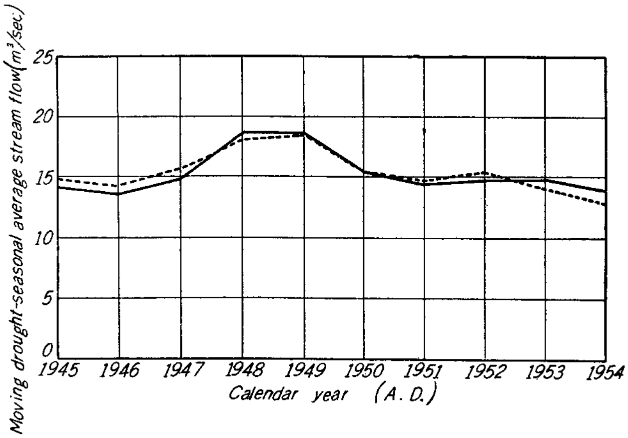

Fig. 9 (a) (the River A)

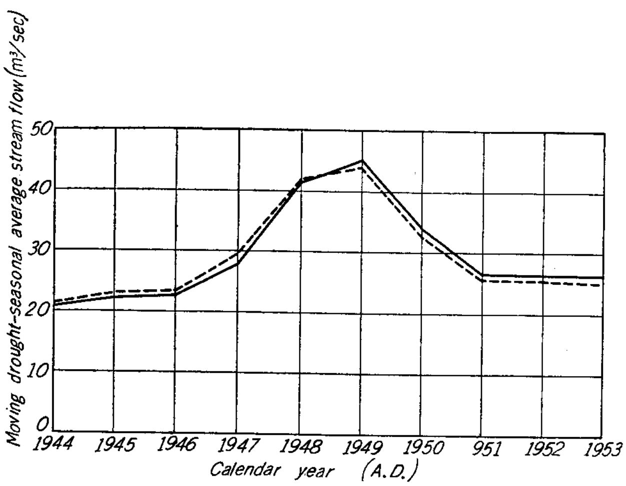

Fig. 9 (b) (the River B)

The comparison between the estimate and data of the moving seasonal average stream flow during winter.

Note: ——— Data
- - - - Estimate

average stream flow during the coming winter by means of the above-mentioned method in **III** (**1**), by employing the data of the moving seasonal average stream flow during winter and autumn obtained by applying the 3-year moving average of Eq. (2) to the respective seasonal average stream flows.

When we represent $W(i)$ as the estimate of the i-th year's moving seasonal average stream flow during winter, and $x_w(i)$ as the data of the seasonal average stream flow during winter, the conditional estimate of the $(i+1)$-th year's seasonal average stream flow during winter, $\chi_w(i+1)$, will be obtained as follows,

$$\chi_w(i+1) = 4W(i) - x_w(i-1) - 2x_w(i) . \qquad (14)$$

As a result of applying this method to the examples for the Rivers A and B, first we obtained the estimated value of the moving seasonal average stream flow during winter (Figs. 9 (a), (b) show the estimated values together with the data for both rivers). Furthermore, we can estimate the seasonal average stream flow during winter from the above-mentioned equation (14) by making use of the estimated value just obtained. Figs. 10 (a), (b) show the estimated values together with the data.

The estimation of the seasonal average stream flow during winter obtained by this method is also as precise as that obtained in the above-mentioned **III (1)**.

When we compare these two methods (cf. Figs. 8 and 10), we can easily find the different results, because the method for finding the periodic components is different.

Therefore, if we consider both methods together, it is possible to get a much more precise estimation of the seasonal average stream flow than when either is used separately.

As is shown in the above-mentioned **II,** we can take the moving seasonal average stream flow as defined by Eq. (2) to be approximately the periodic components. In this case, the estimated result thus obtained is as precise as that in the above-mentioned case. Examples of this estimation are given in Fig. 6 (a), (b).

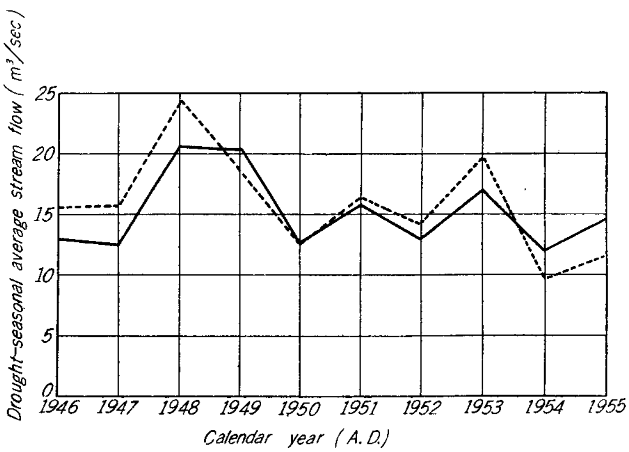

Fig. 10 (a) (the River A)

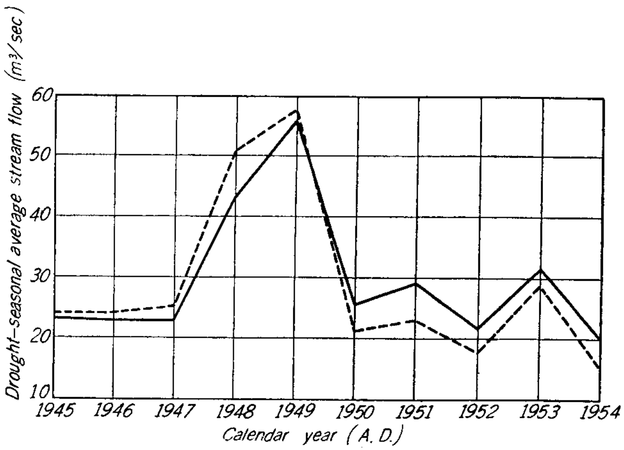

Fig. 10 (b) (the River B)

The comparison between the estimate and data of the seasonal average stream flow during winter.

Note: By means of the estimated result of the moving seasonal average stream flow

——— Data

- - - - Estimate

IV. Conclusion

In making an economical study on the control of a hydro-plant in a large combined hydro-steam power system, when it is necessary to estimate the annual average stream flow for any given year in the immediate future, we find the length of the fundamental fluctuation period which is contained in the fluctuation of the stream flow, and we obtain the mean containing a trend and the periodic component by means of harmonic analysis (the trend is found by fitting the $\frac{1}{2}$

harmonic component) or by the analysis of the 3-year moving average stream flow (the trend is found by fitting a straight line or a simple curve by least squares).

When we apply the method of statistical extrapolation to the residual series which was obtained by subtracting these mean and periodic components from the orginal series, and add the periodic component and d.c. component (mean) again which was already subtracted, we then obtain the estimated value. Thus it is possible to estimate by a theoretically sound method, by considering external factors such as the meteorological conditions.

Moreover, when it is necessary to estimate the drought-seasonal average stream flow during the coming winter for the control of a plant of pondage or storage type, first we apply the extrapolation formula for a time series with two members, to the residual series which was obtained by subtracting the respective periodic components and d.c. components (containing the trends) from the seasonal average stream flows during winter and autumn over a number of recent years including that for autumn of the estimated year. Then it is possible to find the estimate of the drought-seasonal average stream flow during the coming winter by adding to this estimate the periodic component and d.c. component of the drought-seasonal average stream flow which was already subtracted. It is also possible to get a conditional expectation of the seasonal average stream flow for each year, by making use of the result obtained by means of the extrapolatory estimation with regard to the 3-year moving average stream flow in the same way as in the above-mentioned case, and taking it as the estimated value. In this case, the estimation is also as precise as that of the above-mentioned case.

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