

# A Design of a Three-Component Tool Dynamometer

By

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A new three-component tool dynamometer was designed. It consists of an elastic disc which is supported at its periphery. A cutting tool is installed in the center of the disc. Strain gages are attached to the disc for the force measurements. The three components of a cutting force are measured independently without any interference among them. It was found from the experimental results that this tool dynamometer has sufficient sensitivity, rigidity, and stability for practical use. This tool dynamometer is very simple in shape, and its characteristic was analyzed theoretically. The best position for the strain gages was determined, and nomographs were prepared for the design of tool dynamometers of this type.

## 1. Introduction

A tool dynamometer is used in order to measure the cutting force in machining. It is usually most convenient to measure this force relative to a set of two- or three-dimensional rectangular coordinates. The important requirements for the tool dynamometer are as follows:

- a. High sensitivity
- b. High rigidity
- c. Linear relationship between magnitudes of the cutting force and readings in the recorder with no hysteresis phenomenon
- d. No cross sensitivity among cutting force components
- e. Stability relative to time, temperature, and humidity.

Several types of tool dynamometer satisfying the above requirements have been invented and used in metal cutting research.

In this paper the authors present a simply designed three-component tool dynamometer with strain gages. It consists of an elastic disc, in the center of which a cutting tool is installed. The characteristic of this tool dynamometer was analyzed theoretically from the standpoint of the theory of elasticity. This paper describes this analysis and presents nomographs for design of the tool dynamometer.

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## 2. Principle and Construction of the Tool Dynamometer Designed

The whole construction of a newly designed tool dynamometer is shown in Fig. 1. The main part of this tool dynamometer is an elastic disc which is supported at its periphery. A cutting tool is installed on a circular boss in the center of the disc. Twelve strain gages are attached to the two surfaces of the disc as illustrated in the figure.  $A$ ,  $B$ ,  $A'$ , and  $B'$  are the strain gages for the principal cutting forces,  $C$ ,  $D$ ,  $C'$ , and  $D'$  for the feed cutting force, and  $E$ ,  $F$ ,  $E'$ , and  $F'$  for the radial cutting force. This disc type tool dynamometer is set on the carriage of a lathe or the ram of a shaper or a planer with a suitable support.

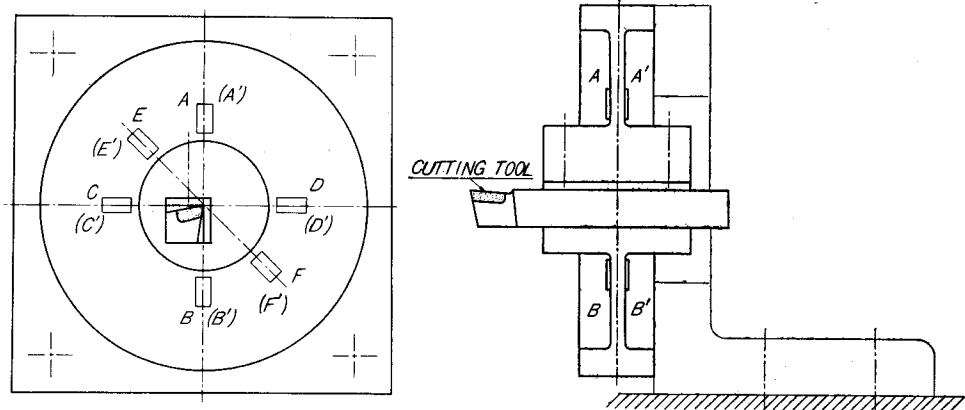


Fig. 1. Construction of the tool dynamometer designed.

The principle of independently measuring three components of a cutting force with this tool dynamometer is explained as follows.

The principal and feed cutting forces actually cause the same type of deformation of the disc due to the bending moment as shown in Fig. 2 (b) and (c). On the other hand, the radial cutting force deforms the disc due to the thrust as shown in Fig. 2(d).

The connection of the wires for the strain gages and deflection of the disc during cutting are shown in Figs. 3 and 4, which correspond to measurement of the principal or feed cutting force and the radial cutting force, respectively. Strain gages for the principal or feed cutting force  $A$ ,  $B$ ,  $A'$ , and  $B'$  in Fig. 3(a) are connected as shown in Fig. 3(b). According to the deflection of the disc due to the bending moment caused by the principal or feed cutting force acting at the cutting edge of a tool, as shown in Fig. 3(c), strain gages  $A$  and  $B'$  are put in tension and  $A'$  and  $B$  are put in compression. This change in resistance of strain gages causes an imbalance in the Wheatstone bridge shown in Fig. 3(b)

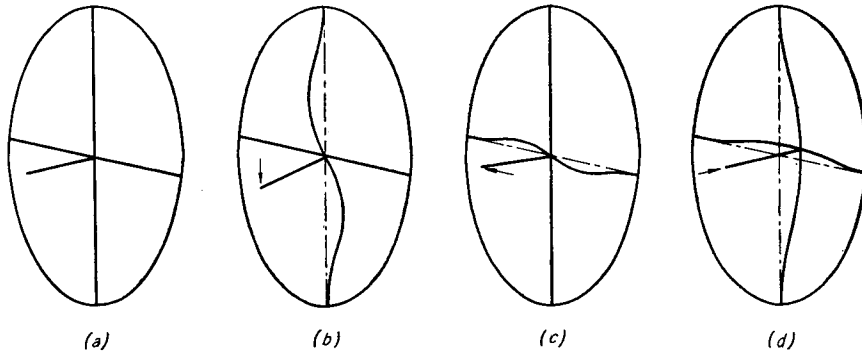


Fig. 2. Deflection of the tool dynamometer due to three components of a cutting force.

- (a) No load
- (b) Principal cutting force
- (c) Feed cutting force
- (d) Radial cutting force

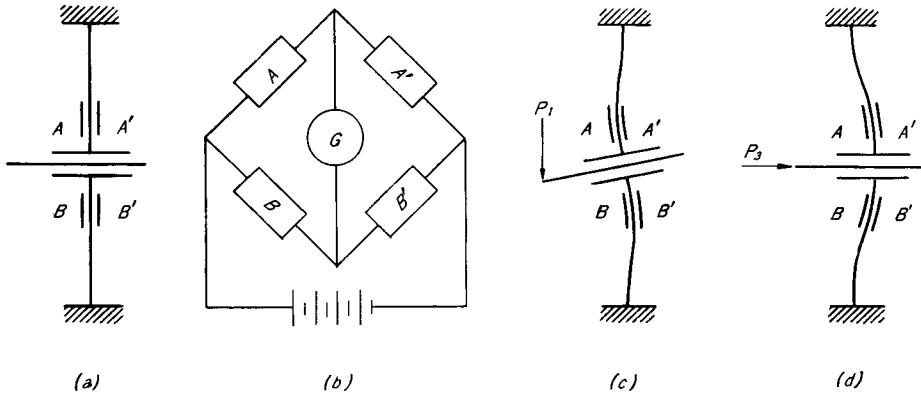


Fig. 3. Connection and operation of the strain gages for the principal or feed cutting force.

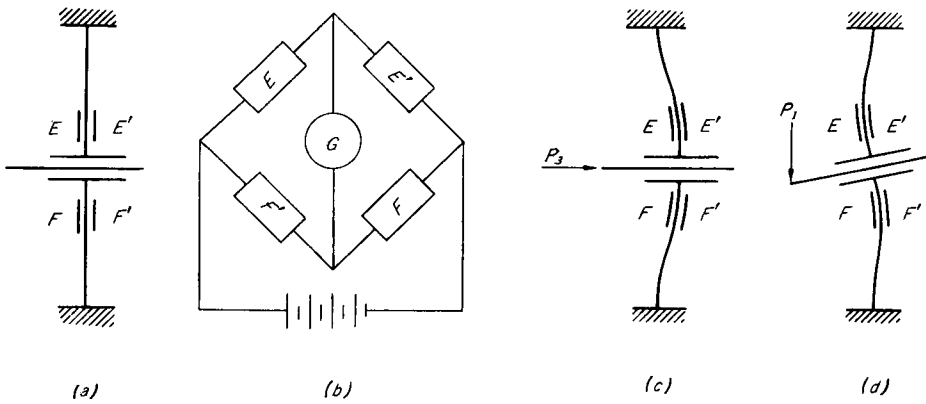


Fig. 4. Connection and operation of the strain gages for the radial cutting force.

and a current proportional to the magnitude of the principal cutting force  $P_1$  or feed cutting force  $P_2$  flows through the galvanometer or the recorder. Even if the radial cutting force deforms the disc in this case, strain gages  $A$  and  $B$  or  $A'$  and  $B'$  are subjected to the same magnitude of tension or compression as shown in Fig. 2(d), and the balance in the bridge is not affected. In other words, the radial cutting force does not affect the reading of the principal or feed cutting force.

A similar explanation holds for the strain gages of the feed cutting force  $C$ ,  $D$ ,  $C'$ , and  $D'$ .

Both principal and feed cutting force components do not interfere in strain gages for each component, because strain gages for both components are attached to the different neutral orthogonal axes (refer to Figs. 1 and 2).

Strain gages for the radial cutting force  $E$ ,  $F$ ,  $E'$ , and  $F'$  of Fig. 4(a) are connected as shown in Fig. 4(b). According to the deflection of the disc due to the thrust caused by this force component  $P_3$ , as shown in Fig. 4(c), strain gages  $E$  and  $F$  are put in compression and  $E'$  and  $F'$  are put in tension. This causes an imbalance in the bridge of Fig. 4(b) and a current proportional to the magnitude of the feed cutting force  $P_3$  flows through the galvanometer or the recorder. Even if the principal or feed cutting force deforms the disc in this case, strain gages  $E$  and  $F'$  or  $E'$  and  $F$  are subjected to the same magnitude of tension or compression, and the balance in the bridge is not affected. In other words, the principal or feed cutting force component does not affect the reading of the radial cutting force.

Thus three components of a cutting force can be measured independently with the tool dynamometer explained above.

The three force components can also be measured with only six strain gages  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ , and  $F'$  in place of the twelve strain gages, but the sensitivity of measurement is reduced.

The bridges fully compensated for any change in resistance due to temperature because of the symmetrical connection of the bridges.

Further, this tool dynamometer involves no friction.

Connecting this tool dynamometer to the amplifier and the recorder, a pen deflection proportional to cutting force components is recorded on an oscillograph paper.

### 3. An Analysis of the Tool Dynamometer Designed

The type of deformation of the tool dynamometer designed was analyzed theoretically and the best position for attaching the strain gages was determined from the theory of elasticity.

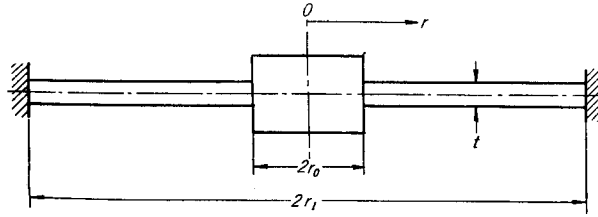


Fig. 5. Schematic view of the tool dynamometer designed.

The schematic view of this tool dynamometer is shown in Fig. 5. The diameter of the disc is denoted as  $2r_1$ , and the thickness of the disc is denoted as  $t$ . The periphery of the disc is supported rigidly. The cutting tool is installed on a rigid circular boss of radius  $r_0$  in the center of the disc.

**a. Deflection due to Principal or Feed Cutting Force**

Principal or feed cutting force produces a bending moment  $M$ , and the disc is deformed as shown in Fig. 6.

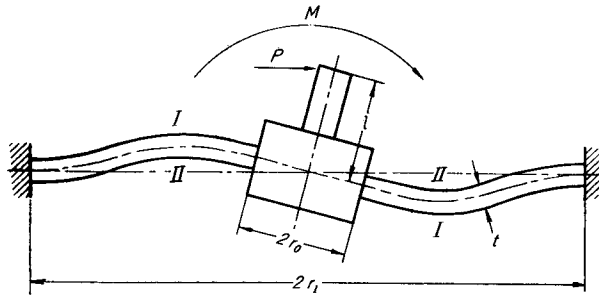


Fig. 6. Schematic view of deflection of the tool dynamometer due to the principal or feed cutting force.

Tensile or compressive strain at an arbitrary point on the surfaces of the disc, whose distance from the the center is  $r$ , is expressed as follows:

$$\epsilon_{r_0} = \mp \frac{m^2 - 1}{m^2 E} \frac{3}{2\pi t^2 (r_0^2 + r_1^2)} \left[ 3r - (r_0^2 + r_1^2) \frac{1}{r} - r_0^2 r_1^2 \frac{1}{r^3} \right], \quad (1)$$

where the minus sign is for tensile strain at surface I and the plus sign is for compressive strain at surface II, and  $E$  is Young's modulus of elasticity and  $m$  is Poisson's ratio.

Considering the tensile strain at surface I, this function of the strain is a decreasing function in respect to radius  $r$ , because

$$\frac{d\epsilon_{r_0}}{dr} = - \frac{m^2 - 1}{m^2 E} \frac{3}{2\pi t^2 (r_0^2 + r_1^2)} \left[ 3 + (r_0^2 + r_1^2) \frac{1}{r^2} + r_0^2 r_1^2 \frac{3}{r^4} \right] < 0. \quad (2)$$

Therefore, the maximum value of the tensile or compressive strain on the surfaces of the disc exists at the edge of the rigid boss, namely, at  $r=r_0$ , and is expressed in the following.

$$\epsilon_{r0 \max} = \pm \frac{m^2-1}{m^2E} \frac{3}{2\pi} \frac{M}{t^2 r_0} \frac{r_1^2-r_0^2}{r_1^2+r_0^2}, \quad (3)$$

where the plus sign is for the maximum tensile strain at surface I and the minus sign is for the maximum compressive strain at surface II.

If the cutting tool is set so that the length from the cutting edge to the center of the disc is  $l$ , and the principal cutting force  $P_1$  or the feed cutting force  $P_2$  acts on the cutting edge, the bending moment  $M$  is expressed as follows:

$$M_1 = lP_1 \quad \text{for principal cutting force} \quad (4)$$

$$M_2 = lP_2 \quad \text{for feed cutting force} \quad (5)$$

#### b. Deflection due to Radial Cutting Force

Radial cutting force  $P_3$  deforms the disc of the tool dynamometer as shown in Fig. 7.

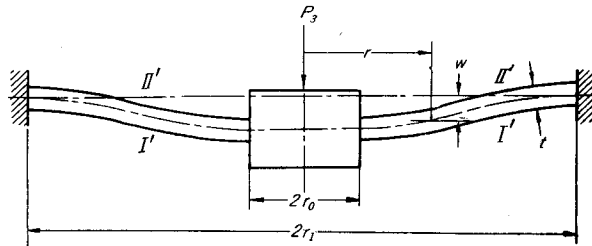


Fig. 7. Schematic view of deflection of the tool dynamometer due to the radial cutting force.

Tensile or compressive strain at an arbitrary point on the surfaces of the disc, whose distance from the center is  $r$ , is expressed as follows:

$$\epsilon_{r0'} = \mp \frac{m^2-1}{m^2E} \frac{3}{2\pi} \frac{P}{t^2} \left[ \ln r - \frac{r_0^2 r_1^2}{r_1^2 - r_0^2} \ln \frac{r_1}{r_0} \cdot \frac{1}{r^2} + \frac{r_0^2 \ln r_0 - r_1^2 \ln r_1}{r_1^2 - r_0^2} + 1 \right], \quad (6)$$

where the minus sign is for tensile strain at surface I' and the plus sign is for compressive strain at surface II'.

Considering the tensile strain at surface I', this function of the strain is a decreasing function in respect to radius  $r$ , because

$$\frac{d\epsilon_{r0'}}{dr} = - \frac{m^2-1}{m^2E} \frac{3}{2\pi} \frac{P}{t^2} \left[ \frac{1}{r} + \frac{r_0^2 r_1^2}{r_1^2 - r_0^2} \ln \frac{r_1}{r_0} \cdot \frac{2}{r^3} \right] < 0. \quad (7)$$

Therefore, the maximum value of the tensile or compressive strain on the surfaces of the disc exists at the edge of the rigid boss, namely, at  $r=r_0$ , and is expressed in the following.

$$\epsilon_{r_0'} \max = \pm \frac{m^2-1}{m^2E} \frac{3}{2\pi} \frac{P}{t^2} \left[ \frac{2r_1^2}{r_1^2-r_0^2} \ln \frac{r_1}{r_0} - 1 \right], \quad (8)$$

where the plus sign is for the maximum tensile strain at surface I' and the minus sign is for the maximum compressive strain at surface II'.

When the disc is subjected to the radial cutting force, it is deformed as shown in Fig. 7. The deflection at an arbitrary point on the center surface of the disc, whose distance from the center is  $r$ , is expressed as follows:

$$w = \frac{m^2-1}{m^2E} \frac{3}{2\pi} \frac{P}{t^3} \left[ \left( \frac{r_0^2 \ln r_0 - r_1^2 \ln r_1}{r_1^2 - r_0^2} - \frac{1}{2} \right) r^2 + r^2 \ln r + \frac{2r_0^2 r_1^2}{r_1^2 - r_0^2} \ln \frac{r_1}{r_0} \ln r + \frac{r_1^2}{2} - \frac{r_0^2 r_1^2}{r_1^2 - r_0^2} (2 \ln r_1 - 1) \ln \frac{r_1}{r_0} \right]. \quad (9)$$

This deflection is a maximum at the boss of the tool dynamometer, or at the edge of the boss. The equation for the maximum deflection of the disc or the recession of the cutting tool due to the radial cutting force is obtained from the above equation, putting  $r=r_0$ .

$$w_{\max} = \frac{m^2-1}{m^2E} \frac{3}{4\pi} \frac{P}{t^3} \left[ r_1^2 - r_0^2 - \frac{4r_0^2 r_1^2}{r_1^2 - r_0^2} \left( \ln \frac{r_1}{r_0} \right)^2 \right]. \quad (10)$$

**c. Discussion**

It is concluded from the above theoretical analysis of the disc type tool dynamometer that the tensile or compressive strain on the surfaces of the disc due to the tangential or feed cutting force and the radial cutting force is a maximum at the periphery of the rigid circular boss which holds the cutting tool. Therefore, the best position for attaching the strain gages is the periphery of the boss in the center of the disc for both the tangential or feed and radial cutting forces. It is recommended to attach the strain gages as near the periphery of the boss as possible.

Using Equations (3) and (8), nomographs for strains due to the tangential or feed cutting force and the radial cutting force are made and shown in Figs. 8 and 9 with respect to the diameter and thickness of the disc and the diameter of the rigid boss. These nomographs are for unit amount of bending moment  $M=1 \text{ kg}\cdot\text{cm}$  or radial cutting force  $P_3=1 \text{ kg}$ , assuming Poisson's ratio  $m=\frac{10}{3}$  and Young's modulus of elasticity  $E=2,100,000 \text{ kg/cm}^2$ .

The disc of the tool dynamometer deflects due to the radial cutting force so that a cutting tool recesses. The amount of the recession of the cutting tool is expressed by Equation (10) and its nomograph is made as shown in Fig. 10 with respect to the diameter and thickness of the disc and the diameter of the rigid boss. This is for unit amount of radial cutting force  $P_3=1 \text{ kg}$ , assuming Poisson's ratio  $m=\frac{10}{3}$  and Young's modulus of elasticity  $E=2,100,000 \text{ kg/cm}^2$ .

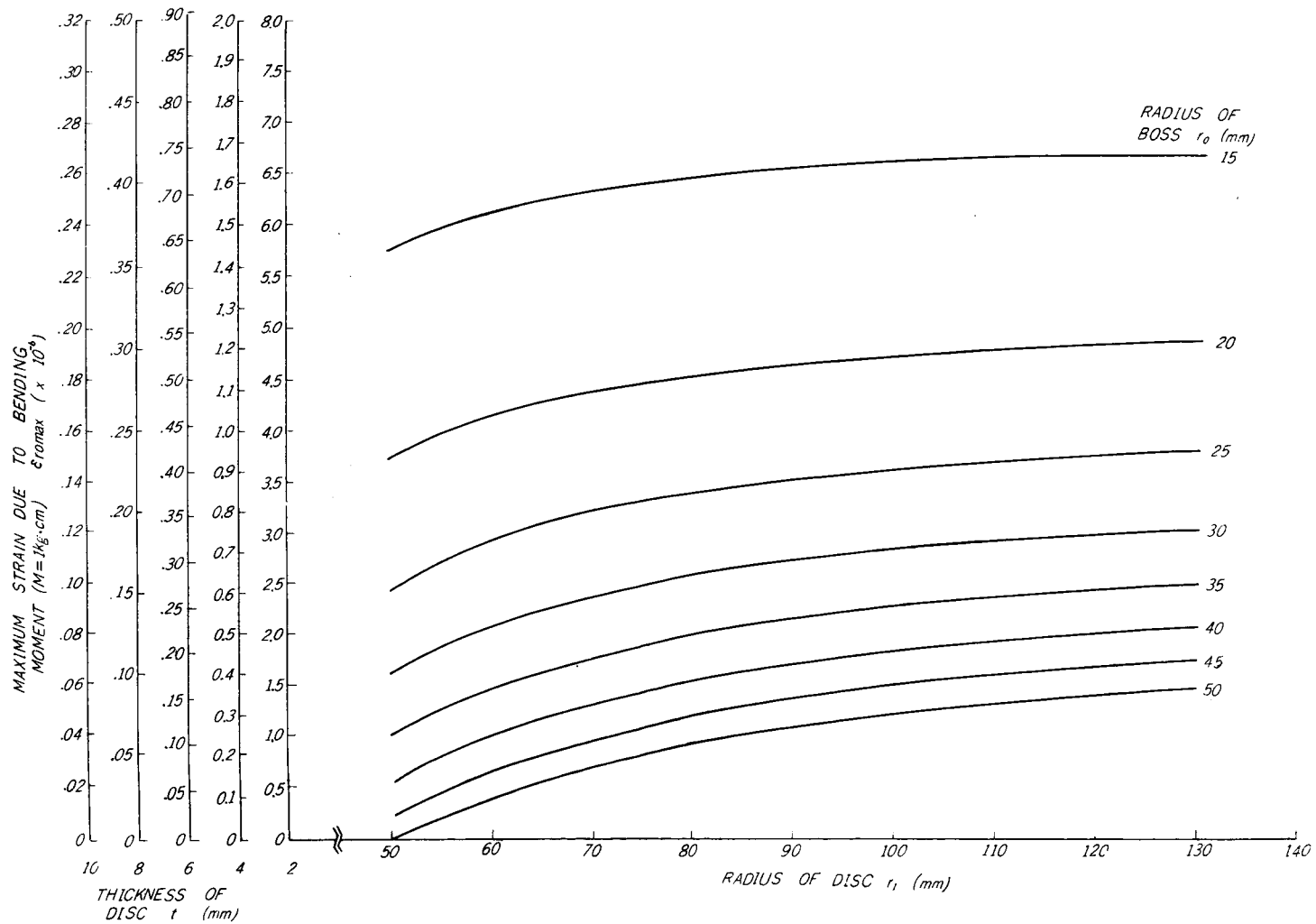


Fig. 8. Nomograph for the principal or feed cutting force.



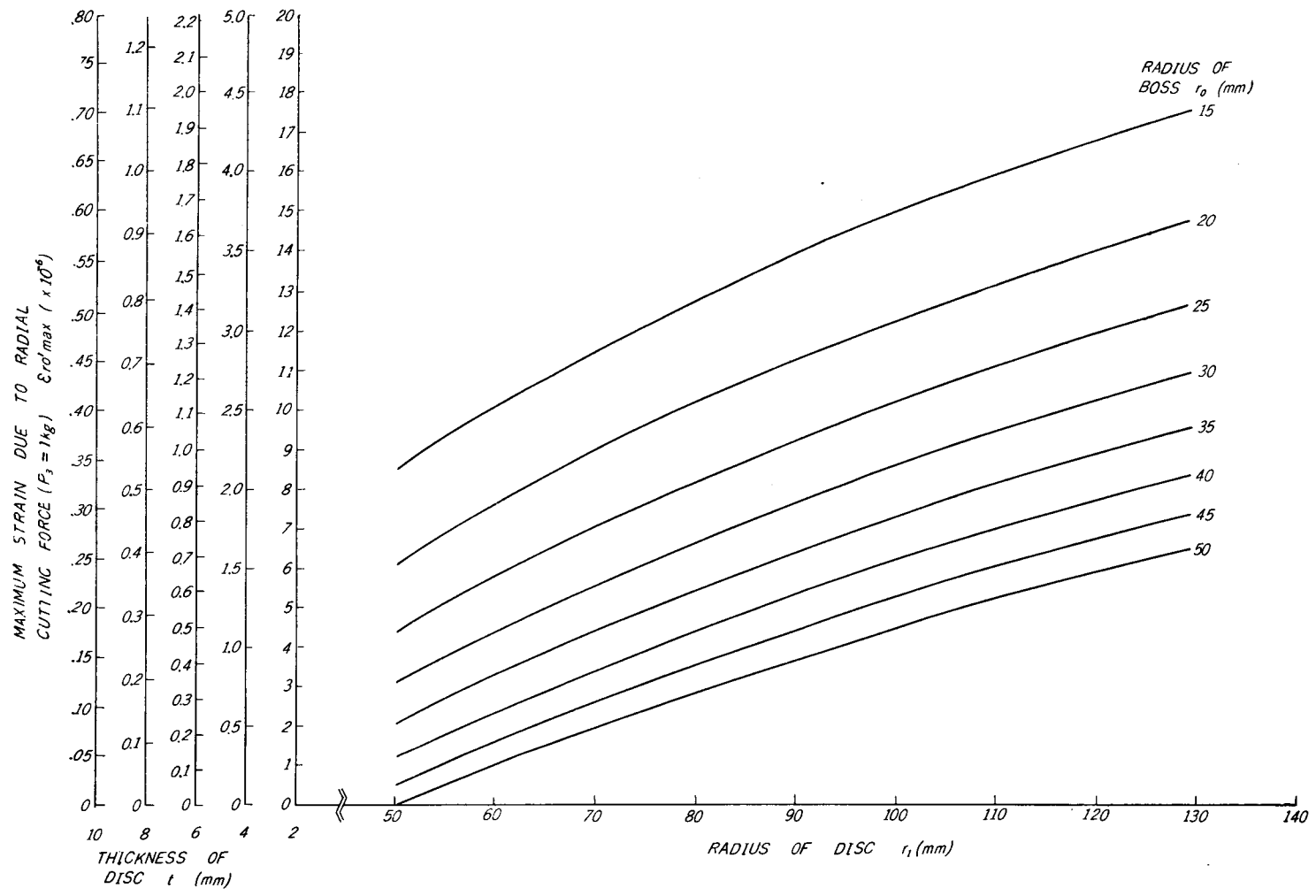


Fig. 9. Nomograph for the radial cutting force.

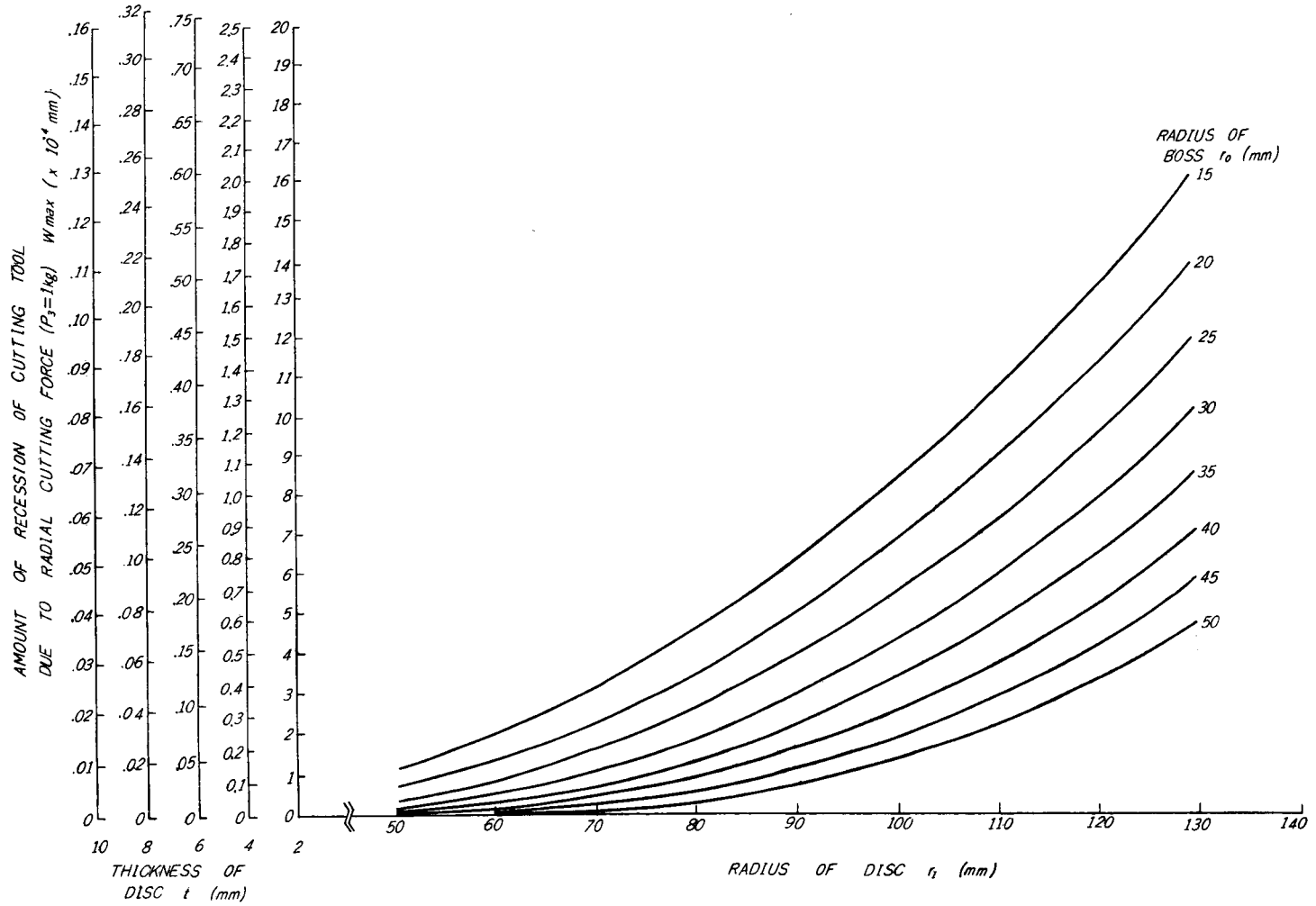


Fig. 10. Nomograph for the recession of the cutting tool due to the radial cutting force.

The sensitivity of a tool dynamometer of this type depends upon the diameter and thickness of the disc and the diameter of the rigid circular boss. The larger the diameter of the disc and the smaller the thickness of the disc and the diameter of the boss, the more sensitive the tool dynamometer. However, in this manner, the deflection of the disc or the amount of the recession of the cutting tool increases. Therefore, there are some limitations as to size for the diameter and thickness of the disc and the diameter of the boss from the standpoint of the rigidity of the tool dynamometer. Nomographs of Figs. 8 to 10 are helpful when a tool dynamometer of this type is designed.

#### **4. Characteristics of the Tool Dynamometer Designed and an Example of the Cutting Force Measurement**

The tool dynamometer designed has the elastic disc of 120 mm diameter and 6 mm thickness and the rigid circular boss of 36 mm diameter for installing a cutting tool. The material is hard steel.

Bakelite-cement base bonded wire resistance strain gages of 4 mm length and 1.86 gage factor were attached to the disc.

It was found that both the static and dynamic rigidities of this tool dynamometer are satisfied and that a frequency response is obtained sufficiently high to follow significant fluctuations of force with a natural frequency of more than 1,000 cycles per second.

An example of calibration curve is shown in Fig. 11. The relationships between the three cutting force components and their respective readings in the recorder are linear, and no hysteresis phenomenon appears.

There is no interference among the readings of the three force components from the theoretical point of view as explained in Section 2. But in practice cross sensitivity appears to some degree. With the above tool dynamometer a feed or radial cutting force of 500 kg gave a deflection of 1 kg in the principal cutting force, and hence cross sensitivity was almost negligible.

The sensitivity of this tool dynamometer depends upon time, temperature, and humidity. This also depends upon the amount of amplification. The error of force measurement due to the change of sensitivity was within 5% for the largest amount of amplification and within 1% for a smaller amount of amplification. The change of sensitivity is not so important if the calibration is checked every time that the tool dynamometer is used.

An example of three components of a cutting force measured with this tool dynamometer is shown in Fig. 12. The tool dynamometer was set on the carriage

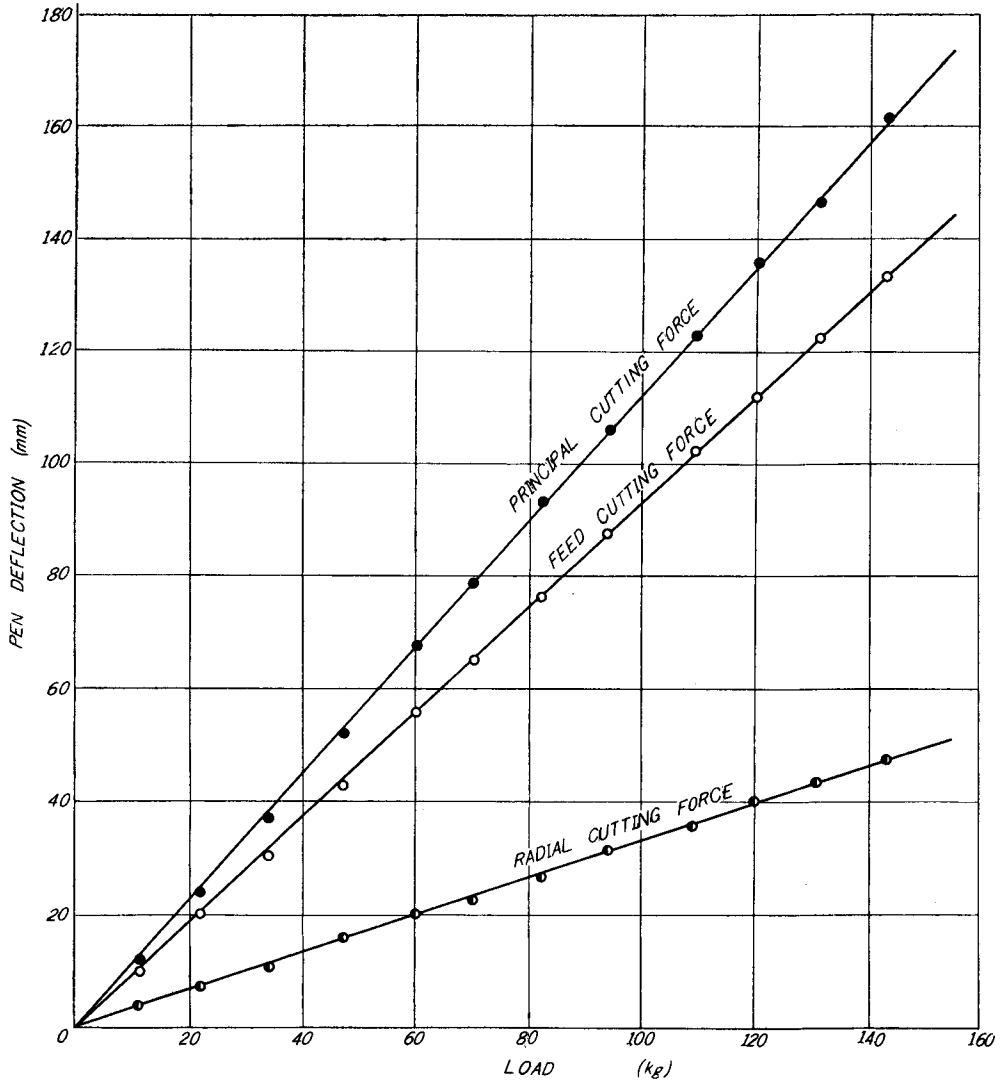


Fig. 11. An example of calibration curve for the tool dynamometer designed.

of a lathe (Kärger type) and mild steel (0.17% carbon, hardness Rockwell B scale 70) was machined under the following cutting conditions.

Depth of cut	0.5 mm
Feed	0.1 mm/rev.
Cutting speed	20 m/min.

The following values for the three cutting force components were obtained from Fig. 12.

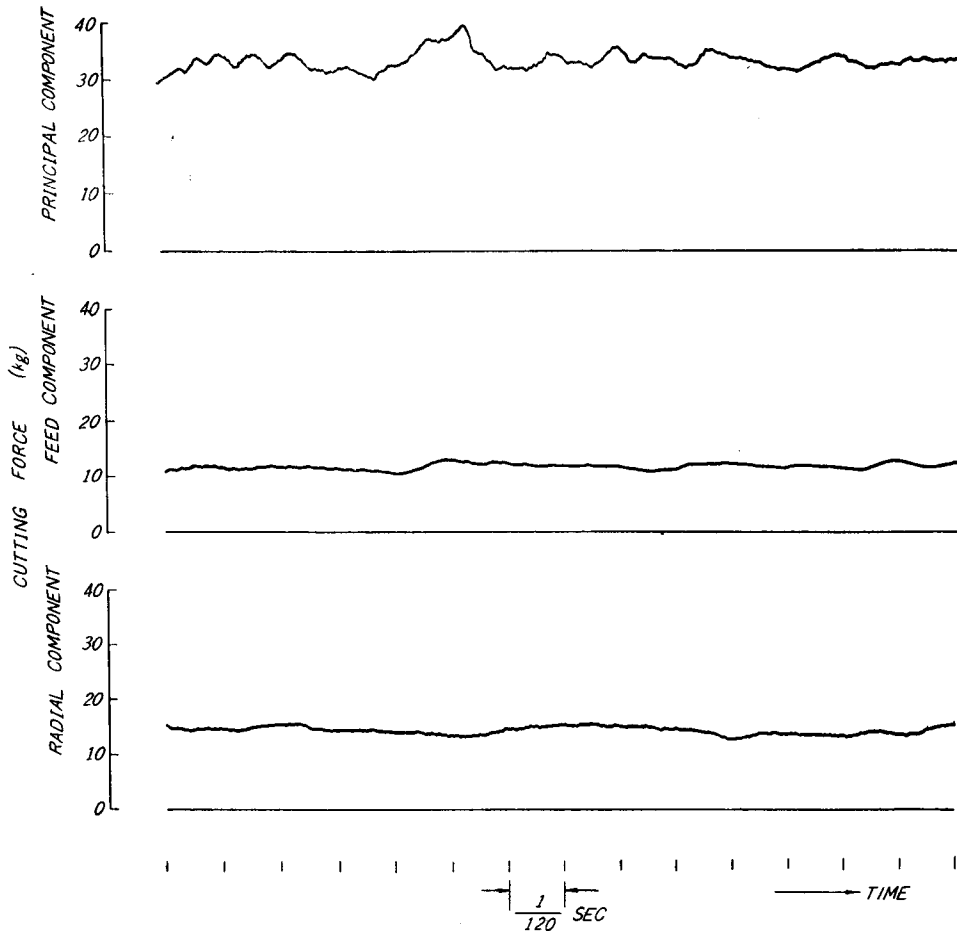


Fig. 12. An example of the three components of a cutting force recorded on an oscillograph paper.

Principal cutting force	33 kg
Feed cutting force	11 kg
Radial cutting force	14 kg

**5. Conclusion**

A new three-component tool dynamometer, consisting of an elastic disc supported at its periphery and a cutting tool installed at its center, was designed. Strain gages are attached to the disc, and the three components of a cutting force are measured independently without any interference among them. It was found from the experimental results that this tool dynamometer has sufficient sensitivity, rigidity, and stability for practical use. Because of its simple shape,

the characteristic of this tool dynamometer was analyzed theoretically without difficulty, and the best position for the strain gages was determined. Nomographs for design of tool dynamometers of this type were prepared for the determination of the diameter and thickness of the disc and the diameter of the rigid circular boss for the support of a cutting tool.