Studies on Through Flow Drying. II. Simultaneous Heat and Mass Transfer in the Decreasing Drying Rate Period

By

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Part II of this paper reports an investigation of the mechanism of the decreasing drying rate period for through flow drying. The moisture distribution in the packed bed during the decreasing rate period was obtained by measuring the moisture content of particles at arbitrary drying times. A theoretical discussion on simultaneous heat and mass transfer in the decreasing drying rate period is presented and basic rate equations are derived from theoretical considerations under several assumptions. The authors solved numerically the simultaneous equations by digital computer. The calculated results generally agreed with the experimental date of moisture distribution.

1. Introduction

In the previous report part I, the constant drying period was studied and correlation of heat and mass transfer between fluids and solids were obtained. The mechanism of the decreasing rate period for through flow drying is a complex phenomenon of simultaneous heat and mass transfer between the particles and the air flowing through the packed bed, and of water diffusion and heat conduction in the particles. Many studies on unsteady heat transfer without mass tranfer in the packed bed have been published[1)2)3)], but works on unsteady simultaneous heat and mass transfer in packed beds are still few. Van Arsdel[4)] reported a theoretical analysis of the decreasing rate period for through flow drying in the low moisture range and obtained numerical solutions of the differential equations under several assumptions. He calculated the variation of air temperature and the water content of materials in a packed bed of half-dice white potato, but the conclusions drawn from the theory have not been verified by experimental measurements. The purpose of this report is to discuss these problems more generally and to compare experimental data with theoretical calculations.

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2. Experimental procedure

Experimental equipment used was the same as diescribed in the previous report. The particles used in these experiment were spheres of 9.43 mm preformed from silica gel and porcelain spheres of 7.46 mm diameter. The method of weighing the drying chamber and of measuring air temserature were analogous to those illustrated in the previous report. It is very difficult to measure continuously the moisture distributions during a run.

The authors used the following method; under constant drying conditions for which the temperature of inlet air, height of packed bed, properties of particles and other initial conditions were identical, many experiments were performed for different drying times, for example, 1 min, 2 min, 5 min. Each run was stopped at a different time, and the moisture content of the particles at the several heights in the packed bed were measured.

Plotting the data for each run at different drying times, the continuous curve of moisture distribution over a drying period could be obtained. The data which showed reproducibility were selected from many runs.

3. Experimental results

Expeaimental data obtained by the method mentioned above are presented in Table 1 for spheres of silica gel, and Table 2 for spheres of porcelain. Fig. 1 is the plot of water content and temperature of material and of air versus drying time. The moisture distributions of the particles in the direction of the axis

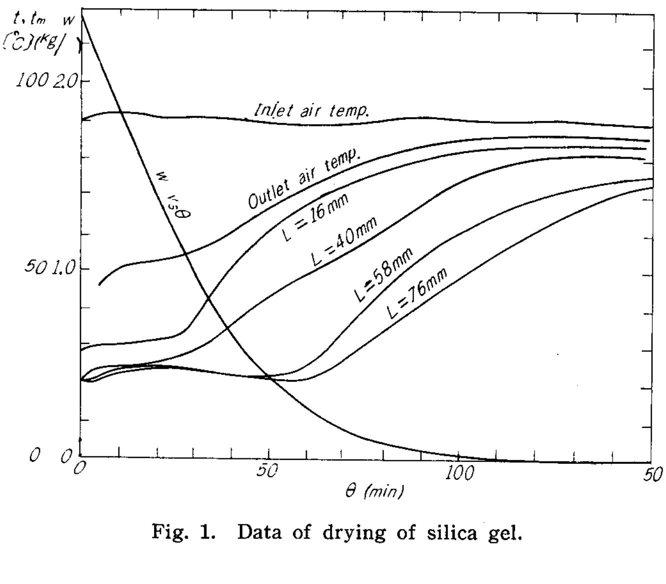

Fig. 1. Data of drying of silica gel.

are plotted in Fig. 2 and Fig. 3. The average critical moisture content over the entire bed obtained from the data presented in Fig. 1 was about 160 percent on the dry base and is shown by the dotted line in Fig. 2. Then, the authors measured the moisture distribution across the radial direction in the packed bed. The moisture distribution versus radius of the drying chamber at 8 minutes of drying time are shown in Fig. 4. It is apparent from Fig. 4 that the moisture content of the particles across the radius of the drying chamber has a

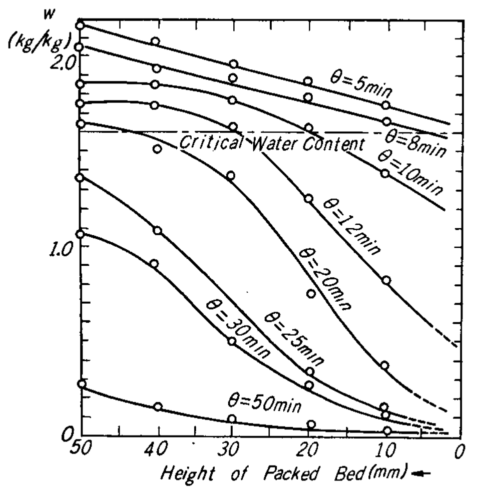

Fig. 2. Moisture distribution of silica gel sphere.

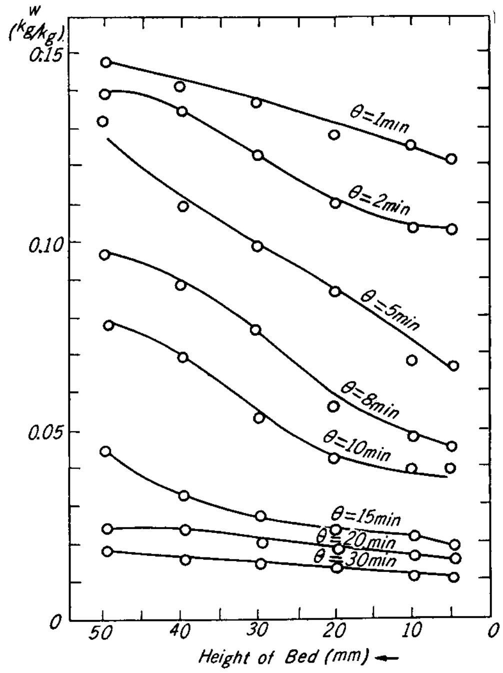

Fig. 3. Moisture distribntion of packed bed of porcelain spheae.

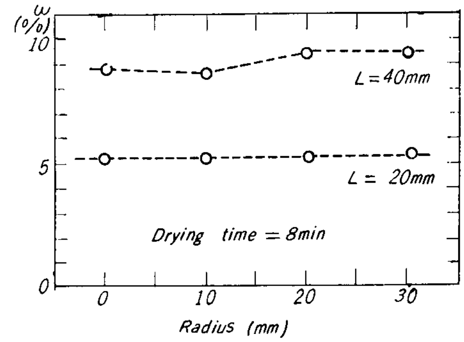

Fig. 4. Radial distribution of moisture content.

uniform distribution. This fact will give experimental support for an assumption which will be made in the following section.

Table 1. Experimental Data of Silica Gel (Air velocity 1.58 m/sec.)

Run No.	105	106	101	107	102	103	109	111	112	average
Drying time (min)	5	8	10	12	15	20	25	30	50	
Height of bed (m)	0.055	0.055	0.052	0.054	0.054	0.053	0.054	0.056	0.055	0.55
Loading wet material (kg)	0.1129	0.1182	0.1126	0.1180	0.1121	0.1138	0.1164	0.1177	0.1121	
Initial water content (kg/kg)	2.33	2.52	2.59	2.25	2.38	2.41	2.31	2.43	2.44	2.41
Loading (dry material kg)	0.0331	0.0346	0.0330	0.0346	0.0329	0.0334	0.0342	0.0346	0.0329	0.0337
Inlet air temperature (°C)	71.4	66.6	75.3	73.1	70.4	72.1	73.1	73.6	73.1	72.1
Inlet air humidity (kg/kg)	0.0067	0.0070	0.0070	0.0075	0.0080	0.0070	0.0070	0.0060	0.0085	0.072
Average particle size (mm)	8.49	7.93	9.91	8.67	8.89	9.42	9.50	8.51	7.68	8.8
Number of particle	230	315	16.7	230	215	240	179	206	308	232
Moisture distribution: Height of bed										
0.05 m	2.18	2.05	1.87	1.76	1.81	1.66	1.39	1.05	0.264	
0.04	2.08	1.92	1.86	1.75	1.63	1.50	1.09	1.02	0.177	
0.03	1.96	1.89	1.78	1.64	1.56	1.29	0.515	0.544	0.0543	
0.02	1.87	1.81	1.62	1.25	0.99	0.741	0.344	0.288	0.0314	
0.01	1.73	1.66	1.39	0.81	0.70	0.366	0.178	0.155	0.0223	

Table 2. Experimental Data of Porcelain Sphere (D_p=9.43 mm)

Run No.	43	27	18	41	37	31	33	39	average
Drying time (min)	1	2	5	8	10	15	20	30	
Height of bed (m)	0.055	0.055	0.055	0.055	0.055	0.055	0.055	0.055	0.055
Loading (dry material kg)	0.166	0.165	0.162	0.167	0.165	0.168	0.161	0.160	0.165
Inlet air temperatuer (°C)	43.2	39.0	44.1	40.0	41.2	40.2	40.7	42.9	41.4
Inlet air humidity (kg/kg)	0.0076	0.0061	0.0078	0.0078	0.0065	0.0070	0.0070	0.0060	0.0070
Air velocity (m/sec)	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39
Initial water content (kg/kg)	0.154	0.153	0.157	0.151	0.154	0.149	0.154	0.155	0.155
Number of partile	227	"	"	"	"	"	"	"	227
Moisture distribution: height of bed									
0.05 m	0.144	0.141	0.133	0.0972	0.0798	0.0458	0.0255	0.0202	
0.04	0.142	0.136	0.110	0.0898	0.0710	0.0334	0.0255	0.0154	
0.03	0.138	0.123	0.100	0.0779	0.0534	0.0282	0.0219	0.0158	
0.02	0.128	0.111	0.087	0.0563	0.0430	0.0239	0.0202	0.0141	
0.01	0.126	0.104	0.068	0.0485	0.0404	0.0234	0.0175	0.0118	
0.005	0.122	0.103	0.067	0.0446	0.0403	0.0195	0.0173	0.0111	

4. Theoretical discussion

The experimental results obtained above were analysed by a theoretical discussion on the mechanism of simultaneous heat and mass transfer in the decreasing rate period similar to that of Van Arsdel. The batch through flow dryer is a cylindrical container of uniform cross section, with a perforated bottom through which a current of air of uniform temperature t_1, absolute humidity H_1 and mass velocity G can be forced into the bed of particles. The particle is initially at the uniform critical moisture content w_c and the wet bulb temperature t_w. The following assumptions are made:

(1) Air flow through the bed is equally distributed through all elements of the cross section. (this condition was proved by experimental data shown in Fig. 4 and described above.)

(2) Heat transfer through the walls of the container is neglegible.

(3) Conduction of heat from one particle of material to another is negligible.

(4) There is no temperature gradient within the particles of materials.

(5) No appreciable volume shrinkage occurs during the drying period.

From mass and heat balances for a differential thickness in the bed, the equations expressing the decreasing rate period can be written as follows.

$$-\rho_m\left(\frac{\partial w}{\partial \theta}\right) = -\rho_m\phi \tag{1}$$

$$G\left(\frac{\partial H}{\partial L}\right) = -\rho_m\phi \tag{2}$$

$$-GC_H\left(\frac{\partial t}{\partial L}\right) = ha(t-t_m) \tag{3}$$

$$\rho_m C_m\left(\frac{\partial t_m}{\partial \theta}\right) = \rho_m \gamma_m \phi = ha(t-t_m) \tag{4}$$

Initial conditions are as follows.

$$\text{for} \quad \left.\begin{array}{llll} \theta=0 & w=w_c & t_m=t_w & \text{at all } L \\ L=0 & H=H_1 & t=t_1 & \text{at all } \theta \end{array}\right\} \tag{5}$$

A further transformation of the independent variable to dimensionless numbers faciliates computation.

$$\left.\begin{array}{l} Z = \dfrac{ha}{GC_H} L = N_t \\ \tau = \dfrac{ha}{\rho_m C_m}\theta \end{array}\right\} \tag{6}$$

Making the substitution and solving for the derivatives, we have

$$\frac{\partial w}{\partial \tau} = \rho_m \alpha C_m \phi \tag{7}$$

$$\frac{\partial H}{\partial Z} = -\alpha C_H \phi \tag{8}$$

$$\frac{\partial t_m}{\partial \tau} = (t-t_m)+\alpha \gamma_m \phi \tag{9}$$

$$\frac{\partial H}{\partial Z} = -(t-t_m) \tag{10}$$

where $\alpha = \rho_m / ha$

Initial conditions therefore are as follows

$$\text{for} \quad \left.\begin{array}{llll} \tau = 0 & w = w_c & t_m = t_w & \text{at all } Z \\ Z = 0 & H = H_1 & t = t_1 & \text{at all } \tau \end{array}\right\} \tag{11}$$

Assuming that the decreasing rate is proportional to the decreasing water content of the material, the drying rate for a layer of differential thickness is

$$\phi = \phi_c \cdot \frac{F}{F_c} \tag{12}$$

where $F = w - w_e$

The constant drying rate for a small element of packed bed is expressed as follows, if the sensible heat for the water vapor is neglesibly small in comparison with the latent heat of vaporization of warer; that is $\gamma_w \gg 0.46\ (t-t_w)$

$$\phi_c = -\frac{GC_{H_1}(1-e^{-\Delta Z})(t_1-t_w)}{\rho_m \gamma_m \Delta Z} \tag{13}$$

Substituting equation (13) into equation (12), the decreasing rate for a layer of differential thickness is

$$\phi_c = -\frac{(1-e^{-\Delta Z})(t_1-t_w)}{\alpha \Delta Z \gamma_w} \cdot \frac{F}{F_c} \tag{14}$$

Again, initial conditions are as follows.

$$\text{for} \quad \left.\begin{array}{llll} \tau = 0 & w = w_c & t_m = t_w & \text{at all } Z \\ Z = 0 & H = H_1 & t = t_1 & \text{at all } \tau \end{array}\right\} \tag{15}$$

Humid heat of wet material C_m, latent heat γ_m and humid heat of gas C_H are expressed respectively as the function of water content, temperature of material and humidity of air, that is

$$\left.\begin{array}{l} C_m = C_0 + w \\ \gamma_m = 595 - 0.55\, t_m \\ C_H = 0.24 + 0.46H \end{array}\right\} \tag{16}$$

5. Calculation results and discusion

The authors solved numerically the simultaneous equations from (7) to (10) under drying rate equation (14) and the initial conditions in (15) by digital computer FACOM 128B and checked the results by hand calculation using the modified Euler method.

The numerical calculations were performed under the same conditions of experiment mentioned above in order to compare with experimental results.

The conditions of calculation are as follows.

$$
\begin{aligned}
D_p &= 9.43\ \text{mm}\\
w_0 &= 0.155\\
w_e &= 0.01\\
a &= 410\ \text{m}^2/\text{m}^3\\
\rho_m &= 1064\ \text{kg/m}^3\\
t_1 &= 41.4^\circ\text{C}\\
H_1 &= 0.007\\
v_g &= 1.39\ \text{m/sec}\\
G &= 5570\ \text{kg/hr}\cdot\text{m}^2\\
Re_0 &= 770
\end{aligned}
$$

The heat transfer coefficient was calculated from equation (3), reported in a previous report, and then the constant drying rate was calculated by using this value. Calculated results were $h=136.3$ kcal/hr·m^2 °C, $ha=5550$ kcal/hr·m^3 °C, $\alpha=\rho_m/ha=0.0192$. For $\Delta Z=0.4$, decreasing drying rate was

$$\phi = -\frac{(1-e^{-0.4})(t_1-t_w)}{(0.0192)(0.4)(\gamma_w)}\cdot\frac{F}{0.145} \tag{17}$$

The numerical calculations were performed at intervals of $\Delta\tau=2.0$ and $\Delta Z=0.4$. The calculated results are also shown in Fig. 5 and Fig. 6. In Fig. 5, distribution curves of moisture and temperature of the particles in the packed bed are plotted aginst time θ, reduced from the dimensionless variable τ. The experimental data of water distribution mentioned above were also plotted with dotted lines in order to compare with the calculated results. It is recongized from Fig. 5 that the calculated results generally agree with experimental data for moisture distribution.

It was difficult to obtain the exact temperature distribution for particles in a packed bed, and so comparisons with the calculated results were not possible.

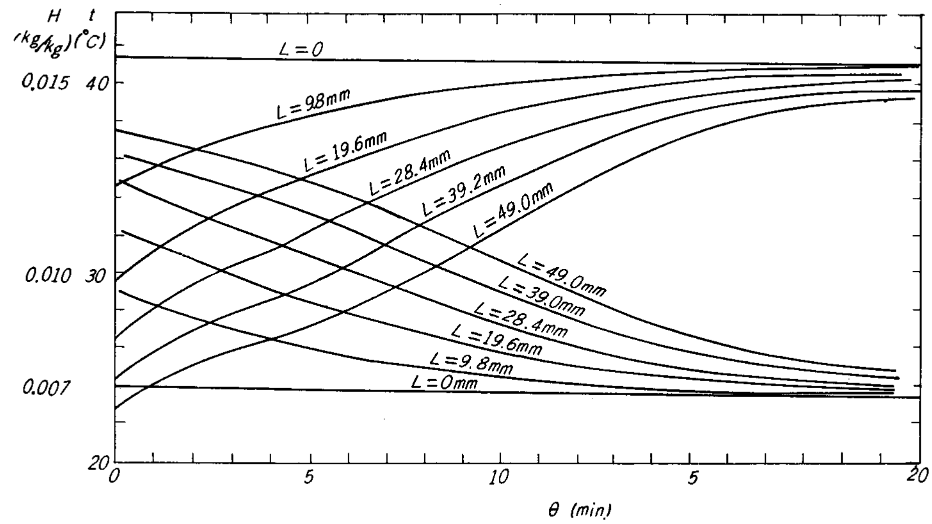

Fig. 5. Variation of air temperature and humidity through packed bed of porcelain sphere (calculation).

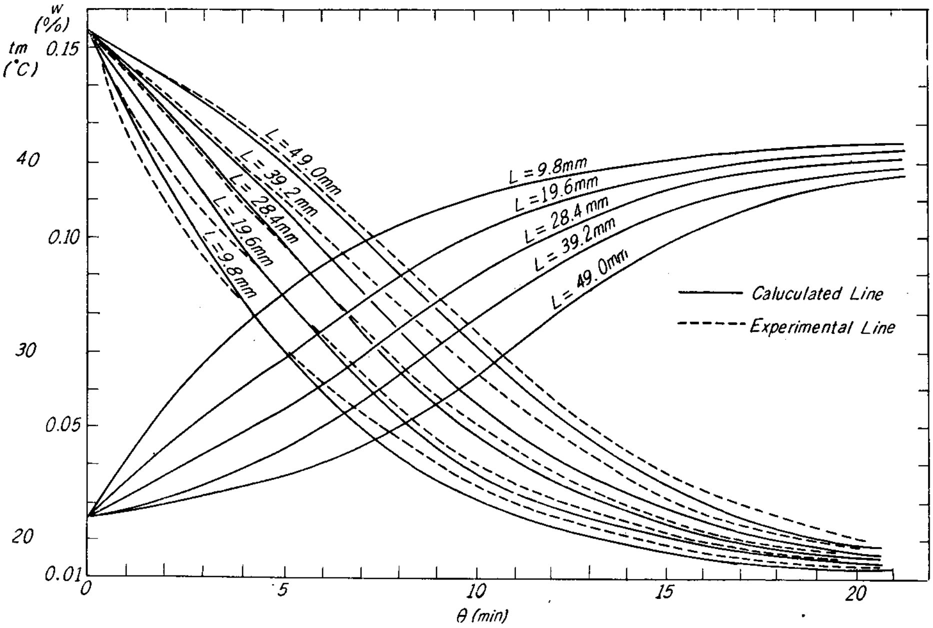

Fig. 6. Distribution curve of moisture and temperature of material (culculation).

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Nomencleature

C_H	: Specific heat of air	(kcal/kg °C)
C_m	: Specific heat of wet material	(kcal/kg °C)
C_0	: Specific heat of dry material	(kcal/kg °C)
G	: Mass velocity of air	(kg/hr·m^2)
H	: Absolute humidity of air	(kg-water/kg dry air)
ha	: Heat transfer capacity coefficient	(kcal/hr·m^3 °C)
L	: Heights of bed	(m)
Nt	: Number of transfer units	
γ_m	: Latent heat at material temperature	(kcal/kg)
γ_w	: Latent heat at wet bulb temperature of inlet air	(kcal/kg)
t	: Temperature of air	(°C)
t_m	: Temperature of material	(°C)
w	: Water content of material	(kg/kg)
w_c	: Critical water content of material	(kg/kg)
Z	: A dimensionless transformed variable related to height of bed	
ρ_m	: Bulk density of the material in packed bed	(kg-dry solid/m^3)
t	: Time	(hr)
ϕ	: Drying rate	(kg-water/kg dry mat·hr)
ϕ_c	: Constant drying rate	(kg-water/kg dry mat·hr)
τ	; A dimensionless transformed variable related to time	