Response Function of a Three-inch Diameter by Three-inch Long NaI(Tl) Scintillator to Gamma-Rays

By

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The response of a three-inch diameter by three-inch long sodium-iodide scintillator to axially incident gamma-rays has been studied and expressed as a 20 by 20 matrix for the energy ranging from 0 to 1.440 Mev. The matrix was inverted on an automatic computer for the purpose of obtaining response-corrected spectra of scattered gamma-rays.

§1. Introduction

In the course of the study of the scattering and absorption of ^{60}Co, ^{137}Cs and ^{198}Au gamma-rays by a scintillation spectrometer, it is indispensable to reduce pulses of Compton electrons caused by the higher energy gamma-rays in the pulse-height distribution. There are three methods for accomplishing this. The first method is the mathematical conversion of pulse-height distribution obtained by a scintillation spectrometer to a photon energy spectrum with the help of a response function matrix.[1-9] The second one is to use an extremely large scintillator[10], and the third is to reduce the Compton electron distribution by an anti-coincidence method.[11]

In our gamma-ray scattering measurements, the scintillation head had to be moved on a circle, and consequently the anti-coincidence method was found to be unsuitable because of its large size of assembly. The energy resolution of the scintillation spectrometer should be high, and the resolution of 3-inch photomultiplier tubes was found to be almost as good as that of 2-inch tubes and better than 5-inch tubes[12], so a 3-inch diameter by 3-inch long NaI (Tl) scintillator with a 6363 photomultiplier tube was chosen in our laboratory. Since the conversion with a finite matrix was particularly suitable for measuring energy spectra of continuous gamma-rays and bremsstrahlung, this method was adopted for this purpose.

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A few response function matrices and their inverse matrices have been calculated in recent years. They are shown in Table I. There is, however, no response function matrix for a 3-inch diameter by 3-inch long NaI (Tl) crystal. The response for this crystal and gamma-ray energy up to 1.440 MeV has been calculated, and in this paper, the authors present a table of the response function matrix composed of 20 rows by 20 columns and a matrix inverted by an automatic computer.

Table I. Response function matrices.

	Maximum energy	Matrix	Scintillator
Starfelt and Koch[3)]	10 MeV	56×56	5″ ϕ×4″
Hubbell[4)]	8 MeV	28×28	5″ ϕ×4″
Kockum and Starfelt[7)]	34 MeV	15×15	5″ ϕ×4″
Rawson and Cormack[9)]	0.4 MeV	20×20	1¾″ ϕ×2″
Present work	1.442 MeV	20×20	3″ ϕ×3″

§2. The Response Function Matrix

If the spectrum of the photons incident on the spectrometer is $N(E)dE$, where E is the photon energy, the pulse-height distribution $P(E')dE'$ obtained from the scintillator and photomultiplier tube system can be expressed as[1)]

$$P(E')dE' = dE' \int_0^{E\max} K(E, E')N(E)[1-e^{-\mu L}]dE, \qquad (1)$$

where E' denotes pulse-height and $K(E, E')$ is the response function of the spectrometer and $E_{\max}$ is the maximum energy of the gamma-ray spectrum. This equation is easily transformed to a matrix equation.[3)]

Integration over the interval $\Delta E'$ of the variable E' and replacement of the integral over E in Eq. (1) by a sum of integrals over intervals E_i, gives

$$\int_{E_j'-\frac{1}{2}\Delta E_j'}^{E_j'+\frac{1}{2}\Delta E_j'} P(E')dE'$$

$$= \int_{E_j'-\frac{1}{2}\Delta E_j'}^{E_j'+\frac{1}{2}\Delta E_j'} dE' \sum_{i=0}^{n} \int_{E_i-\frac{1}{2}\Delta E_i}^{E_i+\frac{1}{2}\Delta E_i} K(E, E')N(E)[1-e^{-\mu(E)L}]dE. \qquad (2)$$

It it is assumed that the true spectral distribution $N(E)$ varies slowly, Eq. (1) is transformed into a matrix form, and it can be written as[1,3−5)]

$$\langle P(E')\rangle = \langle N(E)\rangle M, \qquad (3)$$

where $\langle P(E')\rangle$ and $\langle N(E)\rangle$ are pulse-height distribution and photon spectrum respectively, and M is the response function of the detector represented as a matrix form whose elements are given by

$$M_{ij} = \int_{\Delta E_j'} dE' \int_{\Delta E_i} K(E, E')[1-e^{-\mu L}]dE/\Delta E_j' . \tag{4}$$

If the matrix has been obtained, the response corrected spectrum $\langle N(E) \rangle$ can easily be obtained by an automatic computer as:

$$\langle N(E) \rangle = \langle P(E') \rangle M^{-1} . \tag{5}$$

More than ten meshes are necessary to cover up to 1.332 MeV and each mesh must be uniform in energy because the sources used in the study of backscattering were ^{60}Co, ^{137}Cs and ^{198}Au. A 20 by 20 matrix respesentation of the detector response is most adequate for this purpose. The bin width is 72 keV, and 1.332 MeV is located at the center of the 19th bin.

§3. Input Data

The scintillation head consisted of 3-inch diameter by 3-inch long NaI (Tl) scintillator together with a photomultiplier tube of type 6363 and a cathode follower mounted in a single unit. The high voltage power supply was regulated by twenty 85A2's connected with a 6146 vacuum tube in parallel. The output pulses of the scintillation head were amplified by an Argonne A-61 type amplifier

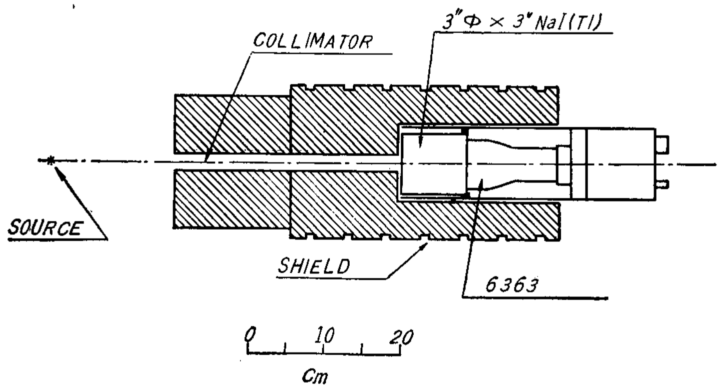

Fig. 1. Experimental arrangement.

and fed to a core memory type, 400 channel pulse-height analyzer. The assembly of scintillation head, collimator and lead shield were arranged as shown in Fig. 1.

The present investigation was concerned with pulse-height distribution as produced by gamma-rays from a point source of ^{60}Co, ^{137}Cs, ^{198}Au, ^{203}Hg and ^{51}Cr, each source having been deposited on a thin mica plate, and ^{65}Zn in zinc metal.

In addition, indium metal irradiated by thermal neutrons and cooled for more than three months as well as ^{42}K produced by the Kyoto University cyclotron were also used. In Fig. 2 are shown some of the observed pulse-height distributions from which the background has been subtracted.

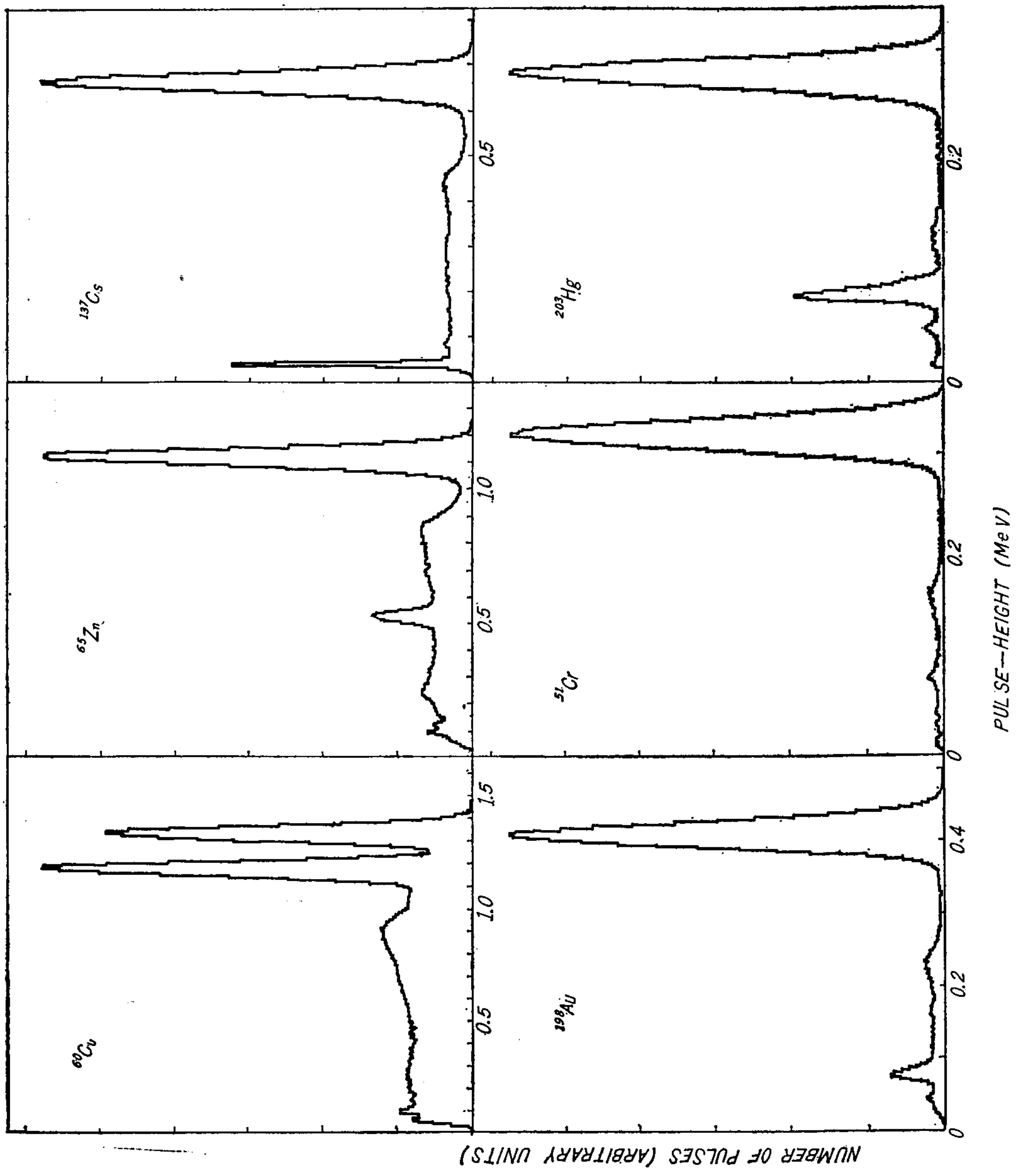

Fig. 2. Experimental pulse-height distributions produced by gamma-rays of different energies used as input data.

The resolution of the spectrometer was 6.1, 6.5 and 7.9 per cent for 1.33, 1.11 and 0.66 MeV gamma-rays respectively. The photopeak of each pulse-height distribution was in good agreement with a Gaussian distribution. It may be concluded that the scattered radiation from the surface of the collimator was not large as compared with the gamma-rays arrived at the scintillator.

Comparing these pulse-height distributions with the Monte Carlo values for the energy distribution of the energy lost by photons in a 3-inch diameter by 3-inch long NaI (Tl) crystal obtained by Davisson and Beach,[13] the lower part of the Compton electron distributions of the experimental data were somewhat larger than the calculated ones. The discrepancy might be originated in the photons backscattered from the enveloping aluminum can, reflector, glass and lead shield. Since they are fundamentally associated with the assembly, pulse-height distributions obtained experimentally were used as input data.

§4. Approximate Calculation of the Response Function Materix Elements

The elements of the response function matrix were calculated by the approximation method, since the amount of input data was not sufficient to give an exact calculation and the response of the scintillator varied slowly with the input gamma-ray energy.

The matrix element M_{ij} is obtained by using an approximation formula:

$$M_{ij}=\frac{F\left(E_i-\frac{\Delta E}{2}, E_j'\right)+4F(E_i, E_j')+F\left(E_i+\frac{\Delta E}{2}, E_j'\right)}{6}\frac{\Delta E}{\Delta E'}, \tag{6}$$

where $F(E, E_j')$ is defined as:

$$E(E, E_j)=\varepsilon(E)\int_{E'_j-\frac{\Delta E'}{2}}^{E_j'+\frac{\Delta E'}{2}} K(E, E_j')\, dE', \tag{7}$$

where $\varepsilon(E)$ is the detection efficiency of the scintillator for monochromatic gamma-rays. For the parallel beam, it may be written as:

$$\varepsilon(E)=[1-e^{-\mu(E)L}], \tag{8}$$

where $\mu(E)$ is the total attenuation coefficient for the incident gamma-rays,[14] and L is the crystal thickness.

The value of the integral in Eq. (7) was obtained by graphical interpolation from the input data. The center of the photopeak was determined with the help of the Gaussian distribution. In the next place, the pulse-height distribution was divided into 72 keV intervals, and two groups of histograms were composed as shown in Fig. 3. In one of the groups, the center of the photopeak was located at the center of an interval, (Graup A), and in the other, at the boundary, (Group B).

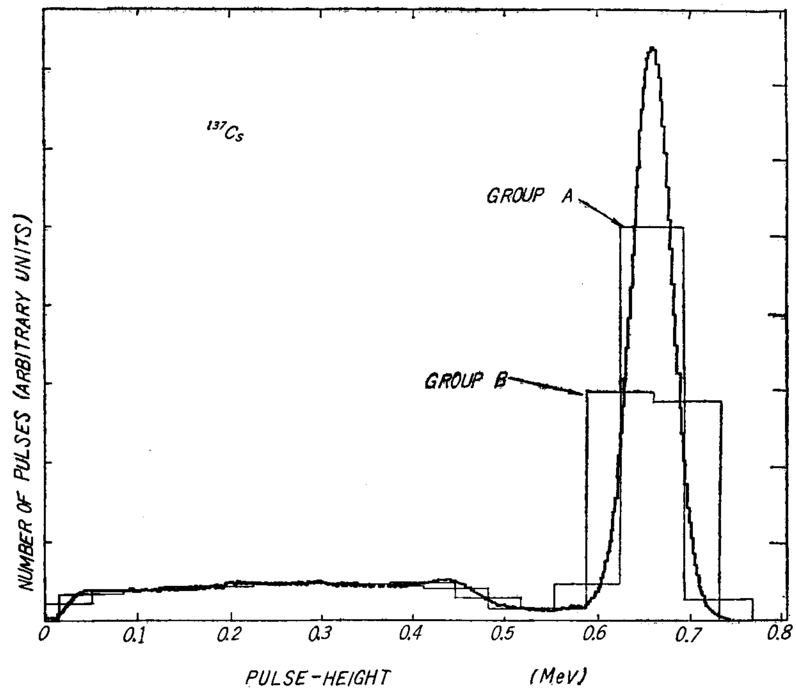

Fig. 3. Division of the pulse-height distribution.

We may write Eq. (2) for monochromatic gamma-rays in the form:

$$\int_{E_l'-\frac{\Delta E'}{2}}^{E_l'+\frac{\Delta E'}{2}} P(E')dE'$$

$$= N(E_0)\,\varepsilon(E_0)\int_{E_l'-\frac{\Delta E'}{2}}^{E_l'+\frac{\Delta E'}{2}} K(E, E')_{E=E_0}dE' , \qquad (9)$$

where E_0 is incident gamma-ray energy, and $P(E')\ \Delta E'$ is the total number of pulses in the interval from $E_l'-\Delta E'/2$ to $E_l'+\Delta E'/2$. When $E_l'=E_0'+l\Delta E'$, where E_0' is the center of photopeak for the incident monochromatic gamma-rays whose energy is E_0, $P(E')\Delta E'$ in Eq. (9) gives the value of histogram Group A, and when $E_l'=E_0'+(2l+1)\Delta E'/2$, it gives that of Group B.

Table II. Response function matrix.

E' \ E	36	108	180	252	324	396	468	540	612	684	756	828	900	972	1044	1116	1188	1260	1332	1404
36	917	83																		
108	90	828	82																	
180	14	96	810	80																
252	16	29	97	778	77															
324	18	30	30	92	736	74														
396	24	31	39	25	90	676	71													
468	28	36	38	43	21	87	613	70												
540	28	39	43	44	40	18	86	541	68											
612	28	37	43	44	43	40	15	85	477	66										
684	29	35	39	43	45	44	39	13	83	431	64									
756	28	34	36	40	43	45	44	37	12	80	389	62								
828	27	31	33	36	40	42	43	43	35	12	77	353	61							
900	26	30	31	33	35	38	41	42	43	33	12	73	323	60						
972	24	29	29	29	31	34	37	40	41	42	32	12	71	296	58					
1044	23	28	28	27	28	30	33	35	38	39	41	31	13	69	272	57	1			
1116	23	27	27	26	25	27	29	33	34	37	39	41	30	14	67	249	55	1		
1188	21	25	25	25	24	24	25	28	32	33	35	38	41	30	16	66	227	51	1	
1260	20	24	24	24	24	23	22	25	27	31	33	34	37	41	30	17	65	206	51	1
1332	20	23	23	23	23	22	22	21	24	27	30	32	33	37	41	30	17	65	186	49
1404	19	23	23	22	22	22	21	21	21	23	26	29	31	33	35	41	31	18	64	168

E represents the center of the photon interval (keV), *E'* the energy corresponding to the center of the pulse-height interval (keV). To yield the matrix elements the numbers in the table shoud be multiplied by 10^{-3}.

Table III. Inverse response function matrix.

E' \ E	36	108	180	252	324	396	468	540	612	684	756	824	900	972	1044	1116	1188	1260	1332	1404
36	1101	−112	12	−1																
108	−119	1234	−127	13	−1															
180	−4	−142	1265	−132	14	−2														
252	−15	−23	−105	1317	−140	16	−2													
324	−17	−35	−22	−159	1393	−155	18	−2												
396	−27	−34	−54	−13	−181	1521	−178	23	−3											
468	−33	−44	−45	−70	−4	−212	1686	−223	32	−5										
540	−35	−52	−62	−64	−77	7	−265	1925	−282	44	−7	1								
612	−38	−51	−64	−71	−74	−97	23	−343	2205	−349	59	−10	2							
684	−40	−49	−57	−71	−85	−86	−118	45	−429	2462	−422	75	−15	3						
756	−40	−47	−51	−67	−80	−100	−104	−139	68	−514	2753	−501	97	−20	4	−1				
824	−39	−41	−46	−53	−75	−92	−110	−127	−161	91	−610	3063	−607	126	−28	7	−2			
900	−36	−38	−40	−49	−58	−81	−107	−126	−154	−179	110	−699	3382	−716	158	−37	9	−2		
972	−32	−36	−35	−36	−48	−65	−86	−125	−142	−181	−200	130	−826	3734	−849	201	−50	13	−4	1
1044	−29	−36	−33	−32	−35	−48	−76	−92	−143	−157	−212	−223	153	−961	4127	−1004	253	−65	19	−5
1116	−28	−34	−31	−30	−26	−37	−48	−87	−104	−155	−179	−253	−246	175	−1123	4583	−1195	306	−89	24
1188	−26	−31	−29	−27	−27	−20	−38	−50	−101	−117	−170	−206	−301	−286	206	−1343	5119	−1370	397	−110
1260	−23	−27	−27	−28	−25	−25	−14	−40	−55	−104	−133	−188	−219	−354	−337	265	−1655	5773	−1745	482
1332	−21	−23	−24	−24	−24	−21	−20	−7	−39	−59	−104	−143	−193	−221	−398	−381	425	−2013	6589	−1904
1404	−27	−37	−33	−35	−36	−36	−32	−30	−16	−46	−81	−132	−188	−278	−298	−574	−668	328	−2378	6648

E denotes the center of the photon interval (keV), E' the energy corresponding to the center of the pulse-heiht interval (keV). To yeild the matrix elements the numbers in the table should be multiplied by 10^{-3}.

The values of $E(E_i, E_j')$ ane $F(E_i \pm \Delta E/2, E_j')$ were consequently obtained from graphical interpolation of the histogram Group A and Group B of the input data respectively.

The matrix M_{ij} shown in Table II is now a response function which converts $N(E)$ into $P(E')$, and the inverted matrix M_{ij}^{-1} was obtained as shown in Table III.

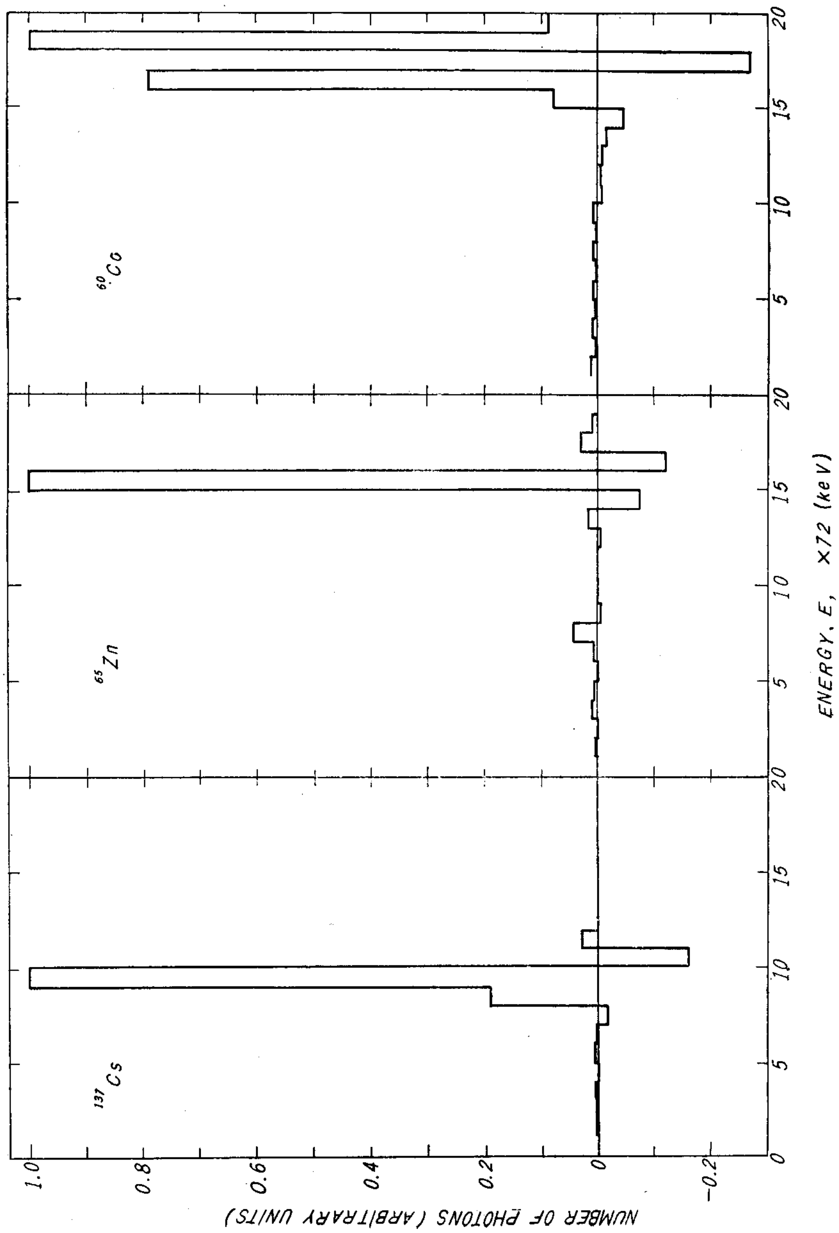

Fig. 4. Typical response corrected spectra for monochromatic gamma-rays of ^{137}Cs, ^{65}Zn and ^{60}Co.

§5. Application of M_{ij}^{-1} to Pulse-height Distribution

For the purpose of correcting a measured pulse-height distribution by the inverse matrix M_{ij}^{-1}, the distribution has to be divided into 72 keV intervals as in the case of the pulse-height interval of the response function matrix, and is designated as $\langle P(E')\rangle$. The response corrected spectrum $\langle N(E)\rangle$ is obtained from the product of $\langle P(E')\rangle$ and M_{ij}^{-1} as:

$$N_i = \sum_{j=1}^{N} P_j M_{ij}^{-1}. \tag{10}$$

The pulse-height distribution caused by monochromatic gamma-rays is not suitable for correction by this matrix. Fig. 4 shows typical response corrected spectra for monochromatic gamma-rays of ^{137}Cs, ^{65}Zn ann ^{60}Co. A response corrected spectrum of backscattered radiation from iron is shown in Fig. 5.

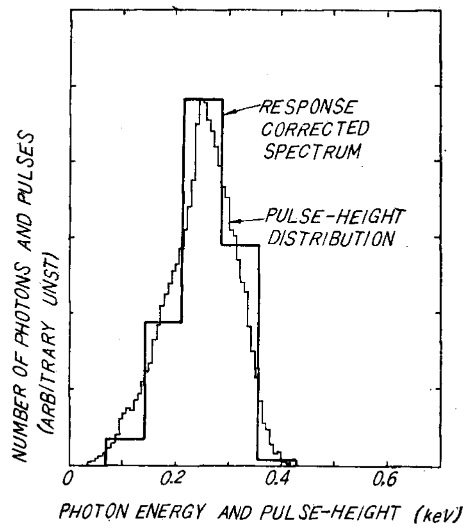

Fig. 5. A response corrected spectrum of backscattered radiation from iron.

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