Analysis of the Drying Process of Granular and Powdered Materials During the Falling Rate Period

By

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The drying rate curves of granular and powdered materials for various drying methods are shown in Figs. 1~4. It is clear from these figures that the falling drying rate of granular material is proportional to the water content of the material. It is also shown that the temperature gradient can be neglected for the granular material which has a diameter below 2~3 mm. According to these facts, we have discussed the relation between the water content and the material temperature during the falling rate period under stationary drying conditions in Chap. I.

In Chap. II, we have discussed the relations among water content, material temperature and air temperature under non-stationary conditions which take place in a continuous (parallel- or counter-current flow) dryer. The drying rate and the drying time in the continuous adiabatic dryer can be easily calculated by using these relations.

The application of this calculation to the dryer design is discussed in Chap. III.

I. Analysis of the drying process under stationary drying conditions:

I-1. The drying rate curves of the granular and powdered materials during the falling rate period:

(a) The drying rate curve of a small sphere which is suspended in an air stream:

A sphere (8 mmϕ) made of $BaSO_4$ powder (100# pass) was suspended in a hot air stream and the weight loss of the sphere was measured by a quartz spiral balance and a cacetometer (the precision being 1 mg/0.05 mm).

The surface temperature of the sphere was measured by a copper-constantan thermocouple which was inserted into the surface of the sample. The air condition was kept constant. The drying characteristic curve of this experiment is shown in Fig. 1.

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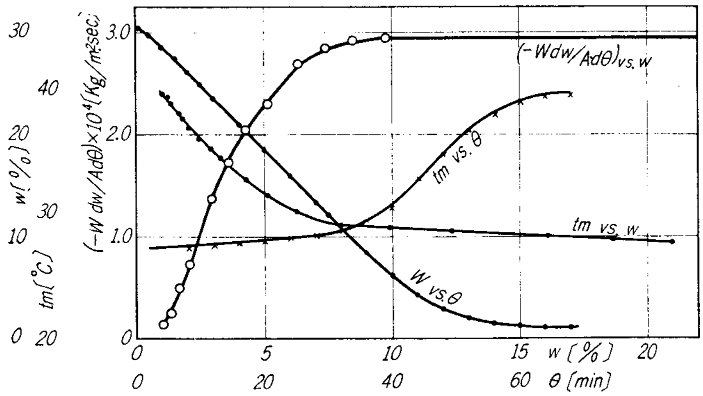

Fig. 1. Drying curve for small sphere. (dia. 0.807 cm, material $BaSO_4$ powder (100 # pass), $t=40°C$, $H=0.0148$, $\bar{u}=1.0$ m/sec).

(b) The drying rate curves of ammonium sulphate salts for trough agitator drying

In a trough agitator dryer[2], the heat is transferred to the material by conduction through the jacketed wall. The drying characteristic curve of this experiment is shown in Fig. 2[3].

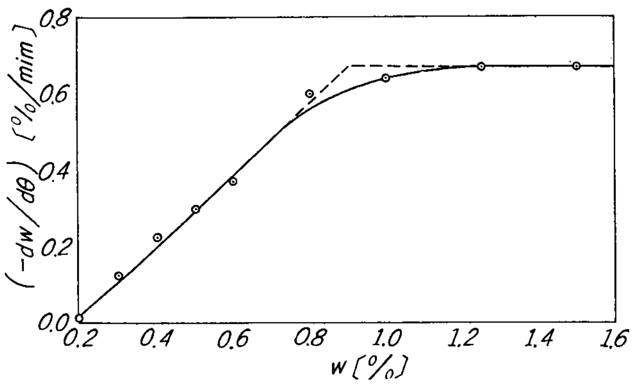

Fig. 2. Drying rate curve for $(NH_4)_2SO_4$ salt in a trough agitator dryer[2] (30 r.p.m, 3 kg-charge, jacket steam press. 1.5 kg/cm^2 gauge, wt. basis median dia. 0.3 mm, initial water content $w_1=0.06$).

(c) The drying rate curve of a droplet of an aqueous suspension of potato starch powder[8]:

A droplet of an aqueous suspension of potato starch was suspended on thermocouple elements in an air stream. The weight loss of the droplet was calculated from the decreasing of the droplet diameter during the surface evaporation period. The drying characteristic curve of this experiment is

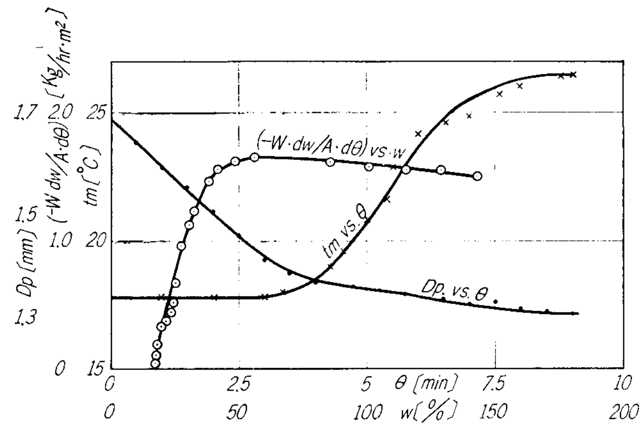

Fig. 3. Drying curve for the droplet containing potato starch[8] (t=26.6°C, H=0.0083, $\bar{u}$=0.99 m/sec, initial conc. 41%, initial dia. 1.685 mm).

shown in Fig. 3. It is observed that the temperature of the droplet remains constant during the surface evaporation period and the decreasing of the droplet diameter ceases at the critical moisture content. We can calculate the drying rate of the droplet from consideration of the heat balance during the falling rate period.

It is also reduced to the fact that transfer coefficients are represented by following equations:

$$\left.\begin{aligned} Nu &= 2(1+0.3\,Re^{1/2}Pr^{1/3}) \\ Sh &= 2(1+0.3\,Re^{1/2}Sc^{1/3}) \end{aligned}\right\}. \qquad (1)$$

(d) The drying rate curve of pyrite particles for pneumatic conveying drying:

In pneumatic conveying drying[4], the particles are conveyed with the hot air stream co-currently and the heat is transferred by convection between the particle and air. The drying rate of pyrite powder[5] (the weight based medium diameter being 0.33 mm) is calculated by the data of experimental values of the moisture content of particles and the gas temperature at each point along the dryer. Furthermore, these values are corrected by the following relation:

$$\text{The corrected drying rate} = (\text{the experimental drying rate}) \times \frac{\text{the constant drying rate under standard condition}}{\text{the constant drying rate under experimental condition}}.$$

At the terminal velocity zone the heat transfer coefficients for the particles are constant. Fig. 4 shows an example of the drying rate curve during

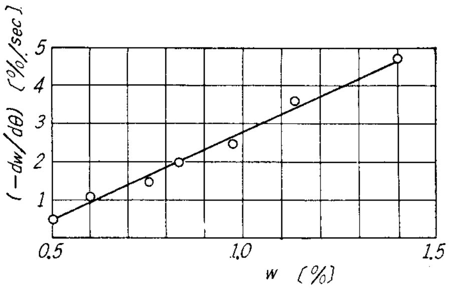

Fig. 4. Corrected decreasing drying rate curve for pyrite particles by pneumatic conveying drying[4,5], at the terminal velocity zone. The drying rate obtained from the experiment is corrected to meet the value at the standard stational drying conditions, where $t=135$, $H=0.0257$ at $w=0.0083$. (pyrite wt. basis median dia. 0.33 mm, $\rho_m=5000$ kg/m^3, initial water content $w_1=0.072$).

the falling rate period in the terminal velocity zone which are corrected by the above relation.

It is observed from the drying rate curves during the falling rate period in Figs. 1~4 that the falling drying rates of granular and powdered materials (of which diameters range from 0.3 mm to 8 mm) roughly decrease in proportion to the moisture content for any method of drying.

Thus the drying rate R_d during the falling rate period at a given free moisture content F is represented by the following equation:

$$R_d = R_c(F/F_c)\,. \tag{2}$$

I-2. The temperature gradient in granular and powdered material under drying operation:

The heat transferred to the material from the surrounding air is represented by equation (3):

$$q_n d\theta = (W_0/A_0)(-dw)r_m + d\left[\int t_m(c+c_w w)d(W_0/A_0)\right] \tag{3}$$

where q_n is the heat quantity transferred to the surface.

When the temperature gradient does not exist in a particle, eq. (3) becomes;

$$q_n d\theta = (W_0/A_0)(-dw)r_m + W_0(c+c_w w)dt_m/A_0 \tag{4}$$

If we ignore the sensible heat of the evaporated vapor for the sake of simplicity, the heat transfer equation can be written as follows:

$$q_n d\theta = h(t-t_{ms})d\theta = h(t-t_m)d\theta \tag{5}$$

From equations (4) and (5) we can obtain;

$$hA_0(t-t_m)d\theta = W_0(-dw)r_m + W_0(c+c_w w)dt_m \tag{6}$$

Fig. 5 shows the relation between the material temperature and the drying time during the falling drying rate period of a small sphere. This is from the same experiment as shown in Fig. 1.

In Fig. 5, the solid curve represents the experimental values and the broken curve represents the calculated values. The calculated values are obtained from the experimental values of the drying rate $(W_0(-dw)/A_0d\theta)$ by applying eq. (6).

From Fig. 5, a good agreement between the experimental value and the calculated value was observed. So, for the conditions of such an experiment, the temperature gradient in the material is negligibly small and eq. (6) can be used satisfactorily. Bloomfield[1] measured the temperature of a marl sphere of which the diameter was 2 cm in air convective drying (the air temperature being 60°C and wet bulb temperature 36°C). He observed that the temperature difference between the surface and the center of the sphere did not exceed 3°C.

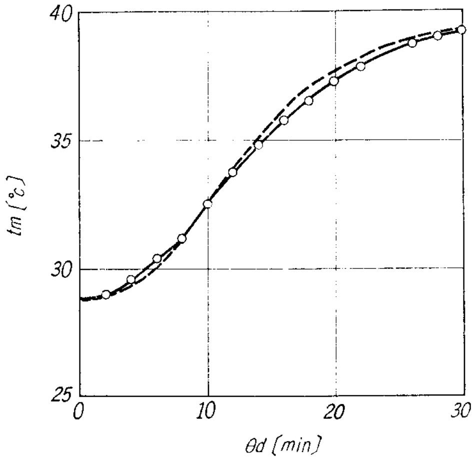

Fig. 5. Relation between material temperature and drying time at the decreasing drying rate period.
This is the same experiment as shown in Fig. 1.
——— experimental value
- - - - calculated value

The temperature difference between the surface and the interior of material must decrease sufficiently as the particle diameter decreases.

Fig. 6 shows the relation between the water content and the temperatures

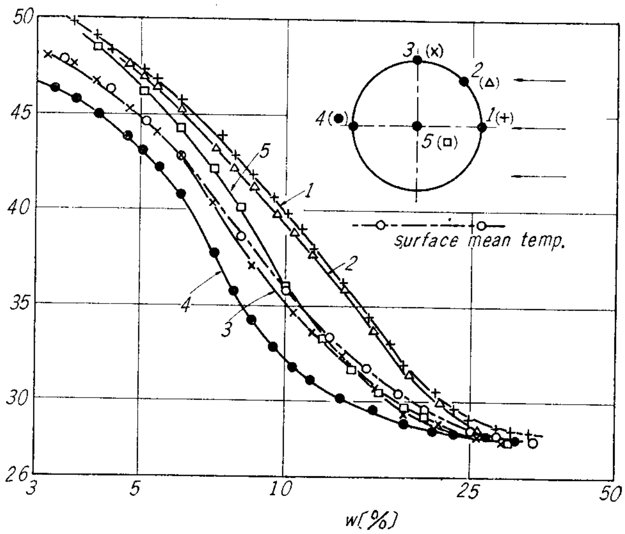

Fig. 6. Relation between the water content and the temperatures of the various points of the sphere (material clay, dia. 38.6 mm, $t=55°C$, $H=0.015$, $\bar{u}=1.0$ m/sec).

at various points of a large clay sphere with a diameter of 3.86 cm.

For such a large sphere, the surface temperatures at various points are different. The average temperature of these surface temperatures is also plotted in Fig. 6. This average temperature does not coincide with the temperature at the center and the maximum difference between these two temperatures becomes 4~5%.

The temperature difference between surface and interior is influenced also by the rising rate of surface temperature, the particle diameter and the heat conductivity. Under the usual drying conditions, the rising rate of surface temperature is several °C/sec. Calculating the temperature difference with this value, 0.5°C is obtained as maximum value in case of the particle diameter being 1 mm.

The rising rate of surface temserature is not so large in the case of the drying of larger particles. Thus the temperature difference between the center and the surface is also small in this case.

In closing this paragraph, it can be concluded that the temperature gradient in the granular materials can be negligible when the particle diamater is below 3 mm. Thereby eq. (4) can be used as a heat transfer equation.

I-3. The relation between the material temperature and the moisture content:

(i) When the sensible heat of the evaporated vapor is not considered:

In this case, the drying rate of the constant rate period (the surface evaporation period) R_c is represented by eq. (7):

$$R_c = W_0(-dw)/A_0 d\theta = h(t-t_w)/r_w \tag{7}$$

Eliminating $d\theta$ from eqs. (2), (6) and (7);

$$(dt_m/dw) = \left[r_m - \frac{(t-t_m)}{(t-t_w)}\frac{r_w F_c}{F}\right]\Big/(c+c_w w) \tag{8}$$

The relation of t_m and w is obtained by solving eq. (8) numerically, e.g. by Runge-Kutta's method. The relation of t_m and θ can be obtained by a similar procedure, but this calculation is more complicated.

When the critical moisture content w_c is small, i.e. $c \gg c_w w$, $c+c_w w \doteqdot c$, the calculation is as follows. This condition is permitted at the drying of granular material dispersed in the dryer, because the critical moisture content is usually below 0.05 and the specific heat of powdered solid is nearly 0.3 kcal/kg°C in such cases.

Furthermore, the following approximation is taken;

$$r_m(=595-0.55t_m) \doteqdot r_w(=595-0.55t_w)$$

With these approximations eq. (8) becomes:

$$(dt_m/dw) = (dt_m/dF) = \left[r_w - \frac{(t-t_m)r_wF_c}{(t-t_w)F}\right]\Big/c \tag{9}$$

Considering the initial condition, $F=F_c$, $t_m=t_w$ at $\theta=0$. The solution of eq. (9) is,

$$(t-t_m) = (t-t_w)\left[\frac{r_wF - c(t-t_w)(F/F_c)^{\frac{F_cr_w}{c(t-t_w)}}}{F_cr_w - c(t-t_w)}\right] \tag{10}$$

Eq. (10) represents the relations of F and t_m in the stationary drying of granules. When the critical moisture content is large and c_ww cannot be negligible, eq. (8) must be solved.

Under the approximation $r_m \fallingdotseq r_w$, we can rewrite eq. (8) in the form of eq. (11).

$$(dt_m/dw) = (dt_m/dF) = \left[r_w - \frac{(t-t_m)r_wF}{(t-t_w)F_c}\right]\Big/[c+c_w(w_e+F)] \tag{11}$$

Eq. (11) cannot be solved analytically, but we can obtain eq. (12) as the approximated solution of eq. (11) for the condition in which

$$F_cr_w/(c+c_ww_e)(t-t_w) \gg 1$$

$$(t-t_m) = (t-t_w)\left[\frac{r_wF}{r_wF_c-(c+c_ww)(t-t_w)} - \left\{\frac{(c+c_ww_c)(t-t_w)}{r_wF_c-(c+c_ww_c)(t-t_w)}\right\}\left\{\frac{F(c+c_ww_c)}{F_c(c+c_ww)}\right\}^{\frac{F_cr_w}{(c+c_ww_e)(t-t_w)}}\right] \tag{12}$$

Eq. (12) is written in a similar form to eq. (10) and shows good agreement to the exact numerical solution for this condition.

The condition of $F_cr_w/(c+c_ww_e)\times(t-t_w) \gg 1$ is given at a large value of F_c and small value of t.

Under the following drying conditions is which $t=80$, $t_w=42$, $c=0.30$, $w_c=0.29$, $w_e=0$;

$$F_cr_w/(c+c_ww_e)(t-t_w) = 15$$

The numerical solution of eq. (8) and the solution with eqs. (10) and (12) are calculated. These calculated results are shown in Fig. 7.

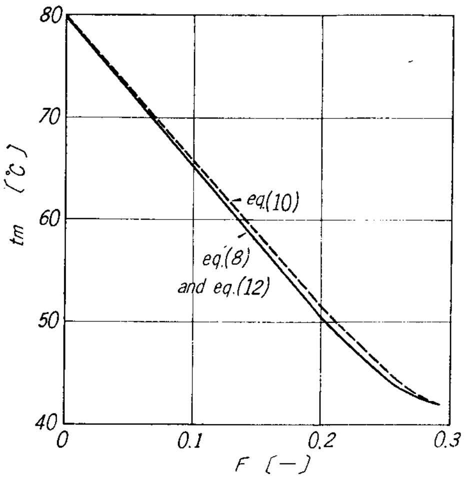

Fig. 7. Relation between water content and material temperature during the falling rate period ($t=80$°C, $t_w=42$°C, $w_c=0.29$, $w_e=0$, $c=0.3$).

For these conditions, eq. (12) shows good agreement with the exact numerical solution. When the drying conditions are $t=400$, $t_w=60$, $c=0.30$, $w_c=0.05$, $w_e=0$,

$$F_c r_w/(c+c_w w_e)(t-t_w) = 0.337.$$

The numerical solution of eq. (8), and the solutions with eqs. (10) and (12) are calculated and shown in Fig. 8.

For these conditions, the solution with eq. (10) shows good agreement with the numerical solution. Thus it is recognized that eq. (10) or (12) can be used without any serious error in order to calculate the relation between the moisture content and the material temperature during the falling drying rate period.

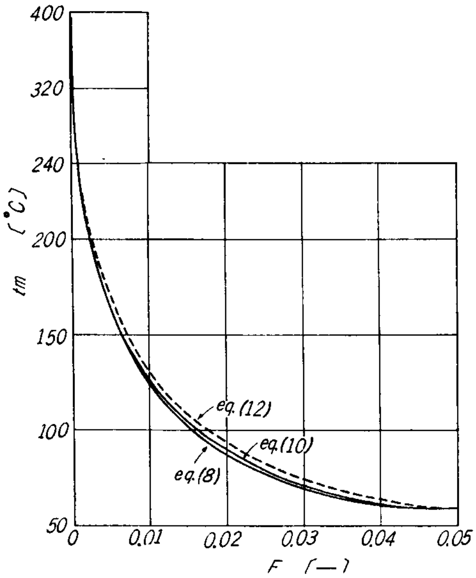

Fig. 8. Relation between water content and material temperature during the falling rate period ($t=400°C$, $t_w=60°C$, $w_c=0.05$, $w_e=0$, $c=0.3$).

(ii) When the sensible heat of the evaporated vapor is considered:

In this case the calculation is as follows. The sensible heat is necessary for heating of the evaporated vapor from the surface temperature to the air temperature in the boundary layer. This heat quantity cannot be ignored at the evaporation of liquids having a small latent heat of evaporation such as organic solvents or the evaporation of water at a high temperature.

This is known as Ackermann's effect. This effect on the drying was discussed by the author in the literature[9].

Heat transferred to the surface q_n is shown by eqs. (13) and (14);

$$q_n = h(t-t_m)a/(e^a-1) \tag{13}$$

$$a = Rc_p/h \tag{14}$$

Heat transferred from the bulk air to the boundary layer q_t is shown by eq. (15):

$$q_t = h(t-t_m)ae^a/(e^a-1) \tag{15}$$

Thereby q_t-q_n is equal to the increase of sensible heat of the evaporated vapor;

$$q_t-q_n = h(t-t_m)a = Rc_p(t-t_m) \tag{16}$$

During the surface evaporating period, q_n is consumed for evaporation only, thus we obtain;

$$R_c = q_n/r_w = h(t-t_w)a/(e^a-1)r_w = (h/c_p)\ln[\{c_p(t-t_w)+r_w\}/r_w] = (h/c_p)\ln D \quad (17)$$

where $D = \{c_p(t-t_w)+r_w\}/r_w$.

During the falling rate period, the heat quantity q_n is used for both the evaporation and heating of materials. Eq. (18) can be obtained from eqs. (4) and (13);

$$q_n d\theta = h(t-t_m)a/(e^a-1)d\theta = (W_0/A_0)r_m(-dw)+(W_0/A_0)(c+c_w w)dt_m \quad (18)$$

and eq. (19) from eqs. (2) and (17);

$$R_d = R_c(F/F_c) = (h/c_p)\ln D(F/F_c) \quad (19)$$

Eliminating $d\theta$ from eqs. (18) and (19),

$$(dt_m/dw) = \left\{r_m - \frac{c_p(t-t_m)}{D^{F/F_c}-1}\right\}\Big/(c+c_w w) \quad (20)$$

On the drying of powders and granules, we can take the following approximations as described before; $(c+c_w w) \fallingdotseq c$, $r_m \fallingdotseq r_w$,

$$(dt_m/dw) = (dt_w/dF) = \left\{r_w - \frac{c_p(t-t_m)}{D^{F/F_c}-1}\right\}\Big/c \quad (21)$$

Eq. (21) is solved in the following form,

$$(t-t_m) = \frac{r_w F_c}{c\ln D}\sum_{k=1}^{\infty}\frac{1}{n-k}\left(\frac{D^{F/F_c}-1}{D^{F/F_c}}\right)^k + \left(\frac{D}{D-1}\right)^n\left(\frac{D^{F/F_c}-1}{D^{F/F_c}}\right)^n \times\left\{(t-t_w) - \frac{r_w F_c}{c\ln D}\sum_{k=1}^{\infty}\frac{1}{n-k}\left(\frac{D-1}{D}\right)^k\right\} \quad (22)$$

where $n = c_p F_c/c\ln D$.

Under usual drying conditions, only the first three terms of the series in eq. (25) may be taken for this calculation.

When A, B and a are taken as follows, these figures and D become constant;

$$A = (r_w F_c/c\ln D),$$
$$B = (D/D-1)^n\left\{(t-t_w) - \frac{r_w F_c}{c\ln D}\sum_{k=1}^{3}\frac{1}{n-k}\left(\frac{D-1}{D}\right)^k\right\}$$
$$a = (D^{F/F_c}-1)/D^{F/F_c}.$$

Eq. (22) becomes eq. (23),

$$(t-t_m) = A\sum_{k=1}^{3}\frac{a^k}{n-k} + Ba^n \quad (23)$$

Table I shows the calculated results of the numerical solution of eqs. (20) and (21) and the results by eq. (23) show better agreement with the numerical solution than by eq. (10).

Table I $t=400°C$, $t_w=60°C$, $F_c=0.05$, $c=0.3$, $w_e=0$

F	t_m			
	eq. (20)	eq. (21)	eq. (23)	eq. (10)
0.05	60	60	60	60
0.04	62.2	62.4	62.5	62.1
0.03	70.2	71.2	71.2	69.9
0.02	88.3	90.8	90.6	87.2
0.01	130.0	133.3	132.9	124.6
0.005	173.0	176.0	175.9	164.1

II. Analysis of the drying process under non-stationary conditions:

When we consider the dryer in which the material and the hot air contact continuously in counter- or parallel-csrrent flow, the drying conditions in the dryer must change with different locations. This state is called non-stationary conditions in this paper. The heat transferred from bulk air in a differential section of the dryer is given as follows;

$$\pm GC_H dt = q_t A d\theta \qquad \begin{matrix} +\text{counter current flow} \\ -\text{parallel current flow} \end{matrix} \tag{24}$$

where q_t is defined by eq. (15).

The drying rate at any differential section is given by the following equation:

$$R = W(-dw)/Ad\theta = (h/c_p)\ln D(F/F_c) \tag{25}$$

The heat transferred to the material in a differential section is given as follows:

$$q_n d\theta = h(t-t_m)a/(e^a-1)d\theta = (W/A)r_m(-dw)+(W/A)(c+c_w w)dt_m \tag{26}$$

Eliminating $d\theta$ from eqs. (24) and (25);

$$\pm\frac{dt}{dw} = (Wc_p/GC_H)\frac{D^{F/F_c}}{D^{F/F_c}-1}(t-t_m) \qquad \begin{matrix} +\text{ parallel c.f.} \\ -\text{ counter c.f.} \end{matrix} \tag{27}$$

and eliminating $d\theta$ from eqs. (25) and (26);

$$\frac{dt_m}{dw} = \left\{r_m - \frac{c_p(t-t_m)}{D^{F/F_c}-1}\right\}\Big/(c+c_w w) \tag{28}$$

These two differential equations, eqs. (27) and (28), are solved by a numerical method, e.g. Runge-Kutta's method. The solution of these equations gives relations among t, t_m and w during the falling rate period under non-stationary conditions. Figs. 9 and 10 show the calculated results of the relations among t, t_m and w during the falling drying rate period in parallel- and counter-current flow operations.

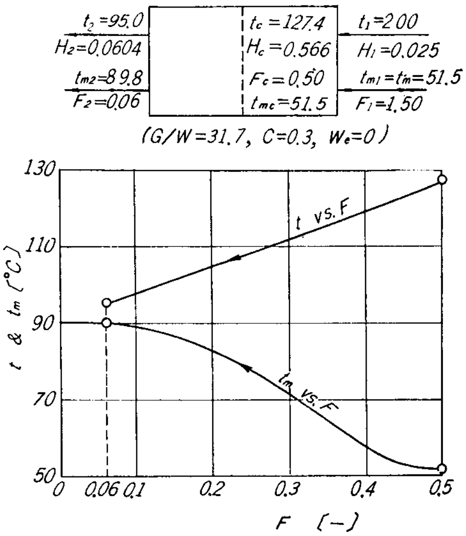

Fig. 9. Variation of t and t_m with F in parallel-current flow dryer during the falling rate period.

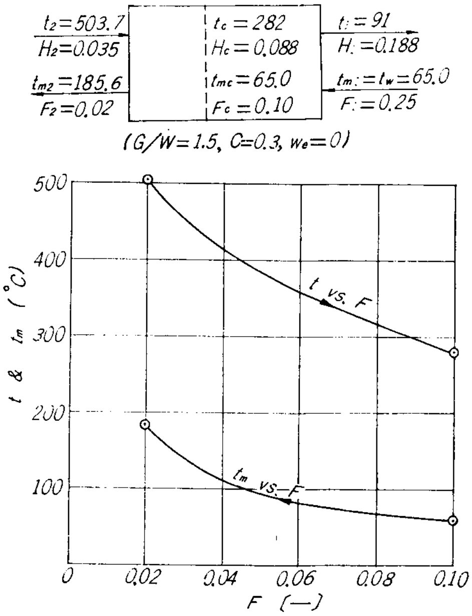

Fig. 10. Variation of t and t_m with F in counter-current flow dryer during the falling rate period.

The humidity of air corresponding to any w between w_c and w_2 is determined from the mass balance immediately.

$$W(w_c - w) = \pm G(H_c - H) \quad \begin{matrix} +\text{c.c.f.} \\ -\text{p.c.f.} \end{matrix} \tag{29}$$

The changes of air conditions in the dryer during this period can be shown on the humidity chart based upon these relations as shown in Fig. 11.

The drying rate at any water content during the falling rate period is expressed by eq. (25) or (30) which is a rewritten form of eq. (25).

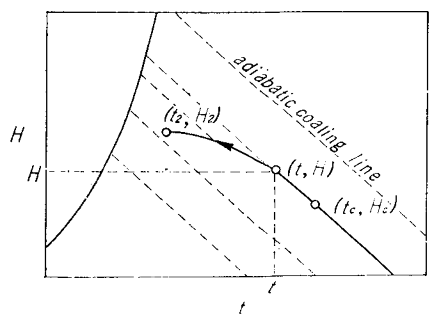

Fig. 11. The change of air conditions in the dryer during the falling rate period. (parallel-current flow)

$$(-dw/d\theta) = (hA/Wc_p) \ln D(F/F_c) . \tag{30}$$

The air temperature and humidity corresponding to any water content are known, hence eq. (30) can be calculated. Calculating the values of $(-dw/d\theta)$ at several values of w between w_c and w_2, and then plotting these w vs. calculated values of $1/(-dw/d\theta)$, the drying time is obtained by the following graphical integration:

$$\int_{F_c}^{F_2} -(d\theta/dw)dw = \int_0^{\theta_d} d\theta = \theta_d \tag{31}$$

When the heat capacity coefficient Ua is used instead of h, the above calculation, that is, eq. (25) becomes,

$$\frac{W(-dw)}{UadV} = \ln D(F/F_c)/c_p \tag{32}$$

and the graphical integration is carried out as follows:

$$\int_{F_c}^{F_2} -(dV/dw)dw = \int_0^{V_d} dV = V_d \tag{33}$$

The enthalpy balance in the dryer between points A and B is;

$$\pm G(i_A - i_B) = W\{(c + c_w w_A)t_{mA} - (c + c_w w_B)t_{mB}\} \quad \begin{matrix} +\text{c.c.f.} \\ -\text{p.c.f.} \end{matrix} \tag{34}$$

Eq. (34) is rewritten in the differential form as follows:

$$\pm GC_H dt = W(-dw)\{r_m + c_p(t - t_m)\} + W(c + c_w w)dt_m \quad \begin{matrix} +\text{c.c.f.} \\ -\text{p.c.f.} \end{matrix} \tag{35}$$

where we neglect the secondary differential term.

Combining eqs. (26) and (35), eq. (36) can be obtained as follows:

$$\pm GC_H dt = \{q_n + c_p(t - t_m) W(-dw)/Ad\theta\} Ad\theta = q_t Ad\theta \quad \begin{matrix} +\text{c.c.f.} \\ -\text{c.p.f.} \end{matrix} \tag{36}$$

Eq. (36) which is reduced from the enthalpy balance is identical with eq. (24) which is reduced by considering the heat transfer to the boundary layer.

III. Application to designing adiabatic continuous dryers using hot gases:

There exist the preheating period, the surface evaporation period and the falling drying rate period in the continuous dryer, and the drying condition changes with location in the dryer. Figs. 12 and 13 show the conditions of hot air and material in the dryer for parallel-current and counter-current flow drying operation, respectively.

Under these conditions, four conditions surrounded by circles in Figs. 12 and 13 are usually unknown. There are two related equations among these four unknown conditions.

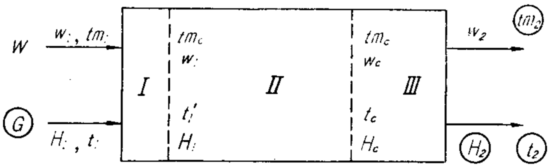

Fig. 12. The conditions of hot air and materials at parallel-current flow dryer.

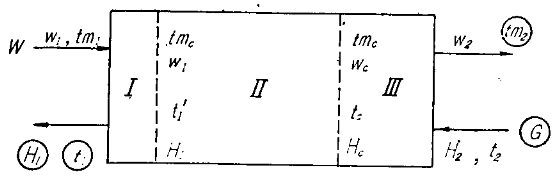

Fig. 13. The conditions of hot air and materials at counter-current flow dryer.

They are:

$$W(w_1-w_2) = \pm G(H_1-H_2) \qquad \begin{matrix}+\text{c.c.f.}\\ -\text{p.c.f.}\end{matrix} \tag{37}$$

$$\pm G(i_1-i_2) = W\{(c+c_w w_1)t_{m_1}-(c+c_w w_2)t_{m_2}\} \qquad \begin{matrix}+\text{c.c.f.}\\ -\text{p.c.f.}\end{matrix} \tag{38}$$

Thereby the other two unknown conditions should be decided by some other proper method.

III-1. Parallel-current flow dryer:

First, the temperature of the exhaust air is to be estimated. The operating cost decreases with decreasing of this temperature, but the cost of construction increases with the decrearing of same. The optimum value is determined from the consideration of an economic balance but actually from empirical relations.

Secondly, for the determination of the dried material temperature t_{m2}, the followihg consideration may be used.

It can be shown that the value of t_{m2} in the numerical solution eqs. (27) and (28) agrees satisfactorily with the value of t_{m2}' calculated by eqs. (10), (12) or (22) using the values of t_2, H_2 and w_2 at the outlet of dryer.

In the case of a parallel-current flow operation, the proper value of t_{m2} may be assumed, then G and H_2 are calculated from eqs. (37) and (38). The value of t_{m2}' may be obtained solving eqs. (10), (12) or (22) with the value of t_2, w_2 and H_2 above calculated. This calculation procedure should be repeated until t_{m2} comes to be in agreement with t_{m2}' by trial and error method.

After determination of t_{m2}, the values of G and H_2 are also determined.

The calculations of the capacity of dryer is then performed as follows:

(i) During the preheating period, the heat quantity is used only for heating of material without vaporization, thereby;

$$q_1 = GC_{H1}(t_1-t_1') = hA_p' \frac{(t_1-t_{m1})-(t_1'-t_{mc})}{\ln\frac{(t_1-t_{m1})}{(t_1'-t_{mc})}} = hA_p'(\Delta t)_{lm} = W(c+c_w w_1)(t_{mc}-t_{m1}) \tag{39}$$

where t_{mc} is the wet bulb temperature at air conditions of t_1' and H_1. The required drying area A_p' is calculated by eq. (39). The required drying time during this period is calculated as follows:

$$\theta_p = A_p'/A$$

When the heat capacity coefficient Ua is given instead of h, eq. (40) must be used instead of eq. (39).

$$q_1 = Ua(\Delta t)_{lm} V_p \tag{40}$$

(ii) During the surface evaporation period, by combining eqs. (24), (15) and (17), the required drying time is calculated as follows:

$$\pm(G/Ahr_w)\int_{t_1'}^{t_c}\frac{c_p C_H}{D\ln D}dt = \int_0^{\theta_c} d\theta = \theta_c \qquad \begin{matrix}+\text{c.c.f.}\\-\text{p.c.f.}\end{matrix} \tag{41}$$

where t_c is calculated by considering the following enthalpy and mass balance:

$$C_{H1}t_1' - C_{Hc}t_c = (H_c-H_1)(r_0-c_w t_w) \tag{42}$$

$$W(w_1-w_c) = \pm G(H_1-H_c) \qquad \begin{matrix}+\text{c.c.f.}\\-\text{p.c.f.}\end{matrix} \tag{43}$$

If the heat capacity coefficient is given instead of h, eq. (44) must be employed.

$$\pm(G/Uar_w)\int_{t_1'}^{t_c}\frac{c_p C_H dt}{D\ln D} = \int_0^{V_c} dV = V_c \qquad \begin{matrix}+\text{c.c.f.}\\-\text{p.c.f.}\end{matrix} \tag{44}$$

(iii) During the falling drying rate period, calculation becomes as follows: The air conditions at critical moisture content have been given in the above calculation. We can thus obtain the relations among t, t_m and w by applying the same calculation procedure as described in Chap. II.

Thereby determined values of t_2 and t_{m2} at w_2 are precise. The required drying time or dryer volume are calculated by graphical integration as described in Chap. II.

Consequently, the total required drying time is $\theta = \theta_p + \theta_c + \theta_d$ and total required dryer volume is $V = V_p + V_c + V_d$.

(iv) Approximated calculation method:

At the surface evaporation period eq. (45) or (46) may be used as the approximated calculation of θ_c or V_c.

$$\theta_c = \pm \frac{GC_{Hav}}{Ah} \ln \left[\frac{\ln [\{c_{pc}(t_c - t_w) + r_w\}/r_w]}{\ln [\{c_{p1}(t_1' - t_w) + r_w\}/r_w]} \right] \quad \begin{matrix} +\text{c.c.f.} \\ -\text{p.c.f.} \end{matrix} \tag{45}$$

$$V_c = \pm \frac{GC_{Hav}}{Ua} \ln \left[\frac{\ln [\{c_{pc}(t_c - t_w) + r_w\}/r_w]}{\ln [\{c_{p1}(t_1' - t_w) + r_w\}/r_w]} \right] \tag{46}$$

where C_{Hav} is the mean value between C_{H1} and C_{Hc}, c_{p1} is the value for the mean temperature of t_1' and t_w and c_{pc} is the value for the mean temperature of t_c and t_w.

During the falling drying rate period, we can suggest an approximated calculation method for θ_d and V_d.

It has been known that the values of t_2, t_{m2} determined by trial and error method initially have little difference with the exact values by numerical calculation of eqs. (27) and (28).

Therefore, we can use the initially calculated values of t_2 and t_{m2}, henceforth G and H_2 in the approximated calculation. It is then assumed that the air conditions change in the dryer along the straight line from point (t_c, H_c) to (t_2, H_2) on the humidity chart, as shown in Fig. 14.

This assumption has no serious error. At any water content during $w_c \sim w_2$, the humidity H corresponding to this w can be calculated by mass balance as eq. (29).

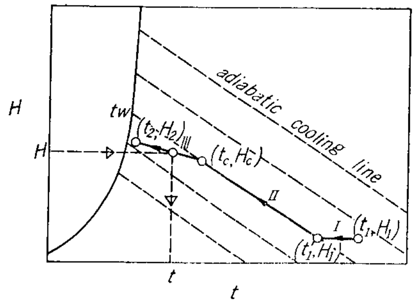

Fig. 14. The approximated change of the air conditions in parallel-current flow dryer.

The air temperature t having this humidity can be decided as shown in Fig. 14. The calculation procedure afterwards is the same as described before. By this method, the numerical calculation of eqs. (27) and (28) can be relieved and the calculation procedure becomes very simple.

III-2. Counter-current flow dryer:

In case of a counter-current flow dryer, t_1 is estimated properly at first as described in the case of the parallel-current flow operation. Then t_{m2} is calculated directly by eqs. (22), (10) or (12). G and H_1 are determined from eqs. (37) and (38) using the values of t_2 and t_{m2}. ($H_1 < H_{w2}$)

The values of t_c and t_{mc} at the critical moisture content w_c can be obtained by a numerical calculation of eqs. (27) and (28) starting from a point (t_2, H_2, w_2, t_{m2}).

Such a calculation is repeated with different t_{m2} by trial and error method until t_{mc} agrees with the wet bulb temperature t_w for the air conditions of (t_c, H_c).

Thus the relations among t, t_m and w during the falling rate period can be determined. The required drying time and dryer volume can be calculated by the same procedure as described in Chap. II.

At the surface evaporating zone, t_1' is determined from eqs. (42) and (43), and the required drying time or dryer volume is determined from eqs. (41) and (44).

At the preheating period,

$$q_1 = GC_{H1}(t_1'-t_1) = hA_p' \frac{(t_1'-t_{mc})-(t_1-t_{m1})}{\ln \frac{(t_1'-t_{mc})}{(t_1-t_{m1})}} = W(c+c_w w_1)(t_{mc}-t_{m1}). \quad (47)$$

From this equation, we can determine t_1 and A_p' and also θ_p. The value of t_1 thus obtained is a precise one.

By the above calculation the total required drying time and dryer volume can be determined.

As the approximed calculation of θ_c or V_c during the surface evaporation period, eqs. (45) and (46) may be used.

As the approximated calculation of θ_d or V_d, we can calculate these values using the same procedure as described in case of a parallel-current flow operation.

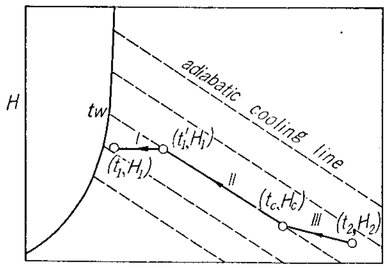

Fig. 15. The approximated change of the air conditions in counter-current flow dryer.

In this case it is assumed that the air conditions change in the dryer along the straight line from point (t_2, H_2) to (t_c, H_c) on the humidity chart as shown in Fig. 15.

We can use satisfactorily the initially calculated values of t_1 and t_{m2}, therefore, G and H_1 in the above approximated calculation, because these values have little difference with the precise values.

III-3. Comparison with other approximated methods already suggested:

There are two approximated calculation methods during the falling rate period already suggested. One[6] is based upon the assumption that the sensible heat of material can be neglected during the falling drying rate period. The change of the air conditions in the dryer is assumed to be adiabatical during the falling rate period, and is shown on the humidity chart, as in Fig. 16.

The other method[7] is based upon the assumption that the evaporation of moisture during the falling rate period is negligible. The change of the air conditions in the dryer is assumed to be isohumiditical during the falling rate period and is shown on the humidity chart, as in Fig. 17. They are considered as the extreme cases. Our method proposed above is more accurrate procedure.

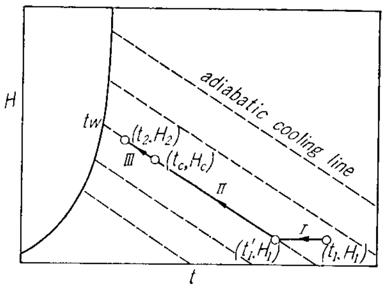

Fig. 16. The approximated change of the air conditions in parallel-current flow dryer.

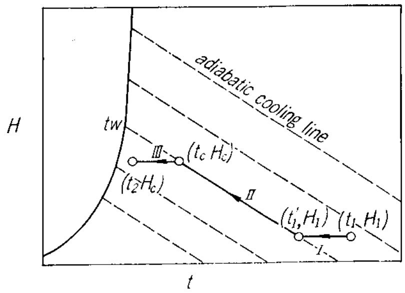

Fig. 17. The approximated change of the air conditions in parallel-current flow dryer.

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Nomenclature

A :	drying area of W	[m²/hr]
A_0 :	drying area of W_0	[m²]
A_p' :	drying area of preheating period	[m²]
a :	$=(RC_p/h)$	[−]
c :	specific heat of bone dried material	[kcal/kg.°C]
c_w :	specific heat of water $=1$	[kcal/kg.°C]
c_p :	specific heat of evaporated vapor	[kcal/kg.°C]
C_H :	humid heat of gas	[kcal/°C kg-dry gas]
D :	$=\{c_p(t-t_w)+r_w\}/r_w$	[−]

e	:	base of natural logarithms	[−]
F	:	free water content $F=w-w_e$	[−]
F_c	:	value of F at critical water content	[−]
G	:	dry gas rate	[kg/hr]
h	:	heat transfer coeff.	[kcal/hr.m^2.°C]
H	:	humidity of gas	[kg/kg-dry gas]
H_c	:	value of H at critical water content	[kg/kg-dry gas]
q_n	:	heat transferred to the drying surface	[kcal/hr.m^2]
q_t	:	heat transferred to the film from main gas stream	[kcal/hr.m^2]
R	:	drying rate	[kg/hr.m^2]
R_c	:	constant drying rate	[kg/hr.m^2]
R_d	:	falling drying rate	[kg/hr.m^2]
r_0	:	heat of vaporization at 0°C=595 (for water)	[kcal/kg]
r_w	:	heat of vaporization at t_w°C	[kcal/kg]
r_m	:	heat of vaporization at t_m°C	[kcal/kg]
t	:	gas temp.	[°C]
t_c	:	gas temp. at critical water content	[°C]
t_w	:	wet bulb temp.	[°C]
t_m	:	material temp.	[°C]
t_{ms}	:	surface temp. of material	[°C]
t_{mc}	:	temp. at critical water content	[°C]
Ua	:	heat capacity coefficient	[kcal/hr.°C.m^3]
V	:	dryer volume	[m^3]
W	:	feed rate of bone dried material	[kg/hr]
W_0	:	wt. of bone dried material	[kg]
w	:	water content	[kg/kg-dry material] [−]
w_c	:	critical water content	[−]
w_e	:	equilibrium water content	[−]
θ	:	time	[hr]