# Strain Measurements of Prestressed Concrete Cylindrical Tank Walls

By

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This paper presents the results of strain measurement on prestressed cylindrical concrete tank walls at the time of transferring of prestress. Freyssinet's method as well as Preload's method are employed to transfer prestress to the prestressed concrete tank walls. By means of Carlson type strainmeters embeded in the concrete, strains due to prestressing and bending moments accompanied by prestressing in the longitudinal direction are measured and compared with the theoretical values for each of the above described methods. The results obtained show that the prestresses applied meet fairly well the design requirements and the measured strains satisfactorily coincide with the theoretical values in the angular (tangential) direction. It is expected that comparatively large strains occur due to the circumferential prestressing, while the measured strains are approximately a half of the theoretical values.

## 1. Introduction

Recently, prestressed concrete techniques have made great development, and prestressed concrete cylindrical tank wall structures also have been designed and constructed successfully as well as beams, sleepers and slabs.

When the cylindrical tank is filled with inner materials, such as water, oil or sand, there occur hoop tension circumferentially in the wall and usually bending moment vertically along the height of the tank wall.

In the prestressed concrete tank, however, the wall is precompressed circumferentially by the prestressing tendon so as to eliminate the hoop tension described above, and is also prestressed or reinforced ordinarily in the vertical direction to resist the bending moment as mentioned above.

It is to be noted that additional bending stresses are caused in the vertical direction as the circumferential prestress is transferred progressively along the height of the cylindrical tank. Therefore, to prevent the formation of tension cracks in the wall concrete during the circumferential prestressing,

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the tank wall should be reinforced or prestressed longitudinally before transferring the prestress circumferentially.

Generally in our country now two methods are used for the circumferential prestressing of a tank: one is Freyssinet's method and one is Preload's or BBRV method. This paper presents the results of strain measurements in prestressed concrete cylindrical tanks at the time of circumferential prestressing, which were specially designed as temporary bulkhead or breakwater structures and both actually used in Kobe Harbour.

## 2. Designs, construction and strain measurements of the cylindrical concrete tank walls

### 2.1. Design[1),2),3)]

Two different types of cylindrical concrete tanks are designed; one for the temporary bulkhead and one for the breakwater structures. The designs of both tanks are as follows.

### 2.1.1. For temporary bulkhead

The tank having no bottom slab is settled on the clean gravel base under the water, and filled with sand and gravel, then covered with the precast concrete lid on the top. The forces considered in design are soil pressure, hydraulic pressure, etc. Dimensions of tank, arrangement of prestressing cables, sequence of prestressing, condition of soil and hydraulic pressure are given in Figs. 1, 2, and 3.

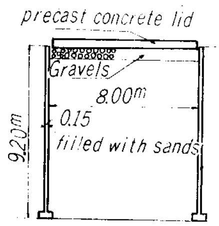

Fig. 1. Dimensions of tank (for temporary bulkhead).

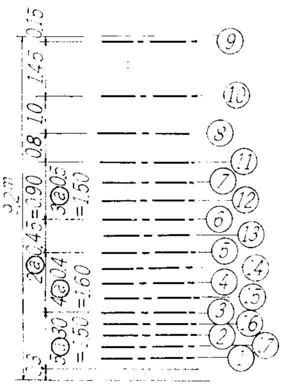

Fig. 2. Arrangement of prestressing cables and sequence of tensioning.

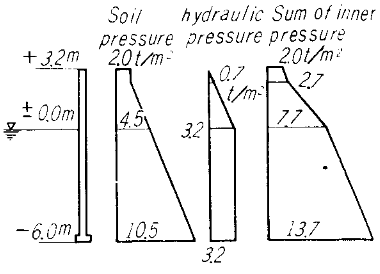

Fig. 3. Condition of soil and hydraulic pressure.

### 2. 1. 2. For breakwater structure

First a tank having no bottom slab is firmly pressed into the sea bottom by Sanken's method (the patented method) and another similar tank is settled just above the lower tank, then the structures are filled with gravel, sand, or soil, and covered with the steel lid, thus composing one element of the whole breakwater structure. In designing these tanks the following are considered or assumed.

(a) In adopting Freyssinet's method, the circumferential prestress can be transferred with little loss due to friction by using six pilars which divide the tank wall into six segmented parts.

(b) The bending stress caused in the longitudinal direction by the circumferential prestress is taken into account in the design.

(c) The loss in the circumferential prestress due to shrinkage, creep of the concrete and relaxation of the prestressing tendon is assumed as much as 15%.

(d) The hydraulic pressure acting on the tank wall and the steel cover plate used during settling the tank beneath the water is assumed as 13 $t/m^2$.

(e) The external forces as assumed to act after the tank has been settled down, and those caused by earthquake are shown in Fig. 4.

Figs. 5 and 6 illustrate dimensions of the tank, layouts of the prestressing tendons and sequence of prestressing.

### 2. 2. Manufacture of tanks and tensioning the prestressing tendons

The wall concrete were placed by using the ordinary form method in the tanks for temporary bulkhead use and by the sliding form method in the tanks for breakwater use. The circumferential prestressings were carried out by Freyssinet's method in the temporary bulkhead tank and by Freyssinet's or Preload's method in the breakwater tank.

In Freyssinet's method, first, a half of the prestressing cables, each with the initial tension of 23.4 ton per one cable, were pulled arround the wall one by one from the bottom up to the top, and second, the remaining half cables were pulled similarly from the top to the bottom in the temporary bulkhead tank and from the bottom to the top in the breakwater tank.

In Preload's method, the circumferential prestress was continuously transferred into the tank wall by the self-traveling winding machine and the effective tension per one wire was assumed as 85 $kg/mm^2$.

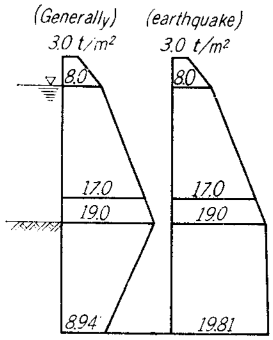

Fig. 4. External forces after the tank has been settled.

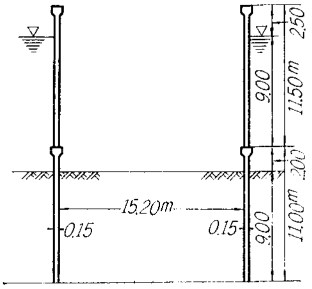

Fig. 5. Dimension of tank (for breakwater structure).

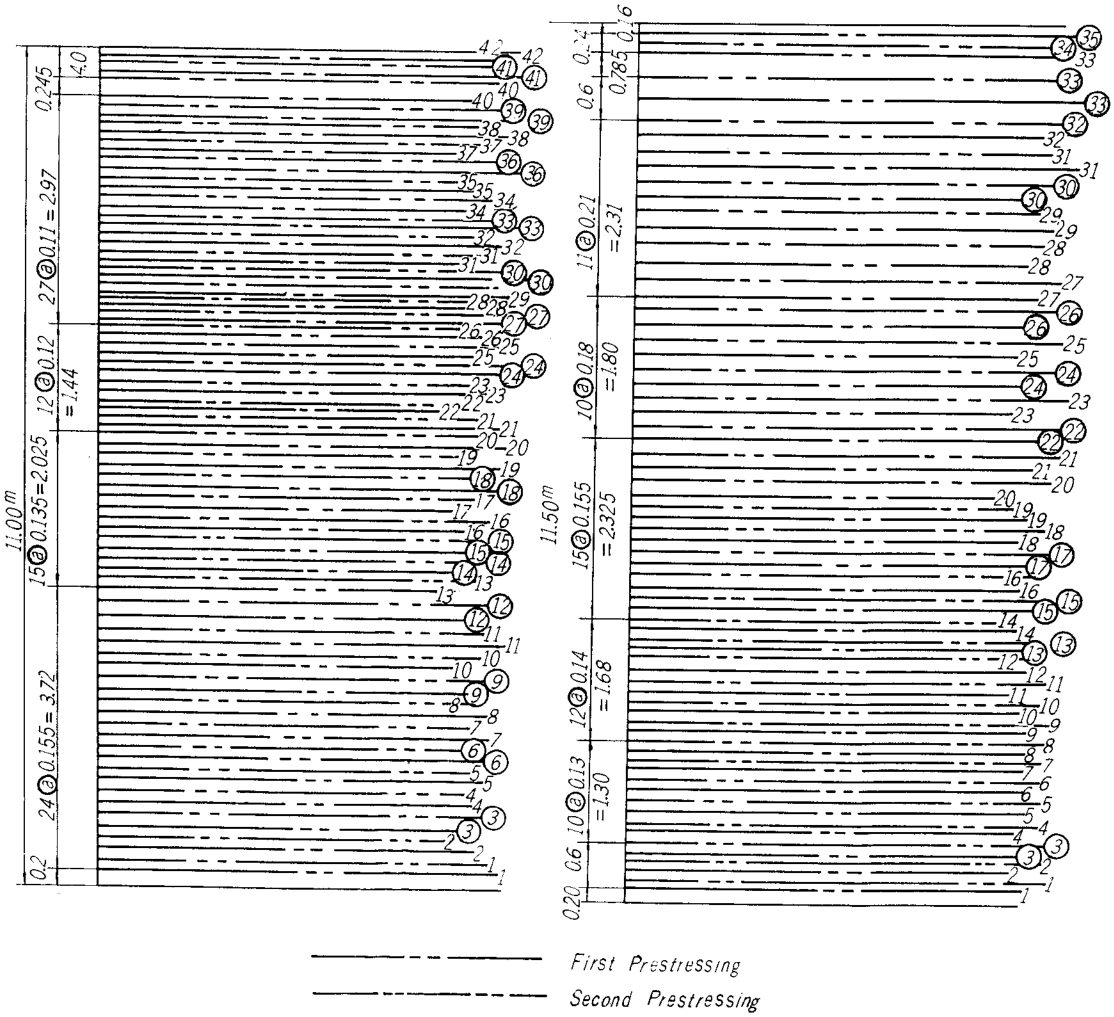

Fig. 6. Layouts of the prestressing tendons and sequence of tensioning. Number shows sequence of prestressing and Ⓝ means cable for which strains are measured.
(Reight : bottom tank, Left : top tank)

## 3. Theoretical analysis on the prestressed concrete tank wall

### 3.1. General solution on the circular cylindrical structure 4), 5), 6)

General solution of the structures with circular cylindrical section such as pipe, cylindrical shell and tank, can be obtained by using the theories given by S. Timoshenko and W. Flügge on the cylindrical shell. It is well known that the deflection of a tank wall is expressed by similar equation to that for a beam with constant flexural rigidity, supported on an elastic foundation and carrying the external load (if the effect of Poisson's ratio is neglected).

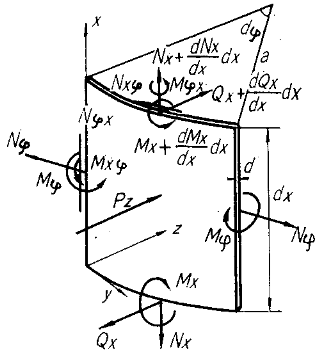

Fig. 7.

Thereafter the following notations are used in the analysis. (Fig. 7)

$w$ : Deflection of the circular cylindrical shell wall

$N_\varphi$ : Membrane force per unit length of axial section of tank

$M_x$ : Bending moment per unit length of section of a plate perpendicular to $x$-axis

$Q_x$ : Shearing force per unit length of a section perpendicular to the axis of a cylindrical wall

$P_z$ : The action of load

$a$ : Tank radius

$d$ : Wall thickness

$\nu$ : Poisson's ratio (assumed as 0)

$E$ : Modulus of elasticity of wall concrete

$$D = Ed^3/12(1-\nu^2) : \text{ Flexural rigidity} \qquad \cdots\cdots\cdots\cdots\cdots (1)$$

$$\beta^4 = \frac{Ed}{4a^2D} = \frac{3(1-\nu^2)}{a^2d^2} \qquad \cdots\cdots\cdots\cdots\cdots (2)$$

The differential equation and the general solution of deflection of beam supported on the continuous elastic foundation are as follows,

$$\frac{d^4w}{dx^4}+3\beta^4w = \frac{P_z}{D} \qquad \cdots\cdots\cdots\cdots\cdots (3)$$

$$w = e^{\beta x}(C_1\cos\beta x+C_2\sin\beta x)+e^{-\beta x}(C_3\cos\beta x+C_4\sin\beta x)+f(x) \qquad \cdots\cdots (4)$$

in which $f(x)$ is a particular solution of Eq (3) and $C_1$, $C_2$, $C_3$ and $C_4$ are integral constants to be determined from the boundary conditions at the ends of the tank. If the tank wall is free at the bottom ends such as is found in

the measured prestressed concrete tank, and if the wall thickness $d$ is small enough compared with the tank height $h$ and radius $a$, both $C_1$ and $C_2$ are 0, and, for transferring prestress, $f(x)$ can be put to $f(x)=0$.
Hence, Eq (4) becomes

$$w = e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x) \qquad \cdots\cdots\cdots\cdots\cdots(5)$$

and we obtain

$$N_\varphi = -\frac{w}{a}Ed = -\frac{Ed}{a}\Big\{e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x)\Big\} \qquad \cdots\cdots\cdots\cdots\cdots(6)$$

$$M_x = -D\frac{d^2w}{dx^2} = -2D\beta^2\Big\{e^{-\beta x}(C_3\cos\beta x - C_4\sin\beta x)\Big\} \qquad \cdots\cdots\cdots\cdots\cdots(7)$$

In this case, if the flexual rigidity $D$ of the beam is constant, it is to be considered on what range of the beam the load is acting and whether the beam is considered as infinitely long or finitely long.

The greater the value of $\beta$, the more restricted is the zone of major flexure , and it is found that the classification of the beam depends upon the quantity $\beta l$, where $l$ is the length of the beam from the loading point to either end. If $\beta l$ exceeds 2.5 the beam may, in most cases, be treated as infinitely long⁷⁾.

The general solutions for various cases of the infinite beam are given as follows.

### 3. 2. In the case of bending moment $M_0$ and shearing force $Q_0$ both acting along one end of the infinite beam. (Fig. 8)

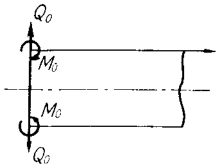

Fig. 8.

In this case, Eq (5) becomes

$$\left.\begin{aligned}(M_x)_{x=0} &= -D\left(\frac{d^2w}{dx^2}\right)_{x=0} = M_0\\ (Q_x)_{x=0} &= \left(\frac{dM}{dx}\right)_{x=0} = Q_0\end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots(8)$$

and we obtain

$$C_3 = -\frac{1}{2\beta^2 D}(Q_0+\beta M_0), \quad C_4 = \frac{M_0}{2\beta^2 D} \qquad \cdots\cdots\cdots\cdots\cdots(9)$$

$$w = \frac{1}{2\beta^3 D}\Big\{\beta e^{-\beta x}M_0(\sin\beta x - \cos\beta x) - e^{-\beta x}Q_0\cos\beta x\Big\} \qquad \cdots\cdots\cdots\cdots\cdots(10)$$

$$N_\varphi = -\frac{Edw}{a} = -2a\beta\Big\{\beta e^{-\beta x}M_0(\sin\beta x - \cos\beta x) - e^{-\beta x}Q_0\cos\beta x\Big\} \qquad \cdots\cdots(11)$$

$$M_x = -D\left(\frac{d^2w}{dx^2}\right) = -\frac{1}{\beta}Q_0e^{-\beta x}\sin\beta x + e^{-\beta x}M_0(\sin\beta x + \cos\beta x)\}\Big\} \qquad \cdots\cdots(12)$$

### 2.3. In the case of a concentrated load acting on the infinite beam. (Fig. 9)

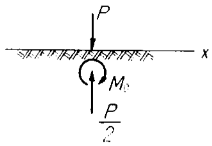

Fig. 9.

In this case $Q_0$ and $M_0$ in Eq (10) are given as follows.

$$\left.\begin{aligned} \left(\frac{dw}{dx}\right)_{x=0} &= \frac{1}{2\beta^2 D}(2\beta M_0+Q_0) = 0 \\ Q_0 = -\frac{P}{2},\quad M_0 &= \frac{P}{4\beta} \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots(13)$$

and thus we obtain

$$w = \frac{P}{8\beta^3 D}e^{-\beta x}(\cos\beta x+\sin\beta x) \qquad \cdots\cdots\cdots\cdots\cdots(14)$$

$$N_\varphi = -\frac{a\beta P}{2}e^{-\beta x}(\cos\beta x+\sin\beta x) \qquad \cdots\cdots\cdots\cdots\cdots(15)$$

$$M_x = \frac{P}{4\beta}e^{-\beta x}(\cos\beta x-\sin\beta x) \qquad \cdots\cdots\cdots\cdots\cdots(16)$$

$$Q_x = -\frac{P}{2}e^{-\beta x}\cos\beta x \qquad \cdots\cdots\cdots\cdots\cdots(17)$$

### 3.4. In the case of uniformly distributed load of intensity *q* acting along a length *l* on the infinite beam.

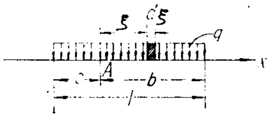

Fig. 10.

(a) In the inner portion of the loaded range (Fig. 10),

$$w = \frac{qd\xi}{8\beta^3 D}e^{-\beta\xi}(\cos\beta\xi+\sin\beta\xi) = \int_0^b \frac{qd\xi}{8\beta^3 D}e^{-\beta\xi}(\cos\beta\xi+\sin\beta\xi)$$
$$+\int_0^c \frac{qd\xi}{8\beta^3 D}e^{-\beta\xi}(\cos\beta\xi+\sin\beta\xi) = \frac{qa^2}{2Ed}(2-e^{-\beta b}\cos\beta b-e^{-\beta c}\cos\beta c) \quad \cdots(18)$$

$$N_\varphi = -\frac{qa}{2}(2-e^{-\beta b}\cos\beta b-e^{-\beta c}\cos\beta c) \qquad \cdots\cdots\cdots\cdots\cdots(19)$$

$$M_x = \frac{q}{4\beta^2}(e^{-\beta b}\sin\beta b+e^{-\beta c}\sin\beta c) \qquad \cdots\cdots\cdots\cdots\cdots(20)$$

$$M_{\max} = \frac{q}{2\beta^2}e^{-\beta l/2}\sin\frac{\beta l}{2} \qquad \cdots\cdots\cdots\cdots\cdots(21)$$

(b) In the outer portion of the loaded range (Fig. 11),

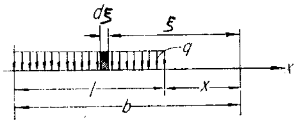

Fig. 11.

$$w = \int_c^b \frac{qd\xi}{8\beta^3 D}e^{-\beta\xi}(\sin\beta\xi+\cos\beta\xi) = \frac{q}{2Ed}(e^{-\beta c}\cos\beta c-e^{-\beta b}\cos\beta b) \quad \cdots(22)$$

$$N_\varphi = -\frac{qa}{2}\left(e^{-\beta c}\cos\beta c - e^{-\beta b}\cos\beta b\right) \qquad \cdots\cdots\cdots\cdots\cdots(23)$$

$$M_x = \frac{q}{4\beta^2}\left(e^{-\beta b}\sin\beta b - e^{-\beta c}\sin\beta c\right) \qquad \cdots\cdots\cdots\cdots\cdots(24)$$

### 3.5. The point where the bending moment becomes maximum or zero when prestressing tendons are tensioned.

(a) In the case of both $M_0$ and $Q_0$ acting at the end of the infinite beam.

The distance $x'$ from the origin where $M_{\max}$ is given, is expressed by

$$x' = \frac{1}{\beta}\tan^{-1}\frac{1}{Q_0 - 2\beta M_0} \qquad \cdots\cdots\cdots\cdots\cdots(25)$$

and for the distance $x''$ for $M=0$ is,

$$x'' = \frac{1}{\beta}\tan^{-1}\frac{M_0}{\frac{1}{\beta}Q_0 - M_0} \qquad \cdots\cdots\cdots\cdots\cdots(26)$$

(b) In the outer portion of loaded range ($\beta l < 2.5$) when $x'$ is taken as shown in Fig. 12.

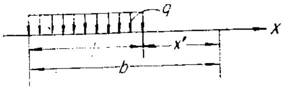

Fig. 12.

$$M_{x'} = \frac{q}{4\beta^2}\left\{e^{-\beta(l+x')}\sin\beta(l+x') - e^{-\beta x'}\sin\beta x'\right\} \qquad \cdots\cdots\cdots\cdots\cdots(27)$$

Thus is obtained the location $x'$ where $M_{\max}$ is given

$$x' = \tan^{-1}\frac{-e^{-\beta l}\sin\beta l + e^{-\beta l}\cos\beta l - 1}{e^{-\beta l}\sin\beta l + e^{-\beta l}\cos\beta l - 1} \qquad \cdots\cdots\cdots\cdots\cdots(28)$$

and $x''$ for $M=0$ is

$$x'' = \frac{1}{\beta}\tan^{-1}\frac{-e^{-\beta l}\sin\beta l}{e^{-\beta l}\cos\beta l - 1} \qquad \cdots\cdots\cdots\cdots\cdots(29)$$

For the case where loaded range is infinite ($\beta l \geqq 2.5$) we obtain

$$M_x = \frac{q}{4\beta^2}e^{-\beta x}\sin\beta x \qquad \cdots\cdots\cdots\cdots\cdots(30)$$

$$x' = \frac{1}{\beta}\tan^{-1}(1) = \frac{1}{\beta}\left(\frac{\pi}{4} + n\pi\right) \qquad \cdots\cdots\cdots\cdots\cdots(31)$$

$$x'' = \frac{n\pi}{\beta} \qquad (n = 0, 1, 2, 3, \cdots) \qquad \cdots\cdots\cdots\cdots\cdots(32)$$

When the equations as derived above for various loading conditions and the value of $\beta l$ are combined suitably, $N_\varphi$, $M_x$, $M_{\max}$ etc. for any loading can be calculated.

## 4. Measurements and observation on the results

The theoretical values of strain in the wall concrete during prestressing are calculated for the following conditions and are compared with the measurements. On the tank for temporary bulkhead use in which the magnitude of prestress is small and the distance between prestressing cables is fairly far (30 cm), the strains are each measured after one cable has been tensioned, assuming that the prestressing force acts on the wall as a concentrated load.

On the breakwater tank to which Freyssinet's method is applied, the prestressing forces of prestressing cables are converted into an equivalent umiform load $p$ as shown below to make simple the computation.

$$p = PN/al \qquad \cdots\cdots\cdots\cdots\cdots(33)$$

where, $p$ is the equivalent uniformly distributed load, $P$ the tensioning force per prestressing cable, $a$ the radius of tank, $l$ the depth of wall an which prestressing cables are tensioned with the same pitch and $N$ the numbers of cables tensioned.

On the tank to which Preload's method is applied, prestressing wires are tensioned arround the wall with small and nearly uniform pitch, the prestressing forces are assumed to act uniformly along the depth of the tank wall.

### 4.1. Tank for temporary bulkhead (Figs. 13 and 14)

One example of strain measurements is shown in Fig. 14. The theoretical

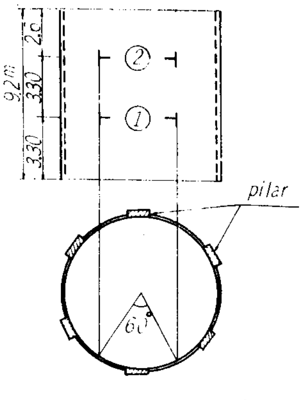

Fig. 13. Locations of strain-meters embeded in concrete (for temporary bulkhead).

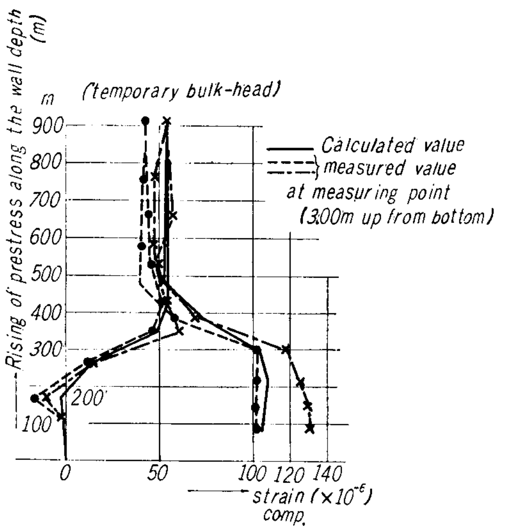

Fig. 14. Strain measurement (temporary bulkhead).

values are computed generally from Eq. (11) and Eq. (15) by using the principle of superposition in order to satisfy the condition that the tank be "free at edges". However it is to be noticed that if tensioned prestressing cables are far from the edge of the wall, for instance, about 1.5 m ($l<2.5/\beta$) from the top or the bottom, bending moment and shearing force hardly occur at all the edge due to the prestressing.

It can be seen from Fig. 14 that the strains measured are in fairly good agreement with the theoretical values which are computed individually for the locations of the tensioned cables and the total tensioning forces given by the prestressing cables then used.

When the measured strains are compared with the design values liniarly varying along the wall height from 10 kg/cm$^2$ at top to 70 kg/cm$^2$ at bottom, the former are smaller by about 40% than the latter. Therefore, it can be considered that the cables should have been laid more closely to obtain the stress condition required in the design.

### 4. 2. Tank for breakwater

### 4. 2. 1. Freyssinet's method (Figs. 15, 16 and 17)

Some examples of the theoretical and measured strain and stress distributions are shown in Figs. 16 and 17. As seen from these results, the prestresses induced theoretically in the wall concrete are 50, 70, 75 kg/cm$^2$ by the first prestressing and 100, 130, 135 kg/cm$^2$ by the second prestressing, at the points 4 m, 7 m, 10 m removed from the bottom of the tank respectively, and each of these distributions satisfies the design condition.

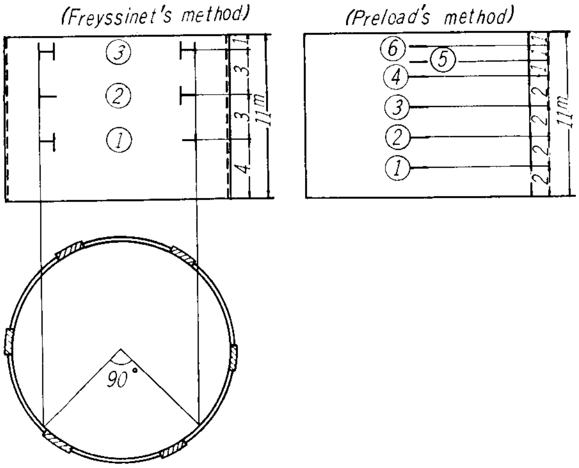

Fig. 15. Locations of strainmeters embeded in tank wall (for breakwater).

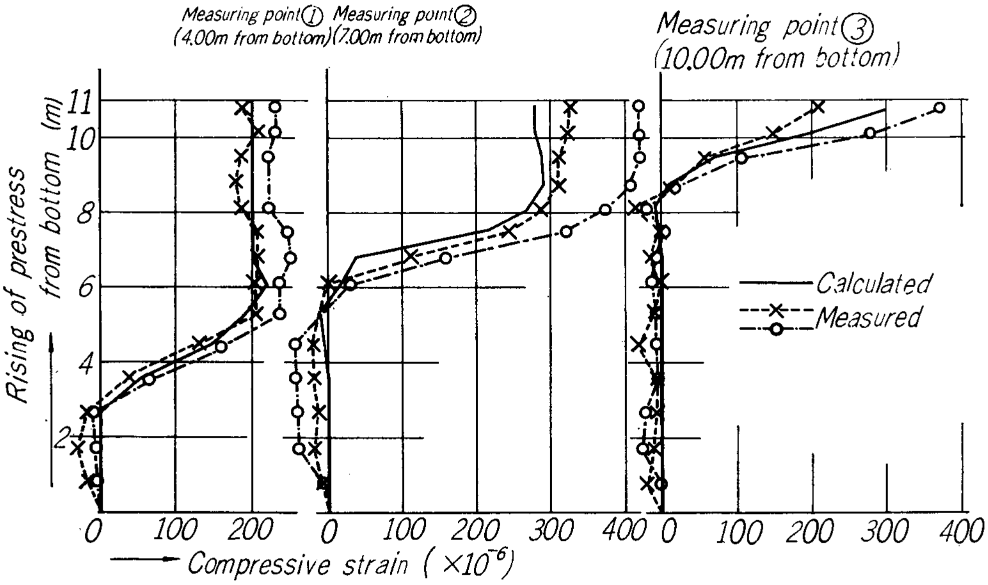

**Fig. 16.** Strain measurements (breakwater: Freyssinet's method).

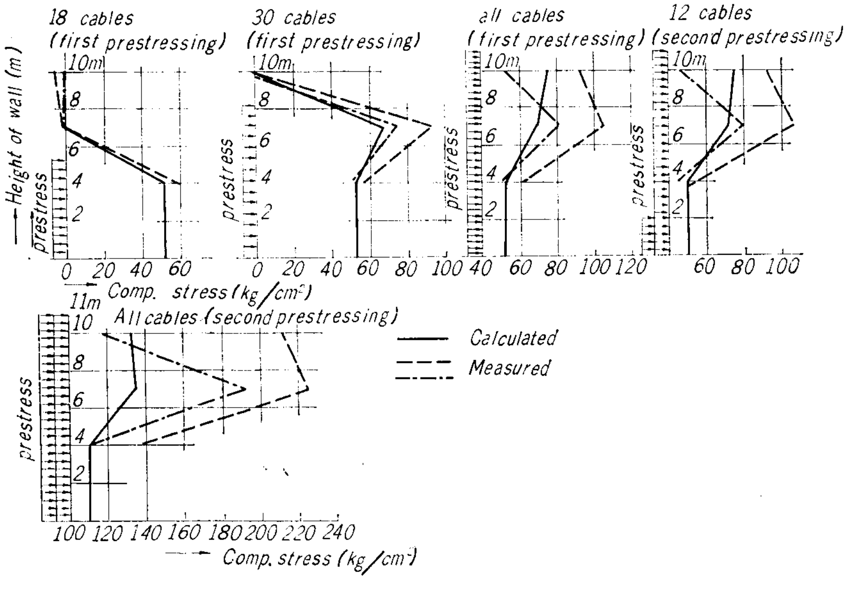

**Fig. 17.** Some examples of stress distribution (breakwater: Freyssinet's method).

The strain measurements are generally in good agreement with the calculation though giving the somewhat large strains observed on the second prestressing.

### 4. 2. 2. Preload's method (Figs. 18 and 19)

The measured strain and stress distributions obtained by the first prestressing up to the whole height and the second only up to 4 m from the bottom are shown in Figs. 18 and 19. They are in close agreement with the calculations, also satisfying the design condition.

The measured stresses are over 70, 110, 140 kg/cm$^2$, at bottom, 4 m from the bottom and top of the tank, respectively, all of which are greater than those obtained by Freyssinet's method. It also can be seen that the prestresses are transferred very smoothly in this method.

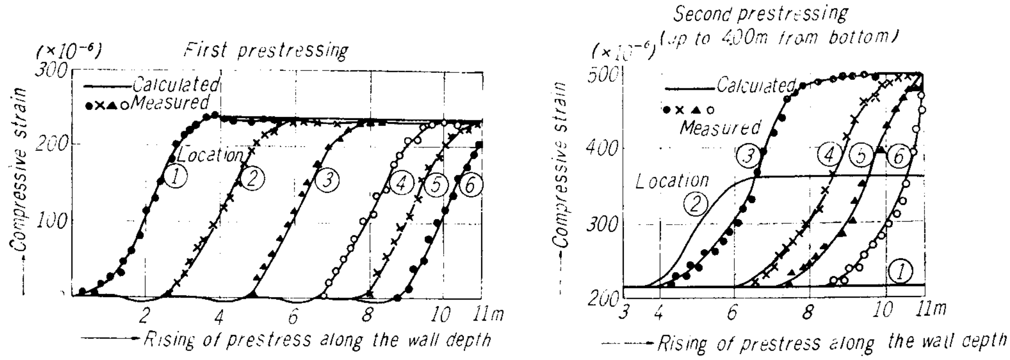

Fig. 18. Strain distributions (breakwater: Preload's method).

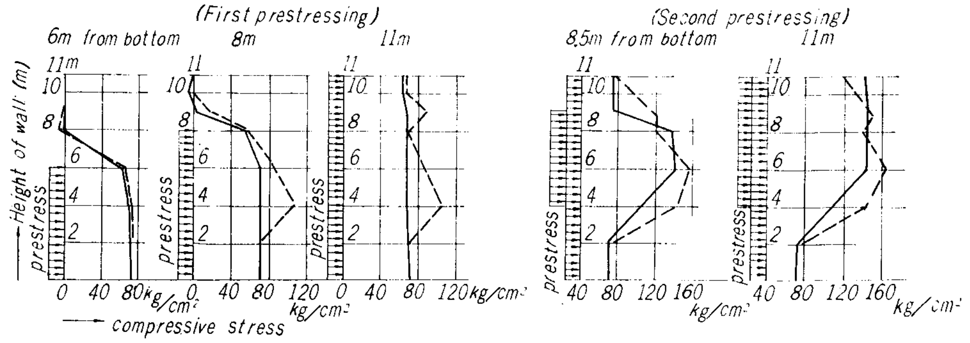

Fig. 19. Stress distributions (breakwater: Preload's method).

## 5. Vertical strains due to tensioning of prestressing tendons (Fig. 20)

When the circumferential prestress are transferred to the wall of the tank, vertical bending moments are yielded. In some cases, the amount of prestress introduced are so large that these bending moments may be large enough to cause horizontal cracking on the wall surface. The vertical strain of the concrete due to the circumferential prestress was measured only on the tank made by Freyssinet's method. In this method, the prestressing cables are, as mentioned before, so wound with gradationally different intervals as to give the equivalent uniform loads also of different gradations. Just as is made in the calculation of the circumferential axial force, the bending moment at a point on the wall should be calculated by superposing all the moments caused by each load. It is also the same as in the case of the calculation of $N_\varphi$. Some examples of the calculated distribution of bending moments are shown in Fig. 20.

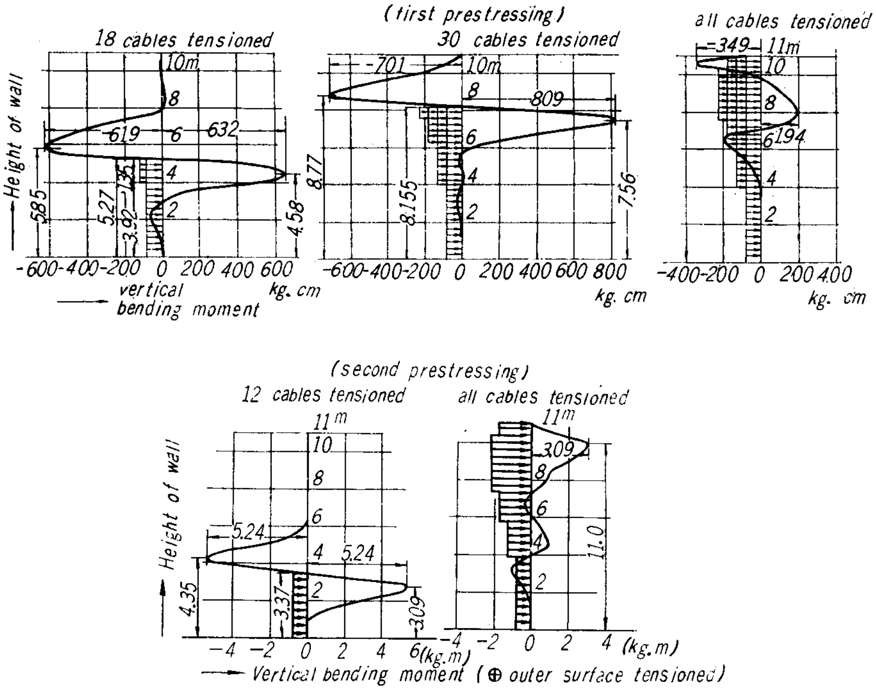

Fig. 20. Distributinn of vertical bending moment (breakwater: Freyssinet's method)

Theoretical values of vertical strain, just after introduction of the prestress, are obtained as the sum of the strain caused by bending moments and and that caused by circumferential axial forces. The stress caused by bending moments is rather small, compared with the one caused by the circumferential axial forces. The measured strains were found to be smaller than the theoretical value. This might be due to the effect of reinforcement inserted in the wall.

## 6. Conclusion

In both construction methods, in general, prestress transferred by the prestressing tendons were sufficient to meet the design requirements. The theoretical values are fairly well agreed with the measurements.

In prestressing, the greatest amount of prestress was introduced at or close to the portion wound by the tendon, and little influence was seen by the tendons tensioned far arart. The length of this influence zone is 150 cm in the case of the temporary bulkhead tank and 200 cm in the case of the breakwater tanks, both values corresponding to $1 \leq 2.5/\beta$.

The calculated vertical stresses are fairly large though the measured strains are about half of these. Only when the bending moment turns positive as the tension is going on is a considerable amount of tensile stress yielded. However, these are only the transient secondary stresses which rapidly decrease as the wiring goes on.

In addition, when the uniform prestress has been introduced all over the wall, the bending moments disappear and only the circumterential axial force remains. Therefore this tensile stress may be resisted either by the conventional reinforcement or the vertical prestress. On these points more consideration should be taken in design hereafter.

This paper describes the detailed analysis of strains in the wall concrete caused by prestressing of the circular structure. The authors hope it will be helpful for the design and construction of prestressed concrete circular structures in the future.

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