The S_4 and Few-Group Diffusion Calculations of Fast Reactors*

By

Masao OHTA**, Hiroshi NISHIHARA** and Yoko DEGUCHI**

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To economize a large amount of numerical work in the calculation of fast breeders, the present work has been done in the hope that few-group diffusion theory might give acceptable results in some cases. A hypothetic ^{233}U-^{232}Th system with large core size ($\sim$670 l) as well as a hypothetic ^{239}Pu-^{238}U fast reactor with small core size ($\sim$50 l) are adopted. These systems are assumed to be of spherically symmetric geometry. One dimensional calculations are applied to obtain the static characteristics of the systems. The results from few-group S_4 and diffusion method are investigated. These results seem to indicate that four- or three-group diffusion calculation might at least be used in place of three-group S_4 computation for both large and small fast reactors.

A new convergence criterion imposed upon the static parameters is proposed. The leakage rate of neutrons from the blanket is selected as the sensitive measure of convergence. This rate is estimated in two ways, i.e. with the aid of neutron current and by neutron inventory. The sufficiently converged state can be reached when these two values coincide with each other. One is also able to infer the necessary number of spatial mesh points by comparing these two values.

1. Introduction

It is of great interest to us to compare the nuclear characteristics of Pu-U and U-Th fast breeders from various points of view. There is a wealth of related work performed so far[1-6].

In fulfilling such researches, a large amount of numerical work must be done. Therefore, economization of computational effort should be carefully considered.

For fast reactors, transport theory calculations are expected to give best results, as far as the accuracy of the theory is concerned. Unfortunately, however, the numerical procedure for solving the neutron transport equation is most time consuming. In general, the transport equation for fast reactors has been numerically analyzed with the aid of Carlson's S_n method and others[7-11]. For a reactor of large size, it is known that multi-group diffusion

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** Department of Nuclear Engineering

theory gives essentially the same results as more refined methods, such as S_n method. Indeed, the nuclear characteristics of the Enrico Fermi Fast Reactor were estimated by means of multi-group diffusion theory[4].

In fast reactors fission and capture occur over a wide range of energies, and the cross sections vary strongly with energy, and it may not be feasible to average the cross sections with the intention of finding few-group nuclear constants. But the multi-group calculation is time consuming even if the diffusion approximation were used. The present work has been done in the hope that few-group diffusion theory may give acceptable results at least in some cases.

Two HYPOTHETIC FAST REACTORs, a ^{233}U–^{232}Th system with large core size ($\sim$670 l) and a ^{239}Pu–^{238}U system with small core size ($\sim$50 l) were adopted. Hereafter, these systems will be abbreviated to REACTOR I and REACTOR II, respectively.

First, sixteen-energy group calculations for these hypothetic systems were carried out by the S_4 method and the neutron spectra in the very near critical state were obtained. Then the flux-weighted few-group constants were obtained by taking the average of the original sixteen-group constants over these spectra. Secondly, three-, two- and one-group calculations were carried out by the S_4 method. At the same time, four-, three-, two- and one-group calculations were made by the diffusion method. In this way, eigenvalues, neutron flux distributions, neutron energy spectra, static and other parameters which characterize the above systems as fast breeders were estimated. A detailed procedure along this line and the results therefrom will be described in the following sections.

2. Multi-Group Calculation

Both REACTORs I and II are of spherical shape consisting of a core and a blanket surrounding the core concentrically. Each region is assumed to be homogeneous. The composition and the configuration of these systems are shown in Fig. 1 and Table 1[3-6]. The sixteen-group data published by Yiftah et al.[12] were used as the multi-group cross sections. The fission spectrum and the energy/lethargy structure for the

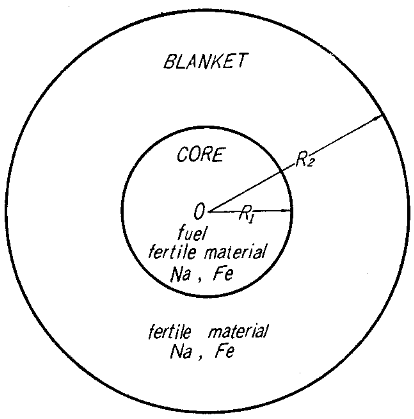

Fig. 1. Geometry of the system,

Table 1. Compositions and configurations.

^{233}U–^{232}Th system			^{239}Pu–^{238}U system		
material	volume fraction		meterial	volume fraction	
	core	blanket		core	blanket
^{233}U	0.032		^{239}Pu	0.080	
^{232}Th	0.218	0.600	^{238}U	0.420	0.400
Na	0.500	0.200	Na	0.167	0.200
Fe	0.250	0.200	Fe	0.333	0.400

Table 2. Fission spectrum and energy grouping structure.

g	E_L(MeV)	Δu_g	χ_g	grouping			
				4g	3g	2g	1g
1	3.668	1.0	0.132				
2	2.225	0.5	0.213	1	1		
3	1.35	0.5	0.232				
4	0.825	0.5	0.179				
5	0.5	0.5	0.116	2		1	
6	0.3	0.5	0.067		2		
7	0.18	0.5	0.034				
8	0.11	0.5	0.017	3			
9	0.067	0.5	0.010				
10	0.0407	0.5	0				
11	0.025	0.5	0				
12	0.015	0.5	0	4	3	2	
13	0.0091	0.5	0				
14	0.0055	0.5	0				
15	0.0021	1.0	0				
16	0.0005	—	0				

In the case of Pu-U system, the lower boundary of the first group of four- and three-group scheme is set at that of the fourth group of original sixteen-group scheme as is indicated by the dotted line.

breakdown to the sixteen- and the few-group scheme are given in Table 2. In the case of REACTOR II, the lower boundary of the first group of the four- and the three-group scheme is set at 0.825 MeV as is indicated by the dotted line in the Table 2.

In this and following sections, details of calculation will be given only for REACTOR I because the same procedures were adopted for both REACTORs. The numerical analyses have been carried out by an S_4 program coded for KDC-I. The program is based on (8), and solves the one-dimensional multi-group transport equation on spherically symmetric geometries with the assumption of isotropic neutron scattering and emission. The program is coded also so as to fit the scheme of the data in (12).

Throughout the entire process of computation, the outer radius of each reactor R_2 was fixed, i.e. $R_2=105$ cm for REACTOR I, $R_2=75$ cm for REACTOR II, while the core radius R_1 was varied around the critical value in order to search for the criticality. The eigenvalue λ, defined by

$$1=\sum_{g=1}^{16}\int_{\substack{\text{reactor}\\\text{volume}}}\frac{\nu_g\Sigma_{f,g}}{\lambda}\phi_g dV \tag{1}$$

is estimated after every iteration, and the fission neutron source is normalized by the factor of $1/\lambda$ for successive iterations.

A sixteen-group S_4 calculation was carried out for $R_1=54.2$ cm, and resulted in the eigenvalue $\lambda=0.999\,87$.

Usually, the convergence criterion imposed upon the eigenvalue gives poorer convergence. It is desirable to have the sufficient convergence of static parameters which characterize the system as a fast breeder. Therefore, a new criterion was set up on the basis of the convergence of these parameters. Of these parameters, especially, the total leakage rate of neutron from blanket to the environment seems to be the suitable measure of convergence.

This leakage rate was estimated by two methods. The one is based upon the neutron current calculated from neutron angular flux and the other is based upon the neutron inventory. Each method is applied according to the formulae (2) and (3) respectively.

$$\text{leakage rate}=\sum_{g=1}^{16}\int \boldsymbol{J}\cdot\boldsymbol{n}\,dA \tag{2}$$

$$\text{leakage rate}=\sum_{g=1}^{16}\int \nu_g\Sigma_{f,g}\phi_g dV-\sum_{g=1}^{16}\int \Sigma_{a,g}\phi_g dV \tag{3}$$

Here, $\boldsymbol{J}\cdot\boldsymbol{n}$ is the outward neutron current estimated from angular flux[8] and the integration on dA should be done over the surface of blanket. The formula (3) is the expression for neutron balance and the integration is performed over the entire volume of the reactor. The total strength of the fission neutron source is normalized to unity.

Because of the relatively rapid convergence of the angular flux, this leakage rate gives a good estimation of the leakage rate of the system in a comparatively early stage of iteration. In contrast to this, as long as the convergence of neutron flux is insufficient the leakage rate obtained after each iteration by neutron inventory undergoes change though λ has reached its stationary value. The state in which values of leakage from these two methods coincide was concluded to be the converged state of static parameters.

Moreover, if the number of spatial mesh points is insufficient, these values of leakage due to two ways never coincide with each other even if the convergence of flux is already attained. This means that the comparison of these two kinds of leakage values is of practical use in deciding the necessary and

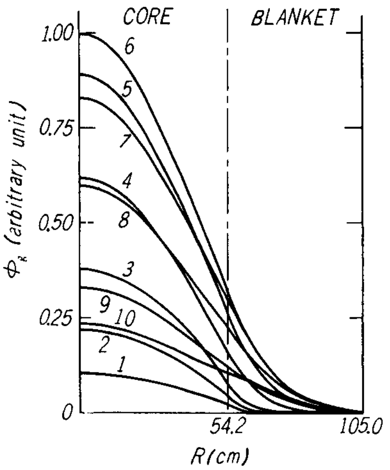

Fig. 2(a). Sixteen-group neutron flux distributions for U–Th system. From 1st to 10th group distributions are indicated.

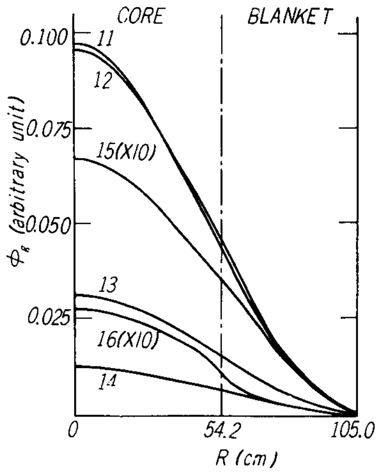

Fig. 2(b). Sixteen-group neutron flux distributions for U–Th system. From 11th to 16th group distributions are indicated.

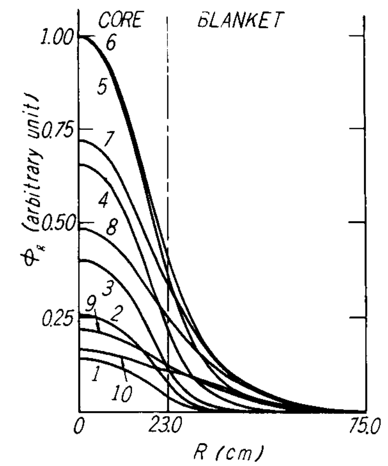

Fig. 3(a). Sixteen-group neutron flux distributions for Pu–U system. From 1st to 10th group distributions are indicated.

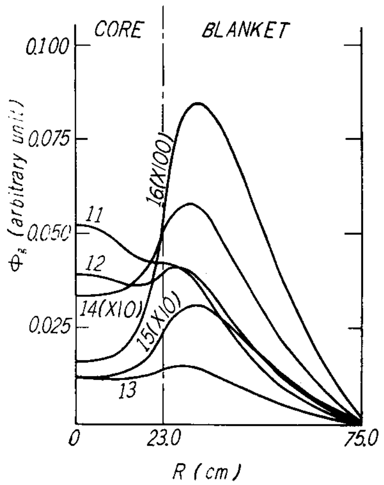

Fig. 3(b). Sixteen-group neutron flux distributions for Pu–U system. From 11th to 16th group distributions are indicated.

sufficient number of mesh points for the attainment of a perfectly balanced state. This is why the eighty mesh points were adopted in the few-group diffusion calculation of REACTOR II.

As this leakage rate itself is of small magnitude and of the order of several per cent of the fission neutrons in the entire system, it might well be said that the present convergence criterion is of very strict character.

The resulting sixteen-group neutron flux distributions are given in Figs. 2(a), 2(b), 3(a) and 3(b). Neutron spectra in the core and blanket are also shown in Figs. 4 and 5. These curves are normalized so that the integral under the respective curve is equal to unity.

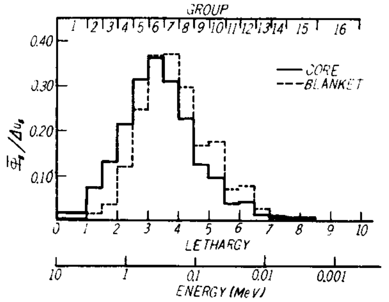

Fig. 4. Sixteen-group neutron energy spectra for U–Th system.
Area under respective curve is normalized to unity.

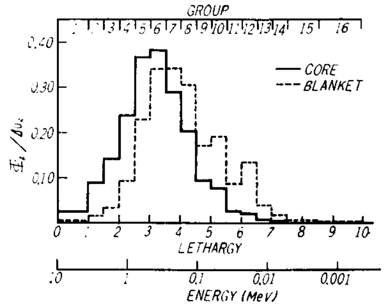

Fig. 5. Sixteen-group neutron energy spectra for Pu–U system.
Area under respective curve is normalized to unity.

3. Few-Group Calculation

The few-group constants for each region, core and blanket, were obtained separately by averaging the original group constants over the sixteen-group spectrum for core and for blanket. These sixteen-group spectra were obtained by the multi-group calculation. As was mentioned above, three-, two-, one- and four-, three-, two-, one-group constants were prepared for the S_4 and the diffusion method respectively. For the three-group S_4 method a full account of the computative procedure will be given below.

To make the standards of comparison clear, first, the value of R_1 and R_2 were fixed to the same values as in the case of sixteen-group S_4 calculations, i.e. 54.2 cm and 105 cm for each. Following the same procedure as was stated in section 2, the stationary value of $\lambda=1.018\,28$ was obtained. Next, the value of R_1 was reduced to a smaller one of 52.6 cm and the steady value of $\lambda=0.998\,90$ was reached. The critical radius of core $R_1^{\mathrm{crit}}=52.69$ cm was obtained by linear interpolation.

In the same way, the calculations for the other few-group cases were carried out. For the diffusion method, the computations were made by a WANDA-type program coded for KDC-I. In the case of diffusion method, the total leakage rate of neutrons was also estimated in two ways. The one is from the outward neutron current estimated as the gradient of neutron flux and the other is from the neutron inventory.

Table 3. Structure of the calculations.

	S_4-16g	S_4-3g	S_4-2g	S_4-1g	Diff.-4g	Diff.-3g	Diff.-2g	Diff.-1g
R_1 (cm)	λ (^{233}U-^{232}Th system)							
55.5					1.00503			
55.0						1.00386		
54.2	0.99987	1.01828	1.03014	1.04864	0.98910	0.99411	1.00230	1.00797
53.9							0.99861	
53.5								0.99943
52.6		0.99890						
51.7			0.99960					
50.0				0.99696				
number of space points	20	40	40	40	40	40	40	40
time per iteration	5^m30^s	2^m10^s	1^m30^s	50^s	55^s	40^s	27^s	14^s
R_1 (cm)	λ (^{239}Pu-^{238}U system)							
24.0					1.00348			
23.9						1.00521		
23.5							1.00686	
23.0	1.00478	1.01483	1.03643	1.06354	0.97152	0.97643	0.98986	1.00568
22.8								0.99880
22.5		0.99927						
21.8			0.99587					
20.9				0.99126				
number of space points	20	40	40	40	80	80	80	80
time per iteration	5^m30^s	2^m10^s	1^m30^s	50^s	1^m44^s	1^m20^s	54^s	28^s

The results of all the calculations are listed in Table 3. The relationship between R_1 and λ is shown in Figs. 6 and 7. As will be seen from Table 3 and Figs. 6 and 7, with the same geometry the fewer-group estimation gives larger value of k_{eff}, whether in the case of the S_4 method or the diffusion method.

It is interesting to see that in the near critical state the points (R_1, λ) are all situated on parallel lines. Therefore, the relation between R_1 and λ for the sixteen-group S_4 case, for example, can be given by the straight line parallel to those obtained for the other few-group cases without any further

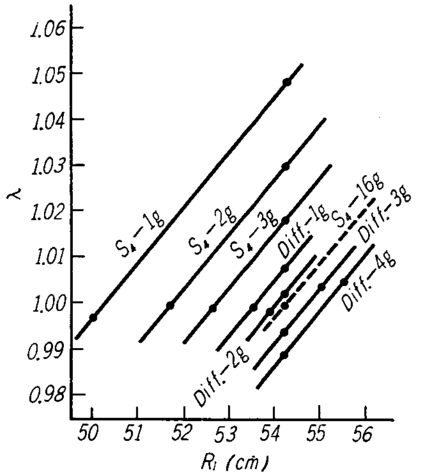

Fig. 6. Eigenvalue versus core radius in the near critical state (U-Th system).

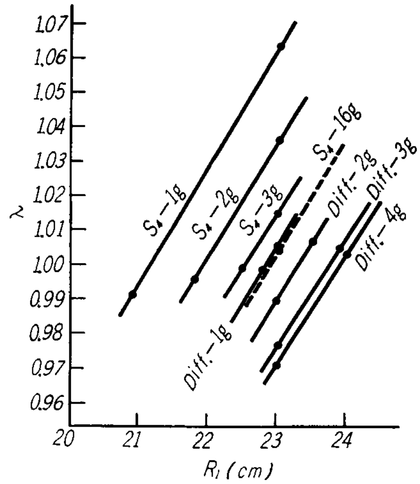

Fig. 7. Eigenvalue versus core radius in the near critical state (Pu-U system).

sixteen-group calculation. The dotted line in Figs. 6 and 7 are obtained in this manner.

The static parameters and integrated flux of REACTORs I and II, each with the dimensions of $R_1=54.2$ cm, $R_2=105$ cm and $R_1=23.0$ cm, $R_2=75.0$ cm are tabulated in Tables 4, 6 and 5, 7. The quantities associated with the critical state are given in Table 8. In these tables, the magnitude of the quantities characterizing the neutron inventory are represented relative to the strength of the fission source in the entire system which is normalized to unity.

In these tables the values in the column denoted L. O. are the results of an eleven-group calculation reported by Loewenstein and Okrent[3,4]. The composition of REACTOR I is the same as that of theirs but their system has an 800 litre core ($R_1=57.59$ cm) and a blanket thickness of 45 cm. In contrast to this, REACTOR I has a blanket thickness of 50.8 cm. The values in the parentheses of the row, "*escape from the blanket*", are obtained from neutron inventory while the values not in the parentheses are ones estimated from outward neutron current.

As will be seen from Tables 4 and 5, all the diffusion few-group calculations resulted in larger values of leakage rate. These values may be reduced somewhat by introducing the idea of extrapolated length.

For the sake of comparison, four-group cross section are tabulated in Tables 9 and 10. But these cross sections were estimated according to the following procedure. An average spectrum of the entire system was estimated by mixing the integrated fluxs of core and blanket in proportion to volume fraction. This average spectrum, in turn, was used in reducing the original sixteen-group microscopic cross sections[12] to the few-group ones.

Table 4. Static parameters for U-Th system (R_1=54.2 cm, R_2=105 cm)

region	static parameter	L.O.*	S_4-16g	S_4-3g	S_4-2g	S_4-1g	Diff.-4g	Diff.-3g	Diff.-2g	Diff.-1g
core	fuel fissions	0.386	0.376	0.384	0.388	0.395	0.372	0.374	0.378	0.380
	fuel captures	0.0326	0.0315	0.0323	0.0326	0.0331	0.0312	0.0315	0.0317	0.0318
	Th fissions	0.00836	0.00873	0.00873	0.00897	0.00916	0.00852	0.00852	0.00872	0.00880
	Th captures	0.171	0.146	0.149	0.151	0.153	0.144	0.145	0.147	0.147
	coolant (Na) captures	0.00116	0.00113	0.00117	0.00118	0.00119	0.00112	0.00114	0.00115	0.00114
	structural (Fe) captures	0.0178	0.0125	0.0128	0.0129	0.0131	0.0124	0.0125	0.0126	0.0126
	fuel captures / fuel fissions	0.0845	0.0837	0.0841	0.0839	0.0837	0.0839	0.0841	0.0340	0.0837
	fuel absorptions	0.419	0.408	0.416	0.421	0.428	0.403	0.406	0.409	0.412
	initial internal B. R.	0.408	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358
blanket	Th fissions	0.00346	0.00379	0.00378	0.00364	0.00356	0.00409	0.00409	0.00366	0.00359
	Th captures	0.310	0.351	0.342	0.335	0.329	0.343	0.340	0.336	0.332
	coolant (Na) captures	0.000304	0.000597	0.000580	0.000568	0.000560	0.000579	0.000573	0.000565	0.000565
	structural (Fe) captures	0.00896	0.00851	0.00830	0.00814	0.00799	0.00835	0.00827	0.00816	0.00805
	escape from the blanket	0.069	(0.0596)** 0.0592	(0.0578) 0.0578	(0.0566) 0.0566	(0.0533) 0.0533	(0.0731) 0.0732	(0.0738) 0.0739	(0.0742) 0.0742	(0.0748) 0.0748)
initial total B. R.		1.15	1.22	1.18	1.16	1.13	1.21	1.20	1.18	1.16

* The values in the column denoted L. O. are the results of the 11-group calculation reported by Loewenstein and Okrent (3).
** The values in parentheses are obtained from the neutron balance.

Table 5. Static parameters for Pu-U system (R_1=23.0 cm, R_2=75.0 cm)

region	static parameter	S_4-16g	S_4-3g	S_4-2g	S_4-1g	Diff.-4g	Diff.-3g	Diff.-2g	Diff.-1g
core	fuel fissions	0.254	0.258	0.265	0.272	0.244	0.245	0.252	0.256
	fuel captures	0.0358	0.0367	0.0376	0.0383	0.0346	0.0350	0.0359	0.0360
	U fissions	0.0567	0.0567	0.0586	0.0607	0.0539	0.0539	0.0555	0.571
	U captures	0.121	0.123	0.126	0.129	0.116	0.118	0.121	0.121
	coolant (Na) captures	0.000167	0.000175	0.000180	0.000179	0.000164	0.000168	0.000173	0.000169
	structural (Fe) captures	0.00286	0.00292	0.00299	0.00307	0.00276	0.00278	0.00285	0.00288
	fuel captures / fuel fissions	0.141	0.142	0.142	0.141	0.142	0.143	0.143	0.141
	fuel absorptions	0.290	0.295	0.302	0.311	0.278	0.280	0.287	0.292
	initial internal B. R.	0.416	0.419	0.419	0.416	0.418	0.419	0.420	0.416
blanket	U fissions	0.0299	0.0298	0.0284	0.0276	0.0322	0.0322	0.0288	0.0278
	U captures	0.421	0.410	0.398	0.389	0.416	0.411	0.400	0.392
	coolant (Na) captures	0.00193	0.00187	0.00182	0.00178	0.00187	0.00186	0.00181	0.00179
	structural (Fe) captures	0.0114	0.0111	0.0108	0.0105	0.0113	0.0111	0.0108	0.0106
	escape from the blanket	(0.0652)* 0.0656	(0.0703) 0.0704	(0.0701) 0.0707	(0.0669) 0.0672	(0.0878) 0.0874	(0.0888) 0.0886	(0.0914) 0.0911	(0.0935) 0.0932
initial total B. R.		1.97	1.91	1.74	1.67	1.91	1.88	1.81	1.76

* The values in parentheses are obtained from the neutron balance.

Table 6. Integrated flux for U-Th system (R_1=54.2 cm, R_2=105 cm)

region	grouping	group	L.O.*	S_4-16g	S_4-3g	S_4-2g	S_4-1g	Diff.-4g	Diff.-3g	Diff.-2g	Diff.-1g
core	4g	1	11.13	12.50				12.20			
		2	42.51	46.55				45.87			
		3	35.11	34.67				34.44			
		4	14.60	10.31				10.25			
	3g	1	11.13	12.50	12.51			12.20	12.20		
		2	77.62	81.22	82.66			80.31	80.56		
		3	14.60	10.31	10.70			10.26	10.46		
	2g	1	88.75	93.72	95.16	96.39		92.51	92.77	93.69	
		2	14.60	10.31	10.70	10.78		10.25	10.46	10.52	
	1g		103.35	104.03	105.86	10.717	109.23	102.77	103.22	104.21	104.95
blanket	4g	1	1.71	2.17				2.34			
		2	19.95	27.13				27.11			
		3	23.05	30.86				30.29			
		4	11.83	13.67				13.21			
	3g	1	1.71	2.17	2.16			2.34	2.34		
		2	43.00	57.99	56.72			57.39	56.69		
		3	11.83	13.67	13.26			13.21	13.09		
	2g	1	44.71	60.16	58.88	57.71		59.73	59.03	58.10	
		2	11.83	13.67	13.26	13.01		13.21	13.09	12.90	
	1g		56.54	73.83	72.14	70.72	69.31	72.94	72.12	71.00	69.89

* The values in the column denoted L. O. are estimated from the results of the 11-group calculation reported by Loewenstein and Okrent (3).

Table 7. Integrated flux for Pu-U system (R_1=23.0 cm, R_2=75.0 cm)

region	grouping	group	S_4-16g	S_4-3g	S_4-2g	S_4-1g	Diff.-4g	Diff.-3g	Diff.-2g	Diff.-1g
core	4g	1	9.70				9.22			
		2	13.97				13.27			
		3	10.93				10.54			
		4	2.58				2.59			
	3g	1	9.70	9.71			9.22	9.22		
		2	24.90	25.18			23.81	23.93		
		3	2.58	2.78			2.59	2.70		
	2g	1	34.60	34.89	35.77		33.03	33.15	33.93	
		2	2.58	2.78	2.88		2.59	2.70	2.80	
	1g		37.18	37.67	38.65	33.82	35.62	35.85	36.73	37.46
blanket	4g	1	7.27				7.82			
		2	26.14				26.60			
		3	37.52				37.07			
		4	20.79				20.12			
	3g	1	7.27	7.25			7.82	7.82		
		2	63.66	62.04			63.67	62.54		
		3	20.79	20.12			20.12	19.97		
	2g	1	70.94	69.29	67.36		71.49	70.35	68.17	
		2	20.79	20.12	19.57		20.12	19.97	19.46	
	1g		91.73	89.40	86.92	84.72	91.61	90.33	87.64	85.34

Table 8. Static parameters in the critical state.

static parameter	L. O.*	S_4-16g	S_4-3g	S_4-2g	S_4-1g	Diff.-4g	Diff.-3g	Diff.-2g	Diff.-1g
	^{233}U-^{232}Th system								
radius of core (cm)	57.59	54.21	52.69	51.73	50.25	55.09	54.68	54.01	53.55
volume of core (l)	800	667	613	580	531	700	685	660	643
mass of fuel (kg)	475	405	372	352	322	425	416	400	390
initial internal B. R.	0.408	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358
initial total B. R.	1.15	1.22	1.20	1.23	1.25	1.18	1.18	1.19	1.18
fuel captures/fuel fissions	0.0845	0.0837	0.0840	0.0839	0.0837	0.0840	0.0841	0.0840	0.0837
	^{239}Pu-^{238}U system								
radius of core (cm)		22.86	22.52	21.92	21.15	23.89	23.74	23.30	22.84
volume of core (l)		50.1	47.9	44.1	39.7	57.1	56.0	53.0	49.9
mass of fuel (kg)		79.0	75.6	69.7	62.6	90.2	88.5	83.7	78.8
initial internal B. R.		0.415	0.418	0.419	0.416	0.419	0.420	0.420	0.416
initial total B. R.		1.88	1.86	1.86	1.87	1.80	1.80	1.78	1.78
fuel captures/fuel fissions		0.141	0.142	0.142	0.141	0.143	0.143	0.143	0.141

* The values in the column denoted L. O. are the results of the 11-group calculation reported by Loewenstein and Okrent (1).

Table 9. Four-group cross sections for U-Th system.

material	G	E_L(MeV)	σ_t	ν	σ_{tr}	$\sigma_{n,\gamma}$	σ_{er}	$\sigma_{n,n'}$	$\sigma_{n,n'G\to G+k}$			
									k=0	1	2	3
^{233}U	1	1.35	1.86	2.81	4.76	0.031	0.014	1.350	0.114	0.929	0.307	0
	2	0.3	2.11	2.58	6.17	0.135	0.028	0.390	0.173	0.176	0.041	
	3	0.067	2.46	2.52	10.16	0.240	0.0245	0.169	0.147	0.022		
	4	0.0005	3.78	2.50	14.57	0.569	—					
^{232}Th	1	1.35	0.105	2.53	4.59	0.0568	0.0172	2.727	0.466	1.983	0.278	0
	2	0.3			5.94	0.166	0.0486	1.311	1.144	0.167	0	
	3	0.067			9.59	0.261	0.0301	0.648	0.544	0.104		
	4	0.0005			12.92	0.494	—					
Fe	1	1.35			2.06	0.0021	0.044	0.971	0.431	0.521	0.019	0
	2	0.3			2.57	0.0042	0.0915	0.060	0.028	0.032	0	
	3	0.067			3.30	0.0075	0.0667					
	4	0.0005			5.74	0.0110	—					
Na	1	1.35			1.73	0.00016	0.123	0.562	0.353	0.201	0.008	0
	2	0.3			3.40	0.00038	0.241	0.166	0.129	0.033	0.004	
	3	0.067			3.35	0.00031	0.0931					
	4	0.0005			4.88	0.00829	—					

Table 10. Four-group cross sections for Pu–U system.

material	G	E_L(MeV)	σ_f	ν	σ_{tr}	$\sigma_{n,\gamma}$	σ_{er}	$\sigma_{n,n'}$	$\sigma_{n,n'}$ G→G+k			
									k=0	1	2	3
^{239}Pu	1	0.825	1.87	3.09	5.01	0.0964	0.0201	0.973	0.336	0.452	0.185	0
	2	0.3	1.70	2.95	6.80	0.161	0.0425	0.688	0.531	0.149	0.008	
	3	0.067	1.72	2.91	9.83	0.385	0.0249	0.297	0.291	0.006		
	4	0.0005	2.20	2.90	13.81	0.932	—					
^{238}U	1	0.825	0.217	2.66	4.87	0.0929	0.026	2.129	1.034	0.935	0.160	0
	2	0.3			6.36	0.132	0.049	1.401	1.129	0.272	0	
	3	0.067			9.72	0.213	0.0293	0.757	0.599	0.158		
	4	0.0005			13.53	0.473	—	0.045	0.045			
Fe	1	0.825			2.01	0.0026	0.0606	0.595	0.299	0.182	0.114	0
	2	0.3			2.70	0.0045	0.113					
	3	0.067			3.35	0.0076	0.0700					
	4	0.0005			5.80	0.0111	—					
Na	1	0.825			2.42	0.00018	0.222	0.514	0.251	0.260	0.003	0
	2	0.3			3.51	0.00042	0.299	0.095	0.051	0.040	0.004	
	3	0.067			3.34	0.00029	0.0978					
	4	0.0005			4.96	0.00940	—					

4. Summary

In order to obtain the few-group constants for REACTORs I and II, sixteen-group neutron flux spectra were estimated from Carlson's S_4 method. The neutron spectra in the near critical state were derived. It is not necessarily important to pursue the sectra in the exact critical state because of inherent uncertainties in nuclear data. Especially, the values of ν_g are somewhat questionable and if one adopts another value for this quantity within the error of empirical data[12], the reactivity of the system will be subjected to an appreciable change.

The few-group computation gives the larger eigenvalue k_{eff}, i.e. smaller critical mass, for both S_4 and diffusion methods. These discrepancies in critical size arise from the following facts. In reducing the sixteen-group constants to the few-group ones, the above method of averaging ignored differences in effectiveness of neutrons in the various groups and subregions in core and blanket. A new method remedying this result was proposed by Novozhilov and Shikhov[13]. They took into account the neutron weighting and reduced the multi-group constants to one-group ones without asking for the spatial calculations. Their method can be applied to one-group estimation of critical size of a two-region fast reactor. However, it might be possible to extend their method to, for example, four- or three-group calculation and to a fast reactor with more than two regions.

Judging from the results obtained so far, it may well be said that four- or three-group diffusion calculation will at least be used in place of three-group S_4 computation for both large and small fast reactors. The authors have the intention of carrying out the few-group diffusion calculations for fast reactors in order to investigate and survey the change of nuclear characteristics as the burn-up of fuel material and the buil-up of its higher isotopes proceed.

In the above estimations special attention was paid to the neutron balance. A new convergence criterion imposed upon the static parameters are proposed. Of these parameters, the leakage rate of neutrons from the blanket was selected as the most sensitive medium to the degree of convergence. The computed value of breeding ratio depends upon the extent to which threse parameters converge. Therefore, in investigating a system of low breeding, such as the U–Th system, careful consideration must be given to this point.

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Nomenclature

B.R.	:	breeding ratio
E_L	:	lower boundary of energy group
$\boldsymbol{J}\cdot\boldsymbol{n}$	:	outward neutron current. $\boldsymbol{n}$ is the outward normal to the surface of blanket
k_{eff}	:	effective multiplication constant
R_1	:	radius of core
R_1^{crit}	:	critical radius of core
R_2	:	outer radius of reactor
u	:	lethargy
Δu	:	group lethargy interval
$\lambda(=k_{\text{eff}})$	:	eigenvalue
ν	:	average number of neutrons per fission
Σ_f	:	macroscopic fission cross section
Σ_a	:	macroscopic absorption cross section
σ	:	microscopic cross section in barns

σ_{tr} : transport cross section
σ_f : fission cross section
$\sigma_{n,\gamma}$: radiative capture cross section
σ_{er} : cross section for neutrons removed to the next lowest energy group by elastic scattering
$\sigma_{n,n'}$: total inelastic scattering cross section
$\sigma_{n,n'G\to G+k}$: cross section for neutrons transferred from group G to group $G+k$ by inelastic scattering
ϕ : neutron flux
Φ : integrated neutron flux
χ : fiission spectrum

(Subscripts)
g : group index for the case of sixteen-group
G : group index for the case of few-group

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