A Study on the Control Performance of Relay Control Systems Subjected to a Random Input

By

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The present problem in an on-off relay control system is to construct the operation of a controller generating the control signal which determines the on-off position in such a way that the mean squared value of the error response becomes minimum. First, when the input to the system is random with gaussian amplitude probability distribution and the control signal is of some non-linear function of the error and its derivative, the general procedure of analysis for the evaluation of the error probability density function is described by solving the Fokker-Planck equation. Second, the determination of parameters of the controller is proposed so that the mean squared error is minimized. Finally, how the results obtained here are incorporated in a predictive-controller is presented. The remainder of this paper is devoted to the analytical consideration of a simple predictor-relay control system subjected to a gaussian random input.

List of Principal Symbols

$v(t)$: a stationary random input to the system

$\varepsilon(t)$ and $x(t)$: error signal and control variable of the system respectively

$z(t)$ and $y(t)$: input and output of an on-off relay element respectively

$f(\varepsilon, \dot{\varepsilon})$: a non-linear function with respect to the error signal $\varepsilon(t)$ and it derivative $\dot{\varepsilon}(t)$

$p_2(\varepsilon, \dot{\varepsilon})$: joint probability density function of $\varepsilon(t)$ and $\dot{\varepsilon}(t)$

ψ_ε : mean squared value of the error signal $\varepsilon(t)$

$S_v(\omega)$: spectral density of the input signal $v(t)$

$V(t)$: effective white gaussian noise which is a linear conbination of the gaussian random input $v(t)$

$2D$: power of $V(t)$ per unit frequency

a, b and T: circuit parameters

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N, a_1 : clipping level of an on-off relay element
λ : adjustable parameter of a controller
t : time variable
t_p : prediction time
s : Laplace operator

1. Introduction

An on-off relay controller has originally been used in automatic control systems not only because of its simple construction but also because of the effective improvement of performance superior to linear counterparts in the sense of minimum response time.[1] Moreover, on-off relay control systems with a random input have also optimum properties in the sense of the least mean squared error whenever the magnitude of the manipulating signal may be restricted by a system constraint.[2] The present synthetical problem of an on-off relay control system is to determine the optimum switching function in such a way that the mean squared value of the error response becomes minimum. However, there seems to be no general method for the determination of the optimum switching function in the random excitation because the optimum switching function depends upon the statistical properties of the input signal as well as upon the dynamic characteristic of the controlled system. It is therefore useful to specialize to the case of a gaussian random input and the second order controlled system which simplifies the calculation enough to permit more extensive results. The most simple optimization technique is to select the parameters of the controller so that the mean squared value of the error is minimized. For this purpose, unfortunately, the application of the statistical equivalent linearization technique[3] may be inadequate, first because of its ignorance of imformation concerning the error probability distribution and second because of severe non-linearity of the switching function. Our attention must, therefore, be directed to the specification of the error probability distribution.

There are two methods of calculation of the error probability density function in closed loop, i.e.,

(1) method of Fokker-Planck equation[4]

(2) method of successive calculation of higher order cumulants[5]

In the present discusion, the former may be suitable while the later requires much tedious calculation. The principal purpose of this paper is therefore to extend the method of Fokker-Planck equation and then to discuss the operation of the controller generating the switching signal. The remainder of this paper is devoted to the consideration of a simple predictor-relay control system subjected to a gaussian random input.

2. Determination of the Error Probability Density Function

The block diagram of on-off relay control system to be considered here is shown in Fig. 1. The signal $z(t)$ which will determine the on-off position of the manipulating signal $y(t)$ depends upon the error signal $\varepsilon(t)=v(t)-x(t)$ and its first derivative $\dot{\varepsilon}(t)=d\varepsilon(t)/dt$ in a certain nonlinear fashion and $z(t)$ is denoted by

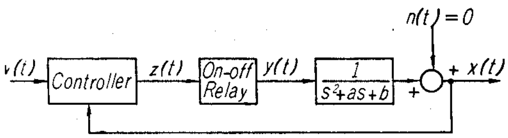

Fig. 1. An on-off relay control system subjected to a stationary random input.

$$z(t) = f(\varepsilon, \dot{\varepsilon}) , \tag{2.1}$$

where

$$\left.\begin{aligned} \varepsilon(t) &= v(t)-x(t) \\ \dot{\varepsilon}(t) &= \frac{d\varepsilon(t)}{dt} \end{aligned}\right\} . \tag{2.2}$$

The on-off relay element is characterized by

$$\begin{aligned} y(t) &= N \operatorname{sgn}[f(\varepsilon, \dot{\varepsilon})] \\ &= \begin{cases} N & \text{for } f(\varepsilon, \dot{\varepsilon}) > 0 \\ -N & \text{for } f(\varepsilon, \dot{\varepsilon}) < 0 . \end{cases} \end{aligned} \tag{2.3}$$

The control equation of the system shown in Fig. 1 is expressed as

$$\frac{d^2\varepsilon}{dt^2}+a\frac{d\varepsilon}{dt}+b\varepsilon+N \operatorname{sgn}[f(\varepsilon, \dot{\varepsilon})] = V(t) , \tag{2.4}$$

where

$$V(t) = \frac{d^2v}{dt^2}+a\frac{dv}{dt}+bv . \tag{2.5}$$

Since the function $V(t)$ is a linear combination of a gaussian random input $v(t)$, it becomes also a gaussian random function. If the auto-correlation function of $V(t)$ is given by

$$\phi_V(\tau) = 2D\delta(\tau) , \qquad D: \text{const.} \tag{2.6}$$

that is, $V(t)$ is effectively white gaussian noise, then as stated in our previous paper,[6] the Fokker-Planck equation becomes

$$0 = -\frac{\partial \dot{\varepsilon} p_2}{\partial \varepsilon}+\frac{\partial\{a\dot{\varepsilon}+b\varepsilon-N \operatorname{sgn}[f(\varepsilon, \dot{\varepsilon})]\}p_2}{\partial \dot{\varepsilon}}+D\frac{\partial^2 p_2}{\partial \dot{\varepsilon}^2} \tag{2.7}$$

from which the joint probability density function $p_2(\varepsilon, \dot{\varepsilon})$ can be obtained as a solution of Eq. (2.7).

The assumption of Eq. (2.6) for the auto-correlation function of $V(t)$ is never strictly true. However, the spectral density $S_V(\omega)$ of $V(t)$ is related to the spectral density $S_v(\omega)$ of $v(t)$ in the form

$$S_V(\omega) = |(j\omega)^2+a(j\omega)+b|^2 S_v(\omega)\ , \tag{2.8}$$

which may conceivably be flat over any range of interest. In order to solve the partial differential equation given by Eq. (2.7), the following assumptions are made the form of function $f(\varepsilon, \dot{\varepsilon})$:

(1) the existence of $f(\varepsilon, \dot{\varepsilon})=0$ only in the second- and the fourth-quadrant of the $\varepsilon-\dot{\varepsilon}$ phase plane
(2) a symmetric function with respect to the origin of the phase plane
(3) a single-valued function of both variables ε and $\dot{\varepsilon}$

From practical viewpoints, the above three assumptions can be applied to the present analysis without much loss of generality.

The solution of Eq. (2.7), $p_2(\varepsilon, \dot{\varepsilon})$, can be easily obtained as

$$p_2(\varepsilon, \dot{\varepsilon}) = K\exp\left(-\frac{a}{2D}\dot{\varepsilon}^2\right)\exp\left\{-\frac{aN}{D}\mathrm{sgn}\,[f(\varepsilon, \dot{\varepsilon})]\varepsilon-\frac{ab}{2D}\varepsilon^2\right\}, \tag{2.9}$$

where the normalized factor K is determined by

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_2(\varepsilon, \dot{\varepsilon})d\varepsilon d\dot{\varepsilon} = 1\,. \tag{2.10}$$

The first probability density function $p(\varepsilon)$ of the error signal $\varepsilon(t)$ is found by

$$p(\varepsilon) = \int_{-\infty}^{\infty} p_2(\varepsilon, \dot{\varepsilon})d\dot{\varepsilon}\,. \tag{2.11}$$

To carry out further discussion, we confine our consideration to the case where $b=0$. However, we can easily extend the present simple case to the case where $b\neq 0$ without any formal change. Eq. (2.11) becomes

$$p(\varepsilon) = K\int_{-\infty}^{\infty}\exp\left(-\frac{a}{2D}\dot{\varepsilon}^2\right)\exp\left\{-\frac{aN}{D}\varepsilon\,\mathrm{sgn}\,[f(\varepsilon, \dot{\varepsilon})]\right\}d\dot{\varepsilon}\,, \tag{2.12}$$

which can be separately evaluated corresponding to the respective half phase plane as

$$p(\varepsilon) = K\exp\left(-\frac{aN}{D}\varepsilon\right)\left[\frac{1}{2}\sqrt{\frac{2\pi D}{a}}-\int_0^{f(\varepsilon,\dot{\varepsilon})=0}\exp\left(-\frac{a}{2D}\dot{\varepsilon}^2\right)d\dot{\varepsilon}\right] \quad \text{for}\ \ f(\varepsilon, \dot{\varepsilon})>0 \tag{2.13}$$

and

$$p(\varepsilon) = K\exp\left(\frac{aN}{D}\varepsilon\right)\left[\frac{1}{2}\sqrt{\frac{2\pi D}{a}}+\int_0^{f(\varepsilon,\dot{\varepsilon})=0}\exp\left(-\frac{a}{2D}\dot{\varepsilon}^2\right)d\dot{\varepsilon}\right] \quad \text{for}\ \ f(\varepsilon, \dot{\varepsilon})<0\,. \tag{2.14}$$

From Eqs. (2.13) and (2.14), the error probability density function is written as

$$p(\varepsilon) = K\sqrt{\frac{2\pi D}{a}}\cosh\left(\frac{aN}{D}\varepsilon\right) + 2K\sinh\left(\frac{aN}{D}\varepsilon\right)\int_0^{f(\varepsilon,\dot{\varepsilon})=0}\exp\left(-\frac{a}{2D}\dot{\varepsilon}^2\right)d\dot{\varepsilon}. \quad (2.15)$$

On the other hand, from the three assumptions listed above, the switching function, $f(\varepsilon, \dot{\varepsilon})=0$, can be solved in terms of $\varepsilon(t)$ as

$$\dot{\varepsilon}(t) = -g[\varepsilon(t)]. \quad (2.16)$$

By using Eq. (2.16), Eq. (2.15) becomes

$$p(\varepsilon) = K\sqrt{\frac{2\pi D}{a}}\cosh\left(\frac{aN}{D}\varepsilon\right) - 2K\sinh\left(\frac{aN}{D}\varepsilon\right)\int_0^{g(\varepsilon)}\exp\left(-\frac{a}{2D}\dot{\varepsilon}^2\right)d\dot{\varepsilon}, \quad (2.17)$$

where the normalized factor K is determined as

$$K = \frac{aN}{4D}\cdot\frac{1}{\int_0^\infty \frac{d[g(\varepsilon)]}{d\varepsilon}\cosh\left(\frac{aN}{D}\varepsilon\right)\exp\left\{-\frac{a}{2D}g^2(\varepsilon)\right\}d\varepsilon}. \quad (2.18)$$

Eq. (2.17) can not be simplified essentially for the general case. Some typical examples are presented in the following section.

3. Evaluation of Mean Squared Error

The mean squared error ψ_ε is given by

$$\psi_\varepsilon = \int_{-\infty}^{\infty}\varepsilon^2 p(\varepsilon)d\varepsilon \quad (3.1)$$

by using the error probability density function obtained in the preceding section. It is however not feasible to go much beyond these results without specifying the form of $f(\varepsilon, \dot{\varepsilon})$. In this section, the following two typical switching functions are considered in detail.

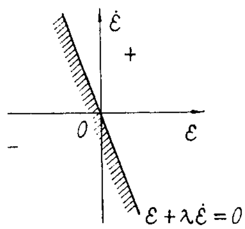

Fig. 2. Linear switching function.

Case 1 *Linear Switching Function*

The linear switching function as illustrated in Fig. 2 is given by

$$f(\varepsilon, \dot{\varepsilon}) = \varepsilon + \lambda\dot{\varepsilon} = 0, \quad (3.2)$$

where λ is an arbitrary positive constant. Since, from Eq. (3.2), we have

$$g(\varepsilon) = \frac{\varepsilon}{\lambda}, \quad (3.3)$$

$$p(\varepsilon) = K\sqrt{\frac{2\pi D}{a}}\cosh\left(\frac{aN}{D}\varepsilon\right) - 2K\sinh\left(\frac{aN}{D}\varepsilon\right)\int_0^{\varepsilon/\lambda}\exp\left(-\frac{a}{2D}\dot{\varepsilon}^2\right)d\dot{\varepsilon}, \quad (3.4)$$

where from Eq. (2.18), we have

$$K=\frac{1}{\sqrt{2\pi}}\cdot\frac{a}{D}\cdot\sqrt{\frac{a}{D}}\cdot\frac{N}{2}\exp\left(-\frac{aN^2\lambda^2}{2D}\right). \tag{3.5}$$

Substitution of Eq. (3.4) for $p(\varepsilon)$ in Eq. (3.1) gives the mean squared error as

$$\psi_\varepsilon=N^2\lambda^4-\frac{D}{a}\lambda^2+\frac{2D^2}{N^2a^2}. \tag{3.6}$$

Case 2 Quadratic Switching Function

The quadratic switching function as illustrated in Fig. 3 is given by

$$f(\varepsilon,\dot{\varepsilon})=\varepsilon+\lambda\dot{\varepsilon}|\dot{\varepsilon}|=0. \tag{3.7}$$

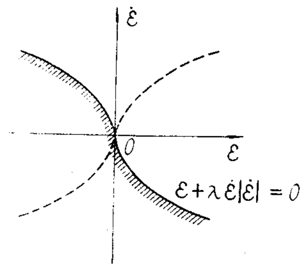

Fig. 3. Quaratic switching function.

First, we consider the case of $\varepsilon>0$. From Eq. (3.7), we have

$$g(\varepsilon)=\sqrt{\frac{\varepsilon}{\lambda}}. \tag{3.8}$$

The error probability density function can thus be obtained as

$$p(\varepsilon)=K\sqrt{\frac{2\pi D}{a}}\cosh\left(\frac{aN}{D}\varepsilon\right)-2K\sinh\left(\frac{aN}{D}\varepsilon\right)\int_0^{\sqrt{\varepsilon/\lambda}}\exp\left(-\frac{a}{2D}\dot{\varepsilon}^2\right)d\dot{\varepsilon}. \tag{3.9}$$

Next, for $\varepsilon<0$, the error probability density function can be easily obtained with the help of its symmetric form. The constant K in Eq. (3.9) is

$$K=\sqrt{\frac{a}{2\pi D}}\cdot\frac{a}{D}N\Big/\left[\frac{1}{\sqrt{1-2\lambda N}}+\frac{1}{\sqrt{1+2\lambda N}}\right]. \tag{3.10}$$

By using Eq. (3.9), the mean squared error can therefore be obtained as

$$\psi_\varepsilon=\frac{3D^2\lambda^2}{a^2}\cdot\frac{C_1^3+C_2^3}{C_1+C_2}-\frac{2D^2\lambda}{Na^2}\cdot\frac{C_1^3-C_2^3}{C_1+C_2}+\frac{2D^2}{N^2a^2}, \tag{3.11}$$

where

$$C_1=\sqrt{\frac{1}{1-2\lambda N}},\qquad C_2=\sqrt{\frac{1}{1+2\lambda N}}. \tag{3.12}$$

4. Numerical Examples

The block diagram of the nonlinear control system to be considered here is shown in Fig. 4, which is excited by a stationary gaussian random signal $v(t)$ with the spectral density

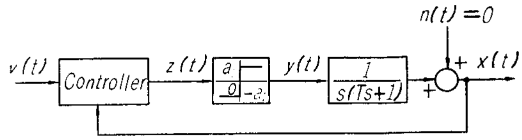

Fig. 4. An on-off relay control system subjected to a stationary random input.

$$S_v(\omega) = \frac{A}{(\omega^2+\alpha^2)(\omega^2+\beta^2)}. \quad (4.1)$$

For the convenience of analysis, assuming $\alpha^2 \ll \beta^2$, D may be expressed as

$$D = \frac{A}{2\beta}. \quad (4.2)$$

First consider the case of linear switching function, $f(\varepsilon, \dot{\varepsilon}) = \varepsilon + \lambda\dot{\varepsilon}$. By replacing a and N by $1/T$ and a_1/T in the previous results, we readily have

$$p(\varepsilon) = K\sqrt{2\pi DT}\cosh\left(\frac{a_1}{DT^2}\varepsilon\right) - 2K\sinh\left(\frac{a_1}{DT^2}\varepsilon\right)\int_0^{\varepsilon/\lambda}\exp\left(-\frac{1}{2DT}\dot{\varepsilon}^2\right)d\dot{\varepsilon} \quad (4.3)$$

where

$$K = \frac{1}{\sqrt{2\pi DT}}\cdot\frac{a_1}{2DT^2}\exp\left(-\frac{a_1^2\lambda^2}{2T^2D}\right). \quad (4.4)$$

The mean squared error is obtained as

$$\psi_\varepsilon = \frac{a_1^2}{T^2}\lambda^4 - DT\lambda^2 + \frac{2D^2T^4}{a_1^2}. \quad (4.5)$$

From Eq. (4.5), the optimum value of the parameter λ which minimizes the mean squared error becomes

$$\lambda_0 = \sqrt{\frac{DT^3}{2a_1^2}} = \frac{T}{a_1}\sqrt{\frac{DT}{2}}. \quad (4.6)$$

The error probability density function with the optimum value of λ is shown in Fig. 5 by the solid curve with the value of D as a parameter in which the dotted curve expresses the case where $\lambda=0$. Relation between λ and ψ_ε is illustrated in Fig. 6.

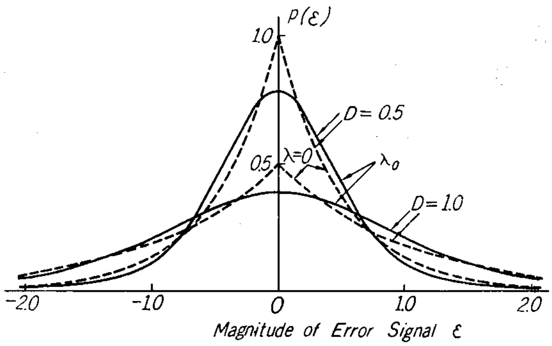

Fig. 5. Error probability density function.

By a precisely similar method, for the quadratic switching function, $f(\varepsilon, \dot{\varepsilon}) = \varepsilon + \lambda\dot{\varepsilon}|\dot{\varepsilon}|$, we have

$$p(\varepsilon) = K\sqrt{2\pi DT}\cosh\left(\frac{a_1}{DT^2}\varepsilon\right) - 2K\sinh\left(\frac{a_1}{DT^2}\varepsilon\right)\int_0^{\sqrt{\varepsilon/\lambda}}\exp\left(-\frac{1}{2DT}\dot{\varepsilon}^2\right)d\dot{\varepsilon}, \quad (4.7)$$

where

$$K = \sqrt{\frac{1}{2\pi DT}}\cdot\frac{a_1}{DT^2}\Bigg/\left[\frac{1}{\sqrt{1-\frac{2a_1\lambda}{T}}} + \frac{1}{\sqrt{1+\frac{2a_1\lambda}{T}}}\right]. \quad (4.8)$$

The mean squared error is given by

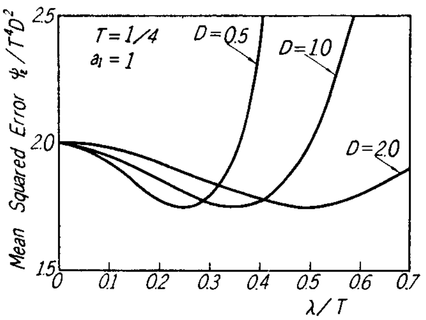

Fig. 6(a). Evaluation of mean squared error—Linear switching function.

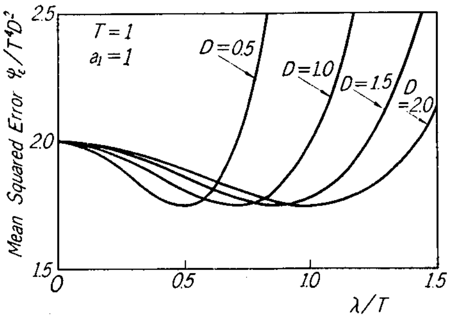

Fig. 6(b). Evaluation of mean squared error—Linear switching function.

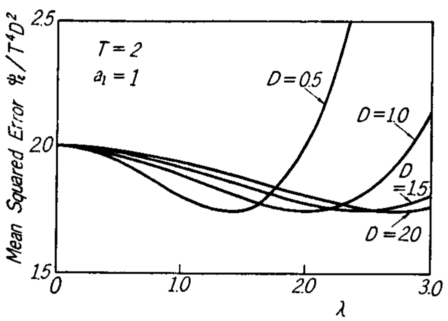

Fig. 6(c). Evaluation of mean squared error—Linear switching function.

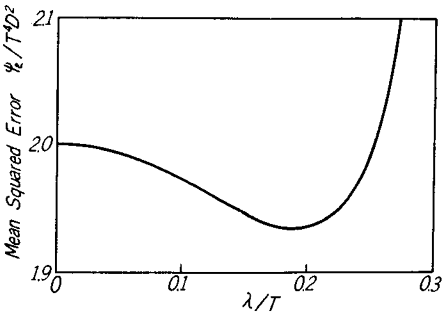

Fig. 7. Evaluation of mean squared error—Quadratic switching function.

$$\psi_\varepsilon = 3D^2T^2\lambda^2 \cdot \frac{C_1^3+C_2^3}{C_1+C_2} - \frac{2D^2T^3\lambda}{a_1} \cdot \frac{C_1^3-C_2^3}{C_1+C_2} + \frac{2D^2T^4}{a_1^2} \tag{4.9}$$

where

$$C_1 = \frac{1}{\sqrt{1-\frac{2a_1\lambda}{T}}}, \qquad C_2 = \frac{1}{\sqrt{1+\frac{2a_1\lambda}{T}}}, \tag{4.10}$$

which is illustrated in Fig. 7.

5. Predictor-Relay Control System

It should be noted here that the linear switching function, $f(\varepsilon, \dot{\varepsilon}) = \varepsilon + \lambda\dot{\varepsilon}$, is regarded as the first two terms of Taylor's expansion of the future error;

$$\varepsilon(t+\lambda) = \varepsilon(t) + \lambda\dot{\varepsilon}(t) + \frac{\lambda^2}{2}\ddot{\varepsilon}(t) + \cdots. \tag{5.1}$$

Such an important fact may suggest the possibility of the remarkable improvement of control performance by introducing a predictor with appropriate prediction time which, in general, may not be fixed. Since it is however impossible to design a predictive controller with varying predication time, a simple predictor-relay servo system as shown in Fig. 8 is considered in this section. This means the replacement of $f(\varepsilon, \dot{\varepsilon})$ by the future error $\varepsilon(t+\lambda)$ as the switching function. It is easily seen that the linear switching function

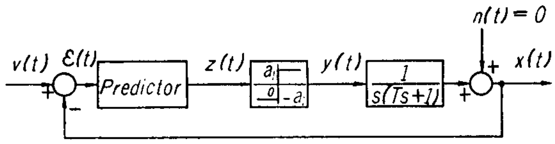

Fig. 8. A simple predictor-relay control system subjected to a stationary random input.

agrees with the predictive compensation of the Taylor's expansion type. Now we consider the predictive compensation in the Wiener sense. For the system shown in Fig. 8, when it is excited by a random signal with the spectral density given by Eq. (4.1), the spectral density of the error signal can be approximately assumed as

$$S_{\varepsilon}(\omega) = \left|\frac{1}{T(j\omega)^2+j\omega+1}\right|^2. \tag{5.2}$$

The transfer function $Y(s)$ of the pure predictor can easily be obtained as

$$Y(s) = \exp\left(-\frac{t_p}{2T}\right)\left[\cos\gamma t_p+\frac{1}{2\gamma T}\sin\gamma t_p + \frac{s}{\gamma}\sin\gamma t_p\right], \tag{5.3}$$

where t_p is the prediction time and γ is given by

$$\gamma = \sqrt{4T-1}/2T. \tag{5.4}$$

When the predictor specified by Eq. (5.3) is used as a controller in the system shown in Fig. 8, the switching function, $f(\varepsilon, \dot{\varepsilon})$, becomes

$$f(\varepsilon, \dot{\varepsilon}) = \exp\left(-\frac{t_p}{2T}\right)\left[\varepsilon(t)\cos\gamma t_p+\frac{\varepsilon(t)}{2\gamma T}\sin\gamma t_p+\frac{\dot{\varepsilon}(t)}{\gamma}\sin\gamma t_p\right]. \tag{5.5}$$

By expanding $\exp(-t_p/2T)$, $\cos\gamma t_p$ and $\sin\gamma t_p$ and rearranging the order of power of t_p, Eq. (5.5) can be expressed as

$$\varepsilon(t+t_p) \fallingdotseq \varepsilon(t)+t_p\dot{\varepsilon}(t)-\frac{t_p^2}{2}\left[\frac{\varepsilon(t)}{T}-\frac{\dot{\varepsilon}(t)}{T}\right]+\cdots. \tag{5.6}$$

It is interesting that Eq. (5.6) can be considered as the approximated expression of the first three terms in Eq. (5.1) by replacing $\ddot{\varepsilon}(t)$ in Eq. (5.1) by $-[\varepsilon(t)/T-\dot{\varepsilon}(t)/T]$.

From Eq. (5.5), it follows that

$$f(\varepsilon, \dot{\varepsilon}) = \lambda_1\varepsilon(t)+\lambda_2\dot{\varepsilon}(t), \tag{5.7}$$

where

$$\left.\begin{aligned}\lambda_1 &= \exp\left(-\frac{t_p}{2T}\right)\left[\cos \gamma t_p + \frac{1}{2\gamma T}\sin \gamma t_p\right] \\ \lambda_2 &= \exp\left(-\frac{t_p}{2T}\right)\cdot\frac{\sin \gamma t_p}{\gamma}\end{aligned}\right\}. \tag{5.8}$$

Thus, in the present example, the predictor in the relay servo system serves the same effect as the linear switching function described above and the evaluation of the control performance can be carried out in the same manner as described in the preceding sections. The numerical results are illustrated in Figs. 9 and 10.

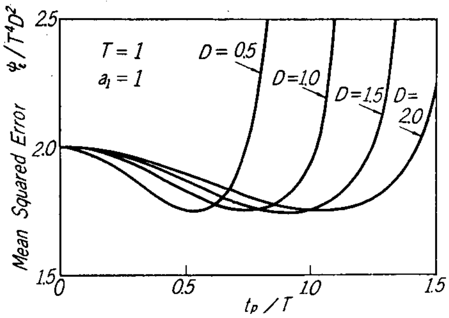

Fig. 9(a). Evaluation of mean squared error—Case of predictive controller.

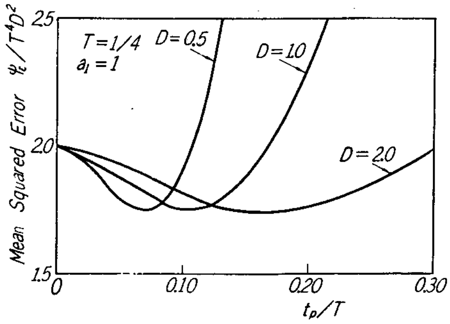

Fig. 9(b). Evaluation of mean squared error—Case of predictive controller.

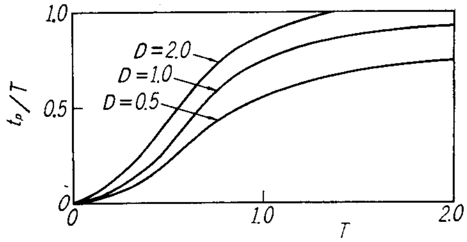

Fig. 10. Relation between the optimum prediction time and the time constant of the controlled system.

We have pointed out that the linear predictive controller reduces to the linear switching function. However it is true that the optimum predictor must have a non-linear characteristic because the error signal is of the non-gaussian probability distribution corresponding to the on-off relay element and the dynamic characteristic of the controlled system. Moreover, we should take the future error derivatives into account. Therefore, although the system shown in Fig. 8 may not be optimum, we can present the possibility of a predictive controller in a relay system from on analytical point of view.

6. Conclusion

In the relay control system, the optimum switching function depends not only upon statistical properties of the input signal but also upon the dynamic characteristics of the controlled system. In this paper, the general procedure of the statistical evaluation of the control performance is presented in the case of a second order controlled system with a random input. Linear and quadratic switching functions are also discussed in detail from a certain synthetical viewpoint. From results of the analytical study, the possibility of improvement of the control performance by introducing a predictor with the appropriate prediction time corresponding to the input level is considered.

Although our discussions are confined to the second order control system, it is clear that a similar set of considerations in this paper can be applied to any problem in both system analysis and system optimization.

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