# A Study of Economical Machining
## (An Analysis of the Maximum-Profit Cutting Speed)

By

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Apart from the conventional theory of the minimum-cost or maximum-production cutting speed, a new concept of the machining conditions for maximizing the profit for the manufacturing enterprise was presented.

## 1. Introduction

The first step for economical machining is to select the most suitable type of machine tool for the production purpose. The next problem is to discuss the economics of machining itself, and fast and low-cost tooling should be considered. Regarding the optimal machining conditions, Gilbert** and other investigators analyzed the minimum-cost cutting speed at which the production cost per piece of products is minimum and the maximum-production cutting speed at which the rate of production is maximum.

In this paper, apart from the above concepts, the authors present a new concept of the maximum-profit machining conditions at which the profit for manufacturing enterprise becomes maximum. Based upon this concept, the economics of machining was analyzed and developed. The expression for the maximum-profit cutting speed was deduced theoretically.

## 2. A Concept of the Maximum-Profit Machining Conditions

Gilbert** and other investigators analyzed the machining conditions for the minimum-cost per piece of products, especially the minimum-cost cutting speed. At this speed, the production cost per piece including idle and loading costs, cutting cost, tool changing cost, tool cost, tool regrinding cost, etc., is minimum,

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** W.W. Gilbert : Economics of Machining ; Machining—Theory and Practice, American Society for Metals, Cleveland, Ohio, 1950, pp. 465–485 ; Machining with Carbides and Oxides, American Society of Tool and Manufacturing Engineers, McGraw-Hill Book Co., New York, N.Y., 1962, pp. 500–516.

and the machining at this speed is the most economical. When the production quantity is limited, this is true, since the profit the manufacturing enterprise obtains is maximum when machining at this speed. However, in the case of continuous mass production the profit obtained when machining at the minimum-cost cutting speed is not always maximum. This is because the production rate is small at a certain constant time interval when machining at this speed, and so the profit obtained is small even if all the products are sold. In this case, it is better that the production rate is increased by increasing the cutting speed, then the profit is increased even if the wear rate of the cutting tool, hence the total expense per piece of product, is increased. Thus, aiming at the profit after sales of products rather than the production cost itself, the authors try to theoretically analyze the profit per piece produced and maximize it. Increasing the cutting speed to the utmost, the tool cost and the tool changing cost are greatly increased due to a marked increase of wear of a cutting tool. Hence, the optimal cutting speed for the maximum profit should exist. Expanding this idea, the machining conditions for the maximum profit are determined.

In the following section, the maximum-profit cutting speed, which is the most important factor for the machining conditions, was theoretically analyzed.

### 3. An Analysis of the Maximum-Profit Cutting Speed

In the case where a turning is run in a lathe with a single point tool, the maximum-profit cutting speed at which the profit per piece of product or sales good in a certain constant time interval is obtained theoretically in the following.

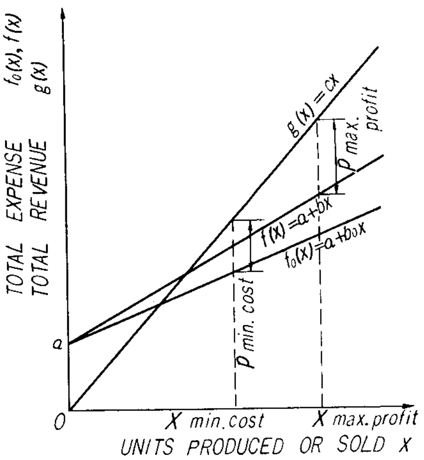


Fig. 1. Analysis of the maximum-profit machining conditions

Fig. 1 represents the comparison between profits obtained at the conventional minimum-cost cutting speed and at the maximum-profit cutting speed in question. In this break-even chart, $f_0(x)$ and $f(x)$ are the total expense curves at the minimum-cost cutting speed and at the maximum-profit cutting speed, respectively, and $g(x)$ is the total revenue curve. Assuming linearity for these three curves,

$$f_0(x) = a + b_0 x \qquad [1]$$

$$f(x) = a + bx \qquad [2]$$

$$g(x) = cx \qquad [3]$$

where $x$ is a variable expressing units produced or sold and $c$ is selling price.

$a$ is a fixed cost including cost of machine tool, its setup and depreciation costs, indirect cost, and others. This value is the same for machining at both minimum-cost and maximum-profit cutting speeds. The second term in Eqs. [2] and [3] is a variable cost. $b_0$ and $b$, variable costs per piece produced at the minimum-cost and the maximum-profit cutting speeds, are not the same. A variable cost per piece produced is simply considered to be the sum of the material cost and production cost. The material cost is the same for both cutting speeds, but the production cost for the minimum-cost cutting speed is smaller than that for the maximum-profit cutting speed. Hence, $b_0$ (for $V_{\text{min. cost}}$)$<b$ (for $V_{\text{max. profit}}$). Therefore, the slope of the total expense curve is smaller for the minimum-cost cutting speed than for the maximum-profit cutting speed. It is clear, however, that the production quantity is larger for the maximum-profit cutting speed than for the minimum-cost cutting speed: $x_{\text{min. cost}} < x_{\text{max. profit}}$. Therefore, the profit is greater for the maximum-profit cutting speed than for the minimum-cost cutting speed: $p_{\text{min. cost}} < p_{\text{max. profit}}$, as shown in Fig. 1.

In analyzing the maximum-profit cutting speed $V_{\text{max. profit}}$, the profit for units produced or sold $x_0$ in a certain constant time interval $t_0$ is obtained from Eqs. [2] and [3] as follows:

$$p = g(x_0) - f(x_0) = (c-b)x_0 - a \qquad [4]$$

In this expression, the fixed cost $a$ and the selling price per piece $c$ are considered to be constants. But the variable cost per piece $b$ and units produced $x_0$ in a constant time interval depend upon the cutting speed. Hence, both $b$ and $x_0$ should be determined in terms of cutting speed in order to deduce the cutting speed maximizing the profit expressed in Eq. [4].

In obtaining units produced $x_0$ in the time interval $t_0$ as a function of cutting speed, factors relating to the total production time per piece produced are as follows:

( i ) Preparation time ...... This is the time necessary to prepare for machining, such as loading and unloading of workpieces, approach of a cutting tool to the workpiece, etc., and independent of cutting speed. It is denoted as $t_p$ (min/pc).

(ii) Machining time ...... This is the time necessary for machine handling. It decreases with an increase in cutting speed. In a simple case that a billet, $L$ (cm) long and $D$ (cm) in diameter, is machined in a lathe with cutting speed $V$ (m/min) and feed rate $f$ (mm/rev.), the machining time per piece $t_m$ (min/pc) is expressed

$$t_m = \frac{\pi DL}{10fV} = \frac{\lambda}{V}, \tag{5}$$

where
$$\lambda = \frac{\pi DL}{10f} \tag{6}$$

(iii) Tool changing time ...... This is the time necessary to exchange a worn cutting tool or insert for a new one. Increasing cutting speed decreases the tool life, hence increases the tool changing time. Denoting the tool life as $T$(min) and the tool changing time per tool as $t_c$(min/pc), the tool changing time per piece produced $t_e$(min/pc) is expressed

$$t_e = \frac{t_m}{T} t_c, \tag{7}$$

since the machining time per piece is $t_m$(min/pc).

Generally, the tool life $T$(min) decreases with an increase in cutting speed $V$(m/min). Assuming the Taylor tool-life equation,

$$VT^n = C, \tag{8}$$

where $C$ and $n$ are constants depending on work material, tool material, tool geometry, machine tool used, cutting conditions, and others.

From Eqs. [5] and [8], Eq. [7] is:

$$t_e = \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}} t_c \tag{9}$$

Therefore, the total production time per piece produced $t_r$(min/pc) is expressed as follows:

$$t_r = t_p + t_m + t_e = t_p + \frac{\lambda}{V} + \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}} t_c \tag{10}$$

Production quantity $x_0$ in a constant time interval $t_0$ is given:

$$x_0 = \frac{t_0}{t_r} = \frac{t_0}{t_p + \frac{\lambda}{V} + \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}} t_c} \tag{11}$$

The variable cost per piece produced $b$(¥/pc) is considered to be the sum of the material cost per piece $m$(¥/pc) and the production cost per piece $c_r$(¥/pc). Factors relating to the production cost per piece are: (refer to factors relating to the production time mentioned previously.)

(i) Preparation cost

$$c_p = k_d t_p, \tag{12}$$

where $k_d$ is direct labor cost plus overhead rate (¥/min).

(ii) Machining cost

$$c_m = k_d t_m = k_d \frac{\lambda}{V}, \quad \text{(from Eq. [5])} \qquad [13]$$

(iii) Tool changing cost

$$c_e = k_d t_e = k_d \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}} t_c, \quad \text{(from Eq. [9])} \qquad [14]$$

(iv) Tool cost ...... This is a cost per cutting edge of a tool. It includes purchasing and depreciation costs of the tool, direct labor and overhead costs for regrinding worn tools, setup and depreciation costs for the tool grinder and the grinding wheel, and others. Since wear rate of a cutting tool increases with cutting speed, this cost increases with cutting speed. Denoting this cost per cutting edge as $k_t$ (¥/tool), tool cost per piece produced $c_t$ (¥/pc) is given as follows:

$$c_t = k_t \frac{t_m}{T} = k_t \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}}, \quad \text{(from Eqs. [5] and [8])} \qquad [15]$$

Therefore, the total production cost per piece is obtained from Eqs. [12] through [15].

$$c_r = c_p + c_m + c_e + c_t = k_d t_p + k_d \frac{\lambda}{V} + k_d \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}} t_c + k_t \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}} \qquad [16]$$

Since the sum of this cost $c_r$ (¥/pc) and the material cost $m$ (¥/pc) is the variable cost per piece $b$ (¥/pc),

$$b = m + c_r = m + k_d t_p + k_d \frac{\lambda}{V} + k_d \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}} t_c + k_t \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}} \qquad [17]$$

Using Eqs. [11] and [17], Eq. [4] is expressed as follows:

$$p = \frac{c - m - k_d t_p - k_d \frac{\lambda}{V} - k_d \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}} t_c - k_t \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}}}{t_p + \frac{\lambda}{V} + \frac{\lambda V^{\frac{1}{n}-1}}{C^{\frac{1}{n}}} t_c} t_0 - a \qquad [18]$$

Thus, the profit per piece is expressed in the form of a function of the cutting speed, the maximum-profit cutting speed is determined by differentiating the above equation with respect to $V$ and setting it equal zero:

$$\frac{dp}{dV} = 0$$

Thus,

$$(1-n)[k_t t_p + (c-m) t_c] V_{\text{max. profit}}^{\frac{1}{n}} + \lambda k_t V_{\text{max. profit}}^{\frac{1}{n}-1} - n(c-m) C^{\frac{1}{n}} = 0 \qquad [19]$$

This is a formulation for the maximum-profit cutting speed. It is difficult to explicitly express $V_{\text{max. profit}}$. However, denoting the tool life for this speed as $T_{\text{max. profit}}$, from [8]

$$V_{\text{max. profit}}^{\frac{1}{n}} = \frac{C^{\frac{1}{n}}}{T_{\text{max. profit}}}, \tag{20}$$

Substituting this into Eq. [19],

$$V_{\text{max. profit}} = \frac{\lambda k_t}{n(c-m)T_{\text{max. profit}}-(1-n)[k_t t_p+(c-m)t_c]} \tag{21}$$

It is found from Eqs. [19] and [21] that the maximum-profit cutting speed depends on tool cost $k_t$, constants $C$ and $n$ determining the tool-life equation, preparation time $t_p$, size of the workpiece $D \times L$, feed rate $f$, selling price $c$, and material cost $m$.

When a constant $n$ expressing the slope of the tool-life curve becomes certain particular values, the maximum-profit cutting speed is determined in simple explicit forms as follows:

Using symbols:

$$\alpha \equiv k_t t_p+(c-m)t_c \tag{22}$$

$$\beta \equiv \lambda k_t \tag{23}$$

$$\gamma \equiv c-m \tag{24}$$

Eq. [19] is

$$(1-n)\alpha V_{\text{max. profit}}^{\frac{1}{n}} + \beta V_{\text{max. profit}}^{\frac{1}{n}-1} - n\gamma C^{\frac{1}{n}} = 0 \tag{25}$$

$n=0.50$ (for ceramic tool):

$$\alpha V_{\text{max. profit}}^2 + 2\beta V_{\text{max. profit}} - \gamma C^2 = 0 \tag{26}$$

$$V_{\text{max. profit}} = \frac{\sqrt{\beta^2+\alpha\gamma C^2}-\beta}{\alpha} \tag{27}$$

$n=0.33$ (for ceramic and carbide tools):

$$2\alpha V_{\text{max. profit}}^3 + 3\beta V_{\text{max. profit}}^2 - \gamma C^3 = 0 \tag{28}$$

$$V_{\text{max. profit}} = \sqrt[3]{A} + \sqrt[3]{B} - \frac{\beta}{2\alpha} \tag{29}$$

$$A, B = \frac{1}{4\alpha}\left(\gamma C^3 - \frac{\beta^3}{2\alpha^2} \pm \sqrt{\gamma^2 C^6 - \frac{\beta^3 \gamma C^3}{\alpha^2}}\right) \tag{30}$$

$n=0.25$ (for carbide tool):

$$3\alpha V_{\text{max. profit}}^4 + 4\beta V_{\text{max. profit}}^3 - \gamma C^4 = 0 \tag{31}$$

$$V_{\text{max. profit}} = \begin{cases} \dfrac{1}{3\alpha}\left(-\beta+\sqrt{3\alpha\kappa+\beta^2} \pm \sqrt{2\beta^2-3\alpha\kappa+\dfrac{2\beta^3}{\sqrt{2\alpha\kappa+\beta^2}}}\right) \\ \dfrac{1}{3\alpha}\left(-\beta-\sqrt{3\alpha\kappa+\beta^2} \pm \sqrt{2\beta^2-3\alpha\kappa-\dfrac{2\beta^3}{\sqrt{3\alpha\kappa+\beta^2}}}\right) \end{cases} \tag{32}$$

$$\kappa = \sqrt[3]{M} + \sqrt[3]{N} \tag{33}$$

$$M, N = \frac{\gamma C^4}{8}\left(-\beta^2 \pm \sqrt{\beta^4 + \alpha^3 \gamma C^4}\right) \tag{34}$$

## 4. Conclusion

Aiming at the maximum-profit in a constant time interval, the optimal cutting speed was analyzed and formulated. Machining at this cutting speed is the most economical for mass production. Expanding this analysis and deducing the optimal feed rate and depth of cut, the optimal machining conditions for maximizing the profit for manufacturing enterprise, i.e., the maximum-profit machining conditions are determined for economical machining.