A Study of the Transfer Function of Chopper Amplifiers

By

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The chopper amplifier is frequently used in several types of *d.c* and *a.c* amplifiers as well as in *d.c* servomechanisms employing *a.c* control elements.

In this paper the transfer function of the chopper amplifier is derived for a sinusoidal input signal. The chopper amplifier has a chopper, or, a contact modulator at the input element of an *a.c* coupled amplifier to provide a means of converting *d.c* and low frequency signals to signals which lie within the pass band of the amplifier. Therefore, we first consider on the general analysis of the circuits containing a periodically operated switch such as a chopper. This analytical method is based on the theories of the Periodically Interrupted Electric Circuits and the complex Fourier series. The demodulation of the amplified signal is obtained by employing a synchronizing rectifier circuit, or a 2-phase induction motor at the output of the amplifier.

In these systems we can easily derive the transfer function of some types of chopper amplifiers and of which the steady-state performance can be clearly stated in detail.

1. Introduction

Choppers or contact modulators play an important role in many circuit designs where modulation (or demodulation) at a fixed frequency is required.

Their advantages for this service include extremely high ratio of open-circuit to closed-circuit resistance, almost negligible internal noise and drift, and excellent stability and reproducibility.

These properties make the chopper particularly suitable for use in several types of d.c and a.c amplifiers as well as in d.c servomechanism employing a.c control elements.

As the input element of an a.c coupled amplifier, the chopper modulator, which is driven at a fairly low audio rate such as a 60 c/s, converts d.c and low-frequency signals to signals which could be applied to the conventional amplifier. After one or more stages of voltage amplification, the signal may be demodulated in a second converter such as a synchronizing rectifier circuit to produce a high level output having essentially the input wave form.

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There are various applications of the chopper amplifier stated above, for example, the amplification of very small direct voltages with a minimum of drift such as a d.c microvoltmeter, the operational amplifier in the analog computer, and the elements of the carrier-type servomechanism in which the demodulation is usually obtained by making use of a 2-phase induction motor.

In recent years, several results[1~4] have served to clarify the performance of the chopper amplifiers. Especially the analysis of the circuits[5] containing a periodically operated switch such as the chopper-modulated circuits has been progressed as well as the study on the time division multiplex system and on the theory of the time varying parameter circuit.

However, these procedures may be not sufficient to show the over-all characteristics, the steady-state and transient characteristics of the chopper amplifier.

Fig. 1. Chopper amplifier.

The present paper introduces an approach[6] for the analysis of the chopper amplifier shown in Fig. 1. For the purposes of this analysis, first the chopper-modulated circuits, which are the fundamental circuits in the configurations of the chopper amplifier, are discussed more generally on the basis of the theories of the Periodically Interrupted Electric Circuits[7] and next in order to show the steady-state behaviour, the transfer function of the chopper amplifier is derived for a sinusoidal input signal through some approximations of neglecting the higher harmonics at the output of the synchronizing rectifier circuit in the chopper amplifier.

2. Analysis of the Chopper-Modulated Circuit

As was mentioned above, the chopper-modulated circuit plays an important role of the modulation which converts input signals to signals which lin within the pass band of the amplifier.

Now let us consider the general form of the chopper-modulated circuit containing one chopper and the passive network with l storage elements of L, C shown in Fig. 2, where the chopper operates ideally taking two modes (close and open) as illustrated in Fig. 3.

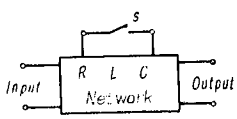

Fig. 2. Chopper-modulated circuit.

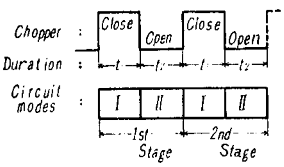

Fig. 3. Chopper operation and circuit modes.

Hence the input signal is supposed to be the voltage signal of the form as a function of the frequency ω.

$$E(t) = e^{j\omega t} \tag{1}$$

which is connected with a loop containing the j-th element of the l storage elements of L, C when the chopper is close (the 1st circuit mode), while which is connected with a loop containing the j'-th element of that when the chopper is open (the 2nd circuit mod). Then the differential equations can be set up in the matrix form, for the 1st circuit mode

$$\begin{pmatrix} z_{111} & z_{112} & \cdots & z_{112} \\ z_{121} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ z_{1l1} & \cdots & \cdots & z_{1ll} \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_j(t) \\ \vdots \\ y_l(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ e^{j\omega t} \\ \vdots \\ 0 \end{pmatrix} \tag{2}$$

and for the 2nd circuit mode

$$\begin{pmatrix} z_{211} & z_{212} & \cdots & z_{21l} \\ z_{221} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ z_{2l1} & \cdots & \cdots & z_{2ll} \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_{j'}(t) \\ \vdots \\ y_l(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ e^{j\omega t} \\ \vdots \\ 0 \end{pmatrix} \tag{3}$$

where $y_1(t), \cdots, y_l(t)$ represent the unknown currents or voltages such that if the element is a inductance L, $y(t)$'s is a current flowing through the inductance L, and if the element is a capacitance C, $y(t)$'s is a voltage at terminals of the capacitance C. The origin of time is placed at time when each circuit mode begins. z_1's and z_2's are given by

$$\begin{aligned} z_{rij} &= L\frac{d}{dt}+R \\ &= C\frac{d}{dt}+Q \qquad (r=1, 2, \quad i, j=1, 2, \cdots, l) \end{aligned} \tag{4}$$

or

$$\begin{aligned} z_{rij} &= 0 \\ &= \pm 1. \end{aligned}$$

By use of the Laplace transformation*, Eqs. (2) and (3) become respectively

* $F(p)=p\int_0^\infty f(t)\,e^{-pt}\,dt.$

$$\begin{pmatrix} Y_1(p) \\ Y_2(p) \\ \vdots \\ \vdots \\ Y_l(p) \end{pmatrix} = \begin{pmatrix} Z_{111}(p) & Z_{112}(p) & \cdots & Z_{11l}(p) \\ Z_{121}(p) & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ Z_{1l1}(p) & \cdots & \cdots & Z_{1ll}(p) \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \frac{p}{p-j\omega} \\ \vdots \\ 0 \end{pmatrix} + p \begin{pmatrix} Z_{111}(p) & Z_{112}(p) & \cdots & Z_{11l}(p) \\ Z_{121}(p) & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ Z_{1l1}(p) & \cdots & \cdots & Z_{1ll}(p) \end{pmatrix}^{-1} \cdot \begin{pmatrix} z^0_{111} & z^0_{112} & \cdots & z^0_{11l} \\ z^0_{121} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ z^0_{1l1} & \cdots & \cdots & z^0_{1ll} \end{pmatrix} \begin{pmatrix} y_1^{-0} \\ y_2^{-0} \\ \vdots \\ \vdots \\ y_l^{-0} \end{pmatrix} \tag{5}$$

and

$$\begin{pmatrix} Y_1(p) \\ Y_2(p) \\ \vdots \\ \vdots \\ Y_l(p) \end{pmatrix} = \begin{pmatrix} Z_{211}(p) & Z_{212}(p) & \cdots & Z_{21l}(p) \\ Z_{21l}(p) & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ Z_{221}(p) & \cdots & \cdots & Z_{2ll}(p) \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \frac{p}{p-j\omega} \\ \vdots \\ 0 \end{pmatrix} + p \begin{pmatrix} Z_{211}(p) & Z_{212}(p) & \cdots & Z_{21l}(p) \\ Z_{221}(p) & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ Z_{2l1}(p) & \cdots & \cdots & Z_{2ll}(p) \end{pmatrix}^{-1} \cdot \begin{pmatrix} z^0_{211} & z^0_{212} & \cdots & z^0_{21l} \\ z^0_{221} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ z^0_{1l1} & \cdots & \cdots & z^0_{2ll} \end{pmatrix} \begin{pmatrix} y_1^{-0} \\ y_2^{-0} \\ \vdots \\ \vdots \\ y_l^{-0} \end{pmatrix} \tag{6}$$

where z^0_{rij} are constants composed of L, C and 0, and y^{-0}'s are initial values of $y(t)$'s in each circuit mode at time $t=0$.

Now for the sake of abbreviation, Eqs. (5) and (6) can be written, respectively,

$$[Y(p)] = [\phi_1(p)] + [X_1(p)][y^{-0}] \tag{7}$$

and

$$[Y(p)] = [\phi_2(p)] + [X_2(p)][y^{-0}]. \tag{8}$$

By use of the inverse Laplace transformation**, Eqs. (7) and (8) are transformed into the time function, respectively, as, for the 1st circuit mode (chopper s: close)

$$[y(t)] = [\varphi_1(t)] + [x_1(t)][y^{-0}] \tag{9}$$

and for the 2nd circuit mode (chopper s: open)

$$[y(t)] = [\varphi_2(t)] + [x_2(t)][y^{-0}]. \tag{10}$$

In these equations the origin of time is at time when each circuit mode

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** $f(t) = \frac{1}{2\pi i} \lim_{\beta \to \infty} \int_{r-i\beta}^{r+i\beta} \frac{F(p)}{p} e^{pt}\, dp.$

begins, here for generality, we change it to the point that the 1st circuit mode in the 1st stage begins, and we suppose the input voltage $E(t)$ to be

$$E(t) = Ee^{j(\omega t+\theta)}. \tag{11}$$

By the change of the orign of time, the input $E(t)$ is rewritten as

$$\left.\begin{aligned} &E(t) = Ee^{j(\omega nT+\theta)}\cdot e^{j\omega(t-nT)} \\ \text{and} \\ &E(t) = Ee^{j(\omega \overline{nT+t_1}+\theta)}\cdot e^{j\omega(t-nT-t_1)} \end{aligned}\right\} \tag{12}$$

which can be supposed to be applied at $t=nT$ and at $t=nT+t_1$, respectively,

where $T=t_1+t_2=2\pi/\omega_c$: one period of the chopper s,
t_1 : duration of the chopper s closed,
t_2 : duration of the chopper s opened,
$n=0, 1, 2, \cdots\cdots\cdots,$.

Therefore the solutions of Eqs. (9) and (10) may be rewritten as follows to the input $E(t)$ of the form shown in Eq. (11) during the n-th stage, respectively,

$$[y(t)] = Ee^{j(\omega nT+\theta)}[\varphi_1(t-nT)]+[x_1(t-nT)][y(nT)] \qquad nT\leq t\leq nT+t_1 \tag{13}$$

and

$$[y(t)] = Ee^{j(\omega nT+t_1+\theta)}[\varphi_2(t-nT-t_1)]+[x_2(t-nT-t_1)][y(nT+t_1)] \qquad nT+t_1\leq t\leq nT+T. \tag{14}$$

To determine the unknown matrices $[y(nT)]$ and $[y(nT+t_1)]$, setting $t=nT+t_1$ in Eq. (13), we have

$$[y(nT+t_1)] = Ee^{j(\omega nT+\theta)}[\varphi_1(t_1)]+[x_1(t_1)][y(nT)]. \tag{15}$$

Substituting this value into Eq. (14) and setting $t=nT+T$, we have

$$[y(nT+T)] = Ee^{j(\omega nT+\theta)}[H]+[B][y(nT)] \tag{16}$$

where

$$\left.\begin{aligned} [H] &= e^{j\omega t_1}[\varphi_2(t_2)]+[x_2(t_2)][\varphi_1(t_1)] \\ [B] &= [x_2(t_2)][x_1(t_1)]. \end{aligned}\right\} \tag{17}$$

Hence we find that Eq. (16) is the difference equation having the difference T with respect to nT. The solution $[y(nT)]$ of this equation is easily obtained according to the general method for the difference equation, described in Appendix 1, then we have

$$[y(nT)] = [B]^n[I]+Ee^{j(\omega nT+\theta)}\{e^{j\omega T}[U]-[B]\}^{-1}[H] \tag{18}$$

where $[U]$ is a unit matrix and the initial value matrix $[I]$ is given by

$$[I] = [y(0)]-Ee^{j\theta}\{e^{j\omega T}[U]-[B]\}^{-1}[H]. \tag{19}$$

The matrix $[B]^n$ is calculable by use of the Sylvester expansion theorem (see Appendix 2) as follows.

$$[B]^n = \sum_{r=1}^{l} a_r^u \frac{\prod_{\substack{s=1,\cdots,l \\ r\neq s}} \{a_s[U]-[B]\}}{\prod_{\substack{s=1,\cdots,l \\ r\neq s}} (a_s-a_r)} \tag{20}$$

where $a_1, \cdots, a_l$ are the distinct latent roots of $[B]$.

From Eqs. (16) and (18), we can determine the unknown matrices, therefore the voltage and current of the R.L.C network in any circuit modes of any stages are calculated by Eqs. (13) and (14) step by step, starting with arbitrary initial conditions.

The stability of this system could be treated by examining the latent roots of $[B]$ as follows[7].

Now assume that the maximum absolute value of the latent roots $a_1, \cdots, a_l$ is a_i, then if $|a_i|\geq 1$, the circuit is unstable and if $|a_i|<1$, the circuit is stable.

Next consider the case of the steady-state phenomena for a sufficiently great value of n when the circuit is stable.

From Eq. (20), we have

$$\lim_{n\to\infty}[B]^n = 0\,. \tag{21}$$

In this case Eq. (18) becomes

$$[y(nT)] = Ee^{j(\omega nT+\theta)}\{e^{j\omega T}[U]-[B]\}^{-1}[H] \tag{22}$$

and substituting Eqs. (15) and (22) into Eqs. (13) and (14) yield finally

$$[y(t)] = Ee^{j(\omega nT+\theta)}([\varphi_1(t-nT)]+[x_1(t-nT)]\{e^{j\omega T}[U]-[B]\}^{-1}[H]) \qquad nT\leq t\leq nT+t_1 \tag{23}$$

and

$$[y(t)] = Ee^{j(\omega nT+\theta)}(e^{j\omega T_1}[\varphi_2(t-nT-t_1)]+[x_2(t-nT-t_1)]\{[\varphi_1(t_1)]+[x_1(t_1)] \cdot\{e^{j\omega T}[U]-[B]\}^{-1}[H]\}) \qquad nT+t_1\leq t\leq (n+1)T\,. \tag{24}$$

Hence in order to obtain the components of the different frequency in the solutions, the steady-state solution may be expressed in the matrix Fourier series expansion

$$[y(t)] = Ee^{j(\omega t+\theta)}\sum_{n=-\infty}^{\infty}[K_m]e^{jm\omega_c t} \tag{25}$$

where $[K_m]$ is the $1\times l$ column matrix having the elements $K_{m1}, K_{m2}, \cdots, K_{ml}$ defined as follows and the harmonics of the input signal frequency do not appear in the solutions which are linealy related to the input.

Setting

$$\frac{[y(t)]}{Ee^{j(\omega t+\theta)}} = [g(t)] = \sum_{m=-\infty}^{\infty} [K_m] \tag{26}$$

we have

$$[K_m] = \frac{1}{T}\int_{nT}^{(n+1)T} [g(t)]e^{-jm\omega_c t}dt. \tag{27}$$

From Eqs. (23) and (24), the matrix $[K_m]$ is rewritten in the form

$$[K_m] = \frac{1}{T}\left\{\int_0^{t_1}([\varphi_1(t)]+[x_1(t)][D])e^{-j(\omega+m\omega_c)t}dt+\kappa\int_0^{t_2}(e^{j\omega t_1}[\varphi_2(t)] + [x_2(t)][P])e^{-j(\omega+m\omega_c)t}dt\right\} \tag{28}$$

where

$$\left.\begin{aligned}[D] &= \{e^{j\omega T}[U]-[B]\}^{-1}[H]\\ [P] &= [\varphi_1(t_1)]+[x_1(t_1)]\{e^{j\omega T}[U]-[B]\}^{-1}[H]\\ \kappa &= e^{-j(\omega+m\omega_c)t_1}\end{aligned}\right\}$$

Substituting Eq. (28) into Eq. (25), we can obtain in general the steady-state solution without any approximations which is complex but of which numerical evaluation may be readily done by such as a digital computer.

3. Transfer Function of the Chopper Amplifier

Consider the general system of the chopper amplifier shown in Fig. 1.

In this system, the synchronizing rectifier circuit is assumed to be the 2-phase induction motor which converts a.c voltages to torques, and with no loss of generality the a.c amplifier is assumed to have no attenuation and phase shift. It is right to consider that high-order harmonic components of the chopper frequency may be remarkably attenuated in this system, therefore it is sufficient to consider only those components in the output of the system having the same frequency as the fundamental frequency $\omega+\omega_c$ and $\omega-\omega_c$ of the input signal, that is, in Eq. (25) we take only $m=\pm1$ and other components may be negligibly small at the system output.

Accordingly, in one case, the output voltage of the a.c amplifier is expressed in the form

$$y_i(t) = Ee^{j(\omega t+\theta)}\{K_{1i}e^{j\omega_c t}+K_{-1i}e^{-j\omega_c t}\} \qquad (i=1, 2, \cdots, l) \tag{30}$$

where K_{1i} and K_{-1i} is derived from Eq. (27).

Considering the condition of the synchronizing rectifier circuit, the complete system output becomes

$$v(t) = \frac{1}{2}Ee^{j(\omega t+\theta)}\{K_{1i}e^{-j\phi}+K_{-1i}e^{j\phi}\}, \tag{31}$$

multiplying the 1st term and 2nd term of (30) by $\frac{1}{2}e^{-j(\omega_c t+\phi)}$ and $\frac{1}{2}e^{j(\omega_c t+\phi)}$, respectively, where ϕ is the phase angle shifted in phase with respect to the input modulater which have to be determined to maximize the value $v(t)$.

The transfer function is the relationship between the input and output voltages in the system, then we have

$$|G(\omega)|e^{j\phi'} = \frac{v(t)}{Ee^{j(\omega t+\theta)}} = 0.5\ (K_{1i}e^{-j\phi}+K_{2i}e^{j\phi}). \tag{32}$$

In the remainder of this paper we will show the characteristics of two types of the chopper amplifier by the use of above procedures.

4. The Transformer Coupled Chopper Amplifier

The type considered in this section is widely used in many applications, of which the chopper-modulated circuit consists of the transformer type shown in Fig. 4. The equivalent circuit of this type is derived as Fig. 5.

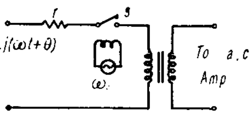

Fig. 4. Chopper-modulated circuit of transformer type.

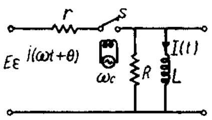

Fig. 5. Equivalent circuit of Fig. 4.

From Eqs. (2) and (3), the following differential equations can be set up, when the chopper is closed

$$L\left(\frac{r}{R}+1\right)\frac{d}{dt}y(t)+ry(t) = e^{j\omega t} \tag{33}$$

and when the chopper is open

$$L\frac{d}{dt}y(t)+Ry(t) = 0 \tag{34}$$

where $y(t)\equiv I(t)$.

Similarly, from Eqs. (22) and (23), the steady-state solution results in

$$y(t) = Ee^{j(\omega nT+\theta)}\{\varphi_1(t-nT)+x_1(t-nT)(e^{j\omega T}-B)^{-1}H\} \qquad nT\leq t\leq nT+t_1 \tag{35}$$

$$y(t) = Ee^{j(\omega nT+\theta)}x_2(t-nT-t_1)\{\varphi_1(t_1)+x_1(t_1)(e^{j\omega T}-B)^{-1}H\} \qquad nT+t_1\leq t\leq(n+1)T \tag{36}$$

where

$$\left.\begin{aligned}&\varphi_1(t) = \frac{a}{r}\cdot\frac{1}{a+j\omega}(e^{j\omega t}-e^{-\alpha t}),\quad x_1(t) = e^{-\alpha t},\quad x_2(t) = e^{-\beta t}\\&B = x_2(t_2)x_1(t_1),\quad H = x_2(t_2)\varphi_1(t_1),\quad a = Rr/r(r+R),\quad \beta = R/L,\end{aligned}\right\} \tag{37}$$

Then the output voltage $L\frac{d}{dt}y(t)$ is easily found from Eqs. (35) and (36). According to the same process as before, we find

$$K_m = \frac{L}{T}\left\{\int_0^{t_1}(\varphi_1'(t)+x_1'(t))De^{-j(\omega+m\omega_c)t}dt + \kappa\int_0^{t_2}x_2'(t)Pe^{-j(\omega+m\omega_c)t}dt\right\} \tag{38}$$

wher $\kappa = e^{-j(\omega+m\omega_c)t_1}$, $D=(e^{i\omega T}-B)^{-1}H$, $P=\varphi_1(t_1)+x_1(t_1)D$ and accent notations mean differentiation.

For simplicity, setting the duty-ratio to be 0.5, that is,

$$t_1 = t_2 = 0.5$$

we have

$$\left.\begin{aligned} K_1 &= \frac{L}{T}\cdot\frac{a}{r(a+j\omega)}\left\{\frac{2\omega}{\omega_c}+\frac{a}{a+j(\omega+\omega_c)}+\frac{\beta}{\beta+j(\omega+\omega_c)}+0(o)\right\} \\ K_{-1} &= \frac{L}{T}\cdot\frac{a}{r(a+j\omega)}\left\{-\frac{2\omega}{\omega_c}+\frac{a}{a+j(\omega+\omega_c)}+\frac{\beta}{\beta+j(\omega-\omega_c)}+0(o)\right\} \end{aligned}\right\} \tag{39}$$

where the term $0(o)$ is almost negligible comparing with other terms.

Since the frequency ω of the input signal is less than the chopper frequency ω_c and in general a, $\beta \ll \omega_c$ is satisfied, therefore taking into account these conditions we can finally obtain the transfer function as follows by substituting Eq. (39) into Eq. (32), where some negligible terms are removed,

$$\dot{G}(\omega) = |G(\omega)|e^{i\phi'} = k\frac{1}{1+j\frac{\omega}{a}} \tag{40}$$

where

$$k = \frac{R(L+r+R)}{2\pi r(r+R)}\sin(-\phi). \tag{41}$$

In this case the maximam gain of $\dot{G}(\omega)$ is given when $\phi=-\pi/2$ and the amplitude and phase characteristics as a function of the ratio ω/a or ω/ω_c are shown in Fig. 6.

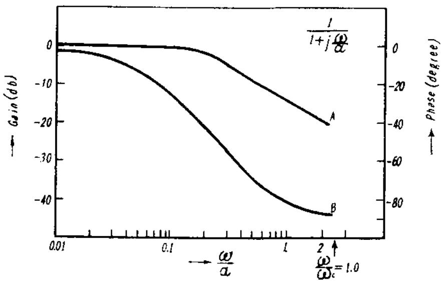

Fig. 6. Transfer function of the transformer coupled chopper amplifier (A : amplitude characteristic) (B : phase characteristic)

Throughout these procedures, the duty-ratio t_2/t_1 was assumed to be 0.5, now we consider the relation between the duty-ratio and output voltage. This relation is readily found by checking the coefficient K_1 or K_{-1} for some duty-ratio. The relative result is shown in Fig. 7 which shows that the optimam duty-ratio becomes 0.5 as expected

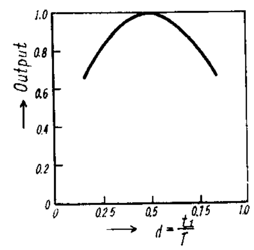

Fig. 7. Relation between the duty-ratio and output amplitude.

5. The Resistance-Capacitane Coupled Chopper Amplifier

The resistance-capacitance type shown in Fig. 8 has a high impedance characteristic in contrast with the transformer type having a low impedance one.

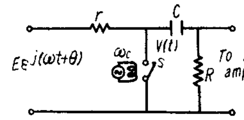

Fig. 8. R.C Chopper-modulated circuit.

Similar to the previous section, the derivation of the transfer function is given in the following without careful discussions. The fundamental differential equation is represented in the form as follows, for the chopper s closed

$$C(r+R)\frac{d}{dt}y(t)+y(t)=e^{j\omega t} \tag{42}$$

and for the chopper s opened

$$RC\frac{d}{dt}y(t)+y(t)=0 \tag{43}$$

where $y(t)\equiv V(t)$.

Similarly, the steady-state solution is

$$y(t)=Ee^{j(\omega nT++\theta)}\{\varphi_1(t-nT)+x_1(t-nT)(e^{j\omega T}-B)^{-1}H\} \qquad nT\leq t\leq nT+t_1 \tag{44}$$

$$y(t)=Ee^{j(\omega nT+\theta)}\cdot x_2(t-nT-t_1)\{\varphi_1(t_1)+x_1(t_1)(e^{j\omega T}-B)^{-1}H\} \tag{45}$$
$$nT+t_1\leq t\leq (n+1)T$$

where

$$\left.\begin{aligned}&\varphi_1(t)=\frac{\alpha}{\alpha+j\omega}(e^{j\omega t}-e^{-\alpha t}),\quad x_1(t)=e^{-\alpha t},\quad x_2(t)=e^{-\beta t},\\&B=x_2(t_2)x_1(t_1),\quad H=x_2(t_2)\varphi_1(t_1),\quad \alpha=1/CR,\quad \beta=1/C(R+r).\end{aligned}\right\} \tag{46}$$

The input voltage $RC\frac{d}{dt}y(t)$ to the a.c complifier is readily obtained from Eqs. (44) and (45), and obviously because of the same form as that in the previous case except for several constants, we can straightforwardly write the transfer function in the form, setting the duty-ratio to be 0.5,

$$\dot{G}(\omega)=|G(\omega)|e^{j\Phi'}=k\frac{1}{1+j\dfrac{\omega}{\alpha}} \tag{47}$$

where

$$k = \frac{2R+r}{2\pi(R+r)} \sin(-\phi).$$

Setting ϕ to be $-\pi/2$, the gain of $\dot{G}(\omega)$ becomes the maximum value too and the frequency characteristic in this case is quite evident to be the same one as shown in Fig. 6, but the value k and a are different.

6. Conclusion

The purpose of this paper is to explain briefly the transfer function of chopper amplifiers. First, we have discussed in general the chopper-modulated circuit by the modified theories of the Periodically Interrupted Electric Circuits. This modified analytical method has been established for the complex domain, therefore it is more general and will be applied in various fields.

In this establishment, the solution for the difference equation has played an important role and the usefulness of the complex Fourier series has been demonstrated for the steady-state analysis of the system.

The transient behaviour of the circuit also could be worked out in detail, although here it has not been examined.

Based on the modified theories presented in this paper, and provided that somewhat negligible terms could be removed, the transfer function has been determined for the two types of the chopper amplifier.

The frequency characteristics have resulted in the same function in the two configurations which were expressed in the form of the 1st order attenuation characteristic.

It is hoped that the results given in the 3rd section will not in any sense represent the limit of the application of this analysis, but it will be widely available in other fields, and that the transfer function presented in this paper may serve as a guide to the circuit designer to obtain good system performance.

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Appendix 1

Solutions of the difference equation

The linear difference equation is written in general in the form

$$\sum_{i=0}^{n} P_{xi} y_{x+i} = K_x \tag{A.1}$$

where P_{xi} and K_x represent constants or functions of x.

The general solution of this equation becomes

$$y_x = \sum_{i=1}^{n} \eta_i(x) \int \Delta c_i(x) \Delta x + c_1\eta_1(x) + c_2\eta_2(x) + \cdots + c_n\eta_n(x) \tag{A.2}$$

where the 1st term on the right side is one particular solution of the nonhomogeneous equation of (A.1) by Lagrange's method and the term $c_1\eta_1(x)+\cdots+c_n\eta_n(x)$ is the general solution of the homogeneous equation of (A.1):

$$\sum_{i=0}^{n} P_{xi} y_{x+i} = 0. \tag{A.3}$$

In the special case where K_x is b^x and P_{xi} is constants a_i, the perticular solution $\eta_0(x)$ of the nonhomogeneous equation can be readily derived as follows

$$\eta_0(x) = \gamma b^x/\varphi(b) \tag{A.4}$$

where $\varphi(b)$ is defined by the fact that if setting $y_x=\lambda^x$ in the following homogeneous equation

$$\sum_{i=0}^{n} a_i y_{x+i} = 0, \tag{A.5}$$

we have

$$\varphi(\lambda) \equiv a_0 + a_1\lambda + a_2\lambda^2 + \cdots + a_n\lambda^n = 0 \tag{A.6}$$

and when we put $\lambda=b$ in (A.6), $\varphi(b)$ is determined.

For our case treated in section 3, it is clear that, by the above method

$$\left.\begin{aligned} \varphi(\lambda) &= \lambda^T[U]-[B] \\ \lambda &= [B]^{1/T} \\ b &= e^{j\omega} \\ \gamma &= Ee^{j\theta}[H] \end{aligned}\right\} \tag{A.7}$$

and the particular solution becomes

$$\eta_0(nT) = Ee^{j(\omega nT+\theta)}\{e^{j\omega T}[U]-[B]\}^{-1}[H] \tag{A.8}$$

and the general solution of the homogeneous equation becomes

$$c_1\eta_1(nT) = [B]^n[I], \tag{A.9}$$

therefore the solution $[y(nT)]$ can be written in the form as Eq. (18).

Appendix 2

Sylvester expansion theorem

This theorem states that, if α_r is an s_r-ple latent root of a square matrix

$[A]$ of order m, and provided that the corresponding characteristic matrix has full degeneracy for that root, and

$$s_1+s_2+\cdots+s_i = m$$

then

$$F([A]) = \sum_{r=1}^{i} F(a_r)[K(a_r)] \tag{A.10}$$

where

$$[K(a_r)] = \prod_{k \neq r}^{k=1,2,\cdots,i} \frac{(a_k[U]-[A])}{(a_k-a_r)} \tag{A.11}$$

For our case treated in section 3, the function of matrix is

$$F([A]) = [A]^n.$$

therefore the Sylvester expansion theorem for this is written as Eq. (20).

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