

# Mixing and Microbial Kinetics of Activated Sludge Process

By

Takeshi GODA\* and Hiroshi NAKANISHI\*

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In the activated sludge process, the characteristics of qualitative response observed in the aerator effluent depends largely on the induced microbiological reaction and the pattern of mixing. The latter controls the "age" distribution of mixing liquor. Authors introduced first two kinds of mixing model, one of which is modified complete mixed type and another is to apply for any intermediate mixing condition between complete mixing and plug flow, in order to explain the pattern of flow in the aeration tank. This serves especially for the investigation of microbiological characteristics of conventional and step aeration process.

Next, the kinetics of microbial reaction was studied, referring to the previous formulation treating the metabolism of single substrate-organism combination, considering the significance of BOD<sub>5</sub> while using it as index of amount of compounded substrates in the sewage; consequently the over-all effect of miscellaneous sludge organisms was attributed to the term of activity, by which the modified mathematical representation of reaction for such mixed-culture system was proposed.

Finally, the authors showed the possibility of calculating exact change in substrate concentration of the effluent by means of computations combining any proposed mixing model with reaction formula. For example, in case of zero- and first-order microbial reaction, the response in effluent substrate concentration was computed and summarized.

## 1. Introduction

One of the major problems in the activated sludge process is how to select the physical condition of the aeration tank in order to maintain the optimum degree of biochemical degradation with respect to the quality of the effluent. Hydraulic characteristics in the tank must be discussed for this purpose, considering its effect for the actual metabolism of the micro-organisms cultivated. In this approach, to use the appropriate mathematical models, both for the mixing condition and microbial kinetics, seems convenient for theoretical explanation of the phenomena. In section 2, new mathematical models for the mixing pattern are proposed, and in section 3, the kinetics of activated

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\* Department of Sanitary Engineering

sludge growth is discussed, then in section 4, the mixing models and microbial kinetics are theoretically combined according to the type of nutrient utilization by micro-organisms under a given condition of mixing and substrate concentration. As a result, the response which is signified in the effluent quality may also be cleared by a series of computation.

## 2. Mathematical Model of Mixed Aeration Tank<sup>1)</sup>

Various types of the activated sludge plant in present use can be classified with regard to their flow patterns into a) plug flow type, and b) complete mixing type, as shown in Table 1. But, some modifications must be necessary because every process can not be adequately described by either type. It is

Table 1. Flow patterns of activated sludge process.

a) Plug flow type	Conventional aeration process Modified aeration process
b) Complete mixing type	Total oxidation process Contact stabilization process Aero-Accelator Step aeration process

obvious that to introduce both the plug flow and the complete mixing must correspond to extremely idealized conditions of mixing. In practice, therefore, a certain intermediate pattern of mixing must exist.

For the purpose of representing the above mentioned intermediate mixing condition, we introduced the following two kinds of model.

### 1) Model of complete mixing with retardation (Model A)

The schematic figure of this model is illustrated in Fig. 1.

Assume that the volume of an aeration tank is  $V (= V_n)$ , the influent flow discharge including recycle is  $q$ , and the influent substrate concentration at

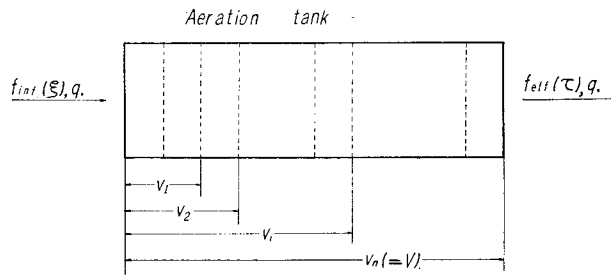


Fig. 1. Model of complete mixing with retardation.

time  $\xi$  is  $f_{inf}(\xi)$ , then the amount of substrate flowing into the tank for a time interval  $d\xi$  at  $\xi$  is equal to  $f_{inf}(\xi)qd\xi [=M(\xi)]$ , which may be completely mixed within a fraction of volume  $V_1$  when the time passed is  $\Delta t$  after inflow. Then, in the same manner afterwards, it may be mixed with a liquid volume of  $V_i$  at time  $\xi+i\Delta t$ , and finally with  $V_n [=V]$  at  $\xi+n\Delta t = \xi+t_s$ , where  $n\Delta t$  or  $t_s$  is the shortest time for a portion of inflow substrate  $M(\xi)$  to spread over the whole of the aeration tank.

The first substrate concentration,  $C_\xi(t_s)$ , in the effluent is expressed as  $M(\xi)/V$ , and at the time  $\xi+t_s+\Delta t$ ,  $C_\xi(t_s+\Delta t)$  is expressed as  $M(\xi)(1-q\Delta t/V)/V$ , where  $C_\xi(t)$  denotes the substrate concentration of the effluent originated from  $M(\xi)$ , which means an instantaneous contribution of substrate at time  $t$ .

$$C_\xi(t) = \frac{f_{inf}(\xi)qd\xi}{V} \left(1 - \frac{q\Delta t}{V}\right)^{(t-t_s)/\Delta t} = \frac{f_{inf}(\xi)d\xi}{T} \left(1 - \frac{\Delta t}{T}\right)^m \dots\dots\dots(1)$$

in which,  $T$  is the nominal detention time of aeration tank, and  $m=(t-t_s)\Delta t$ ,  $t \geq t_s$ .

If  $\Delta t$  is close to 0, Eq. (1) becomes to

$$C_\xi(t) = \frac{f_{inf}(\xi)qd\xi}{V} e^{-q/V(t-t_s)} = \frac{M(\xi)}{V} e^{-q/V(t-t_s)} \dots\dots\dots(2)$$

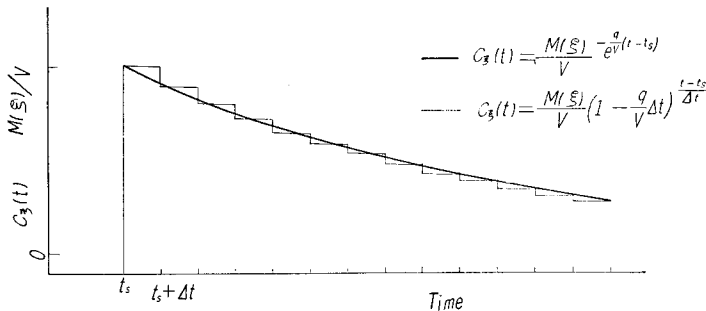


Fig. 2. Effluent concentration computed by Model A.

2) Model of plug flow with partial complete mixing (Model B)

Schematic diagram of this model is shown in Fig. 3.

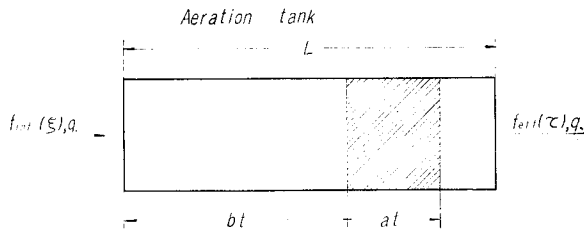


Fig. 3. Model of plug flow with partial complete mixing.

Let  $V$  and  $L$  be the volume of rectangular aeration tank and its length, respectively, and  $M(\xi) = \int_{inr}(\xi) q d\xi$  be the amount of substrate added to the tank during the time interval  $d\xi$  at time  $\xi$ ; then this fraction of added substrate is dispersed to the shaded part ( $at$ ) of the aeration tank at time  $\xi+t$  uniformly, as in the complete mixing tank. After that, the fraction moves straight downwards at a constant rate of displacement  $bt$ .  $b$  and  $a$  may be assumed as constants signifying the flow velocity and the rate of longitudinal dispersion, respectively.

The time of first appearance of substrate  $M(\xi)$  in the effluent  $t_s$ , must be computed by

$$t_s = L/(a+b),$$

and the time when  $M(\xi)$  is completely flowed away from the tank,  $t_e$ , is

$$t_e = L/b.$$

For a portion of  $M(\xi)$ ,  $Q_\xi(t)$ , flowing out between  $t_s$  and  $t$  ( $t_s \leq t \leq t_e$ ) is expressed by

$$Q_\xi(t) = M(\xi) \frac{(a+b)t - L}{at} = M(\xi) \left( \frac{a+b}{a} - \frac{L}{a} t^{-1} \right),$$

By the differentiation of  $Q_\xi(t)$ , we get

$$\frac{dQ_\xi(t)}{dt} = M(\xi) \frac{L}{a} t^{-2}, \dots\dots\dots(3)$$

which gives the excluding rate of  $M(\xi)$  from the tank. The substrate concentration due to  $M(\xi)$  in the effluent at time  $t$ ,  $C_\xi(t)$ , is given by

$$C_\xi(t) = \frac{1}{q} \frac{dQ_\xi(t)}{dt} = M(\xi) \frac{L}{aq} t^{-2} \dots\dots\dots(4)$$

When  $a=0$ , and  $b=Lq/V$  (nominal flow velocity), this model becomes equal to the ideal plug flow model.

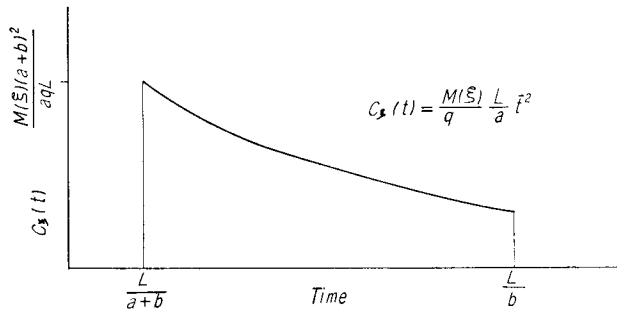


Fig. 4. Effluent concentration computed by Model B.

**3) Application of Model A to the step aeration operation**

Step aeration operation involves the addition of sewage influent along the course of aeration for mixed liquor through the aeration tank and the influent may be added at multiple points along the aeration tank. Assume that the

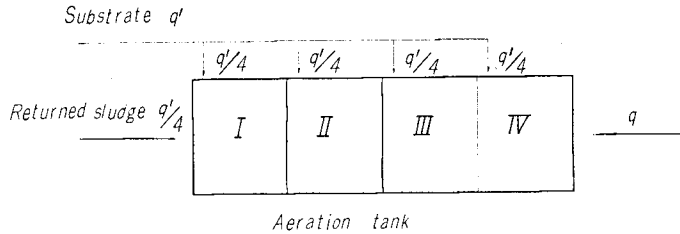


Fig. 5. Typical step aeration operation.

Model A is applicable for such a kind of step aeration, for example in the condition as the influent substrate is added equally at four points along the aeration channel, and the rate of recycle is a quarter of inflowing sewage, then the effluent substrate concentration,  $C_{\xi}(t)$ , due to  $M(\xi)$ , is expressed by

$$\begin{aligned}
 C_{\xi}(t) = & \frac{M(\xi)}{V} \{ e^{-(4q/V)(t-t_{s4})} + 4 [ e^{-(16q/5V)(t-t_{s4}-t_{s3})} - e^{-(4q/V)(t-t_{s4}-t_{s3})} ] \\
 & + 6 [ e^{-(12q/5V)(t-t_{s3}-t_{s2})} - 2e^{-(16q/5V)(t-t_{s4}-t_{s3}-t_2)} + e^{-(4q/V)(t-t_{s4}-t_{s3}-t_{s2})} ] \\
 & + 4 [ e^{-(8q/5V)(t-t_{s4}-t_{s3}-t_{s2}-t_{s1})} - 3e^{-(12q/5V)(t-t_{s4}-t_{s3}-t_{s2}-t_{s1})} \\
 & + 3e^{-(16q/5V)(t-t_{s4}-t_{s3}-t_{s2}-t_{s1})} - e^{-(4q/V)(t-t_{s4}-t_{s3}-t_{s2}-t_{s1})} ] \}, \dots\dots\dots(5)
 \end{aligned}$$

where,  $t_{s1}$ ,  $t_{s2}$ ,  $t_{s3}$ , and  $t_{s4}$  are time lags due to the stepwise adding of sewage in the divided section I, II, III and IV of the channel, respectively.

**4) Application of Model B to the step aeration operation**

A rectangular aeration tank is divided into four equal sections by the separated addition of sewage. Then, the amount of influent substrate added to a section, which may be written as  $M(\xi)/4$ , will disperse with a constant velocity along the course of flow in the same manner as stated in 2). Next, this diffused substrate will flow down with some velocity  $2b$ , in compartment I,  $3b$ , in II,  $4b$ , in III and  $5b$ , in compartment IV, respectively, according to the flow rate.

The amount of influent substrate inflowed at time  $\xi$  at an arbitrary compartment  $i$  is  $M(\xi)/4$ , and at time  $t_{ix}$ , when  $(x/a) \times 100\%$  of  $M(\xi)/4$  inflowed to  $i$ , a part of this substrate begins to flow out of the tank. Corresponding time,  $t_{ix}$ , is computed by

$$t_{ix} = \sum_{i=1}^4 \frac{L}{4 \left\{ (i+1)b + \frac{a}{2} - x \right\}}, \quad \begin{matrix} (t_{is} \leq t_{ix} \leq t_{ie}) \\ (0 \leq x \leq a) \end{matrix} \dots\dots\dots(6)$$

where,  $t_{is}$  is the time required until the first appearance of added substrate is recognized in the effluent, and  $t_{ie}$  is the time of diminishing for supposed substrate.

Besides above, the substrate concentration in the effluent due to  $M(\xi)/4$ , added to  $i$ -th compartment, is expressed as

$$C_{\xi i}(t_{ix}) = \frac{M(\xi)}{4q} \frac{dx/a}{dt} = \frac{M(\xi)}{aLq \sum_{i=1}^4 \left\{ (i+1)b + \frac{a}{2} - x \right\}^{-2}}, \quad \dots\dots\dots(7)$$

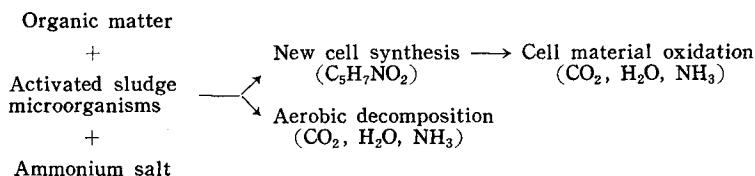
and the total effluent substrate concentration, due to  $M(\xi)$ , is given by

$$C_{\xi}(t) = C_{\xi 1}(t) + C_{\xi 2}(t) + C_{\xi 3}(t) + C_{\xi 4}(t). \quad \dots\dots\dots(8)$$

### 3. Kinetics of Activated Sludge Synthesis

The mode of biological reaction caused by the activated sludge is roughly illustrated in Fig. 6.

Fig. 6. Biological reaction by activated sludge.



#### 1) Kinetic model of general bacterial growth and metabolism

Three different quantitative relationships between the growth rate of sludge and nutrient concentration were used by Grieves et al<sup>2)</sup>. The first relationship assumes that the over-all growth rate is proportional to the population density:

$$\frac{dS}{dt} = K_1 S, \quad \dots\dots\dots(9)$$

where,  $S$  is the population density and  $K_1$  is the specific growth rate. The specific growth rate is considered as a function of a number of environmental factors; however, it might be regarded as a single function of the oxidative substrate concentration, if all other factors are not significant.

The second relationship proposed is

$$K_1 = K_{1\max} \frac{X}{N + X}. \quad \dots\dots\dots(10)$$

where,  $X$  is the concentration of oxidative substrate and  $K_{1\max}$  is the maximum specific growth rate. When  $X$  is sufficiently large,  $K_1$  becomes independent of  $X$ , and  $N$  is the value of  $X$  when  $K_1 = 0.5K_{1\max}$ .

The third relationship is based on the concept that increase in population density is proportional to the decrease in substrate concentration, so

$$dS = -YdX, \dots\dots\dots(11)$$

where  $Y$  is a yield constant.

All three of these relationships are often used by researchers, for example, Monod<sup>3)</sup> used them to describe the growth in batch culture and Herbert et al<sup>4)</sup> used them in the development of the chemostat model.

**2) Discussion on the application of kinetic model**

**a) General**

Before we apply such kinetic models as mentioned in 1) to the analysis of sludge growth, the following problems must be solved :

- i) The most suitable index which expresses the amount of oxidizable substrate
- ii) The most suitable index which expresses the amount of microorganisms in activated sludge

It is common to treat the BOD of a raw waste as the amount of available substrate. On this basis generally, the expression of BOD is used to represent the substrate in system. It must be examined, however, whether BOD is suitable for our analysis or not. If the rates of BOD removal by an activated sludge are the same for several kinds of wastes having same BOD values, in spite of the differences of their sources, BOD may regarded as suitable index for the amount of oxidizable substrate.

On the other hand, it is also common to regard MLSS or MLVSS as the quantitative index for the amount of micro-organisms in the activated sludge. In the same way as in the previous discussion, SS or VSS may be substituted for micro-organisms in the system. However, the species and amounts of micro-organisms are quite changeable with the environmental conditions, so the activity of the sludge of unit weight, which is concerned directly with the potential ability of substrate removal, may vary to some extent.

Strictly speaking, therefore, there seems no sound reason for the assumption that SS or VSS represents the amount of active micro-organisms. By this concept, Equations (9), (10) and (11), based on the assumption that just a single species is attacking the definite kind of substrate, must be corrected or modified in applying to such compound microbial cultivating process as actual activated sludge operation.

**b) Experimental results and discussions**

Although BOD has a significance of showing the combined effects of all

the degradable substances included in a waste, we must pay attention to the difference between the observed microbial reactions in the BOD bottle and the actual biochemical reactions in a biological treatment system. An inclusive schematic explanation for the reduction of organic matter in the activated sludge plant is illustrated in Fig. 7.

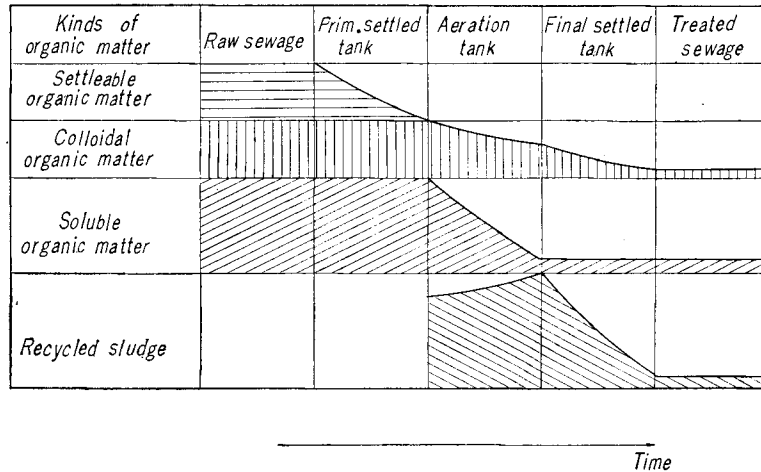


Fig. 7. Reduction process of organic matter in activated sludge.

The mechanism of removal of organic suspended matter is different from that of soluble organic matter. The latter is removed or converted by the biological sludge, while the solid matter is removed not only by the biological oxidation but by the physical process such as sedimentation and flocculation. Therefore, if BOD is substituted with the amount of total organic matters, we should separate it first to soluble and solid, to use as the concentration of available substrate. Furthermore, as seen in the results of experiments, it was found that the reduction rate of soluble BOD varies when the constituents

Table 2. Comparison of removal rates for various pure substrates by activated sludge using index of BOD<sub>5</sub>.

Substrates	BOD <sub>5</sub> values (ppm.)		BOD <sub>5</sub> removal rate (ppm./hr)
	Initial	After 1.5 hr	
D-glucose	150	90	40.0
D-galactose	150	145	3.3
Na-glutamate	150	105	30.0
L-arginine, HCl	150	140	6.7
Na-palmitate	150	75	50.0
Na-acetate	150	15	90.0



change their proportion even if those BOD values are kept same. Table 2<sup>5)</sup> is a part of the verification for this.

In the above, if oxidizable substrate is expressed simply by BOD, we may not recognize that the maximum growth rate  $K_{1\max}$  or yield rate  $Y$  in the kinetic model has a constant value, and only in the case that the kind of substrate is definite, BOD will have a greater significance.

On the other hand, the activity of unit activated sludge was defined and measured by authors with a Warburg respirometer of larger size, in order to estimate the potential ability of substrate removal. Example of measured activity is summarized in Table 3.

Table 3. Activities of various activated sludge.

Kinds of activated sludge	Kinds of substrate	Substrate Concentration as BOD (ppm.)	Activities	
			Rate of net O <sub>2</sub> uptake (mg O <sub>2</sub> /gr SS. hr)	Rate of BOD removal (mg BOD/gr SS. hr)
Sludge in Kyoto plant	D-glucose	1000	9.0	—
Sludge cultured with synthetic sewage A*	D-glucose	1000	14.5	—
"	D-xylose	1000	4.2	—
Return sludge in Kyoto plant	L-arginine HCl	1000	2.8	—
"	Sewage and urine	1000	11.5	—
"	Synthetic sewage A	1000	19.0	—
"	Synthetic sewage A	1000	15.0	—
Sludge in test plant	Synthetic sewage A	1000	12.0	—
Sludge cultured with pepton and glucose	Synthetic sewage A	1000	41.0	—
Sludge in anaerobic condition	Synthetic sewage A	1000	40.0	—
Sludge cultured with phenol	Synthetic sewage A	1000	60.0	—
Sludge cultured with brewery waste	Synthetic sewage A	1000	11.5	—
Sludge in Kyoto plant	Synthetic sewage B**	1000	—	42.5
"	Synthetic sewage B	1000	—	51.0
"	Synthetic sewage B	1000	—	88.0

\* Synthetic sewage (1,000 ppm. as BOD)

D-glucose : 1,009 mg  
 Na-glutamate : 329 "  
 NH<sub>4</sub>-acetate : 430 "  
 Nutrient salt : Fe, Mn, Mg, Ca, K, etc.  
 distilled water : 1,000 ml

\*\* Synthetic sewage B (1000 ppm. as BOD)

Na-acetate : 958.8 mg  
 Na-propionate : 243.8 "  
 Urea : 2,196 "  
 Tap water : 1,000 ml

From the results of our measurements, it is recognized that the activity of sludge changes according to the kind of activated sludge and substrate, and that the expression of SS or VSS seems not so suitable for the index of intending analysis. Here, in the strict sense, it is said that Eq. (10) can hardly be applicable for actual sludge growth phenomena under high substrate concentration; in this case, the activity decreases. However, Eq. (10) seems still available approximately in such a low substrate concentration zone in which usual modified or conventional activated sludge process are operated (Fig. 8).

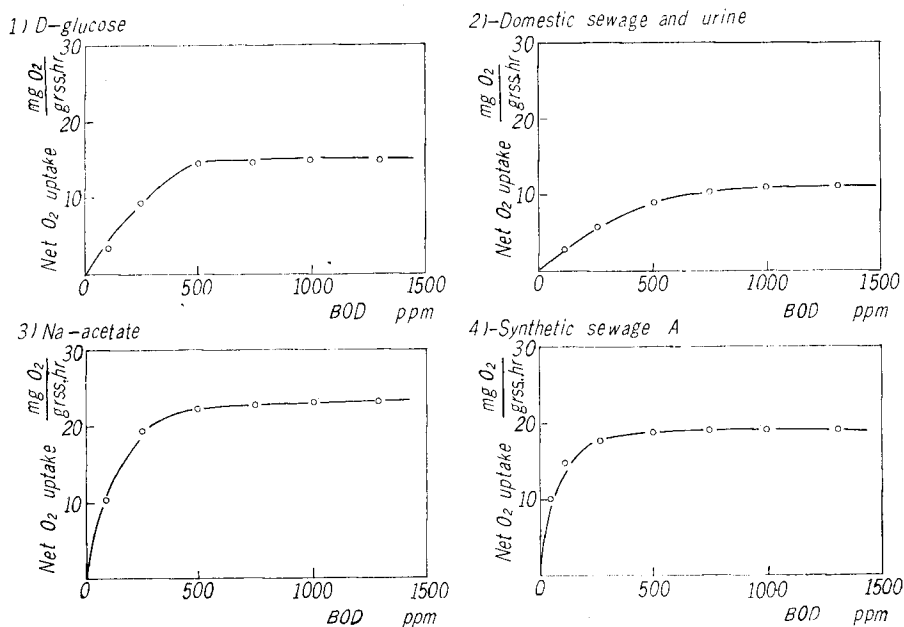


Fig. 8. Relationship between substrate concentration and O<sub>2</sub> uptake rate of sludge.

Consequently, the followings may be concluded by the arguments in this article:

- 1) It is desirable in practice to use BOD for the quantitative measure of oxidizable substrate present, as long as the other better index is not detected, however, it must be noted that the removal rate of BOD really changes with the kind of substrate concerned, even if original BOD values are the same. Therefore, the growth rate, yield constant etc. used in the microbial kinetic model change with the kind of substrate.
- 2) The activity of unit weight of sludge changes due to the kind of sludge. So it is often unreasonable to equate SS or VSS with the amount of micro-

organisms in activated sludge. Then finally, the following modified model is proposed for the metabolic reaction of activated sludge.

$$-\frac{dX}{dt} = \frac{K_{1\max}X}{N+X} \frac{S_a}{Y}, \dots\dots\dots(12)$$

where,  $S_a$  is the concentration of total active mass in the aerator, which may be computed by

$$S_a = SA/A_0. \dots\dots\dots(13)$$

in which  $S$  is the suspended solid concentration in the aerator,  $A$  is the activity of unit weight of sludge which is defined as milligram of removed BOD per gram of activated sludge on the dry basis per hour, measured at the standard BOD of 1,000 ppm for each substrate, and  $A_0$  is the activity of the standard activated sludge used to measure the growth rate and yielding constant.

Also,  $K_{1\max}$ ,  $N$  and  $Y$  are to be determined according to the kind of substrate when all other environmental factors affecting the reaction are kept constant.

- 3) Eq. (12) can be applied approximately for various types of sewage-based microbial reactions which will not contain any significant inhibiting substances. If  $X \gg N$ , Eq. (12) becomes a zero order reaction and the velocity of substrate removal is independent of substrate concentration. In this case Eq. (12) reduces to

$$-\frac{dX}{dt} = \frac{K_{1\max}SA}{YA_0} = \frac{K'_0SA}{A_0}. \dots\dots\dots(14)$$

The integration form of which is

$$X_t = X_0 - \frac{K'_0SA}{A_0} t. \dots\dots\dots(15)$$

where  $K'_0 = \frac{K_{1\max}}{Y}$ ,  $X_0$  is the initial substrate concentration and  $X_t$  is the substrate remaining at time  $t$ .

If  $X \ll N$ , Eq. (12) becomes a first order reaction and reduces to

$$-\frac{dX}{dt} = \frac{K_{1\max}SA}{NYA_0} X = \frac{K'_1SA}{A_0} X, \dots\dots\dots(16)$$

where  $K'_1 = \frac{K_{1\max}}{NY}$ .

And the integrated form of Eq. (16) is

$$X_t = X_0 e^{-(K'_1SA/A_0)t}, \dots\dots\dots(17)$$

**4. Mathematical Expression of BOD Reduction derived from the Combination of Mixing and Kinetic Model**

**1) Dynamic correspondence between influent quality and effluent quality**

The substrate concentration of effluent at time  $\xi$  can be predicted by the following equation when the removal of substrate is expressed by the first order reaction.

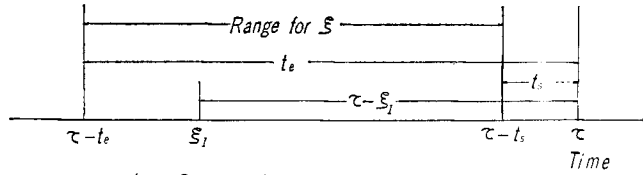


Fig. 9. Definition of time  $\xi$ ,  $\tau$ ,  $t_s$  and  $t_e$ .

$$f_{eff}(\tau) = q \int_{-\infty}^{\tau} f_{inf}(\xi)g(\tau - \xi)h(\tau - \xi)d\xi, \quad \dots\dots\dots(18)$$

Next, if the removal of substrate is expressed by the zero order reaction the effluent substrate concentration is expressed by

$$f_{eff}(\tau) = q \int_{-\infty}^{\tau} [f_{inf}(\xi) - h(\tau - \xi)]g(\tau - \xi)d\xi, \quad \dots\dots\dots(19)$$

where,  $f_{eff}(\tau)$  is the effluent substrate concentration at time  $\tau$ ,  $f_{inf}(\xi)$  is the influent substrate concentration at time  $\xi$ ,  $g(t)$  is called the distribution function, which expresses the timely change of effluent substrate concentration by  $M(\xi)$  due to the pattern of dispersion; for example in complete mixing,  $g(t)$  is given by  $\frac{1}{V} e^{-(q/V)t}$ .  $h(t)$  is the same kind of function due to the biological reaction; for example, in a first order microbial reaction for  $M(\xi)$ ,  $h(t)$  is expressed by  $e^{-(K_1'SA/A_0)t}$ .

When the function is continuous and meaningful only in the region of  $t_s \leq t \leq t_e$ , Equation (18) can be written:

$$f_{eff}(\tau) = q \int_{\tau - t_e}^{\tau - t_s} f_{inf}(\xi)g(\tau - \xi)h(\tau - \xi)d\xi, \quad \dots\dots\dots(20)$$

and the equation (19) becomes

$$f_{eff}(\tau) = q \int_{\tau - t_e}^{\tau - t_s} [f_{inf}(\xi) - h(\tau - \xi)]g(\tau - \xi)d\xi. \quad \dots\dots\dots(21)$$

where  $t_s$  is the time passed until the first appearance of supposed substrate  $M(\xi)$  in the effluent and  $t_e$  is the time needed for disappearance of  $M(\xi)$ .

2) Example of calculation for  $f_{eff}(\tau)$ 

## a) Steady state

For steady state,

 i) If  $f_{inf}(\xi) = 4/5 C_0$  (constant),  $g(t) = \frac{1}{V} e^{-(5q'/4V)t}$  (complete mixing) and  $h(t) = e^{-(K_1' S_0 A / 5 A_0)t}$  (first order reaction),

$$\begin{aligned} f_{inf}(\tau) &= \frac{5}{4} q' \int_{-\infty}^{\tau} \frac{4}{5} C_0 \frac{1}{V} e^{-(5q'/4V)(\tau-\xi)} e^{-(K_1' S_0 A / 5 A_0)(\tau-\xi)} d\xi \\ &= \frac{4C_0}{5 + K_1' \frac{A}{A_0} S_0 T} \end{aligned}$$

 ii) If  $f_{inf}(\xi) = 4/5 C_0$ ,  $g(t) = \frac{1}{V} e^{-(5q'/4V)t}$  and  $h(t) = -\frac{K_0' S_0 A}{5 A_0} t$ , then

$$\begin{aligned} f_{inf}(\tau) &= \frac{5}{4} q' \int_{-\infty}^{\tau} \left[ \frac{4}{5} C_0 - \frac{K_0' S_0 A}{5 A_0} (\tau - \xi) \right] \frac{1}{V} e^{-(5q'/4V)(\tau-\xi)} d\xi \\ &= \frac{4}{5} \left( C_0 - \frac{1}{4} K_0' \frac{A}{A_0} S_0 T \right) \end{aligned}$$

where  $q'$  is the rate of sewage inflow and the sludge recycle rate is assumed a quarter of inflowed sewage discharge  $q'$ , then, inflow rate including the recycled sludge,  $q$  equal to  $5/4q'$ ;  $C_0$  is influent substrate concentration and  $4/5 C_0$  is influent substrate concentration diluted with returned sludge. And  $S_0$  is the suspended solid concentration in the returned sludge;  $T$  is the nominal detention time of the aerator.

In Table 4, the summary of those formulas, derived for various conditions, i.e. supposed combinations of mixing models with the biochemical kinetic equation, are shown for both conventional and step aeration.

## b) Unsteady state

In case if  $f_{inf}(\xi)$  is expressed by such a periodic function as  $f_{inf}(\xi) = 5 \left( \sin \frac{\pi}{12} \xi + 1 \right)$ , which is a functional modification for actual timely change of sewage BOD influent observed,  $f_{eff}(\tau)$  can be calculated by following equations.

i) Plug flow condition and  $h(t)$  is the zero order reaction, assumed, then for the conventional aeration process;  $f_{eff}(\tau)$  is expressed as

$$f_{eff}(\tau) = 4 \left[ \sin \frac{\pi}{12} (\tau + T) + 1 \right] - \frac{1}{5} K_0' \frac{A}{A_0} S_0 T$$

and for step aeration process, it becomes

$$\begin{aligned} f_{eff}(\tau) &= \sin \frac{\pi}{12} \left( \tau + \frac{77}{48} T \right) + \sin \frac{\pi}{12} \left( \tau + \frac{47}{48} T \right) + \sin \frac{\pi}{12} \left( \tau + \frac{27}{48} T \right) + \sin \frac{\pi}{12} \left( \tau + \frac{12}{48} T \right) \\ &\quad + 4 - \frac{77}{240} K_0' \frac{A}{A_0} S_0 T \end{aligned}$$

Table 4. Summary of  $f_{eff}(\tau)$  derived for various conditions in steady state.

Process	Mixing models	Microbial kinetics	Formulas of $f_{eff}(\tau)$
C.A.	Complete mixing	Zero order reaction	$\frac{4}{5} \left( C_0 - \frac{1}{4} K_0' \frac{A}{A_0} S_0 T \right)$
S.A.	"	"	$\frac{4}{5} C_0 - \frac{77}{240} K_0' \frac{A}{A_0} S_0 T$
C.A.	"	1st order reaction	$\frac{4 C_0}{5 + K_1' \frac{A}{A_0} S_0 T}$
S.A.	"	"	$\frac{C_0 \left[ 2880 + 720 \left( 4 + K_1' \frac{A}{4A_0} S_0 T \right) + 80 \left( 4 + K_1' \frac{A}{4A_0} S_0 T \right) \left( 9 + K_1' \frac{A}{4A_0} S_0 T \right) + 5 \left( 4 + K_1' \frac{A}{4A_0} S_0 T \right) \left( 9 + K_1' \frac{A}{4A_0} S_0 T \right) \left( 16 + K_1' \frac{A}{4A_0} S_0 T \right) \right]}{\left( 4 + K_1' \frac{A}{4A_0} S_0 T \right) \left( 9 + K_1' \frac{A}{4A_0} S_0 T \right) \left( 16 + K_1' \frac{A}{4A_0} S_0 T \right) \left( 25 + K_1' \frac{A}{4A_0} S_0 T \right)}$
C.A.	Plug flow	Zero order reaction	$\frac{4}{5} \left( C_0 - \frac{1}{4} K_0' \frac{A}{A_0} S_0 T \right)$
S.A.	"	"	$\frac{4}{5} C_0 - \frac{77}{240} K_0' \frac{A}{A_0} S_0 T$
C.A.	"	1st order reaction	$\frac{4}{5} C_0 e^{-K_1' \frac{A}{(5A_0)} S_0 T}$
S.A.	"	"	$\frac{C_0}{5} \left[ e^{-5K_1' \frac{A}{(4A_0)} S_0 T} (1/2^2 + 1/3^2 + 1/4^2 + 1/5^2) + e^{-5K_1' \frac{A}{(4A_0)} S_0 T} (1/3^2 + 1/4^2 + 1/5^2) + e^{5K_1' \frac{A}{(4A_0)} S_0 T} (1/4^2 + 1/5^2) + e^{-5K_1' \frac{A}{(4A_0)} S_0 T} (1/5^2) \right]$

Note : C.A. : conventional aeration process, S.A. : step aeration process ; it is assumed that no substrate is contained in return sludge.

ii) Plug flow, and  $h(t)$  is the first order reaction, for the conventional aeration process ; we obtain

$$f_{eff}(\tau) = 4 \left[ \sin \left( \frac{\pi}{12} \tau + T \right) + 1 \right] e^{-(1/5)K_1'(A/A_0)S_0T}$$

iii) Assuming complete mixing and  $h(\tau)$  is the first order reaction, shown for the conventional aeration process.

$$\begin{aligned} f_{eff}(\tau) &= \frac{5}{4} q' \int_{-\infty}^{\tau} 4 \left( \frac{\pi}{12} \xi + 1 \right) \frac{1}{V} e^{-(5q'/4V)(\tau-\xi)} e^{-(K_1'S_0A/5A_0)(\tau-\xi)} d\xi \\ &= \frac{4}{T \left[ \left( \frac{\pi}{12} \right)^2 + \left( \frac{1}{T} + \frac{K_1'S_0A}{5A_0} \right)^2 \right]} \left[ \left( \frac{1}{T} + \frac{K_1'S_0A}{5A_0} \right) \sin \frac{\pi}{12} \tau - \frac{\pi}{12} \cos \frac{\pi}{12} \tau \right] + \frac{20}{5 + K_1' \frac{A}{A_0} S_0 T} \end{aligned}$$

Although the above calculations are applicable simply for the mathematical models of such types as the ideal complete mixing model and ideal plug flow model combined with the first order or zero order microbial kinetic model, it may be recognized that the higher efficiency in substrate removal can be expected in case of plug flow than in complete mixing. Also, the higher efficiency of substrate removal can be expected for step aeration process than in the conventional aeration process.

In unsteady state, the complete mixing will effect to make the effluent substrate concentration more regardless of the change in influent substrate concentration. Not only the increase of removed polluted organic matter but also the stabilization of effluent quality are to be said very important in actual sewage treatment practices.

### 5. Conclusion

1) New concept for the hydraulic models on the pattern of flow in the aeration tank was presented. Two of the introduced model correspond to the modified complete mixing and modified plug flow respectively.

2) The kinetics of activated sludge growth and substrate removal was formulated making some modification which describes the microbial growth and metabolism for the cultivation of single bacteria with single substrate.

3) The expression of BOD is capable of substitution with the concentration of oxidizable substrate, but BOD is not always the favourable index because the rate of removal of BOD should vary considerably due to the kind of substrate fed. It is important, therefore, to make a proper prescription for substrate composition while we perform each microbial reaction test.

4) Substituting the suspended solid concentration with that of sludge organisms should also reinvestigated. For this, we can make correction by

using the expression of "activity" of sludge to estimate the potential ability of substrate removal.

5) Those formulas, predicting the effluent substrate concentration of activated sludge aerator in the case of usual conventional and step-aeration operation, were derived by using the above mentioned mathematical models and they are summarized on Table 4.

6) Complete mixing has certainly the effect of making the effluent sewage quality even.

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