Study on Repetitional Consolidation of Clay

By

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The actual phenomena for settlements due to repetitional consolidation of the clay layers are observed frequently, but there remains a great number of unsolved problems in the present situation.

In this paper the authors propose an approximate method for calculating the amount of consolidation when the repetitional, uniformly distributed surcharge is applied on a saturated clay layer. By means of this method they analized some results of experimental tests using a triaxial apparatus. Although in this approximate method some simplified conditions are assumed in order to facilitate analysis, the test results obtained by a series of triaxial consolidation tests for saturated clay specimens are in considerably good agreement with those of the numerical analysis.

1. Introduction

If a saturated clay is placed under repeated pressures with some time intervals in the condition where water escapes from and enters the voids of the clay, the total volume of the clay decreases gradually with residual volume changes. The phenomenon of the ground subsidence would be a typical example of this problem. The ground subsidence is mainly based on the consolidation of clay layers by the depression and the recovery of the artesian pressures in the adjoining aquifers. Respecting the ground subsidence, though many qualitative results have been obtained by the field observations and the experimental studies in the labolatory[1)2)3)], a fundamental investigation under simplified conditions seems to be necessary to estimate them quantitatively.

2. The Approximate Calculation Method

(1) First process of consolidation

A clay layer with the thickness of h which is contiguous to the aquifer at the top and to the impervious foundation at the bottom is treated, and the analysis is based on the following fundamental assumptions ; 1) the voids of

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the clay are completely saturated with water which flows upwards one-dimensionally when the uniformly distributed surcharge p is applied on the clay layer ; 2) the initial hydrostatic excess pressure u_{01} is equal to the uniform surcharge p throughout the whole height of the layer as shown BC in Fig. 1. Maintaining p constant, the distributions of hydrostatic excess pressures change along the isochrones such as AEC, AFC, and AG as time goes on. The following equations have been given by Terzaghi, based on the assumption that the curves AE, AF and AG can be replaced approximately by parabolas[4)];

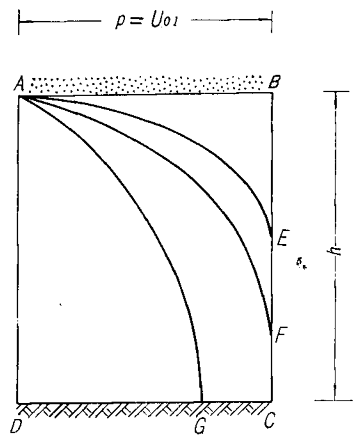

Fig. 1. Isochrones representing progress of consolidation of clay layer after drainage.

for $0 \leqq t \leqq h^2/12c_v$ and $0 \leqq z \leqq 2\sqrt{3c_vt}$,

$$u(z, t) = u_{01}\left\{1 - \frac{(2\sqrt{3c_vt} - z)^2}{12c_vt}\right\}, \tag{1-a}$$

for $0 \leqq t \leqq h^2/12c_v$ and $2\sqrt{3c_vt} \leqq z \leqq h$,

$$u(z, t) = u_{01}, \tag{1-b}$$

and for $h^2/12c_v \leqq t \leqq \infty$ and $0 \leqq z \leqq h$,

$$u(z, t) = u_{01}e^{-(3c_vt/h^2 - 1/4)}\left\{1 - \frac{(h-z)^2}{h^2}\right\}, \tag{1-c}$$

where $u(z, t)$ is the hydrostatic excess pressure at the depth z from the top of the clay layer at the time t and c_v is the coefficient of consolidation. The average hydrostatic excess pressure u_m over the height of the layer at any time during the consolidation can be calculated by the equations (1-a), (1-b) and (1-c) and the average effective stress σ'_m can be obtained as follows ;

for $0 \leqq t \leqq h^2/12c_v$,

$$\sigma'_m = p - u_{01}\left(1 - \frac{2\sqrt{3c_vt}}{3h}\right), \tag{2-a}$$

and for $h^2/12c_v \leqq t \leqq \infty$,

$$\sigma'_m = p - \frac{2}{3}u_{01}\, e^{-(3c_vt/h^2 - 1/4)}. \tag{2-b}$$

The volumetric strain $\Delta V/V$ at any time, which is the average value over the height of the layer, is considered as an index for the average degree of consolidation U_m at the same time. Hence it seems to be rather reasonable to assume the correlation between $\Delta V/V$ and σ'_m.

Immediately after the uniformly distributed surcharge p is applied on the clay layer which has been completely consolidated by the preconsolidation pressure σ'_{01} ($p > \sigma'_{01}$), p is equal to u_{01}. As time goes on, the hydrostatic excess pressure decreases gradually and the effective stress increases at the same time. In this case, in order to facilitate analysis, it is assumed that the clay layer is in a state of overconsolidation in a region $\sigma'_m < \sigma'_{01}$ and is in a state of normal consolidation in a region $\sigma'_m \geq \sigma'_{01}$, although both states actually exist until a finite time because the effective stress may change with depth within the clay layer at any time.

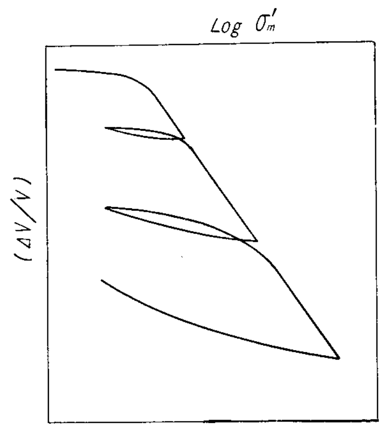

Fig. 2. Correlation between the volumetric strain and the average effective stress.

Plotting $\Delta V/V$ to the logarithm of the average effective stress σ'_m, the curve such as shown in Fig. 2 is obtained in general. Hence $\Delta V/V$ versus t curve in first process of consolidation by p can be plotted by reading $\Delta V/V$ in Fig. 2 which corresponds to σ'_m at t calculated by the equation (2-a) or (2-b). While the relation bewteen $\Delta V/V$ and σ'_m can be written approximately as follows ;

for $\sigma'_m < \sigma'_{01}$, $\quad \Delta V/V = a\sigma'^{b}_m$, (3-a)

and for $\sigma'_m \geq \sigma'_{01}$, $\quad \Delta V/V = c \log d\sigma'_m$. (3-b)

The approximate relationship between $\Delta V/V$ and t can therefore be obtained by substituting the equation (2-a) or (2-b) into the equation (3-a) or (3-b). In the equations (3-a) and (3-b) a, b, c and d are the experimental constants determined by the consolidation curve in Fig. 2.

(2) First process of swelling

Removing p instantly at $t = t_0$ when consolidation by p is in progress, the clay layer begins to swell gradually. In this process the magnitude of the coefficient of swelling c_s is assumed to be equal to that of the coefficient of consolidation c_v in order to simplify the analysis.

The distribution of hydrostatic negative pressure in the clay layer in this process is determined by superposing the hydrostatic excess pressure by p upon the hydrostatic negative pressure by negative surcharge $-p$. However, it must be kept in mind that the question remains whether S is equal to unity in the equation $u_{t0} = -S\sigma'_{t0}$ immediately after the surcharge p is removed from the clay layer. Generaly S in the test results is less than unity. This should be considered to be caused mainly by the inherent characters of the clay. But it is assumed here that S is equal to unity.

Fig. 3 shows the distributions of hydrostatic excess or negative pressures by p or $-p$ and their superposed distribution curves in the case of $t_0 \geq h^2/12c_v$. In this figure the curve marked (o) shows the distribution of hydrostatic excess pressure by p at $t=t_0$, and the curves marked (1′), (2′), (3′), etc. show changes of the distributions by p after the time t_0, while the curves marked (1″), (2″), (3″), etc. are those by $-p$ corresponding to (1′), (2′), (3′), etc., respectively. The average effective stress σ_m' in this process of swelling is calculated from these superposed curves (1), (2), (3), etc. as follows ;

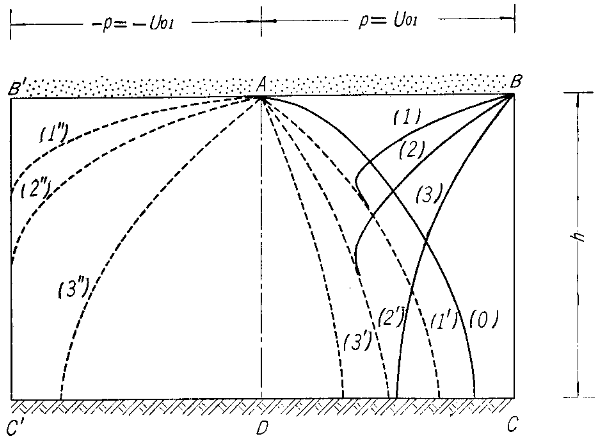

Fig. 3. Isochrones representing progress of swelling of a clay layer after remove of the surcharge.

for $t_0 \leqq t \leqq h^2/12c_v$,

$$\sigma_m' = \frac{2u_{01}\sqrt{3c_v}}{3h}\{\sqrt{t}-\sqrt{(t-t_0)}\}, \tag{4-a}$$

for $t_0 \leqq h^2/12c_v \leqq t \leqq (h^2+12c_vt_0)/12c_v$

or $h^2/12c_v \leqq t_0 \leqq t \leqq (h^2+12c_vt_0)/12c_v$,

$$\sigma_m' = u_{01}\left\{1-\frac{2}{3}e^{-(3c_vt/h^2-1/4)}-\frac{2\sqrt{3c_v(t-t_0)}}{3h}\right\}, \tag{4-b}$$

and for $(h^2+12c_vt_0)/12c_v \leqq t$,

$$\sigma_m' = \frac{2u_{01}}{3}e^{-(3c_vt/h^2-1/4)}\{e^{3c_vt_0/h^2}-1\}. \tag{4-c}$$

Therefore the correlation between $\Delta V/V$ and t in first process of swelling can be obtained using $\Delta V/V$ which corresponds to σ_m' calculated by the equation (4-a), (4-b) and (4-c), or substituting those equations into the following approximate equation ;

$$\Delta V/V = a_1\sigma_m'^{b_1}+c_1 \tag{5}$$

where a_1, b_1 and c_1 are the experimental constants determined from the swelling curves in Fig. 2.

(3) N-th process of consolidation or swelling

It is assumed that the process of consolidation or swelling is repeated at a constant time interval t_0. At $t=2t_0$ in first process of swelling, some amount of hydrostatic negative pressures still remains. The distributions of these residual pressures are shown in Fig. 3 by the parabola with or without an inflexion point. It can easily be seen that the pore water flows downward in the vicinity of the top and upwards near the bottom of the clay layer in the cases shown by the curves (1) and (2). Since deformation characters in the process of drainage are different from those in the process of suction, the clay layer in the actual state is already in the state of nonhomogeneity. However in order to simplify the calculations, this method is based on the following assumptions ; homogeneity within the clay layer is kept ; c_v is kept in constant ; changes of the initial thickness of the clay layer in each process are negligible as compared with the initial thickness h in first process ; and the N-th process of consolidation starts after the residual hydrostatic negative pressure in (N$-$1)th process of swelling distributes uniformly within the clay layer.

If the increment in the effective stress during t_0 in first process of consolidation is Au_{01} and the increment due to the surcharge p from the initial state is $(A+B)u_{01}$ at $2t_0$, the total increment produced in first cycle of consolidation and swelling is equal to Bu_{01}, because the hydrostatic negative pressure due to the negative surcharge $-p$ also changes by $-Au_{01}$ during first process of swelling (Fig. 4). Thus the initial hydrostatic excess pressure u_{02} produced immediately after the beginning of the second process of consolidation is equal to $(1-B)u_{01}$ and the effective stress increases by $A(1-B)u_{01}$ during the time interval t_0 in the second process of consolidation. Since the additional effective stress by p and the decrement in the hydrostatic negative pressure by $-p$ during t_0 after the beginning of second process of swelling is equal to $B(1-B)u_{01}$ and $-A(1-B)u_{01}$, respectively, the magnitude

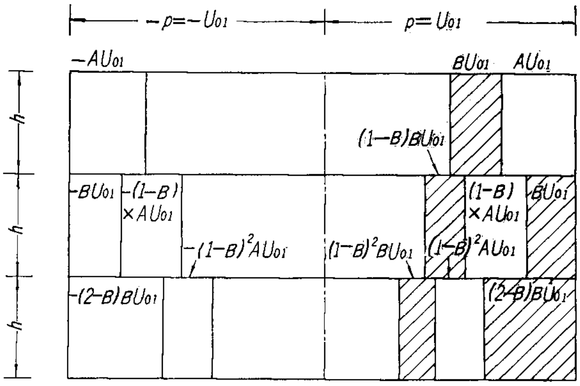

Fig. 4. Diagram illustrating increments in the average effective stress.

of the increased effective stress through the second cycle of consolidation and swelling is eventually equal to $B(1-B)u_{01}$. Similarly the increment in the effective stress through (N−1)th cycle becomes to be equal to $B(1-B)^{N-2}u_{01}$. Hence the total effective stress produced during the total process to (N−1)th cycle and the initial hydrostatic excess pressure u_{0N} of N-th process of consolidation can be expressed with the following equations, respectively ;

$$B\{1+(1-B)+(1-B)^2+\cdots+(1-B)^{N-2}\}u_{01}=\{1-(1-B)^{N-1}\}u_{01} \tag{6}$$

and

$$u_{0N}=(1-B)^{N-1}u_{01}\,. \tag{7}$$

Thus the changes of the average effective stress σ'_m in N-th process of consolidation are given as follows ;

for $0\leqq t^{(N)}\leqq h^2/12c_v$,

$$\sigma'_m=p-u_{0N}\left(1-\frac{2\sqrt{3c_vt^{(N)}}}{3h}\right), \tag{8-a}$$

and for $h^2/12c_v\leqq t^{(N)}\leqq t_0$,

$$\sigma'_m=p-\frac{2}{3}u_{0N}\,e^{-\{3c_vt^{(N)}/h^2-1/4\}}, \tag{8-b}$$

in which $t^{(N)}$ is measured time from the beginning of the N-th process of consolidation. The following equation (9) shows the effective preconsolidation pressure σ'_{0N}, which is equal to the sum of the increments in the effective stress at the end of (N−1)th process of swelling.

$$\sigma'_{0N}=\{1-(1-A)(1-B)^{N-2}\}u_{01} \tag{9}$$

The constant A and B are determined by replacing σ'_m in the equation (2-a) or (2-b) with Au_{01} and $(A+B)u_{01}$;

for $0\leqq t_0\leqq h^2/24c_v$,

$$\left.\begin{aligned}A&=\frac{2\sqrt{3\,c_vt_0}}{3h},\\ B&=\frac{2(\sqrt{2}-1)\sqrt{3c_vt_0}}{3h},\end{aligned}\right\} \tag{10-a}$$

for $h^2/24c_v\leqq t_0\leqq h^2/12c_v$,

$$\left.\begin{aligned}A&=\frac{2\sqrt{3c_vt_0}}{3h},\\ B&=1-\frac{2}{3}e^{-(6c_vt_0/h^2-1/4)}-\frac{2\sqrt{3c_vt_0}}{3h},\end{aligned}\right\} \tag{10-a}$$

and for $h^2/12c_v\leqq t_0\leqq\infty$,

$$\left.\begin{aligned}A&=1-\frac{2}{3}e^{-(3c_vt_0/h^2-1/4)},\\ B&=\frac{2}{3}e^{-(3c_vt_0/h^2-1/4}\{1-e^{-3c_vt_0/h^2}\}\,.\end{aligned}\right\} \tag{10-c}$$

Thus the correlation between $\Delta V/V$ and t in N-th process of consolidation can be determined using the equations (8-a), (8-b), and (9) with the curves in Fig. 2.

Subsequently, the changes of the average effective stress σ'_m in N-th process of swelling can be obtained by using the similar procedure with the case of the first process of swelling, as shown in the next equations;

for $t_0 \leqq t^{(N)} \leqq h^2/12c_v$,

$$\sigma'_m = \frac{2u_{0N}\sqrt{3c_v}}{3h}\left(\sqrt{t^{(N)}} - \sqrt{(t^{(N)}-t_0)}\right) + (u_{01}-u_{0N}), \tag{11-a}$$

for $t_0 \leqq h^2/12c_v \leqq t^{(N)} \leqq (h^2+12c_vt_0)/12c_v$

or $h^2/12c_v \leqq t_0 \leqq t^{(N)} \leqq (h^2+12c_vt_0)/12c_v$,

$$\sigma'_m = u_{0N}\left\{1-\frac{2}{3}e^{-(3c_vt^{(N)}/h^2-1/4)} - \frac{2\sqrt{3c_v(t^{(N)}-t_0)}}{3h}\right\} + (u_{01}-u_{0N}), \tag{11-b}$$

and for $(h^2+12c_vt_0)/12c_v \leqq t^{(N)}$,

$$\sigma'_m = \frac{2}{3}u_{0N}e^{-(3c_vt^{(N)}/h^2-1/4)}\{e^{3c_vt_0/h^2}-1\} + (u_{01}-u_{0N}). \tag{11-c}$$

Thus $\Delta V/V$ *versus* t curve in N-th process of swelling can be described by using equations (11-a), (11-b) and (11-c) with the swelling curves in Fig. 2.

3. Test Results Using a Triaxial Apparatus

In this section some experimental results for the saturated clay specimens obtained from triaxial consolidation tests are compared with those obtained from the analytical method.

In these tests the remoulded clay artificially consolidated under the pre-compression pressure 0.5 kg/cm^2 was used. The physical properties are: LL=75.0%, PL=40.5% and the clay fraction=43.5%. In order to obtain the same stress and boundary conditions with those of the numerical analysis, the constant cell pressure 2.0 kg/cm^2 was applied on and removed from the cylindrical clay specimen which had a diameter of 3.5 cm and a height of 6.0 cm. The drainage was allowed only at the top of the specimen.

Before the repetitional cosolidation tests, the following two series of tests were carried out in order to obtain the correlation between $\Delta V/V$ and σ'_m and c_v value which were necessary for analysis. In the first series of test, the relationship between $\Delta V/V$ and σ'_m were determined as shown in Fig. 5. In this test the cell pressure σ_3 was applied on a clay specimen, increasingly or decreasingly in a region 0.1 kg/cm$^2 \leqq \sigma_3 \leqq$ 2.0 kg/cm^2. Each increment or decrement in σ_3 was applied with an interval of 48 hours which was considered to be enough for complete dissipation of the hydrostatic excess pressure within a specimen. Hence σ_3 values at 48 hours after the increase or the decrease in the cell pressure were used as σ'_m in Fig. 5.

The second series of test was carried out to obtain c_v value, the effective stress, the pore water pressure and the volumetric strain *versus* time curves under the pressure condition without repetition. In this test the clay specimen was consolidated for a long time under the constant cell pressure 2.0 kg/cm^2 with the pore pressure measurement at the bottom of the specimen. Fig. 6 shows the test results and the changes of σ'_m calculated by the equations (2-a) and (2-b). From the curve of $\Delta V/V$ *versus* t in this figure c_v values were calculated. It is well known that c_v value decreases with progress of consolidation, and in general cosolidation problems, therefore it seems to be proper that c_v is dependent of time t as in R. L. Schiffman's method[5]. In the following calculations, however, the coefficient of consolidation at 50 per cent consolidation $c_v = 3.33 \times 10^{-2}$ cm^2/min was used. The dotted curve in Fig. 6 shows the $\Delta V/V$ *versus* t curve which was calculated with the procedure mentioned in 2(1) by using $c_v = 3.33 \times 10^{-2}$ cm^2/min. It can be seen that this calculated

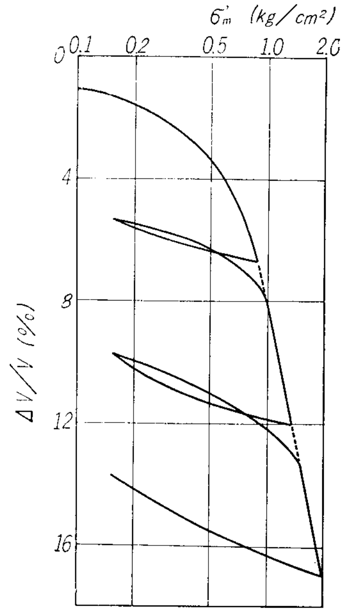

Fig. 5. Correlation between the volumetric strain and the average effective stress.

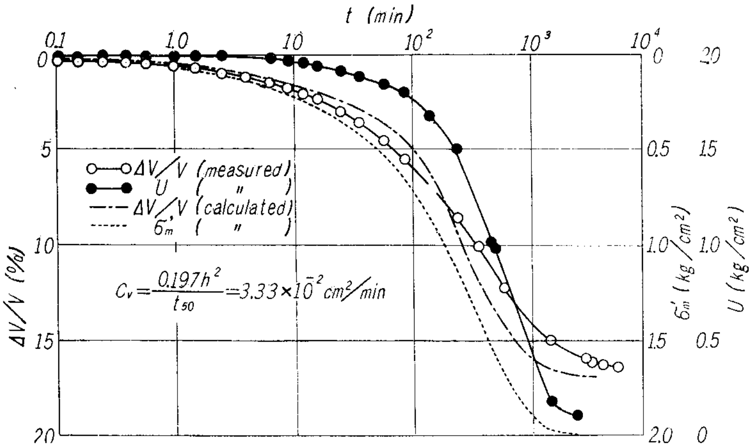

Fig. 6. The volumetric strain, the average effective stress, the pore water pressure at the bottom *versus* time curves.

result is in good agreement with the measured one.

Subsequently, the repetitional consolidation tests were carried out. In these tests the constant cell pressure 2.0 kg/cm^2 was applied on and removed from the clay specimens with various time intervals i.e. 3, 6, and 12 hours. The typical measured and calculated curves of the volumetric strain *versus* time for the case of 12 hours interval are shown in Fig. 7. From this figure it can be seen that the volumetric strain decreases gradually with the increase in the number of repetitions and these two curves are in considerably good agreement. The following characteristics of these two curves should be noted ;

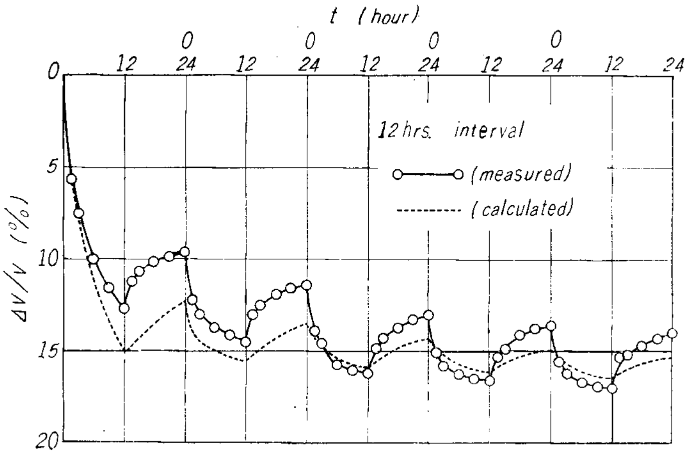

Fig. 7. The volumetric strain *versus* the time curves for the case of 12 hours interval.

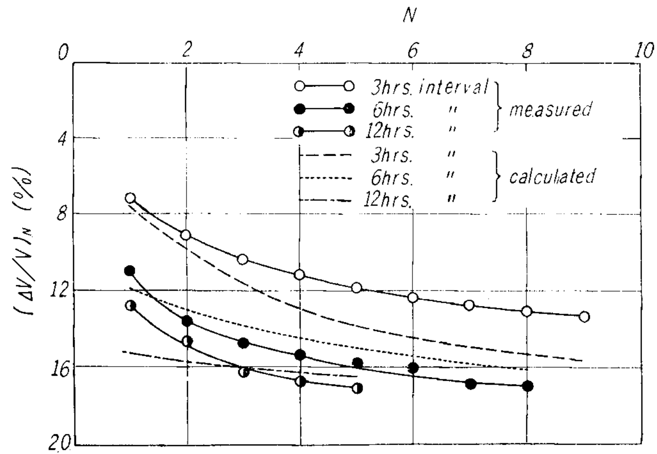

Fig. 8. The volumetric strain at the end of N-th process of consolidation *versus* the number of repetion.

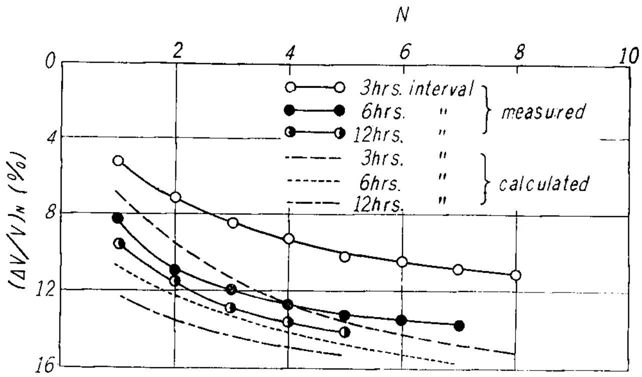

Fig. 9. The volumetric strain at the end of N-th process of swelling *versus* the number of repetition.

the calculated amounts of compression or expansion in each process of consolidation or swelling are smaller than the measured ones with exception of the first consolidation process. Similar results were observed for the cases of 3 hours and 6 hours intervals.

Fig. 8 and Fig. 9 indicate the correlation beteeen the total volumetric strain at the end of N-th process of consolidation or swelling $(\Delta V/V)_N$ and the number of repetition N, respectively. From these figures it can be seen that the longer the cycle of repetition, the greater the total volumetric strain at the same number of repetition, and that the calculated values are in considerably good agreement with the measured ones with the exception of the swelling case with the 3 hours interval.

4. Conclusion

This paper proposes an approximate method of calculating the amount of consolidation when the repetitional, uniformly distributed surcharge is applied on a saturated clay and then some results of experimental tests using a triaxial apparatus are briefly explained.

This approximate method has been developed by the following procedures;

1) Assumption of the saturated clay layer adjoining the aquifer at the top and the impervious foundation at the bottom.

2) Assumption of upward one-demensional flow.

3) Assumption of the initial hydrostatic excess pressure equal to the uniformly distributed surcharge throughout the whole height of the layer and that of the isochrones of parabola.

4) Assumption of the distribution of pore water pressure in the process of swelling which is determined by the superposition of the hydrostatic excess

pressure upon the hydrostatic negative pressure by uniformly distributed negative surcharge.

5) Calculation of the average effective stress over the height of the layer in each process of consolidation or swelling.

6) Determination of the relationship between the average effective stress and the volumetric strain by the experimental tests.

7) Determination of the correlation between the volumetric strain and the time.

Some analytical results of the triaxial consolidation tests by this method are summarized as follows;

1) In the consolidation tests without repetition, the calculated result is in good agreement with the measured one.

2) In the repetitional consolidation tests, the volumetric strain decreases gradually with increase in the number of repetition and the longer the cycle of repetition, the greater the total volumetric strain at the same number of repetiton.

3) In the same tests, the calculated correlations between the volumetric strain and the time are in considerably good agreement with those in measurement, although in calculation there is not so much or rapid increase of expansion in the early stages of the process of swelling as in measurement.

The authors consider that some kinds of phenomena for the ground subsidence can be treated analytically with a similar meteod in a future study.

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