Study on Optimizing Airport Runway Design by Economic Analysis

By

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A rapid increase of air traffic is anticipated by the main airports in Japan. The existing airport facilities of the main airports in Japan are already thought to be insufficient, and it is earnestly requested that these facilities will be reinforced as urgently as possible.

It is vital that we determine the ultimate runway capacity to make a plan more accurately for long range development of adequate airport facilities on a schedule which will meet the projected increase in air traffic.

Mathematical models have been developed that can be used in forecasting the operating rates of runways with associated delays. These models have been devised covering delays under preemptive priority type operations.

The optimum runway design is that which will manage to keep the balance between operating cost and total annual costs of runway. The optimum design then is determined by the economic analysis of the factors that affect runway operations. Economic analysis have been developed that could be used in determining the break-even volume for the improvement of runways.

1. Introduction

It has been found useful to study on assessing accurately the capacity of airport runways by analyzing the delay to operations.

Mathematical models have been developed that can be used in forecasting the operating rates of runways with associated delays. These models that have been devised use observed operating data in relating delay to movement rates for runway operations.

Original mathematical models have been devised covering delays under preemptive priority type operations. These models, together with available queuing models, have been used in typical runway operating problems to determine more accurately optimum runway capacity and a minimum spacing of aircraft to the runway.

It is vital that we determine ultimate runway capacity to more accurately

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plan for long range development of adequate airport facilities on a schedule which will meet the projected increase in traffic.

In the past, airport construction and improvement for runways could not easily be evaluated from an economic standpoint because of a lack of good means of analysis.

Many different airport layouts exist round the country. This fact, together with the growth of air traffic and the increase in operations at these airports, requires more uniform design criteria and more precise economic measures for airport improvements.

In this way, it will be possible to measure more accurately the improvement in delay or capacity obtainable with a given airport design and judge whether or not improvements should be undertaken from the economic point of view.

Applying this method of analysis to the Osaka International Airport, it was made clear that this method is a general procedure which does well with the typical runway configurations and optimum runway capacity from an economic point of view.

Throughout certain abbreviations will be used to simplify this paper:

VFR visual flight rule,
IFR instrument flight rule,
OT over threshold of runway,
OR off runway,
RG ready to go: normally, on piston and turboprop aircraft, **RG** occurs off the runway at end of engine run up (if any),
CTO clear to take-off (controller's call),
SR start roll of take-off.

2. Preparation of Preemptive Arrivals Model for Average Delay

This model is a complex analytical expression that can be used for forecasting operating rates and delays for mixed runway operations. In mixed runway operations, the normal procedure is that landings are given higher priority and departures await a gap in the landing sequence before they can be cleared for take-off.

In the most serious cases of congestion, the controllers will vary this discipline and ask pilots to delay to permit them to handle departures, however, this is the exceptional case. Thus, the preemptive spaced arrivals model is based on the general case where landing aircraft will be given higher priority than departing aircraft, An important subject in waiting lines is that of preemptive priorities. In preemptive priority discipline the arrival of a higher priority while a lower priority is in service requires the return to the queue of the lower priority.

Let us suppose that a fraction α of the arriving units has the right to be served ahead of the remaining $(1-\alpha)$, though it do not have the right of displace a lower priority unit once it has gained the service facility.

This plan assumes again that arrivals are Poisson (rate $\alpha\lambda$ for higher priority units and rate $(1-\alpha)\lambda$ for lower priority), and that the service channel is exponential (mean service rate μ for higher priority units and mean service rate $\beta\mu$ for lower priority).

If one is to distinguish between the two types of arriving units, the state probabilities must have three subscripts; the first denoting whether a higher priority or a lower priority unit is in service; the second, m, indicating how many higher priority units are in the system; and the third, n, giving the number of lower priority units in the system. This assumes that the service channel takes higher priority units from the queue as long as higher priority units are present. The equations of detailed balance are derived as follows:

$$\left.\begin{array}{l}
P_{110}+\beta_{201} = \rho P_0 \\
\alpha\rho P_0+P_{120}+\beta P_{211} = (\rho+1)P_{110} \\
(1-\alpha)\rho P_0+P_{111}+\beta P_{202} = (\rho+\beta)P_{201} \\
\alpha\rho P_{1,m-1,0}+P_{1,m+1,0}+\beta P_{2m1} = (\rho+1)P_{1m0} \\
(1-\alpha)\rho P_{2,0,n-1}+P_{11n}+\beta P_{2,0,n+1} = (\rho+\beta)P_{20n} \\
(1-\alpha)\rho P_{1,1,n-1}+P_{12n}+\beta P_{2,1,n+1} = (\rho+1)P_{1n} \\
\alpha\rho P_{2,m-1,1} = (\rho+\beta)P_{2m1} \\
\alpha\rho P_{1,m-1,n}+(1-\alpha)\rho P_{1,m,n-1}+P_{1,m+1,n}+\beta P_{2,m,n+1}=(\rho+1)P_{1mn}, \quad (m>1\,;n>0) \\
\alpha\rho P_{2,m-1,n}+(1-\alpha)\rho P_{2,m,n-1} = (\rho+\beta)P_{2mn}, \qquad (m>0\,;\ n>1) \\
\text{Where, } \rho = \lambda/\mu
\end{array}\right\} \quad (1)$$

To get any more detail out of our model, we have to solve the generating functions $F_{sm}(y)$, at least enough to obtain an explicit solution for $F_{20}(y)$. By multiplying Eqs. (1) by the appropriate power of y and then adding, we get the following:

$$\left.\begin{array}{l}
[\beta+\rho-(1-\alpha)\rho y]F_{2m}(y) = \alpha\rho F_{2,m-1}(y), \qquad (m>0) \\
\left[\beta+\rho-(1-\alpha)\rho y-\dfrac{\beta}{y}\right]F_{20}(y) = F_{11}(y)-[\rho-(1-\alpha)\rho y]P_0 \\
-F_{12}(y)+[1+\rho-(1-\alpha)\rho y]F_{11}(y)-\alpha\rho P_0 = \dfrac{\beta}{y}F_{21}(y) \\
-F_{1,m+1}(y)+[1+\rho-(1-\alpha)\rho y]F_{1m}(y)-\alpha\rho F_{1,m-1}(y) = \dfrac{\beta}{y}F_{2m}(y), \quad (m>1)
\end{array}\right\} \quad (2)$$

From these we can find that

$$\left.\begin{aligned} F_{1m}(1) &= \frac{\beta(\beta+\alpha\rho-1)+\rho(1-\alpha)}{\beta(\beta+\alpha\rho-1)}(1-\alpha\rho)(\alpha\rho)^m-\frac{\rho(1-\alpha)}{\beta+\alpha\rho-1}\left(\frac{\alpha\rho}{\beta+\alpha\rho}\right)^m \\ F_{2m}(1) &= \frac{\rho(1-\alpha)}{\beta+\alpha\rho}\left(\frac{\alpha\rho}{\beta+\alpha\rho}\right)^m \end{aligned}\right\} \quad (3)$$

From these, by summation, we can obtain the averages we wish. For example, the probability that a higher priority unit is in service is

$$\sum_{m=1}^{\infty} F_{1m}(1) = \alpha\rho$$

and

$$\sum_{m=0}^{\infty} F_{2m}(1) = (1-\alpha)\frac{\rho}{\beta} \quad (4)$$

is the probability of finding a lower priority unit in service. Both of these have the same value as if no priority had been imposed. But the mean number in the system is changed by imposition of priorities. The mean number of higher priority units in the system is

$$\begin{aligned} L_1 &= \sum_{m=1}^{\infty} m[F_{1m}(1)+F_{2m}(1)] \\ &= \alpha\rho+\alpha\rho^2\frac{\alpha+(1-\alpha)(1/\beta^2)}{1-\alpha\rho} \end{aligned} \quad (5)$$

The mean number of lower priority units in the system obtained by the same procedure, is formulated as follows:

$$L_2 = (1-\alpha)(\rho/\beta)+\left[\frac{(1-\alpha)\rho^2}{1-\alpha\rho}\right]\frac{\alpha+(1-\alpha)(1/\beta^2)}{1-\alpha\rho-(1-\alpha)(\rho/\beta)} \quad (6)$$

The average waiting times W_1 for higher priority units and W_2 for lower priority units obtained by the above procedure, are formulated as follows:

$$\left.\begin{aligned} W_1 &= \frac{\rho^2[\alpha+(1-\alpha)(1/\beta^2)]}{\lambda(1-\alpha\rho)} \\ W_2 &= \left[\frac{\rho^2}{\lambda(1-\alpha\rho)}\right]\frac{\alpha+(1-\alpha)(1/\beta^2)}{1-\alpha\rho-(1-\alpha)(\rho/\beta)} \end{aligned}\right\} \quad (7)$$

3. Evaluation of Mean Service Times for Arrivals and Departures

(1) Minimal Arrival Spacing S^*

This is the average minimum spacing between two arrivals on the same runway. It is measured from **OT** of the first aircraft to the **OT** of the second aircraft, when the spacing is at a minimum.

OT to **OT** spacing is affected by:

(a) Mixture of aircraft types,

(b) Arrival rate $\lambda_1=\alpha\lambda$.

A detailed analysis of the observed data was made to extract the minimum **OT** to **OT** times. Generally, this time was measured when two arrivals occurred and a departure was **RG** before **OT** of the first arrival, and the departure was not released between the two arrivals.

It was possible to make up tables showing the variance of **OT** to **OT** time with λ_1 and type of aircraft. Four basic types of aircraft were used:

Jet : Large turbo jet aircraft,

Heavy : Four-engine propeller transports, including turboprops, and heavy two-engine transports such as CV 440,

Medium : Two-engine transports (8,000 to 36,000 pounds),

Light : Two-engine aircraft (2,800 to 8,000 pounds) and single-engine aircraft.

Breaking down these types into their combinations gives a total of 16, that is, Jet followed by Jet, Jet followed by Heavy, etc.

In some cases there was a scarcity of data at the lower movement rates, but the table presented in Table 1 is considered to be a very realistic indication of pilots' performance, particularly at the higher movement rates.

Table 1. Average minimal spacing S^* between successive arrivals at increasing rate. (unit: sec.)

State \ Arrival rate	10	15	20	25	30	35
(1) Jet followed by Jet	103.0	92.5	84.5	80.0	77.0	75.0
(2) Jet followed by Heavy	103.0	92.5	84.5	80.0	77.0	75.0
(3) Jet followed by Medium	106.0	95.5	87.0	81.0	78.0	76.5
(4) Jet followed by Light	102.0	92.0	84.0	76.5	72.0	69.5
(5) Heavy followed by Jet	98.0	87.5	79.5	74.5	71.5	70.0
(6) Heavy followed by Heavy	98.0	87.5	79.5	74.5	71.5	70.0
(7) Heavy followed by Medium	98.5	88.0	79.5	74.0	70.5	69.0
(8) Heavy followed by Light	90.0	80.0	71.5	64.5	60.0	57.5
(9) Medium followed by Jet	78.0	71.0	66.5	63.0	60.5	60.0
(10) Medium followed by Heavy	78.0	70.5	64.5	59.0	54.5	52.0
(11) Medium followed by Medium	75.0	70.0	65.0	61.0	59.0	58.0
(12) Medium followed by Light	98.5	81.5	69.0	58.5	54.5	53.5
(13) Light followed by Jet	88.5	77.0	66.5	58.5	53.0	50.5
(14) Light followed by Heavy	88.5	77.0	66.5	57.5	51.5	49.0
(15) Light followed by Medium	66.0	58.5	52.5	47.0	43.5	41.0
(16) Light followed by Light	57.0	51.5	46.0	41.0	35.5	31.5

(2) Minimal Spacing Between Arrival Followed by Departure S^{**}

This is the average minimum spacing between an arrival followed by a departure. It is measured from **OT** of the arrival to **SR** of the subsequent departure, when the spacing is at a minimum.

Of all the inputs to the spaced arrival model, it is the most difficult to define and therefore requires the most careful attention in its formation.

Since most runway possess good entrance taxiways, the subsequent departure can normally be on the runway before the previous arrival has completed its maneuvers. Therefore, this input is a combination of two easily measured factors—the interval **OT** to **OR** of the arrival plus the interval **CTO** to **SR** of the subsequent departure.

S^{**} varies with λ and with aircraft types. As in **OT** to **OT**, 16 combination of aircraft types were considered.

After some calculation of observed data these minimum spacing times between an arrival followed by a departure at increasing movement rate were obtained and are presented in Table 2:

Table 2. Average minimal spacing S^{**} between arrival followed by departure at increasing movement rate.

(unit: sec.)

State \ Movement rate	20	30	40	50	60	70
(1) Jet followed by Jet	88.5	86.5	85.0	85.5	86.0	86.0
(2) Jet followed by Heavy	58.5	55.5	53.0	53.0	53.0	51.5
(3) Jet followed by Medium	60.0	57.0	54.5	53.5	53.5	52.5
(4) Jet followed by Light	58.0	55.0	53.0	51.0	50.0	48.0
(5) Heavy followed by Jet	86.0	84.0	82.0	82.5	83.0	83.0
(6) Heavy followed by Heavy	56.0	53.0	50.5	50.0	49.5	48.5
(7) Heavy followed by Medium	56.0	53.0	50.5	49.5	49.0	48.0
(8) Heavy followed by Light	52.5	49.0	46.0	44.5	42.5	41.0
(9) Medium followed by Jet	76.5	75.5	75.0	76.0	76.5	76.5
(10) Medium followed by Heavy	46.5	44.5	42.5	41.0	39.5	37.5
(11) Medium followed by Medium	45.5	44.0	43.0	42.5	42.0	41.0
(12) Medium followed by Light	56.5	50.0	45.0	41.0	39.5	38.5
(13) Light followed by Jet	81.5	78.5	75.0	73.5	72.0	71.0
(14) Light followed by Heavy	51.5	47.5	43.5	40.5	37.5	35.5
(15) Light followed by Medium	41.0	38.5	36.0	34.5	33.0	31.0
(16) Light followed by Light	37.0	35.0	32.5	31.0	28.0	25.0

(3) Minimal Departure Spacing S^{***}

This is the average minimum spacing between **CTO** of the first departure and **CTO** of the second. On an average basis this is the same as **SR** to **SR** and was extracted from the data as a minimum where the second departure was **RG** before **SR** of the first.

S^{***} depends upon:

(a) Total movement rate λ,

(b) Aircraft types.

As in **OT** to **OT**, the control procedures can affect S^{***}. Again only **VFR** is considered. All the data used for the compilation of S^{***} were taken under **VFR** conditions at airports having entrance taxiways at the runway threshold.

Under these conditions the second departure normally has time to enter and line up on the runway before the first aircraft has completed the take-off maneuver. At airports where it is necessary to backtrack down the runway before take off, S^{***} can be altered considerably.

As in **OT** to **OT**, 16 combination of aircraft types were considered. Table 3 shows the **CTO** to **CTO** times for a variety of different combinations of aircraft types as a function of movement rate:

Table 3. Average minimal spacing S^{***} between successive departures at increasing movement rate.

(unit: sec.)

State \ Movement rate	20	30	40	50	60	70
(1) Jet followed by Jet	98.5	92.0	86.0	80.0	77.5	76.5
(2) Jet followed by Heavy	96.0	87.5	79.5	72.5	68.0	65.0
(3) Jet followed by Medium	79.5	75.0	71.0	66.5	62.5	60.0
(4) Jet followed by Light	80.5	75.0	70.5	65.5	64.0	63.5
(5) Heavy followed by Jet	103.5	97.0	91.0	85.0	82.0	81.5
(6) Heavy followed by Heavy	83.5	76.5	69.5	62.5	59.0	57.0
(7) Heavy followed by Medium	72.0	67.0	62.5	58.0	53.5	49.5
(8) Heavy followed by Light	68.0	63.0	58.0	52.5	48.0	43.5
(9) Medium followed by Jet	93.0	92.0	89.0	83.0	74.0	69.5
(10) Medium followed by Heavy	70.5	69.0	65.0	58.0	48.0	42.5
(11) Medium followed by Medium	68.0	64.0	59.0	53.5	48.0	43.0
(12) Medium followed by Light	44.5	43.0	41.5	40.0	39.0	37.5
(13) Light followed by Jet	87.0	85.0	84.0	82.0	81.0	80.0
(14) Light followed by Heavy	61.5	59.5	57.0	55.0	52.5	51.0
(15) Light followed by Medium	61.5	59.0	56.0	53.5	50.5	48.0
(16) Light followed by Light	46.0	42.5	39.0	35.5	32.0	29.0

(4) Minimal Spacing Between Departure Followed by Arrival S^{****}

This is the average minimun time required to release and service a departure between the sequence of an arrival followed by an arrival or a departure followed by an arrival. Therefore, S^{****} clearly starts at **SR** of the departure. The time at which S^{****} ends obviously cannot be in excess of the time **OT** of the subsequent arrival.

For the portion **SR** to **OT**, a procedure similar to that used for computing S^* and S^{***} is used.

Table 4 shows the **SR** to **OT** times for a variety of different combinations of aircraft types as a function of movement rate:

Table 4. Average minimal spacing S^{****} between departure followed by arrival at increasing movement rate.

(unit: sec.)

State \ Movement rate	20	30	40	50	60	70
(1) Jet followed by Jet	108.0	94.0	81.0	70.0	61.5	56.0
(2) Jet followed by Heavy	108.0	94.0	81.0	70.0	61.5	56.0
(3) Jet followed by Medium	92.0	76.0	68.0	62.5	58.0	55.0
(4) Jet followed by Light	82.5	76.0	71.5	69.0	67.0	65.5
(5) Heavy followed by Jet	108.0	94.0	81.0	69.0	59.0	51.0
(6) Heavy followed by Heavy	108.0	94.0	81.0	69.0	59.0	51.0
(7) Heavy fyllowed by Medium	92.0	74.5	65.5	57.5	51.5	46.0
(8) Heavy followed by Light	72.5	66.0	61.5	58.5	57.0	55.5
(9) Medium followed by Jet	109.0	96.5	84.0	71.0	59.0	50.0
(10) Medium followed by Heavy	109.0	94.5	80.0	65.0	51.0	43.5
(11) Medium followed by Medium	82.0	75.5	70.0	65.0	61.0	58.0
(12) Medium followed by Light	55.0	47.5	42.5	40.0	38.5	38.0
(13) Light followed by Jet	88.0	74.0	64.0	56.0	51.5	50.0
(14) Light followed by Heavy	81.0	67.0	57.0	49.0	44.5	43.0
(15) Light followed by Medium	68.5	61.0	55.0	50.0	47.0	45.5
(16) Light followed by Light	65.0	63.5	62.0	60.5	59.0	57.5

(5) Calculating Mean Service Times for Arrivals and Departures

To use the Tables 1∼4, it is necessary to know the percentages of aircraft types at a given airport.

Let us assume that the percentages of aircraft types are given in Table 5:

Table 5. Percentages of Aircraft types.

Aircraft type	Percentages
Jet	δ_1
Heavy	δ_2
Medium	δ_3
Light	δ_4

On the other hand, supposing that the environment consists of mixed runway operations, there are 4 possible states, and the occurring probability of each event can be determined as follows:

Table 6. Occurring probability of each event.

Event	State	Probability
E_1	Arrival followed by arrival	α^2
E_2	Arrival followed by departure	$\alpha(1-\alpha)$
E_3	Departure followed by departure	$(1-\alpha)^2$
E_4	Departure followed by arrival	$\alpha(1-\alpha)$

In this Table 6, α denotes the ratio of arrival rate to movement rate (arrival rate plus departure rate).

Next, let us suppose that the environment consists of mixed aircraft operations at a runway, there are 16 possible states, and the occurring probability of each event can be determined as follows:

Table 7. Occurring probability of each event.

Event	State	Probability
e_1	Jet followed by Jet	$\delta_1\delta_1$
e_2	Jet followed by Heavy	$\delta_1\delta_2$
e_3	Jet followed by Medium	$\delta_1\delta_3$
e_4	Jet followed by Light	$\delta_1\delta_4$
e_5	Heavy followed by Jet	$\delta_2\delta_1$
e_6	Heavy followed by Heavy	$\delta_2\delta_2$
e_7	Heavy followed by Medium	$\delta_2\delta_3$
e_8	Heavy followed by Light	$\delta_2\delta_4$
e_9	Medium followed by Jet	$\delta_3\delta_1$
e_{10}	Medium followed by Heavy	$\delta_3\delta_2$
e_{11}	Medium followed by Medium	$\delta_3\delta_3$
e_{12}	Medium followed by Light	$\delta_3\delta_4$
e_{13}	Light followed by Jet	$\delta_4\delta_1$
e_{14}	Ligt followed by Heavy	$\delta_4\delta_2$
e_{15}	Light followed by Medium	$\delta_4\delta_3$
e_{16}	Light followed by Light	$\delta_4\delta_4$

From the above procedure, the probability of each sequence occurring can be determined.

The sum of the individual probabilities multiplied by their individual times gives the mean service time μ for arrivals (higher priority units, arrival rate $\alpha\lambda$), and $\beta\mu$ for departures (lower priority units, movement rate λ):

$$\left.\begin{aligned} \mu &= \alpha\sum_{ij}\delta_i\delta_j S^{*}_{ij} + (1-\alpha)\sum_{ij}\delta_i\delta_j S^{**}_{ij} \\ \beta\mu &= (1-\alpha)\sum_{ij}\delta_i\delta_j S^{***}_{ij} + \alpha\sum_{ij}\delta_i\delta_j S^{****}_{ij} \\ \rho &= \lambda/\mu \qquad\qquad (i, j = 1, 2, 3, 4) \end{aligned}\right\} \tag{8}$$

Calculating the equations (8) by the use of the values given in Table 8, the mean service times for arrivals and departures as a function of movement rate for a variety of different combination of aircraft types were obtained as shown in Table 9, 10:

Table 8. Percentage of aircraft types.

Case	δ_1	δ_2	δ_3	δ_4	Case	δ_1	δ_2	δ_3	δ_4
(1)	40	30	20	10	(5)	80	20		
(2)	20	40	30	10	(6)	40	60		
(3)	10	50	30	10	(7)		40	50	10
(4)	10	40	40	10	(8)		30	60	10

4. Interpretation of Delay and Operational Rates at Airports

The mathematical formulas discussed, when used to evaluate airport configurations, produce an average delay for the selected operating rates. It is important to realize the meaning of this average delay in order to make proper use of it.

In a typical analysis of runway operations, the average delay to an aircraft will increase if the number of landings and take-offs per hour are increased.

Solving the equation (7) by the use of the values given in Table 10, the average delay to an aircraft can be obtained. Figures 1 and 2 show how the average delay builds up as a function of movement rate for a variety of different combinations of aircraft types:

A complete delay analysis must include an analysis of the delay build-up as the operating rate increases to correspond to a typical distribution of daily operations by hours. For example, the hourly distribution of aircraft movements by percentage in the Osaka International Airport is indicated in Figure 3.

The build up in delay may be such that an airport can be temporarily overloaded without exceeding acceptable delay criteria, particularly if the increase in movement rate to the overloaded hour is abrupt. On the other hand, when the practical operating rates are exceeded for any length of time, the delay builds up very rapidly and an intolerable delay situation can very easily develop. This, of course, can be made still worse by equipment failures that temporarily reduce the airport operating capacity. Thus, the higher the airport utilization, the more dependable should be the equipment used for control procedures.

Table 9. Mean service times for arrivals and departures.

μ: Mean service time for arrivals. (sec.)

Case	α \ λ	20	30	40	50	60	70
1	0.3	77.1	73.5	70.4	68.3	66.6	64.8
1	0.4	79.2	74.6	70.2	67.5	65.5	63.9
1	0.5	80.5	74.3	69.3	66.4	64.6	63.2
1	0.6	81.0	73.5	68.2	65.4	64.1	63.2
1	0.7	80.6	72.2	67.2	64.9	64.0	63.7
2	0.3	70.7	67.2	65.0	61.8	59.7	57.9
2	0.4	73.2	68.7	64.4	61.6	59.2	57.6
2	0.5	75.1	69.0	64.1	60.9	58.8	57.5
2	0.6	76.0	68.7	63.4	60.4	59.0	57.8
2	0.7	76.2	67.9	62.9	60.5	59.2	58.9
3	0.3	68.6	64.7	61.3	59.0	56.9	54.8
3	0.4	70.9	66.4	62.0	59.1	56.8	54.8
3	0.5	73.1	67.0	62.0	58.7	56.5	55.0
3	0.6	74.3	66.9	61.6	58.4	57.0	55.7
3	0.7	74.7	66.5	61.3	58.7	57.4	57.0
4	0.3	66.7	63.1	60.3	57.9	56.6	56.0
4	0.4	69.4	65.1	60.9	58.0	55.8	53.8
4	0.5	71.6	65.7	60.9	57.6	55.5	54.0
4	0.6	72.8	65.6	60.5	57.3	56.3	54.4
4	0.7	73.2	64.3	60.1	57.6	56.1	55.7
5	0.3	90.4	86.8	84.9	82.3	81.2	79.8
5	0.4	91.6	88.2	82.5	80.4	78.9	77.9
5	0.5	92.0	85.6	80.8	78.7	77.3	76.3
5	0.6	97.6	84.0	78.8	76.9	75.9	75.2
5	0.7	90.3	81.9	77.3	75.3	74.7	74.7
6	0.3	80.7	77.0	73.5	71.8	70.3	68.6
6	0.4	83.0	78.0	73.5	71.1	69.6	68.1
6	0.5	84.5	78.0	72.8	70.5	68.8	67.8
6	0.6	84.8	77.2	72.2	69.7	68.9	68.0
6	0.7	84.7	76.2	71.8	69.7	69.0	68.8
7	0.3	62.7	59.4	56.4	54.0	52.0	49.8
7	0.4	65.6	61.4	57.4	54.5	52.1	50.1
7	0.5	68.0	62.3	57.6	54.3	52.0	50.5
7	0.6	69.4	62.5	57.4	54.2	52.8	50.9
7	0.7	70.1	62.1	57.3	54.7	52.9	52.5
8	0.3	61.0	58.1	55.3	53.0	51.0	48.9
8	0.4	63.8	60.1	56.3	53.4	51.2	49.2
8	0.5	66.1	61.0	56.5	53.2	51.1	49.6
8	0.6	67.6	61.2	56.3	53.2	51.9	49.8
8	0.7	68.4	60.9	56.2	53.7	51.6	51.2

$\beta\mu$: Mean service time for departures. (sec.)

Case	α \ λ	20	30	40	50	60	70
1	0.3	89.5	82.0	75.2	68.4	66.2	59.9
1	0.4	90.8	82.5	75.1	68.0	62.4	58.8
1	0.5	92.2	83.1	75.1	69.5	61.6	57.7
1	0.6	93.5	83.6	75.1	67.1	60.8	56.6
1	0.7	94.9	84.2	75.0	66.7	60.0	55.5
2	0.3	84.2	76.8	70.2	63.8	57.9	54.6
2	0.4	85.9	77.6	70.5	63.8	57.5	54.0
2	0.5	87.7	78.4	70.7	63.8	57.0	53.5
2	0.6	89.4	79.2	71.0	63.7	56.6	52.9
2	0.7	91.1	80.0	71.3	63.7	56.2	52.3
3	0.3	82.3	75.4	68.4	61.6	55.9	51.9
3	0.4	84.2	76.5	69.0	61.8	55.8	51.6
3	0.5	86.2	77.6	69.6	62.0	55.7	51.3
3	0.6	88.2	78.8	70.2	62.2	55.6	50.9
3	0.7	90.2	79.9	70.9	62.4	55.5	50.6
4	0.3	80.2	73.6	67.3	60.8	55.0	50.9
4	0.4	82.2	74.7	67.9	61.0	55.0	50.8
4	0.5	84.1	75.8	68.5	61.2	55.0	50.6
4	0.6	86.0	76.9	69.0	61.5	55.0	50.5
4	0.7	88.0	78.0	69.6	61.7	55.0	50.3
5	0.3	101.2	92.2	83.9	76.2	71.5	68.8
5	0.4	102.2	92.5	83.5	75.3	70.0	66.8
5	0.5	103.2	92.7	83.1	74.4	68.5	64.8
5	0.6	104.1	93.0	82.6	73.4	67.0	62.9
5	0.7	105.1	93.2	82.2	72.5	65.5	60.9
6	0.3	98.0	88.8	80.1	72.0	66.7	63.4
6	0.4	99.4	89.5	80.2	71.6	65.8	62.0
6	0.5	100.9	90.3	80.4	71.3	64.8	60.5
6	0.6	102.3	91.0	80.5	70.9	63.9	59.0
6	0.7	103.7	91.8	80.6	70.5	62.9	57.5
7	0.3	76.1	70.0	63.8	57.5	51.6	47.4
7	0.4	78.3	71.4	64.7	58.0	52.0	47.7
7	0.5	80.4	72.6	65.5	58.6	52.3	47.9
7	0.6	82.5	74.1	66.4	59.1	52.7	48.2
7	0.7	84.6	75.5	67.2	59.6	53.0	48.5
8	0.3	74.3	68.4	62.7	56.7	51.0	46.8
8	0.4	76.2	69.7	63.6	57.3	51.5	47.3
8	0.5	78.2	70.9	64.4	57.9	52.0	47.8
8	0.6	80.2	72.2	65.3	58.5	52.5	48.2
8	0.7	82.2	73.4	66.1	59.1	53.0	48.7

Table 10. β and ρ as a function of movement rate for a variety of different combination of aircraft types.

(β)

Case	α \ λ	20	30	40	50	60	70
1	0.3	1.161	1.115	1.068	1.002	0.994	0.925
1	0.4	1.147	1.107	1.071	1.007	0.952	0.920
1	0.5	1.144	1.118	1.084	1.047	0.954	0.913
1	0.6	1.155	1.138	1.101	1.026	0.948	0.896
1	0.7	1.176	1.166	1.115	1.028	0.938	0.872
2	0.3	1.191	1.143	1.080	1.033	0.970	0.944
2	0.4	1.174	1.129	1.094	1.077	0.970	0.939
2	0.5	1.167	1.136	1.104	1.047	0.971	0.930
2	0.6	1.176	1.154	1.119	1.055	0.960	0.914
2	0.7	1.196	1.179	1.133	1.054	0.950	0.888
3	0.3	1.199	1.166	1.116	1.045	0.981	0.947
3	0.4	1.188	1.152	1.113	1.046	0.981	0.941
3	0.5	1.179	1.159	1.124	1.057	0.985	0.931
3	0.6	1.187	1.177	1.140	1.065	0.975	0.915
3	0.7	1.208	1.202	1.156	1.064	0.967	0.888
4	0.3	1.202	1.167	1.116	1.049	0.982	0.901
4	0.4	1.184	1.146	1.113	1.051	0.985	0.943
4	0.5	1.175	1.153	1.124	1.062	0.991	0.937
4	0.6	1.182	1.172	1.142	1.072	0.976	0.927
4	0.7	1.202	1.212	1.158	1.071	0.980	0.902
5	0.3	1.120	1.062	0.988	0.925	0.880	0.862
5	0.4	1.116	1.049	1.012	0.936	0.886	0.858
5	0.5	1.121	1.083	1.028	0.945	0.885	0.850
5	0.6	1.067	1.107	1.050	0.955	0.882	0.836
5	0.7	1.164	1.139	1.064	0.963	0.876	0.816
6	0.3	1.214	1.154	1.089	1.003	0.950	0.925
6	0.4	1.198	1.148	1.091	1.007	0.946	0.910
6	0.5	1.193	1.158	1.103	1.011	0.942	0.892
6	0.6	1.206	1.179	1.114	1.017	0.927	0.867
6	0.7	1.225	1.204	1.123	1.012	0.912	0.836
7	0.3	1.214	1.177	1.131	1.064	0.993	0.951
7	0.4	1.193	1.161	1.126	1.066	0.997	0.952
7	0.5	1.188	1.166	1.137	1.078	1.006	0.948
7	0.6	1.189	1.185	1.155	1.090	0.998	0.946
7	0.7	1.207	1.215	1.173	1.090	1.004	0.924
8	0.3	1.217	1.177	1.134	1.070	0.999	0.958
8	0.4	1.196	1.159	1.130	1.073	1.007	0.962
8	0.5	1.183	1.163	1.140	1.088	1.017	0.962
8	0.6	1.186	1.178	1.159	1.099	1.012	0.969
8	0.7	1.201	1.206	1.176	1.100	1.027	0.951

(ρ)

Case	α \ λ	20	30	40	50	60	70
1	0.3	0.428	0.613	0.783	0.949	1.110	1.259
1	0.4	0.440	0.622	0.780	0.938	1.092	1.242
1	0.5	0.447	0.619	0.770	0.923	1.076	1.229
1	0.6	0.450	0.612	0.758	0.909	1.069	1.229
1	0.7	0.448	0.602	0.747	0.901	1.066	1.239
2	0.3	0.393	0.560	0.722	0.858	0.994	1.125
2	0.4	0.407	0.573	0.716	0.855	0.987	1.119
2	0.5	0.417	0.575	0.712	0.846	0.979	1.118
2	0.6	0.422	0.572	0.705	0.839	0.983	1.125
2	0.7	0.423	0.566	0.669	0.840	0.986	1.146
3	0.3	0.381	0.539	0.682	0.819	0.949	1.066
3	0.4	0.394	0.554	0.689	0.821	0.947	1.066
3	0.5	0.406	0.558	0.688	0.815	0.942	1.070
3	0.6	0.413	0.557	0.685	0.812	0.950	1.082
3	0.7	0.415	0.554	0.681	0.815	0.956	1.109
4	0.3	0.371	0.525	0.669	0.805	0.933	1.100
4	0.4	0.386	0.543	0.677	0.806	0.930	1.047
4	0.5	0.398	0.547	0.676	0.800	0.925	1.051
4	0.6	0.404	0.547	0.672	0.796	0.939	1.059
4	0.7	0.407	0.536	0.668	0.800	0.935	1.084
5	0.3	0.502	0.723	0.943	1.143	1.353	1.551
5	0.4	0.509	0.735	0.917	1.117	1.316	1.515
5	0.5	0.511	0.714	0.897	1.092	1.289	1.483
5	0.6	0.542	0.700	0.875	1.068	1.265	1.462
5	0.7	0.501	0.682	0.859	1.046	1.246	1.452
6	0.3	0.448	0.641	0.817	0.997	1.171	1.333
6	0.4	0.461	0.650	0.817	0.988	1.160	1.324
6	0.5	0.469	0.650	0.809	0.978	1.147	1.317
6	0.6	0.471	0.643	0.803	0.968	1.148	1.322
6	0.7	0.470	0.635	0.797	0.967	1.149	1.337
7	0.3	0.348	0.502	0.627	0.751	0.866	0.969
7	0.4	0.364	0.512	0.638	0.756	0.869	0.974
7	0.5	0.378	0.519	0.640	0.754	0.867	0.983
7	0.6	0.386	0.521	0.638	0.753	0.880	0.991
7	0.7	0.389	0.509	0.637	0.760	0.881	1.020
8	0.3	0.339	0.484	0.641	0.736	0.851	0.950
8	0.4	0.354	0.501	0.625	0.742	0.853	0.956
8	0.5	0.367	0.508	0.628	0.739	0.852	0.965
8	0.6	0.376	0.510	0.626	0.740	0.864	0.968
8	0.7	0.380	0.507	0.625	0.746	0.860	0.996

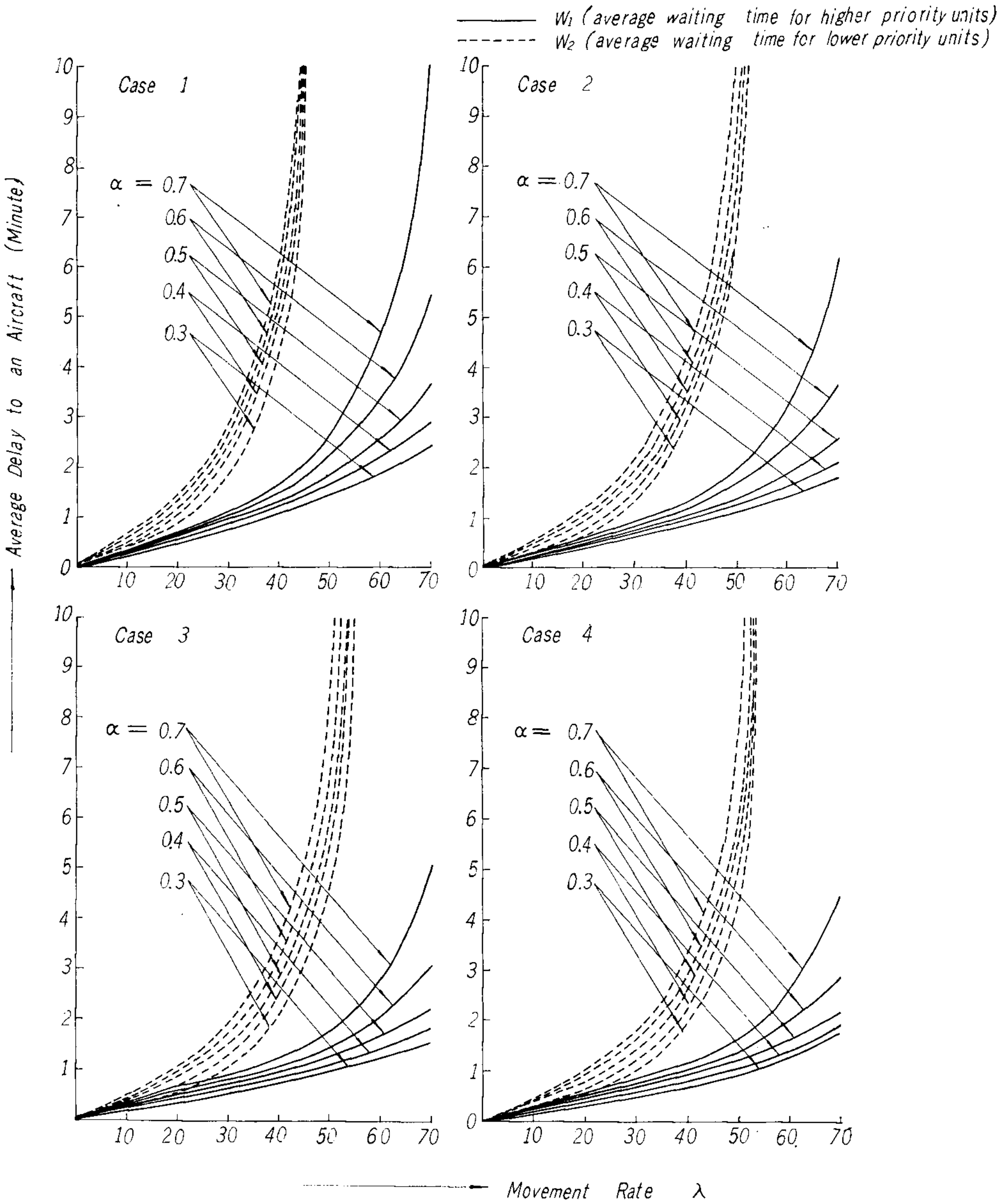

Fig. 1. Average delay as a function of movement rate.

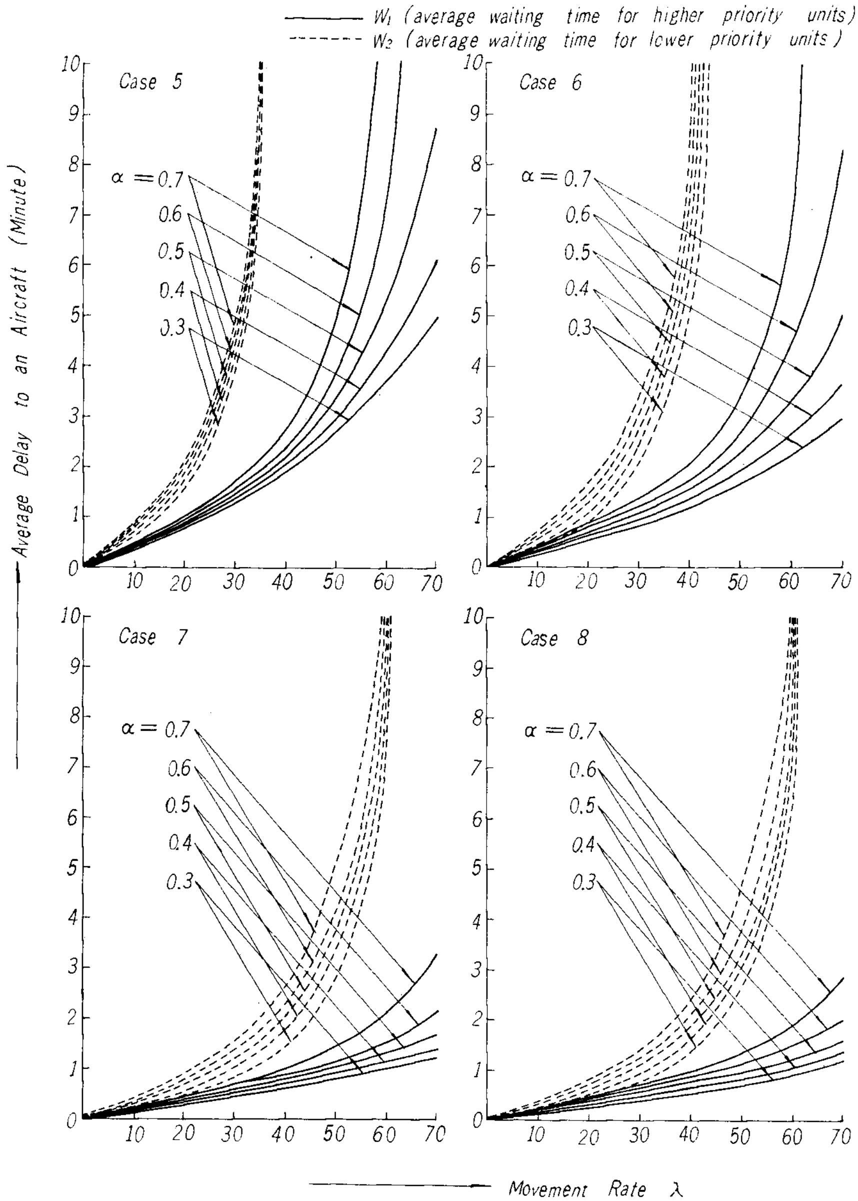

Fig. 2. Average delay as a function of movement rate.

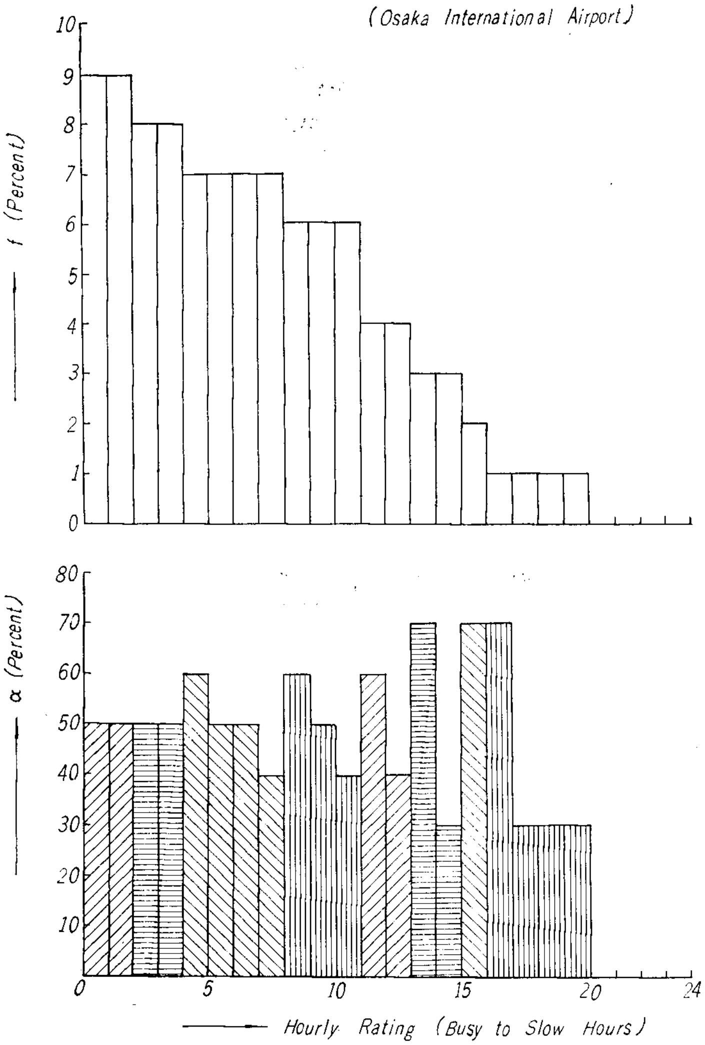

Fig. 3. Hourly distribution of aircraft movements by percentage.

Delay that occurs either to landings or departures is very expensive to the aircraft operations. However, this operating cost does indicate the important economical consideration involved. These costs are discussed in Section 5.

5. Determination of Aircraft Operating Costs

Airport runway services in these analysis are measured by estimated savings in aircraft operating costs. The effort should be paid to estimate

Table 11. Aircraft operating costs per unit time.

Aircraft type	Class	Cost per minute
Boeing 707, Douglas DC 8	Jet	$ 15.00
Medium jet	Heavy	10.00
Electra	Heavy	7.00
Four-engine piston	Heavy	6.00
Viscount	Heavy	4.00
Fairchild F-27	Medium	3.00
Twin-engine piston	Light	1.00
Single engine	Light	1.00

Table 12. Determination of operating costs for average delay.

Case	Operating cost per minute for average delay	Case	Operating cost per minute for average delay
(1)	$ 8.5	(5)	$ 13.2
(2)	6.4	(6)	9.6
(3)	5.5	(7)	4.0
(4)	5.2	(8)	3.7

the aircraft operating costs in the various airlines in Japan. However, for convenience sake, the values used in this paper were developed from data recently reported to the Civil Aeronautics Board, U.S.A. by the various airlines. These data can be properly weighted by the percentages of aircraft types at a given airport to determine aircraft operating cost per minute for an average delay.

6. Estimation of Annual Cost of Construction and Maintenance

Construction, amortization and maintenance costs can be estimated to a degree of accuracy consistent with the purpose of the study. In this paper, however, only the basic cost factors have been considered.

Construction costs are estimated at cost per square meter for pavement, lighting, etc., grading, drainage, etc., and land price. The figures used are:

	Cost in dollars per square meter
Pavement, lighting, etc.	$ 12.50
Grading, drainage, etc.	2.50
Land price	5.00
Total	$ 20.00

Construction costs of runways are estimated as follows;

Table 13. Construction costs of runways.

Runway type	Runway length	Runway width	Cost of construction	Name of international airport	Runway number
(1)	3,150m	60m	$ 3,780,000	Tokyo	C
(2)	3,000	60	3,600,000	Osaka	B
(3)	3,000	45	2,700,000	Tokyo	A
(4)	1,820	45	1,638,000	Osaka	A
(5)	1,650	45	1,485,000	Tokyo	B

Note: Runway type (2) is now in course of construction, and will be open in 1968.

Annual cost of construction, including amortization of capital expenditure, is estimated as follows.

The most commonly used equivalence methods are those of equivalent annual standard value and of present worth. Since both are derived from the same formula, neither has a greater intrinsic value than the other. The application of either method will depend upon the calculation facilities or data available or the aims pursued.

The method of uniform annual equivalent standard cost enables a sum invested on given data to be converted into an equivalent series of equal annual values. For this purpose, the following formula is used:

$$R = P\frac{i(1+i)^n}{(1+i)^n-1} \tag{9}$$

The initial investment P, may be converted into a series of equal payment, R, with n as the period of recovery and i the rate of interest.

Annual cost of maintenance, including repairs and replacement, is assumed equal to the annual cost of construction above mentiond.

Finally, total annual costs obtained by the above procedure, are tabulated as follows:

Table 14. Total annual costs of runways.

Runway type	Cost of construction	Annual cost of construction ($i=7\%$: $n=50$)	Annual cost of maintenance	Total annual cost
(1)	$ 3,780,000	$ 274,000	$ 274,000	$ 548,000
(2)	3,600,000	261,000	261,000	522,000
(3)	2,700,000	196,000	196,000	392,000
(4)	1,638,000	119,000	119,000	238,000
(5)	1,485,000	108,000	108,000	216,000

7. Optimizing Runway Design by Economic Analysis

The only common denominator for a comparison of airport runway services and development is money. The value of the services can be estimated and compared with the total annual costs of runway (construction and maintenance), to determine the economic feasibility of proposed improvements.

The optimum runway design is that which will manage to keep the balance between operating cost and total annual costs of runway. The standards of design, the consideration of any surface restrictions, and the determination of a preliminary runway layout will provide the initial typical configuration for which an economic evaluation can be made. The optimum design then is determined by the economic analysis of the factors that affect runway operations.

In this section, the method of evaluating optimum runway capacity by economic analysis is shown in the examples to follow. The procedure can be extended to an entire airport wherever the input data are available.

The example of the economic analysis is the determination of the stage in airport development at which additional runways are warranted by the increasing traffic. Consider, for example, the Osaka International Airport with a single 1,820 by 45 meters runway, parallel taxiway (2,100 by 15 meters) and terminal apron area (90,000 m^2).

To increase the capacity of the airport, it would be necessary to begin the second phase of construction. Phase II construction might include the addition of a parallel runway system complete with parallel taxiway and connecting taxiway to the terminal.

In determining the break-even volume for "B" runway in the second phase of construction, the previously discussed factors such as time distribution of traffic, average delay as a function of movements rate and cost data are all considered.

Using the values given Figures 1～3 and Table 14, the total delay per day (unit: minute) and total operating costs for total delay per day in dollars as a function of movements rate can be obtained as shown in Table 15:

Table 15. Total delay and total operating costs for a total delay as a function of movement rate.

Movement per day	Total delay per day (minute)	Total operating cost per day in dollars
100	23.66	$ 151.4
200	100.78	645.0
300	239.51	1,532.9
400	457.98	2,931.1
500	857.87	5,490.4

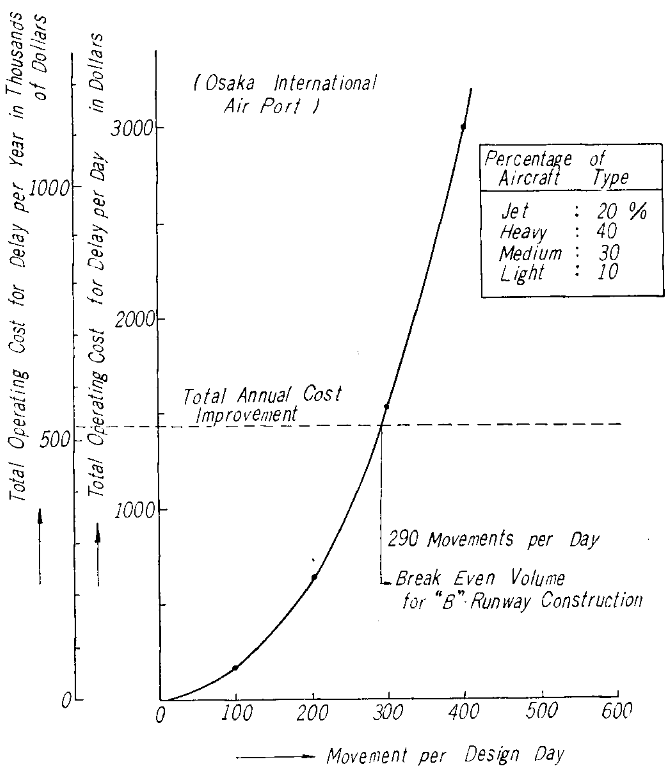

Fig. 4. Economic analysis of parallel runway.

Total operating costs for total delay in dollars to break even would equal $ 522,000—the total annual cost of the improvement. This relation is shown in Figure 4, which indicates the intersection of the curve for the total annual cost with the curve drawn through various points determined by the operating costs for total delay as a function of movement rate. The break-even volume in Figure 4 is thus 290 movements per design day.

8. Concluding Remarks

Using delay prediction for runway operations enables the economic analysis of specific airport designs. Techniques have been suggested for accomplishing this economic analysis.

Analytical models have been developed that can be used to relate operating rates on runways to the resulting delay in a realistic manner. The results of these analysis are considered superior to those obtained in the past by any other means.

The analytical models can also be used to develop basic runway design criteria from factual data.

The theoritical variations of the average delay of a sample queuing operation appear so large that it seems unlikely that runway operations rigidly conform to any simple queuing model. Therefore, it is desirable to make a more complicated model by means of simulation technique.

The next paper to be prepared by the authors in the near future, will deal with the more complicated preemptive spaced arrivals model for average delay by means of simulation technique.

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