On an Optimum Allocation of Workplace and Residence

By

Eiji KOMETANI* and Shogo KAWAKAMI*

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In this paper, we describe a process for obtaining an optimum allocation of workplace and residence to minimize the total commuting hours of workers. We do not consider the people whose workplaces and residences have already been determined, but try to minimize the total commuting hours for work journeys for people who will find a work place or a residence, in the future, by means of the optimum allocation of workplace and residence. Let us suppose that people who want to find work and whose residences have been determined will get a job in the area to which they can commute from their residences, in proportion to the amount of demand for workers. And also let us suppose that people whose residences have not been determined as yet will reside in the area from which they can commute to their workplaces, in proportion to the number of houses. Then, the problem of obtaining the optimum allocation of workplace and residence is amenable to a technique of linear programming. An optimum allocation of workplace and residence in Kyoto City obtained through the above procedure is presented.

1. Introduction

The big cities of today in Japan have developed rapidly since the end of the 2nd World War. The concentration of the population and economic activities in urban areas causes many traffic problems. One of them arises from the great amount of commuting transportation from residential areas to workplaces. In the big cities, the journeys to work are very distant and besides, the traffic facilities are much crowded. Therefore, not only the waste of time and money but the loss of energy is considerable.

Then, too, the concentration of the population and economic activities in urban areas is predicted to continue in the future, and so it is necessary to reform and expand facilities for transportation providing for the future. Therefore, the demand for commuting transportation must be estimated precisely and the most efficient transportation facilities, fitting the demand, must be constructed. At the same time, desirable figure for commuting transporta-

* Department of Transportation Engineering

tion must be given by the rational planning of land use in urban areas (especially an optimum allocation of workplaces and residential areas).

In this paper, from this point of view, we study the process of getting the allocation of workplaces and residential areas to minimize the total commuting hours in urban areas, considering the present land use.

2. Relationship between Workplace and Residence of Commuters

In order to plan facilities for future transportation, as well as the development of employment facilities and residences, we must study how the relationship between workplace and residential place is determined. In this section we take up this problem.

We consider the city and its dependent hinterland, to be the study area. Then the greater parts of the residents and employed persons in the study area must have there their workplaces and residential places. The others we do not study. The reason is that the former is the more important theme of the problem of commuting transportation in urban areas and the latter is generally out of the question.

Now, we divide commuters, finding work in the study area, into two groups as follows: one is W_1, the people whose residences have been determined and the other is W_2. W_1 consists of the people who must live in their own residences in the study area and the professional girls who live with their parents. These people will find their workplaces where they can commute. W_2 consists of the people who move into urban areas to find work and the people who begin their own lives separated from their parents in the study area, and so they will seek their workplaces before determining their residential places. Commuters belonging to W_1 will certainly determine the workplaces, considering the conditions of the journeys to work, (commuting hours, the state of congestion in transportation facilities, times of changing) and the actual working conditions (pay, the contents of work, environment etc.). Commuters belonging to W_2 will surely find the residential places in the zone from which they can commute to their workplaces, considering the conditions of journeys to work, the expenses for residence, environment, etc.

When we investigate the commuters in the future time in the study area by means of dividing them into the present commuters and the others, the commuters may be classified as follows:

(1) Commuters who will remain in the future time as they are commuters in the study areas in the present time, are expressed as L_0. Certainly they are reduced by reasons of retirement, death and moving out until the future

time.

(2) Commuters who will find work in the study area anew. There are two groups as follows:

i) One, expressed as L_1, is the people whose residential places will have been determined before finding work, belonging to W_1.

ii) The other, expressed as L_2, is the people who will have determined the workplaces in the study area first and then will seek their residential places which are suitable for their workplaces, belonging to W_2.

Next, we divide the study area into some zones and investigate the process for estimating inter-zonal movements of commuters in the future time. Now, denoting each zone in the study area as $1, 2, \cdots, i, (j), \cdots, n$, the number of commuters (to zone j) in the year t, E_j^t, is as follows:

$$E_j^t = E_{0j}^t + E_{1j}^t + E_{2j}^t \tag{1}$$

where E_{0j}^t : commuters to zone j, belonging to L_0

E_{1j}^t : commuters to zone j, belonging to L_1

E_{2j}^t : commuters to zone j, belonging to L_2

t : the year t, showing the value in the year t.

Denoting the number of people who commute from zone i in the year t by R_i^t, it consists of R_{0i}^t, R_{1i}^t and R_{2i}^t, which belong to commuters L_0, L_1 and L_2 in zone i in the year t respectively. Thus

$$\left.\begin{aligned} &R_i^t = R_{0i}^t + R_{1i}^t + R_{2i}^t \\ &\sum_{i=1}^n R_{0i}^t = \sum_{j=1}^n E_{0j}^t, \quad \sum_{i=1}^n R_{1i}^t = \sum_{j=1}^n E_{1j}^t, \quad \sum_{i=1}^n R_{2i}^t = \sum_{j=1}^n E_{2j}^t \end{aligned}\right\} \tag{2}$$

In this paper, we forecast the number of future inter-zonal journeys to work of commuters L_0, L_1 and L_2 individually.

In the first place, we consider commuters L_0. As we have the data about the present inter-zonal journeys to work, the proportion of zone i workers commuting to zone j, p_{ij}, is given by

$$p_{ij} = \frac{R_{0ij}^{t_0}}{R_{0i}^{t_0}}, \quad \sum_{j=1}^n p_{ij} = 1 \tag{3}$$

where $R_{0ij}^{t_0}$ is the number of workers who commute from zone i to zone j at the present time t_0. Then the number of inter-zonal journeys $R_{0ij}^{t_0}$ is given by

$$\begin{pmatrix} R_{011}^{t_0}, & R_{012}^{t_0}, & \cdots, & R_{01n}^{t_0} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ R_{0n1}^{t_0}, & R_{0n2}^{t_0}, & \cdots, & R_{0nn}^{t_0} \end{pmatrix} = \begin{pmatrix} R_{01}^{t_0}, & 0, & \cdots\cdots, & 0 \\ 0, & R_{02}^{t_0}, & 0 \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ 0, & \cdots\cdots & 0, & R_{0n}^{t_0} \end{pmatrix} \begin{pmatrix} p_{11}, & p_{12}, & \cdots, & p_{1n} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ p_{n1}, & p_{n2}, & \cdots, & p_{nn} \end{pmatrix} \tag{4}$$

Now, if we assume that p_{ij} of commuters L_0 is unchanged from this time forth, the number of inter-zonal journeys R_{0ij}^{tl} in the future time t_l is given by Eq. (5), when the number of commuters L_0 residing in zone i in the future time t_l, R_{0i}^{tl}, can be forecasted.

$$\begin{pmatrix} R_{011}^{tl}, & R_{012}^{tl}, & \cdots, & R_{01n}^{tl} \\ \vdots & \ddots & & \vdots \\ R_{0n1}^{tl}, & R_{0n2}^{tl}, & \cdots, & R_{0nn}^{tl} \end{pmatrix} = \begin{pmatrix} R_{01}^{tl}, & 0, & \cdots\cdots, & 0 \\ 0, & R_{02}^{tl}, & 0, \cdots, & 0 \\ \vdots & \ddots & \ddots & \vdots \\ & & & 0 \\ 0, & \cdots\cdots & 0, & R_{0n}^{tl} \end{pmatrix} \begin{pmatrix} p_{11}, & p_{12}, & \cdots\cdots & p_{1n} \\ \vdots & \ddots & & \vdots \\ p_{n1}, & p_{n1}, & \cdots\cdots & p_{nn} \end{pmatrix} \tag{5}$$

The process for forecating R_{0i}^{t} by age groups is as under. We denote the annual death-rate by $d_i^{t-1}(x-1)$, the annual efflux-rate from the study area by $e_i^{t-1}(x-1)$ and the annual retirement-rate by $f_i^{t-1}(x-1)$, of age $(x-1)$ in zone i in the year $(t-1)$ respectively. Then commuters L_0 of age x who reside in zone i in the year t, $R_{0i}^{t}(x)$, is given by Eq. (6).

$$R_{0i}^{t}(x) = \{1-d_i^{t-1}(x-1)-e_i^{t-1}(x-1)-f_i^{t-1}(x-1)\}R_{0i}^{t-1}(x-1)\,. \tag{6}$$

Therefore, if $d_i^{t}(x)$, $e_i^{t}(x)$ and $f_i^{t}(x)$ are forecasted, we can obtain $R_{0i}^{tl}(x)$ by Eq. (6). And

$$R_{0i}^{t} = \sum_{x} R_{0i}^{t}(x)\,. \tag{7}$$

In the next place, we consider commuters L_1^{t}, who will find work in the year t in the zone to which they can commute from their residential places which will have been determined. We denote the number of commuters L_1^{t} who commute from zone i to zone j by R_{1ij}^{t}. Then

$$E_{1j}^{t} = \sum_{i=1}^{n} R_{1ij}^{t}\,, \quad R_{1i}^{t} = \sum_{j=1}^{n} R_{1ij}^{t}\,. \tag{8}$$

The commuters L_1^{t} may take into account both the working conditions and the conditions of the journeys to work, when they find work, as has been stated. But they result in determining their employed zones not so much considering both the working conditions and the conditions of the journeys to work as in proportion to the opportunities for employment which take place there, because their employed zones are generally where the opportunity for employment happens to be in finding work, as the generation of demand for workers cannot be communicated to all job-seekers at the same time. Consequently, it is thinkable that R_{1ij}^{t} is in proportion to the opportunities for employment in zone j in the year t, ΔE_j^{t}. Hence,

$$R_{1ij}^{t} = \frac{\Delta E_j^{t}}{\sum_{j=1}^{n} \Delta E_j^{t}} \times R_{1i}^{t} = q_j^{t} R_{1i}^{t}\,. \tag{9}$$

If we assume that q_j^t of commuters L_1^t is unchanged and that $R_{1i}^{t_l(t)}$ is the number of people who will continue to commute in the year t_l, of commuters who will have found work in the year t, the number of inter-zonal journeys of commuters L_1^t in the year t_l, $R_{1ij}^{t_l(t)}$, is given by Eq. (10).

$$R_{1ij}^{t_l(t)} = q_j^t R_{1i}^{t_l(t)} . \tag{10}$$

Consequently, the number of inter-zonal journeys to work of commuters $L_1^{t_0}, L_1^{t_1}, \cdots, L_1^{t_l}$, in the year t_l, $R_{1ij}^{t_l\sim t_0}$, is as follows:

$$R_{1ij}^{t_l\sim t_0} = \sum_{t=t_0}^{t_l} q_j^t R_{1i}^{t_l(t)} . \tag{11}$$

We introduce the assumption that people whose residential places in the study area have been determined decide their workplaces in preference to the people whose residential places have not been determined and the rest of the demands for employees is filled up by the latter. Then $\varDelta E_j^t$ is obtained by Eq. (12), if E_j^t is estimated.

$$\varDelta E_j^t = E_j^t - E_{0j}^t . \tag{12}$$

Now, R_{1i}^t is estimated as under. R_{1i}^t is the residents in zone i who will have found work in the year t and will not change their residential places after doing it. When we denote the ratio of the job-seekers to the population of age x in zone i by $\alpha_i^t(x)$ and the ratio of the people who will not change their residential places after finding work to the job-seekers by $\beta_i^t(x)$, R_{1i}^t is obtained by Eq. (13).

$$R_{1i}^t = \sum_x \alpha_i^t(x) \beta_i^t(x) P_i^t(x) \tag{13}$$

where $P_i^t(x)$ is the zone i population of age x in the year t. $R_{1i}^{t_l(t)}$ is estimated as under. Using $d_i^t(x)$, $e_i^t(x)$ and $f_i^t(x)$, we get

$$R_{1i}^{t+k+1(t)}(x) = \{1 - d_i^{t+k}(x-1) - e_i^{t+k}(x-1) - f_i^{t+k}(x-1)\} R_{1i}^{t+k(t)}(x-1) , \tag{14}$$
$$(k = 0, 1, \cdots, t_l - t - 1)$$

where $R_{1i}^{t+k(t)}(x-1)$ is the number of commuters $R_{1i}^{t+k(t)}$ of age $(x-1)$. From this we get $R_{1i}^{t_l(t)}(x)$ and obtain $R_{1i}^{t_l(t)}$ by the following equation.

$$R_{1i}^{t_l(t)} = \sum_x R_{1i}^{t_l(t)}(x) \tag{15}$$

and

$$R_{1i}^{t_l\sim t_0} = \sum_{t=t_0}^{t_l} R_{1i}^{t_l(t)} . \tag{16}$$

Lastly we consider commuters L_2^t, who will find their residential places after finding work in the study area in the year t. We denote the number of commuters L_2^t who commute from zone i to zone j by R_{2ij}^t. Then we have

$$E_{2j}^t = \sum_{i=1}^{n} R_{2ij}^t, \quad R_{2i}^t = \sum_{j=1}^{n} R_{2ij}^t \tag{17}$$

E_{2j}^t is forecasted as under. The sum of commuters L_1^t and L_2^t employed in zone j, $\varDelta E_j^t$, is given by

$$\varDelta E_j^t = q_j^t \varDelta E^t \tag{18}$$

where

$$\varDelta E^t = \sum_{j=1}^{n} \varDelta E_j^t = \sum_{i=1}^{n} R_{1i}^t + \sum_{j=1}^{n} E_{2j}^t .$$

As the total number of commuters in the study area, E^t, is given by the economic plan, $\varDelta E^t$ is obtained by the following equation.

$$\varDelta E^t = E^t - \sum_{j=1}^{n} E_{0j}^t .$$

Thus

$$E_{2j}^t = q_j^t \varDelta E^t - q_j^t R_1^t = q_j^t E_2^t \tag{19}$$

where

$$R_1^t = \sum_{i=1}^{n} R_{1i}^t , \quad E_2^t = \sum_{j=1}^{n} E_{2j}^t . \tag{20}$$

It seems that commuters L_2 determine their residential places considering the conditions of journeys to work, the expenses for residence and the life-environment. The expenses for residence are high, where the conditions of journey to work are favourable. Therefore it seems that the differences among the conditions of residence in each zone are small, putting these conditions together. When the number increased of employed persons who will be able to reside in each zone in the year t, R_{2i}^t, which is based on vacant houses and new houses, is given, we may suppose that the residential places of employees E_{2j}^t in zone j distribute in proportion to R_{2i}^t. Thus

$$R_{2ij}^t = \frac{R_{2i}^t}{\sum_{i=1}^{n} R_{2i}^t} \times E_{2j}^t = u_i^t E_{2j}^t = u_i^t q_j^t E_2^t . \tag{21}$$

Suppose that u_i^t of commuters L_2^t do not change. Then denoting the people who will have found work in the study area in the year t and will continue to commute in the year t_l by $E_{2j}^{t_l(t)}$, the number of inter-zonal journeys to work of commuters L_2^t in the year t_l, $R_{2ij}^{t_l(t)}$, is as follows.

$$R_{2ij}^{t_l(t)} = u_i^t E_{2j}^{t_l(t)} . \tag{22}$$

Consequently the number of inter-zonal journeys to work of commuters $L_2^{t_0}$, $L_2^{t_1}, \cdots, L_2^{t_l}$ in the year t_l, $R_{2ij}^{t_l \sim t_0}$, is given by

$$R_{2ij}^{t_l \sim t_0} = \sum_{t=t_0}^{t_l} u_i^t E_{2j}^{t_l(t)} . \tag{23}$$

Now, $E_2^{t_l(t)}$ is estimated as follows. Let

$$E_2^{t_l(t)} = \sum_{j=1}^{n} E_{2j}^{t_l(t)} .$$

Then denoting the annual death-rate by $d^t(x)$, the annual efflux-rate from the study area by $e^t(x)$ and the annual retirement-rate by $f^t(x)$ of employees of age x in the year t respectively, the relationship between the employees $E_2^{t+k+1(t)}(x)$ of age x in the year $(t+k+1)$ and the employees $E_2^{t+k(t)}(x-1)$ of age $(x-1)$ in the year $(t+k)$ is as follows:

$$E_2^{t+k+1(t)}(x) = \{1-d^{t+k}(x-1)-e^{t+k}(x-1)-f^{t+k}(x-1)\} E_2^{t+k(t)}(x-1) , \tag{24}$$
$$(k = 0, 1, \cdots, t_l-t-1)$$

And

$$E_2^{t_l(t)} = \sum_x E_2^{t_l(t)}(x) . \tag{25}$$

Thus we get

$$E_{2j}^{t_l(t)} = q_j^t E_2^{t_l(t)} . \tag{26}$$

And the number of commuters $E_2^{t_l \sim t_0}$ in the year t_l of commuters $L_2^{t_0}, L_2^{t_1}, \cdots, L_2^{t_l}$ is given by

$$E_2^{t_l \sim t_0} = \sum_{t=t_0}^{t_l} E_2^{t_l(t)} . \tag{27}$$

3. An Optimum Allocation of Workplace and Residence

In this section, we make the study of the allocation of workplaces and residential areas to minimize the total commuting hours in the study area. Now, as the residential areas of commuters L_0, L_1 and the workplaces of commuters L_0 are determined previously, these cannot be altered in principle. But the workplaces of commuter L_1, and the workplaces and the residential areas of commuters L_2 can be determined suitably, therefore these should be determined so that the total commuting hours is minimum.

Here, the allocation planning of the workplaces and residential areas which minimizes the total commuting hours from the present time t_0 to the future time t_m, is studied. The average commuting hours from zone i to zone j is denoted by τ_{ij}, and then the total commuting hours of commuters L_0, L_1 and L_2 in the study area are presented as follows:

i) The total commuting hours T_0 of commuters L_0

$$T_0 = \sum_{t=t_0}^{t_m} \sum_{i,j} R_{0ij}^{t} \tau_{ij} = \sum_{t=t_0}^{t_m} \sum_{i,j} R_{0i}^{t} p_{ij} \tau_{ij} . \tag{28}$$

ii) The total commuting hours T_1 of commuters L_1

$$T_1 = \sum_{t_l=t_0}^{t_m} \sum_{t=t_0}^{t_l} \sum_{i,j} R_{1ij}^{t_l(t)} \tau_{ij} = \sum_{t_l=t_0}^{t_m} \sum_{t=t_0}^{t_l} \sum_{i,j} q_j^t R_{1i}^{t_l(t)} \tau_{ij} . \tag{29}$$

iii) The total commuting hours T_2 of commuters L_2

$$T_2 = \sum_{t_l=t_0}^{t_m} \sum_{t=t_0}^{t_l} \sum_{i,j} R_{2ij}^{t_l(t)} \tau_{ij} = \sum_{t_l=t_0}^{t_m} \sum_{t=t_0}^{t_l} \sum_{i,j} u_i^t E_{2j}^{t_l(t)} \tau_{ij} . \tag{30}$$

Therefore the total commuting hours is denoted by $T_0+T_1+T_2$. T_0 is a constant value which must not be altered artificially, but T_1 and T_2 are dependent upon q_j^t and u_i^t respectively. Varying the value q_j^t means varying the proportion of employment capacity in zone j to that in the other zone, changing the allocation of employment facilities. And varying the value u_i^t means changing the allocation of houses. After all, in order to determine the allocation of employment facilities and residential areas minimizing the total commuting hours, q_j^t and u_i^t should be calculated to minimize T_1+T_2. The following is the study of this problem. We consider the case that R_i, the number of commuters who can reside in zone i and E_j, the number of employees who can be employed in zone j are given from the zone area and the past trend. It is not desirable that the present allocation of employment facilities and houses will be changed. Then the employees and the residents in each zone should be more than the present. But, if the population and/or employees decrease in all, the above said is not suitable. And the numbers of the commuting residents and of the employees in each zone are not more than R_i and E_j respectively.

In this paper, it is assumed that the numbers of the commuting residents and of the employees do not decrease. Then the conditions of q_j^t and of u_i^t are as follows:

$$\sum_{j=1}^{n} q_j^t = 1 , \quad q_j^t \geqq 0 , \qquad (t = t_0+1, \cdots, t_m, j = 1, 2, \cdots, n) \tag{31}$$

$$E_j^{t_l-1} \leqq \sum_{i=1}^{n} R_{0i}^{t_l} p_{ij} + \sum_{t=t_0}^{t_l} q_j^t R_1^{t_l(t)} + \sum_{t=t_0}^{t_l} E_{2j}^{t_l(t)} \leqq E_j , \tag{32}$$

$$(t_l = t_0+1, \cdots, t_m, j=1, 2, \cdots, n)$$

where

$$R_1^{t_l(t)} = \sum_{i=1}^{n} R_{1i}^{t_l(t)}, \quad E_{2j}^{t_l(t)} = q_j^t E_2^{t_l(t)}, \quad E_j^{t_l-1} = \sum_{i=1}^{n} R_{0i}^{t_l-1} p_{ij} + \sum_{t=t_0}^{t_l-1} q_j^t R_1^{t_l-1(t)} + \sum_{t=t_0}^{t_l-1} E_{2j}^{t_l-1(t)}$$

$$\sum_{j=1}^{n} u_i^t = 1, \quad u_i^t \geqq 0, \qquad (t = t_0+1, \cdots, t_m, \; i = 1, 2, \cdots, n). \quad (33)$$

$$R_i^{t_l-1} \leqq R_{0i}^{t_l} + \sum_{t=t_0}^{t_l} R_{1i}^{t_l(t)} + \sum_{t=t_0}^{t_l} \sum_{j=1}^{n} R_{2ij}^{t_l(t)} \leqq R_i, \quad (t_l = t_0+1, \cdots, t_m, \; i = 1, 2, \cdots, n) \quad (34)$$

where

$$R_{2ij}^{t_l(t)} = u_i E_{2j}^{t_l(t)} = u_i^t q_j^t E_2^{t_l(t)}, \quad R_i^{t_l-1} = R_{0i}^{t_l-1} + \sum_{t=t_0}^{t_l-1} R_{1i}^{t_l-1(t)} + \sum_{t=t_0}^{t_l-1} \sum_{j=1}^{n} R_{2ij}^{t_l-1(t)}.$$

As aforesaid, after q_j^t is calculated, u_i^t is determined. Then, first of all, subject to Eqs. (31) and (32), q_j^t is calculated to minimize the value T_1. Next, using these q_j^t's, u_i^t is calculated to minimize the value T_2 subject to Eqs. (33) and (34). Therefore this problem is Linear Programming and can be solved easily using the Simplex Method.

After the optimum solutions q_j^t and u_i^t are obtained, ΔE_j^t and R_{2i}^t are presented as follows.

$$\Delta E_j^t = q_j^t \Delta E^t = q_j^t (R_1^t + E_2^t) \quad (35)$$

$$R_{2i}^t = u_i^t E_2^t. \quad (36)$$

Therefore, the increase of demands for employees which should be prepared in zone j in the year t is $(E_{0j}^t + \Delta E_j^t - E_j^{t-1})$. And the increse of the commuting residents which should reside in zone i in the year t is $(R_{0i}^t + R_{1i}^t + R_{2i}^t - R_i^{t-1})$.

Now, even if the decreases of population and employees are seen, the optimum solution can be obtained by altering the lower limit value of conditions, Eqs. (32) and (34).

4. Application to Kyoto City

Now, we tried to make the allocation planning of employment facilities and residential areas for Kyoto City in 1975, on the basis of the data in 1960. In this case, the plan to minimize the total commuting hours in 1975 was studied. We divided the study area into 10 zones, 9 administrative districts and the hinter land adjoining Kyoto City.

(1) Forecasting the number of commuters who can be employed and can reside in each zone.

The former was forecasted from the past trend value, and the forecasted results are shown in Table 1.

(2) p_{ij} and R_{0i}^{75}

The value p_{ij} calculated from R_{0ij}^{60} is shown in Table 2. We forecasted

Table 1.

Zone		1	2	3	4	5	6	7	8	9	10	Total
Commuters employed in zone j in 1960	E_j^{60}	15,719	40,113	32,548	78,426	27,100	69,070	33,602	35,970	25,104	41,008	398,660
Commuters employed in zone j	E_j	31,990	52,870	46,820	112,840	35,490	118,090	80,980	72,780	35,130	51,200	638,190
Resident commuters	R_i^{60}	33,869	35,462	52,760	39,559	39,750	36,950	33,140	43,676	41,232	42,259	398,660
Resident commuters	R_i	49,100	35,600	87,600	39,700	76,600	37,000	38,300	95,800	98,400	151,100	709,200

1 : Kita-Ku 2 : Kamigyo-Ku 3 : Sakyo-Ku 4 : Nakagyo-Ku 5 : Higashiyama-Ku
6 : Shimogyo-Ku 7 : Minami-Ku 8 : Ukyo-Ku 9 : Fushimi-Ku

Table 2. The value p_{ij} in 1960.

Origin \ Destination	1	2	3	4	5	6	7	8	9	10	Total
1	0.2602	0.1703	0.0579	0.1837	0.0250	0.1127	0.0296	0.0604	0.0107	0.0895	1.0000
2	0.0470	0.3954	0.0475	0.2030	0.0239	0.1102	0.0287	0.0546	0.0101	0.0796	1.0000
3	0.0289	0.0976	0.3678	0.1804	0.0437	0.1062	0.0292	0.0406	0.0122	0.0935	1.0000
4	0.0186	0.0852	0.0322	0.4517	0.0270	0.1494	0.0383	0.0897	0.0086	0.0993	1.0000
5	0.0115	0.0411	0.0493	0.1653	0.3372	0.1502	0.0577	0.0322	0.0274	0.1281	1.0000
6	0.0106	0.0420	0.0214	0.1648	0.0403	0.4516	0.0775	0.0600	0.0165	0.1152	1.0000
7	0.0067	0.0319	0.0170	0.1016	0.0326	0.1789	0.4340	0.0466	0.0263	0.1246	1.0000
8	0.0223	0.0660	0.0359	0.1868	0.0189	0.1119	0.0385	0.3983	0.0073	0.1142	1.0000
9	0.0074	0.0331	0.0234	0.1055	0.0497	0.1105	0.0708	0.0204	0.3892	0.1899	1.0000
10	0.0148	0.0784	0.0562	0.2149	0.0755	0.2793	0.1037	0.0713	0.1057	0	1.0000

Table 3. The number of resident commuters.

Zone	1	2	3	4	5	6	7	8	9	10	Total
R_{0i}^{60} (persons)	33,869	35,462	52,760	39,559	39,750	36,950	33,140	43,676	41,232	42,259	398,657
R_{0i}^{75} (persons)	25,385	24,529	38,181	26,603	32,990	25,944	23,205	34,638	31,645	34,300	297,420
$R_{1i}^{75\sim60}$ (persons)	5,456	10,933	7,552	12,956	3,635	11,006	6,980	4,609	8,554	16,900	88,581

Table 4. The number of journeys to work, R_{0ij}^{75}.

Origin \ Destination	1	2	3	4	5	6	7	8	9	10	R_{1i}
1	6,605	4,323	1,470	4,663	635	2,861	751	1,533	272	2,272	25,385
2	1,153	9,699	1,165	4,979	586	2,703	704	1,339	248	1,953	24,529
3	1,103	3,726	14,043	6,888	1,669	4,055	1,115	1,550	466	3,570	38,181
4	495	2,267	857	12,017	718	3,974	1,019	2,386	229	2,642	26,603
5	379	1,356	1,626	5,453	11,124	4,955	1,904	1,062	904	4,226	32,990
6	275	1,090	555	4,276	1,046	11,716	2,011	1,557	428	2,989	25,944
7	155	740	394	2,358	756	4,151	10,071	1,081	610	2,891	23,205
8	772	2,286	1,244	6,470	655	3,876	1,334	13,796	253	3,956	34,638
9	234	1,047	740	3,339	1,573	3,497	2,240	646	12,316	6,009	31,645
10	508	2,689	1,928	7,371	2,590	9,580	3,557	2,446	3,626	0	34,300
E_{0j}^{75}	11,679	29,223	24,022	57,814	21,352	51,368	24,706	27,396	19,352	30,508	297,420

the value R_{0i}^{75}, dividing the commuters into 5-year age groups, and R_{0i}^{75} is shown in Table 3. And from the value R_{0i}^{75} we calculated the value R_{0ij}^{75}, shown in Table 4.

(3) $R_{1i}^{75\sim 60}$ and E_2^{75}

On the basis of the population data in 1960, we forecasted value $R_{1i}^{75\sim 60}$, dividing the commuters into 5-year age groups, and the results are shown in Table 3. And forecasting the value E^{75} from the total number of employees in the study area in 1975, we obtained the value $E_2^{75\sim 60}$ by the following equation,

$$E_2^{75\sim 60} = E^{75} - E_0^{75} - R_1^{75\sim 60}, \quad E_2^{75\sim 60} = 97{,}612 .$$

Here we supposed the distribution rates of employment facility and of residence q_j and u_i as follows.

$$q_j = \frac{\Delta E_j^{75\sim 60}}{\Delta E^{75\sim 60}} = \frac{\sum_{i=1}^{10} R_{1ij}^{75\sim 60} + E_{2j}^{75\sim 60}}{R_1^{75\sim 60} + E_2^{75\sim 60}}, \quad u_i = \frac{R_{2i}^{75\sim 60}}{R_2^{75\sim 60}}, \quad (i, j = 1, 2, \cdots, 10) .$$

(4) Total commuting hours

Supposing the average inter-zonal commuting hours as in Table 5, we got following equations expressing the total commuting hours in 1975.

$$T_0 = \sum_{i,j} R_{0ij}^{75} \tau_{ij}$$
$$T_1 = \sum_{i,j} R_{1ij}^{75\sim 60} \tau_{ij} = \sum_{i,j} q_j R_{1i}^{75\sim 60} \tau_{ij}$$
$$T_2 = \sum_{i,j} R_{2ij}^{75\sim 60} \tau_{ij} = \sum_{i,j} u_i q_j E_2^{75\sim 60} \tau_{ij} .$$

Table 5. The inter-zonal commuting hours τ_{ij} (min).

Destination / Origin	1	2	3	4	5	6	7	8	9	10
1	10	15	25	30	45	40	40	30	70	120
2	15	10	15	15	35	25	35	20	55	110
3	25	15	10	15	25	30	40	40	55	110
4	30	15	15	10	20	15	25	30	40	100
5	45	30	20	15	10	15	20	45	40	100
6	40	25	30	15	25	10	15	40	30	95
7	40	35	40	25	25	15	10	40	20	90
8	30	20	40	30	45	40	40	10	70	115
9	70	55	55	40	40	30	20	70	10	85
10	120	110	110	100	100	95	90	115	85	—

(5) Linear Programming

Solving the following two linear-programming problems, we could obtain the amounts of employment facilities and of residences which must be developed in each zone to minimize the total commuting hours.

i) Minimize $T_1=\sum_{i,j} q_j R_{1i}^{75\sim 60}\tau_{ij}$

subject to

$$\sum_{j=1}^{10} q_j = 1, \quad q_j \geqq 0, \qquad (j=1, 2, \cdots, 10)$$

$$E_j^{60} \leqq E_{0j}^{75}+q_j(R_1^{75\sim 60}+E_2^{75\sim 60}) \leqq E_j, \quad (j=1, 2, \cdots, 10)$$

ii) Minimize $T_2=\sum_{i,j} u_i q_j E_2^{75\sim 60}\tau_{ij}$

subject to

$$\sum_{i=1}^{10} u_i = 1, \quad u_i \geqq 0, \qquad (i=1, 2, \cdots, 10)$$

$$R_i^{60} \leqq R_{0i}^{75}+R_{1i}^{75\sim 60}+u_i E_2^{75\sim 60} \leqq R_i, \qquad (i=1, 2, \cdots, 10).$$

The results are shown in Table 6, where the amounts of employment facilities and of residences which must be developed from 1960 to 1975, are expressed as the number of commuters to be admitted. And the number of inter-zonal journeys to work in the study area in 1975, $R_{ij}^{75}(=R_{0ij}^{75}+R_{1ij}^{75\sim 60}+R_{2ij}^{75\sim 60})$, is shown in Table 7.

Table 6.

Zone	1	2	3	4	5	6	7	8	9	10	Total
q_j	0.0217	0.0585	0.0458	0.2955	0.0309	0.3583	0.0560	0.0460	0.0309	0.0564	1.0000
u_i	0.0310	0	0.4204	0	0.4095	0	0.0831	0.0454	0.0106	0	1.0000
Demand for employees (persons)	0	0	0	34,408	0	49,011	1,531	0	0	0	84,950
Demand for houses (persons)	0	0	34,009	0	36,847	0	5,157	0	0	0	76,013

Table 7. The number of journeys to work $R_{ij}^{75}=R_{0ij}^{75}+R_{1ij}^{75\sim 60}+R_{2ij}^{75\sim 60}$.

Origin \ Destination	1	2	3	4	5	6	7	8	9	10	Total
1	6,789	4,819	1,859	7,169	897	5,900	1,226	1,923	534	2,751	33,867
2	1,390	10,339	1,666	8,210	924	6,620	1,316	1,842	586	2,570	35,463
3	2,157	6,568	16,269	21,246	3,170	21,464	3,836	3,785	1,967	6,311	86,773
4	776	3,025	1,450	15,845	1,118	8,616	1,745	2,982	629	3,373	39,559
5	1,325	3,907	3,623	18,339	12,471	20,579	4,346	3,068	2,251	6,686	76,595
6	514	1,734	1,059	7,528	1,386	15,659	2,627	2,063	768	3,610	36,948
7	482	1,623	1,086	6,818	1,223	9,558	10,916	1,775	1,077	3,743	38,301
8	968	2,815	1,658	9,142	934	7,116	1,840	14,212	532	4,466	43,683
9	442	1,608	1,179	6,173	1,869	6,933	2,777	1,087	12,612	6,549	41,229
10	875	3,678	2,702	12,365	3,112	15,635	4,503	3,223	4,148	953	51,194
Total	15,718	40,116	32,551	112,835	27,104	118,080	35,132	35,960	25,104	41,012	483,612

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