Some Features of Turbulent Diffusive Processes in Open Channel Flows

By

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This paper deals with an application of the turbulent diffusion theory by continuous movements of Taylor to the turbulent diffusive processes in open channel flows. It is shown that the mean velocity of the flow can be related to the correlation coefficients of Lagrange by means of the similarity principle of Kolmogoroff and the diffusive characteristics are also estimated by the mean velocity under the condition of constant values in the relative turbulent intensity of uniform flows.

The theoretical analysis derived in this paper is also verified through the experimentation made by fine lucite particles and dilute salt water, from a practical point of view.

The mechanism of mass transfer, which consists of an essential character in the turbulent diffusive processes, will be still studied.

1. Introductory Statement

Most common procedures used to analyse the turbulent diffusive processes in open channel flows are the following two ways ; the one is a mathematical treatment of the basic equation for convective diffusion in a turbulent field derived from the mass conservation law, and the other is a statistical treatment of diffusive processes in an isotropic field initiated by Taylor.[1)]

Engineering practices use generally the former procedure under the assumption of constant values in the eddy diffusivity, which make the equivalence in mathematical analogy between the equation of turbulent convective diffusion and that of heat conduction. In reality, however, the eddy diffusivity is not constant but variable at least within a short diffusion time or distance. The solution is then not obtainable and furthermore a mathematical form of the coefficient can not be expressed by the theoretical approach.

The statistical procedure is a very advantageous way to analyse the turbulent diffusion in an isotropic field, whereas it is not useful for the diffusive processes in a nonisotropic field as the turbulent shear flow.

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The present-day techniques for the turbulent diffusive processes in open channel flows developed by many scientists and engineers are still semi-empirical ones partly provided by the theoretical analysis.

This paper deals with several features of turbulent diffusive processes in open channel flows, from a statistical treatment, as the first phase of research works in the whole program. The study is evidently limited to the central field of the flow itself sufficiently assumed isotropic, but will aim to verify the applicability of results to the diffusive processes in turbulent shear flows.

2. General Description of Turbulent Diffusion in Open Channel Flows

The turbulent diffusive processes in open channel flows are characterized by properties of both substances transferred in the flow and the flow itself as same as in other fields of diffusive processes.

The transferable quantities treated in hydraulics are usually mass, momentum or vorticity, and energy. The mass transfer problem is of real importance in analysis for the disposal of wastes, the intermixing between river water and sea water, and the sand transportation in channels. The momentum or vorticity transfer is known as the basis of the phenomenological treatment to the shear flow. The thermal diffusion at intakes and disposal of water and the turbulent structure in its flow are treated by the energy transfer theory.

On the other hand, the local mean velocity and the turbulent fluctuation characterize several kinds of velocity fields in the flow, among which the uniform flow field with isotropic homogeneous turbulence is the most simplified one to make the analysis of turbulent diffusive processes and the non-uniform flow field with non-isotropic and non-homogeneous turbulence is the most generalized one related to the practical engineering problems.

In view of the above description, hydraulic and rather hydrodynamic studies of the turbulent diffusive processes are quite important in all the fields concerning hydraulic engineering practice, whereas the successful treatments to solve the problem underlied are not still obtained even in a simplified flow pattern.

3. Eddy Diffusivity

The eddy diffusivity in the theory of mass transfer is usually defined in the following two ways. The first, the eddy diffusivity is defined such that[2],

$$\overline{u_i'c'} = -D_i\frac{\partial \bar{c}}{\partial x_i}, \quad \text{or} \quad D_i = -\frac{\overline{u_i'c'}}{\dfrac{\partial \bar{c}}{\partial x_i}} \tag{3.1}$$

in which u_i' is the x_i-wise turbulent fluctuation, $\bar{c}$ is the temporal mean value of concentration and c' is the fluctuation from the mean value. It is evident that Eq. (3.1) is analogous to the expressions of eddy viscosity in momentum transfer. The direct measurement of the eddy diffusivity in the flow can not be made without introducing phenomenological treatment in evaluation of the diffusivity.

The statistical treatment of the turbulent diffusion by continuous movements in the flow with isotropic homogeneous turbulence defines the eddy diffusivity as[3)]

$$D_i = \frac{1}{2}\frac{d\sigma_i^2}{dt} \tag{3.2}$$

in which σ_i^2 is the variance determined from the distribution of concentration in mass transfer. The clear significance of the eddy diffusivity has not been yet established without a case in which the diffusion process is assumed random, but the expression defined by Eq. (3.2) is advantageous to the analysis for the process in a short diffusion time.

4. Turbulent Diffusion in Open Channel Flows

4-1. One-Dimensional turbulent diffusion in unconfined flows

Taking the x_1-axis in the direction of main flow, the x_2-axis horizontal and the x_3-axis vertical in a plane perpendicular to the x_1-axis, and denoting the velocity components by $u_i = \bar{u}_i + u_i'$, the diffusive process of particles ignored the density difference in the steady uniform flow makes the following relationships, through the theory of Taylor[1)] by continuous movements,

$$\begin{aligned} \sigma_i^2(t) &= 2\overline{u_i'^2}\int_0^t dt'\int_0^{t'} dt''\cdot R_{Li}(t'') \\ &= 2\overline{u_i'^2}\int_0^t (t-t')R_{Li}(t')dt' \end{aligned} \tag{4.1}$$

and

$$D_i(t) = \overline{u_i'^2}\int_0^t R_{Li}(t')dt' \tag{4.2}$$

in which $\sigma_i^2(t)$ is the variance determined by the concentration distribution of quantity under consideration, $\overline{u_i'^2}$ is the turbulent intensity, and $R_{Li}(t)$ is the Lagrangian correlation coefficient, which is

$$R_{Li}(t) = \frac{\overline{u_i'(t_0)u_i'(t+t_0)}}{\overline{u_i'^2}} \tag{4.3}$$

The Lagrangian correlation coefficient is usually determined by the direct measurements in turbulent fluctuations, though recent studies on spectral analysis can make a mathematical form of the coefficient to some extent.

It is evident that the diffusion proceeds proportionally with the time for a short diffusion time while it is proportional to the square root of time for a long diffusion time.

G. T. Orlob[4] treated the turbulent diffusion of particles on a free surface under the condition that

$$R_{L2}(t)=\exp\left(-\frac{t}{t_*}\right) \tag{4.4}$$

in which t_* is a constant. The variance and the eddy diffusivity are then expressed as follows,

$$\sigma_2^2(t)=2\overline{u_2'^2}t_*^2\left\{\frac{t}{t_*}+\exp\left(-\frac{t}{t_*}\right)-1\right\} \tag{4.5}$$

or

$$\sigma_2^2(x_1)=2\overline{u_2'^2}t_*^2\left\{\frac{x_1}{u_1t_*}+\exp\left(-\frac{x_1}{u_1t_*}\right)-1\right\} \tag{4.5'}$$

and

$$D_2(t)=\overline{u_2'^2}t_*\left\{1-\exp\left(-\frac{t}{t_*}\right)\right\} \tag{4.6}$$

or

$$D_2(x_1)=\overline{u_2'^2}t_*\left\{1-\exp\left(-\frac{x_1}{u_1t_*}\right)\right\} \tag{4.6'}$$

after transforming the variable from t to x_1, known as the hypothesis of Taylor, though the exponential assumption on the Lagrangian correlation coefficient is not completely valid.

This analysis can make a physical behaviour of diffusive processes of the uniform flow through the direct measurement of variances at two different points, say x_1' and x_1''.

In Eq. (4.5), for short diffusion time, $\sigma_2^2(t)$ becomes approximately $\overline{u_2'^2}t^2$, while for long diffusion time, it becomes $2\overline{u_2'^2}t_*t$, so that two straight lines with the gradient of 1 and 2 expressing both diffusive processes will be obtained at a both log. sheet (See Fig. 1). If these two lines will be used as an approximate value of the variance

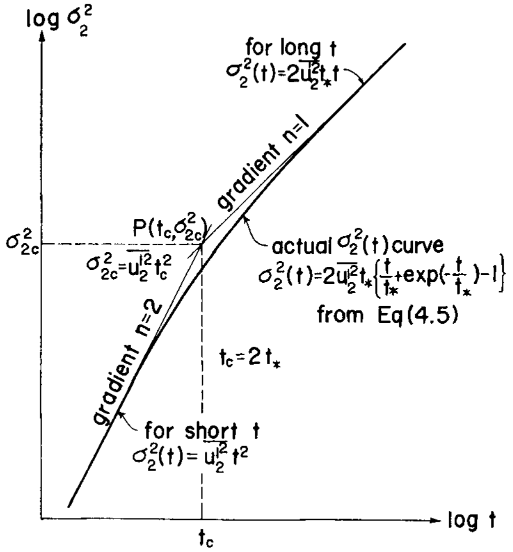

Fig. 1. The definition of characteristic values in turbulent diffusive processes, t_c and σ_{2c}^2.

in the whole, the location of intersection in variance-time plane is

$$t_c = 2t_* \tag{4.7}$$

and

$$\sigma_{2c}^2 = 4\overline{u_2'^2}t_*^2 \tag{4.8}$$

which will be denoted as characteristic values of t_c and σ_{2c}^2.

The characteristic value of t_c will be related to the local mean velocity $\overline{u_1}$ through the similarity principle of Kolmogoroff, which is

$$D \sim E^{1/3}L^{4/3} \tag{4.9}$$

in which D is the eddy viscosity, E is the rate of energy dissipation per unit time, and L is the scale in diffusion. The rate of energy dissipation for the steady uniform flow in open channels is

$$E \sim g\overline{u_1}I_e \tag{4.10}$$

wherein g is the acceleration of gravity, I_e is the energy gradient of flow equivalent to the bottom slope. On the other hand, the definition of the scale of diffusion has not yet been established. In this study, the Lagrangian eddy scale: $L = \Lambda_L = \sigma_{2c} = 2\sqrt{\overline{u_2'^2}}\,t_*$, and the Lagrangian integral scale: $L = L_a = \overline{u_1}\int_0^\infty R_{L2}(t)dt = \overline{u_1}t_*$ will be used. With the use of two expressions of the rate of energy dissipation and the scale of diffusion, the characteristic value of t_c becomes

$$t_c = 2t_* \sim \left(\frac{\sqrt{\overline{u_2'^2}}}{\overline{u_1}}\right)^2 \overline{u_1}, \quad \text{using} \quad \Lambda_L \text{ as } L \tag{4.11}$$

and

$$t_c = 2t_* \sim \left(\frac{\sqrt{\overline{u_2'^2}}}{\overline{u_1}}\right)^6 \overline{u_1}, \quad \text{using} \quad L_a \text{ as } L \tag{4.12}$$

which indicates a linear relationship between t_c and $\overline{u_1}$.

The value of relative turbulent intensity ($\sqrt{\overline{u_2'^2}}/\overline{u_1}$) for the steady uniform flow obtained by past experimental results[5] is approximately 0.03~0.04 and it will be concluded that the use of Λ_L is better in evaluation of D than that of L_a as L.

Actually, the introduction of characteristic value of x_c expressed by $\overline{u_1}t_c$ after the assumption of Taylor will make a physical evidence of the turbulent diffusive processes.

4-2. One-Dimensional turbulent diffusion in confined flows

The flow in open channels is actually a confined flow enclosed by the solid boundaries and the free surface, so that the analysis treated in the preceding paragraph will be used only for a local phenomenon in the turbulent diffusion. On the other hand, the diffusive processes in the open channel flow characterized by the non-isotropic field in turbulence are not examined by the theoretical approach. Only the possible way to make the analysis of the turbulent diffusive processes in the flow will be made by comparing the results obtained theoretically by the one-dimensional analysis for the unconfined flow and the actual behaviours measured directly through experimentations in the confined open channel flow.

Let consider the turbulent diffusive processes in the steady uniform flow in a rectangular channel in Fig. 2 and put the following assumptions: (1) the turbulent field is isotropic and homogeneous, and (2) the transferable quantity will be completely reflected at boundaries.

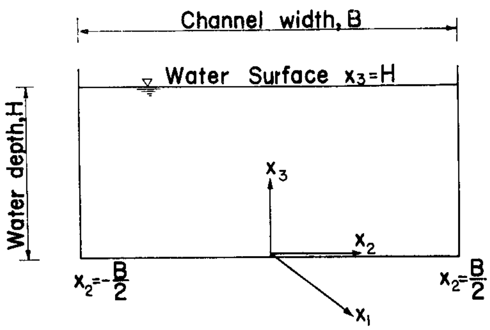

Fig. 2. Co-ordinates in a prismatic open channel flow.

The solution for the mean concentration of transferable quantity $\overline{c_b}$ at a point (x_1, x_2, x_3) is given by the following equation[6],

$$\overline{c_b}(x_1, x_2, x_3) = \frac{q_t}{U_m}\sum_{i=0}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(x_{2i}-x_{2s})^2}{2\sigma^2}\right\}\sum_{j=0}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(x_{3j}-x_{3s})^2}{2\sigma^2}\right\} \tag{4.13}$$

where

$$x_{2i} = \left(\frac{1}{2}+2i\right)B \mp \left(\frac{B}{2}-x_{2s}\right), \quad \text{and} \quad x_{2i} = \left(-\frac{3}{2}+2i\right)B \pm \left(\frac{B}{2}-x_{2s}\right)$$
$$x_{3j} = (1+2j)H \mp (H-x_{3s}), \quad \text{and} \quad x_{3j} = -(1+2j)H \pm (H-x_{3s})$$

in which q_t: the quantity transferred with the flow per unit time, U_m: the mean velocity over the diffusion area, σ^2: the variance of concentration distribution at a distance x_1 from the origin, and the diffusion initiates at $(0, x_{2s}, x_{3s})$.

Putting

$$\left.\begin{aligned}\frac{\sigma}{B} = \sigma_{*B}\,,\ \frac{x_2}{B} = x_{2*}\,,\ f_2(x_{2*}, \sigma_{*B}) = \sum_{i=0}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_{*B}}\exp\left\{-\frac{(x_{2i*}-x_{2s*})^2}{2\sigma_{*B}^2}\right\}\\ \frac{\sigma}{H} = \sigma_{*H}\,,\ \frac{x_3}{H} = x_{3*}\,,\ f_3(x_{3*}, \sigma_{*H}) = \sum_{j=0}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_{*H}}\exp\left\{-\frac{(x_{3j*}-x_{3s*})^2}{2\sigma_{*H}^2}\right\}\end{aligned}\right\} \tag{4.14}$$

Eq. (4.13) can be simplfied in a dimensionless expression:

$$\overline{c_b}(x_1, x_{2*}, x_{3*}) = \frac{q_t}{U_m HB} f_2(x_{2*}, \sigma_{*B}) f_3(x_{3*}, \sigma_{*H}) \tag{4.15}$$

In Eq. (4.15), q_t, U_m, H and B are known in a particular case, and $f_2(x_{2*}, \sigma_{*B})$,

$f_3(x_{3*}, \sigma_{*H})$ are also calculated by given values of the pair of (σ, x_2, x_{2s}) and (σ, x_3, x_{3s}). Here it should be noted that σ is not calculated from the actual distribution of mean concentration but is calculated from the mean concentration in the unconfined flow with isotropy in turbulence by Eq. (4.5). The mean concentration $\overline{c_b}$ at a point (x_1, x_2, x_3) in the open channel flows can be estimated, if a mathematical expression of σ is given.

5. Experimental Verification of Results Analysed

5-1. Turbulent diffusive processes in open channel flows

All the runs in experimentations were made by the use of polyethylene disks of 3 mm in diameter, 1.5 mm in thickness and 0.935 in specific weight, and of 1.5% dilute salt water. The experimental flume is a straight lucite one of 0.25 m in width and 16 m in length, and the bottom slope was set at 1/500.

Fig. 3 is an example of the variance measured by the use of polyethylene disks at free surfaces. The solid line is the Taylor's curve for the turbulent

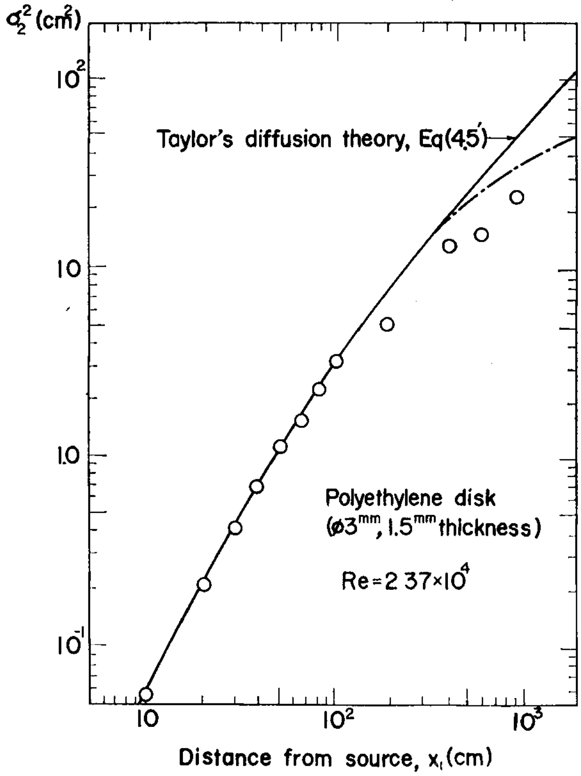

Fig. 3. The relation between the distance from the source and the variance in x_2-direction (Experimentation using polyethylene disks).

diffusion in unconfined flows. For short diffusion distance, the curve quite coincides with actual variance measured, while for long diffusion distance, it deviates from the data obtained, because of the great influence of the boundaries. The chain line is the curve of Eq. (4.15) theoretically derived in this paper and shows a better expression for the turbulent diffusion in open channel flows.

The results obtained by the use of 1.5% dilute salt water as the dispersive tracer are shown in Fig. 4, which indicates the similar behaviours in diffusive processes.

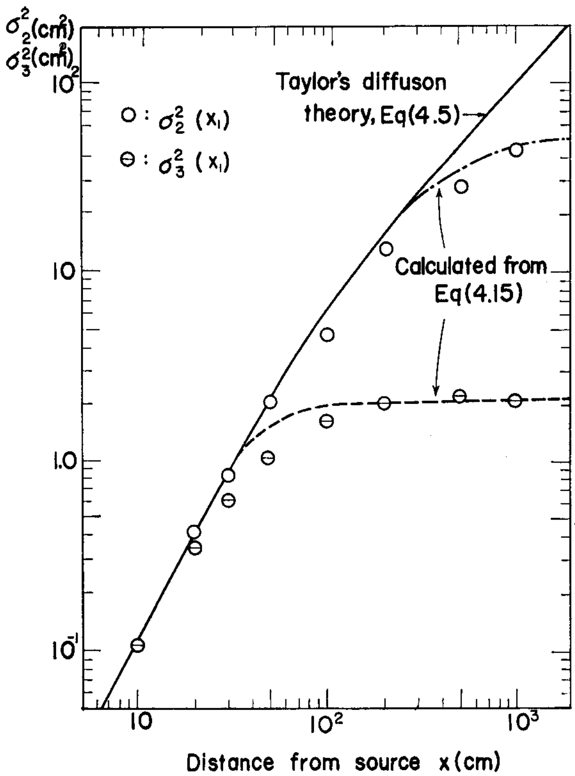

Fig. 4. The relation between the distance from the source and the variance in x_2-direction (Experimentation using 1.5% dilute salt water).

The turbulent diffusive processes in open channl flows, as seen in both figures, are characterized by two behaviours of

$$\sigma^2 \sim t^2 \sim x^2 , \quad D \sim t \sim x , \qquad \text{for short diffusion distance or time}$$

and

$$\sigma^2 \sim t \sim x , \quad D \sim \text{const} \qquad \text{for long diffusion distance or time}$$

On the other hand, for very short diffusion time, the turbulent intensity becomes linearly proportional to the time, so that $\sigma^2 \sim t^3$ and then $D \sim t^2 \sim \sigma^{4/3}$. Emprical relationship, $D \sim \sigma^{4/3}$, widely used in meteorology and oceanography can not be obtained in this analysis based on the Taylor's diffusion theory by continuous movements, but it will result from the use of very short time in measurement.

5-2. Characteristics value t_c

As will be expected in Eq. (4.4), t_* is a function of t for small values of t. Fig. 5 indicates this result and more clearly it increases with the decrease of x_1. Therefore, an exact value of t_*, which is the basic parameter in the analysis of Orlob, can not be estimated.

Fig. 6 indicates the relation between $\overline{u_1}$ and t_c $(=2t_*)$ by the use of Orlob's data[4], and it will be concluded that t_c is linear to $\overline{u_1}$ notwithstanding less accuracies in measurements.

5-3. Distribution of mean concentration

Fig. 7 indicates the theoretical curve of Eq. (4.15) with experimental data of mean concentration of 1.5% dilute salt water injected from a point source of 1 mm in diameter. For short diffusion distance ($x_1 = 50$ cm), the theoretical analysis agrees with experimental value, whereas for long diffusion distance ($x_1 = 500$ cm), it deviates from the plotting of experimental data. The deviation becomes strong in the zone of turbulent shear flow characterized by non-uniformity in local mean velocity.

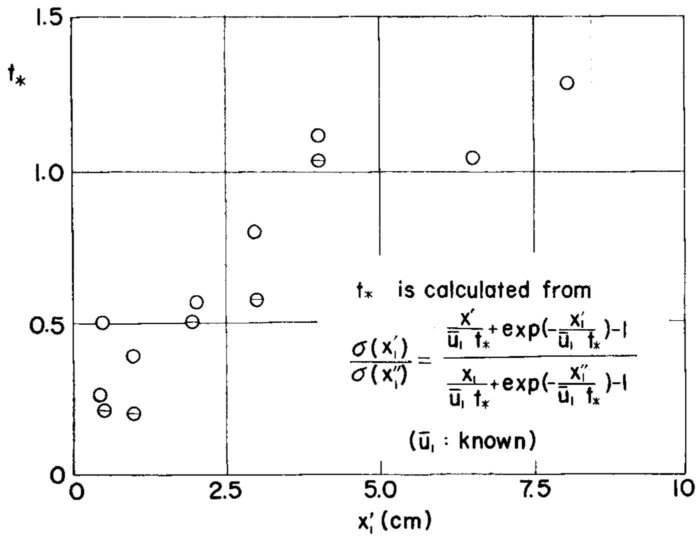

Fig. 5. t_* calculated by Eq. (4.5′) with variable x_1', when $x_1'' = 50$ cm.
(0 : Re $= 2.37 \times 10^4$, θ : Re $= 7.02 \times 10^4$)

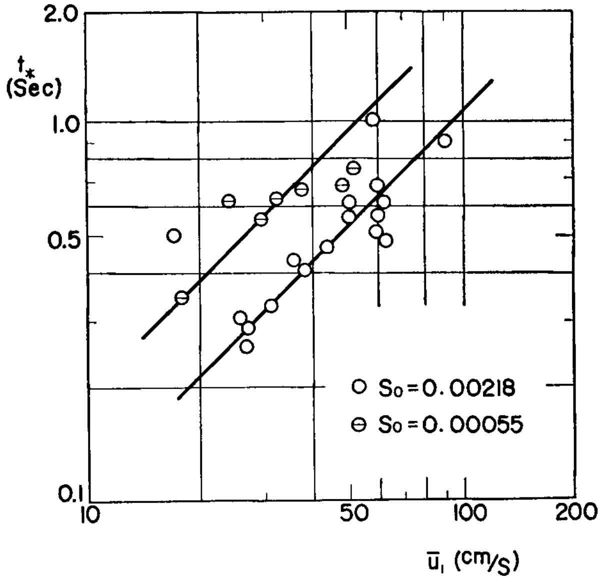

Fig. 6. The relation between $\bar{u}_1$ and t_* from Orlob's data[4]. (S_0: bed slope)

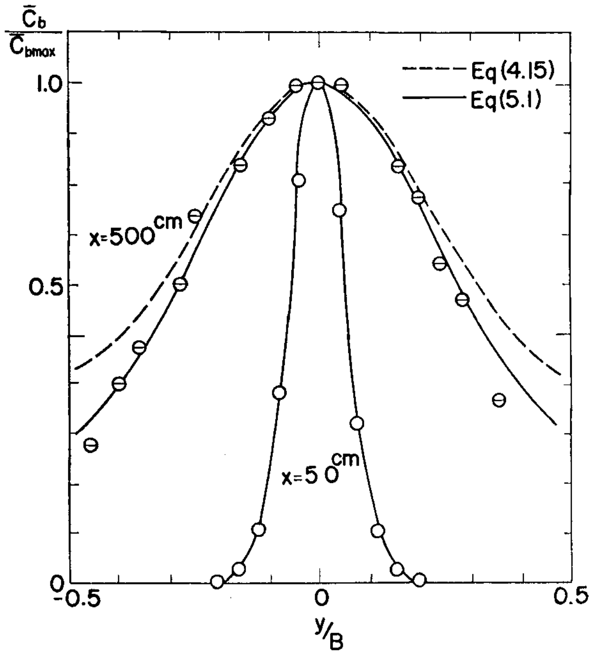

Fig. 7. Mean concentration distribution curve (Experimentation using 1.5% dilute salt water).

In engineering practice, the following correction to use the present analysis for the actual diffusive processes will be made:

$$\overline{c_{b2}} = K\frac{\overline{u_1}}{U_m}\overline{c_b} \tag{5.1}$$

in which $\overline{c_{b2}}$ is the corrected concentration contributive to the engineering use, and K is a constant determined by the mass conservation law of transferable quantity. Eq. (5.1) is shown in Fig. 7 with a good agreement against the mean concentralion of dilute salt water in the turbulent shear flow.

6. Conclusion

The turbulent diffusive processes in open channel flows are so complicated a in their behaviours, that present-day knowledge on turbulence theory is not sufficient to make the analysis possible. The study treated in this paper, as the first phase of the whole research progam, puts the emphasis on the Taylor's theory of diffusion by continuous movements and makes several evidences on actual behaviours with experimental verifications.

The conclusive results obtained are summarized as follows;

(1) The one-dimensional turbulent diffusion in a uniform flow field with homogeneous isotropic turbulence is characterized by the Lagrangian correlation coefficient, the characteristics of which are related to the main flow velocity through the Kolmogoroff's similarity principle under the condition of constant values in relative turbulent intensity. Therefore, the approximate

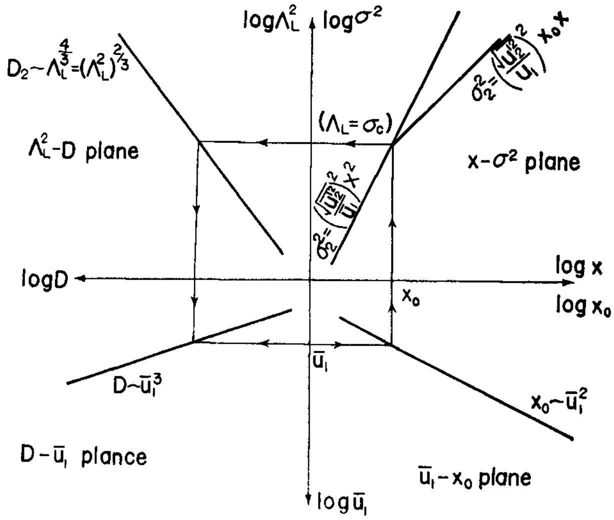

Fig. 8. The estimation of approximate turbulent diffusive processes through $\bar{u}_1$.

turbulent diffusion processes can be estimated by the graphical method as seen in Fig. 8, from the main flow velocity.

(2) The one-dimensional turbulent diffusion in a turbulent shear flow with non-isotropic and non-homogeneous turbulence is more complicated than that in a uniform flow, so the theoretical treatment is not still established. To obtain the distribution curve of concentration in such fields, Eq. (4.15) can be obtained approximately from the concentration in a uniform flow field under the assumptions of: (1) the turbulence is homogeneous and isotropic, and (2) the transferable quantity is completely reflected at boundaries. Eq. (5.1) verified experimentally will be a better practical expressions for actual concentrations in the turbulent shear flow.

It is further necessary to analyse the mechanism of mass transfer, which consists of an essential character in the turbulent diffusive processes, and it is still in progress.

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