The Motion of Particles Caused by a Bubble in Gas-solid Fluidised Bed

By

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The motion of particles caused by a single bubble which is blown into a two-dimensional fluidised bed has been studied. A drift line which is shown when a bubble has passed through the bed is obtained as a generalized curve. The experimental results can be explained fairly well by the authors' model.

The model is as follows.

1. The motion of particles is assumed as the motion of perfect fluid caused by the motion of a circular cylinder in the perfect fluid.
2. There is an imaginary wall at the distance of the diameter of bubble below from the center of the moving bubble and it moves upwards with the same velocity as the bubble.

Introduction

The motion of particles in the gas-fluidised bed has important effects upon such as particle mixing, heat transfer, reaction etc..

Formerly, it was supposed to be similar to diffusional process[1,2,3], but it should be considered that the motion of particles is caused by the motion of bubbles generated in the fluidised bed.

Rowe et al.[4,5,6], Reuter[7], Davidson[8] and Jackson[9] pointed out that the motion of particles in a fluidised bed can be replaced by the motion of perfect fluid caused by the motion of a circular cylinder or a sphere in the perfect fluid in the case of two or three dimensional fluidised bed respectively. The authors have investigated the motion of particles by means of blowing a single bubble into a two dimensional fluidised bed and analysed it by their model.

1. Experimental Apparatus, Testing Materials and Experimental Methods

1.1 Experimental apparatus

The experimental apparatus and the flow sheet is shown in **Fig. 1**. Two kinds

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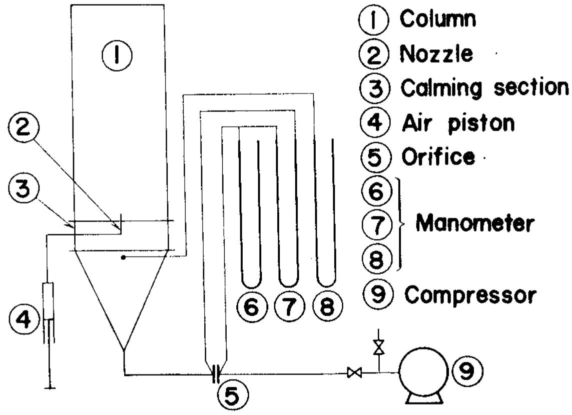


Fig. 1. Schematic diagram of experimental apparatus.

of two dimensional fluidised beds made of rigid transparent resin were used and each dimension was 250 mm wide × 600 mm high × 25 mm thick or 250 mm wide × 800 mm high × 10 mm thick. The calming section was fixed bed of 65～80# glass beads and 80 mm high.

As shown in Fig. 1, a nozzle made of 10 mm dia. copper pipe was installed at the center of the column and just above the calming section to blow a single bubble.

1.2 Testing materials

The glass beads coloured and non coloured were used as fluidising particles, and the sizes of those were 80～100#, 24～28#, 20～24# and 16～20#.

1.3 Experimental method

Filling up the bed with the coloured particles about 18 cm high from the bottom of the bed and non coloured particles about 18 cm high above them, it was fluidised by air at the maximum flow rate by which the bubbles were not found.

Then a single bubble (2～6 cm radius) was blown into it and the motion of the boundary line between coloured and non coloured particles was photographed by 35 mm camera and 16 mm cine camera (about 60 frame/sec). For one experiment the bubbles were blown into three or four times and photographed at each time. The films were analysed by a film analyser. The experiments for different heights of boundary line were also performed.

2. Experimental Results

2.1 The change of drift line during the bubble passing through the bed:

The displacements of the boundary line between coloured and non coloured particles (drift line) were persisted by 16 mm cine camera when the first bubble was

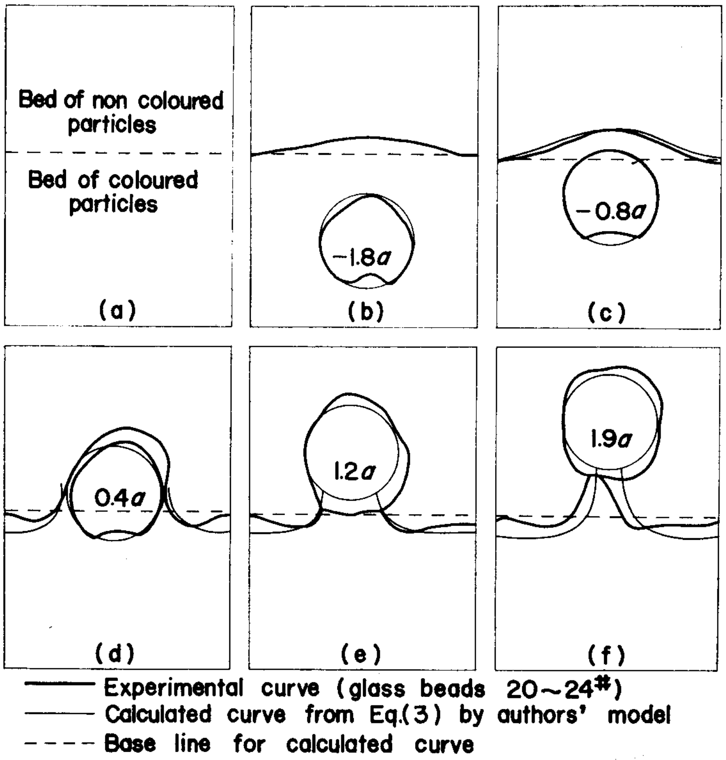


Fig. 2. Change of the drift lines during a bubble passing through the bed.

passing through the bed. An example of the results is shown with bold line in **Fig. 2**.

Fig. 2 (a) shows the state when a bubble was blown into the bed far below the horizontal boundary line and the line rose just the same amount of the bubble volume from the initial height.

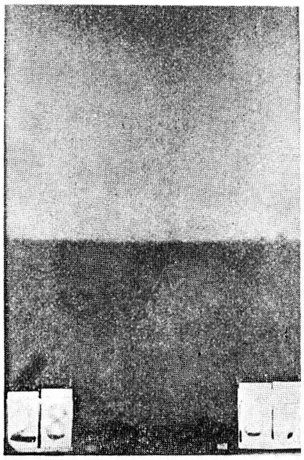
Fig. 2 (b) shows the state when the bubble is approaching the boundary line and the center of the bubble is $1.8 \times a$ lower from the base line, (c) the bubble rises $0.92 \times a$ from (b), (d) $1.0 \times a$ from (c), (e) $0.8 \times a$ from (d) and (f) $0.64 \times a$ from (e). After (f), the particles on the boundary line do not move except near the passing line of the bubble. The numerical values written in the bubble in Fig. 2 mean the distance from the base line. The base line is decided as written later in **2.3.**

From many experiments, it could be concluded that the particles which were located about $2 \times a$ below from the center of the bubble did not move.

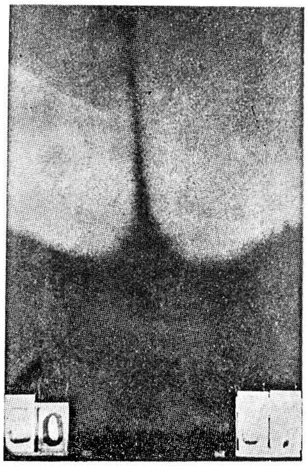
This can be confirmed from the results by Muchi et al.[10] as shown in **Fig. 11.**

2.2 The drift line after the bubble has passed through the bed thoroughly

The examples of the drift lines in this case are shown in **Fig. 3**. Fig. 3 (a) shows the boundary line before blowing the bubble. Fig. 3 (b), (c) and (d) show the drift lines after the first, second and third bubbles have passed through the bed thoroughly respectively.



(a) Initial boundary line



(b) Drift line by the first bubble $a=4.75$ cm

(c) Drift line by the second bubble $a=5.15$ cm

(d) Drift line by the third bubble $a=5.05$ cm

Fig. 3. Drift lines after the bubbles have passed through the bed thoroughly (glass beads 20～24#).

2.3 Generalization of drift line after the bubble has passed through the bed thoroughly

The results of Fig. 3 were summarized in one figure and it is shown in **Fig. 4.** The direction of the rising of the bubble is taken as y and the direction perpendicular to it as x. At first, it is assumed that there is no displacement of particle to x direction before and after a bubble was injected and passed through the bed thoroughly. The displacements to y direction between the drift line ② and ③ at the same value of x can be measured as h_1, h_2 and etc. on Fig. 4.

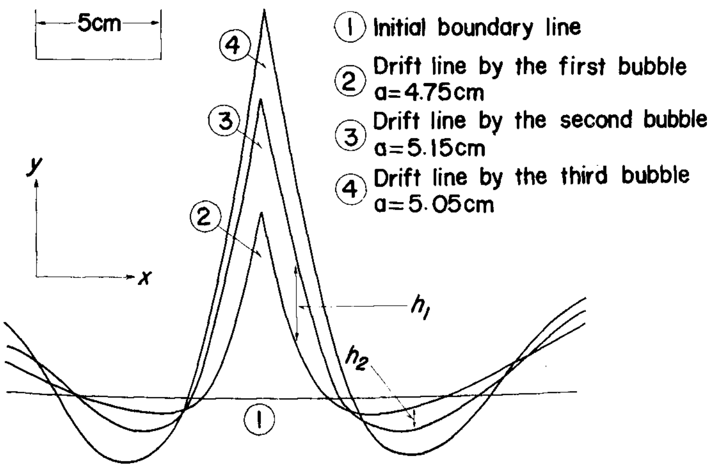


Fig. 4. Drift lines after the bubbles have passed through the bed thoroughly.

When these h_1 and h_2 etc. are taken from the horizontal line, a new drift line for ③ can be obtained. It is equivalent to the drift line after the first bubble has passed through the bed. This procedure is performed for the drift line ④ and so on.

As many results obtained by the above method were considered to be similar to the radius of the bubble, these drift lines were presented by (x/a) and (y/a) co-ordinates instead of x-y co-ordinates. These dimensionless drift lines for various bubbles of different diameter could be shown in one curve. This dimensionless drift line shows the generalized curve for various radii of bubbles and numbers of times of bubble blowing. The same dimensionless drift line was obtained for various particle sizes and initial heights of the boundary line also. So it can be concluded that this dimensionless drift line is a generalized curve for all cases and the assumption described above is appropriate.

This dimensionless drift line is shown as curve ① in **Fig. 5.**

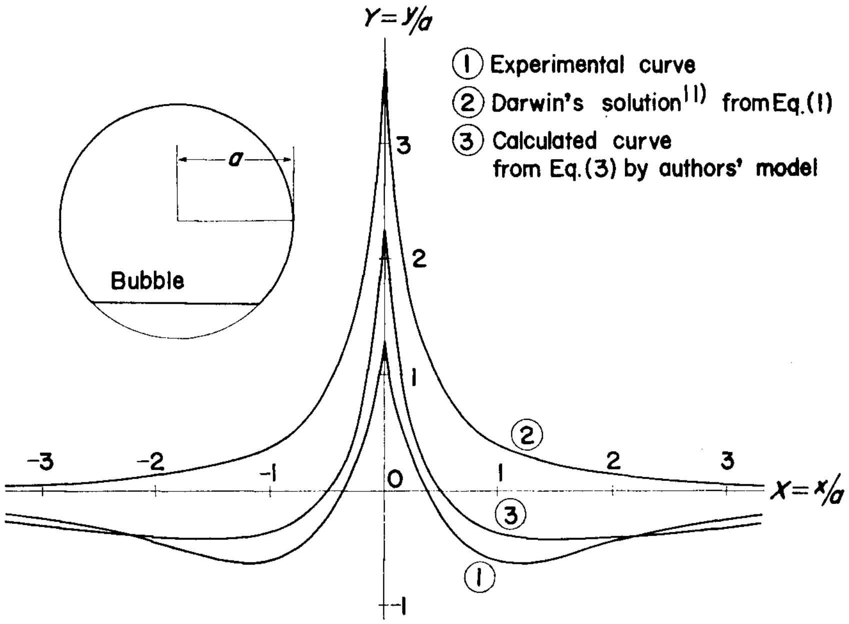


Fig. 5. Generalized drift lines after the bubble have passed through the bed thoroughly.

The horizontal base line in the figure is not the initial boundary line but decided as follows. The experimental apparatus has not infinite width, but for generalization the case of infinite width is considered. The horizontal line ① in **Fig. 6** (base line) is defined as the boundary line before the bubble passes through. And curved line ② is the drift line after the bubble has passed through. The area A_l of non coloured particles lower than line ① should become the sum of the area A_u of coloured particles upper than line ① and the area of bubble A_b, that is $A_l = A_u + A_b$. As the wake of the bubble was accompanied with the bubble, the bubble is considered as a circular cylinder.

Then the base line of Fig. 5 was decided by extending the experimental curve

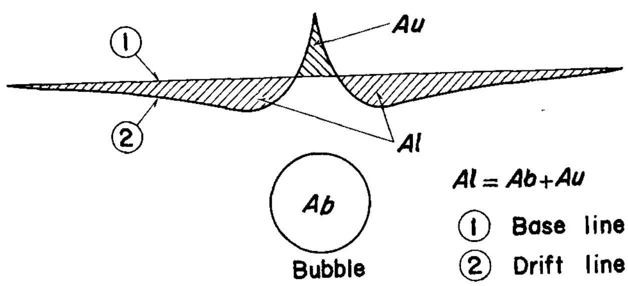


Fig. 6. Relation which must be satisfied in the infinite apparatus.

to the left and right sides so as to satisfy the relation $A_l = A_u + A_b$.

The generalized drift line obtained by experiments has the shape that at $X=0$, $Y \doteqdot 1.3$, at $X=0.4$, $Y=0$; at $X \doteqdot 1.2$, Y has minimum value $Y=-0.61$ and at $X \doteqdot 6$, the drift line approaches the base line.

The amount of particles which moves from the lower part of the initial horizontal line to the upper part and from the upper part to the lower part caused by a bubble of known size in a definite apparatus can be calculated using the curve ① of Fig. 5 and the mass balance of the coloured and non coloured particles to the initial horizontal line.

Though the lifting of particles by wake was also observed, it was not considered in the above results.

3. Consideration

For the motion of particles caused by the bubble Rowe et al.[4,5,6] and Reuter[7] considered that it is very similar to the motion of a perfect fluid caused by the motion of a circular cylinder or a sphere.

The stream functions for these cases considering the relative flow to the bubble are as follows.

For the two dimensional case,

$$\Psi = U_b x\left(1 - \frac{a^2}{x^2+y^2}\right) \tag{1}$$

For the three dimensional case,

$$\Psi = \frac{U_b}{2} x^2\left[1 - \frac{a^3}{(x^2+y^2)^{3/2}}\right] \tag{2}$$

For the two dimensional case, Darwin[11] suggested the calculated drift line from Eq. (1) and it is shown as the curve ② of Fig. 5. This curve does not coincide with the authors' experimental curve ① and also with the photographs of Rowe et al.'s results[5,6]. The difference between curve ① and ② are as follows.

1: The authors' result has the minimum value near $X=1.2$, but curve ② has not. 2: The curve ② appears only on the upper part of the base line, but the curve ① appears on both the upper and lower side. 3: The curve ① shows less amount of drift than the curve ②.

From Eq. (1), it is calculated that the fluid is displaced just the same volume as that of the circular cylinder to the direction of its movement.

But for the fluidised bed as the bubbles rises through the bed, the rear space of the bubble must be filled up by surrounding particles, that is, the relation shown in Fig. 6 should be satisfied. It is the reason why Eq. (1) differes from the experimental results.

3.1 The model for the motion of particles and its stream function

To expalin the experimental results, the following model was considered.

At first, the motion of particles is assumed as the motion of the prefect fluid.

As the rear space of the rising bubble must be filled up by surrounding particles, the flow when the bubble (a circular cylinder or a sphere) goes away from a wall is considered.

This flow of particles can be solved exactly by considering the existence of an imaginary bubble which is located at the same distance from the wall but on the opposite side and moves to a contrary direction.

An aproximate solution can be obtained when the distance between the bubble and the imaginary one is enough large and this solution is used on this paper. By taking the distance from the center of bubble to the wall as y_0 and the original point of coordinate at the center of the bubble, the stream functions of relative flow are shown by Eqs. (3) and (4) for two and three dimensional cases respectively.

For the two dimensional case,

$$\Psi = U_b x\left\{1-\left[1+\left(\frac{a}{2y_0}\right)^2\right]\left[\frac{a^2}{x^2+y^2}-\frac{a^2}{(y+2y_0)^2+x^2}\right]\right\} \qquad (3)$$

For the three dimensional case,

$$\Psi = \frac{U_b x^2}{2}\left\{1-\left[1+\left(\frac{a}{2y_0}\right)^3\right]\left[\frac{a^3}{(x^2+y^2)^{3/2}}-\frac{a^3}{[(y+2y_0)^2+x^2]^{2/3}}\right]\right\} \qquad (4)$$

As y_0 changes with time, Eqs. (3) and (4) show a non steady flow. But as the results such as shown in Fig. 2 presented that the particles located at about $2\times a$ below from the center of the bubble did not move and more over the dimensionless drift line (curve ① of Fig. 5) did not change by the difference of height of the boundary line as described before, y_0 can be considered to be constant, that is, the imaginary wall located at y_0 rises with the same rising velocity as that of the bubble. Then Eqs. (3) and (4) show the steady flow. From Eqs. (3) and (4), the following facts can be concluded: that at $y=+\infty$ the stream lines have the regular interval, and at $y=-y_0$ they have also the regular interval with the same spaces at $y=+\infty$. This result coincides with the assumption to obtain the drift line after the bubble has passed through, that is, the displacement of particles does not occur to x-direction. As up-flow of the bubble the last terms of Eqs. (3) and (4) are very small, they are close to Eqs. (1) and (2). Against this, as at down-flow their values are comparable with the second terms, they differ very much from Eqs. (1) and (2).

From Eqs. (3) and (4) it can be calculated that the relation $A_l = A_u + A_b$ (shown in Fig. 6) is satisfied completly.

Fig. 7 shows the stream lines calculated from Eq. (3) taking $y_0 = 2a$.

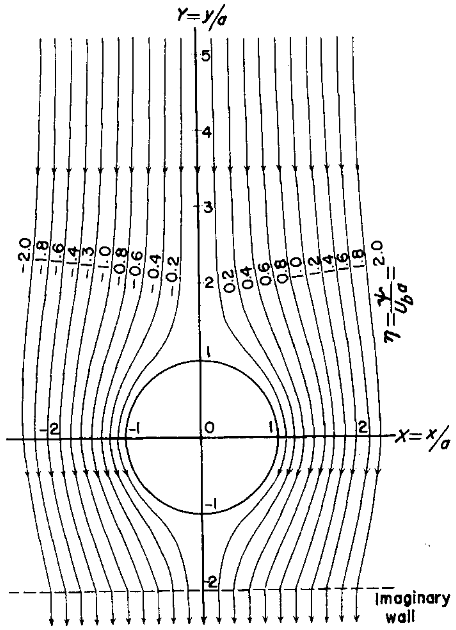


Fig. 7. Stream lines of relative flow of particles calculated from Eq. (3) by authors' model.

3.2 Comparioson between experimental results and the model

As the experiments were performed with a two dimensional fluidised bed, the experimental results were compared with ones calucleated from Eq. (3) taking $y_0 = 2a$.

3.2.1 Path line—As the velocity of particles can be calculated from Eq. (3), the traces of particles located on the horizontal line far above the bubble at the beginning (path line) were obtained as numerical solution. They are shown in **Fig. 8**.

The initial position of bubbles was taken at $8 \times a$ below the horizontal base line. The path line about Eq. (1) was obtained by Darwin[11] analitically and is shown in **Fig. 9**.

Fig. 8 and Fig. 9 do not show as much difference when the bubble approaches the initial horizontal line, but Fig. 8 shows clearly the existence of the flow filling up the rear space of the rising bubble.

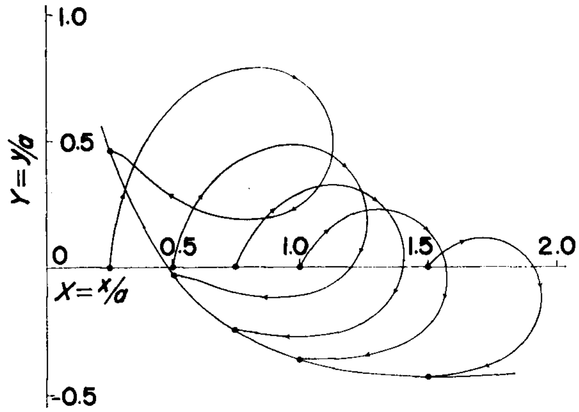


Fig. 8. Path lines calculated from Eq. (3) by authors' model.

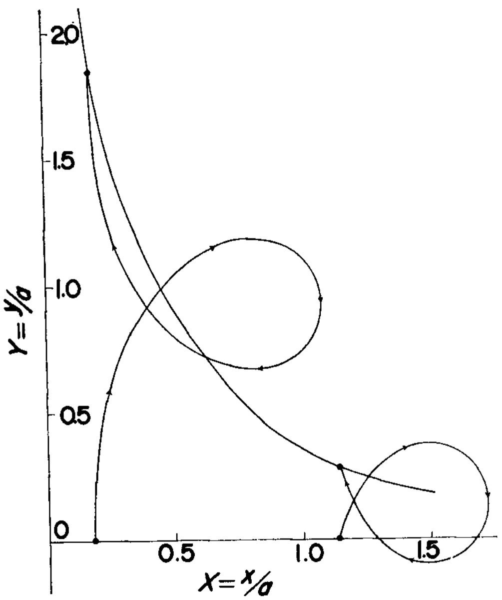


Fig. 9. Path lines calculated from Eq. (1) by Darwin[11].

3.2.2 The drift line during the bubble passing through—The drift lines during the bubble passing through the bed are obtained by linking the position of particles of the same time on the path lines of Fig. 8 and they are shown in Fig. 2 with fine lines. It can be mentioned that experimental curves coincide fairly well with calculated ones. The dotted horizontal line in the figure does not show the initial boundary line between coloured and non coloured particles, but the base line described in **2.3**. As the apparatus is not infinitely wide, the base line gradually goes down. The numerical values in the figures show the distance between the center of the bubble and that base line.

As the width of the apparatus is finite, these numerical values differ from the rising distance of the bubble described in **2.1**.

3.2.3 The drift line after the bubble has pased through—The drift line after the bubble has passed through the bed thoroughly is obtained by linking the last position of particles on the path line and is shown with the curve ③ of Fig. 5. By comparing this curve with the experimental curve ① and the calculated curve ② from Eq. (1), it can be mentioned that the shape of the curve and the amount of drifted particles differ very much between curve ① and ②, but curves ① and ③ resemble each other closely in the existance of minimum value and the numerical relationship.

3.3 Comparison between the model and other author's experimental results

By excluding the first terms of Eqs. (1) and (3), the stream function of absolute

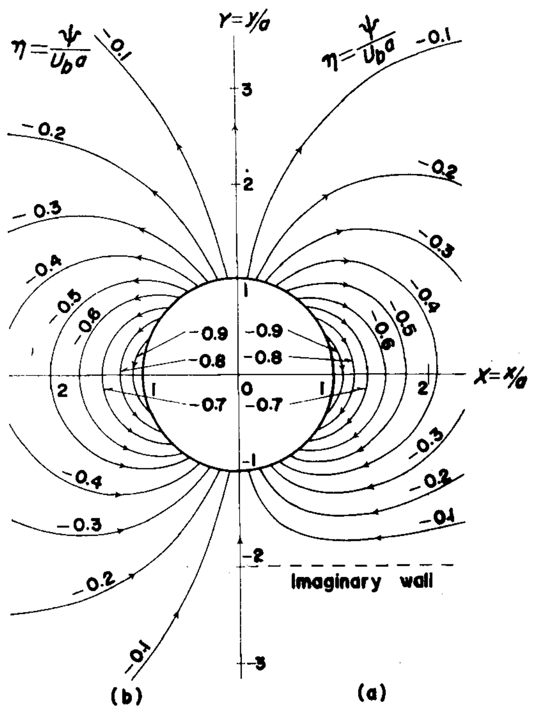


Fig. 10. Stream lines of absolute flow of particles
(a) The ones equivalent to Eq. (3) (authors' model)
(b) The ones equivalent to Eq. (1)

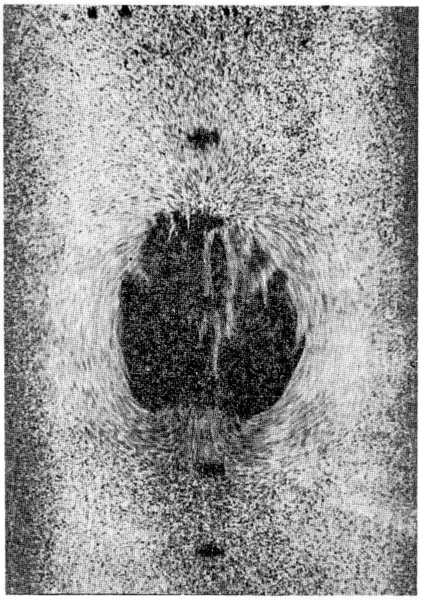


Fig. 11. Stream lines of absolute flow of particles by Muchi et al.'s result[10].

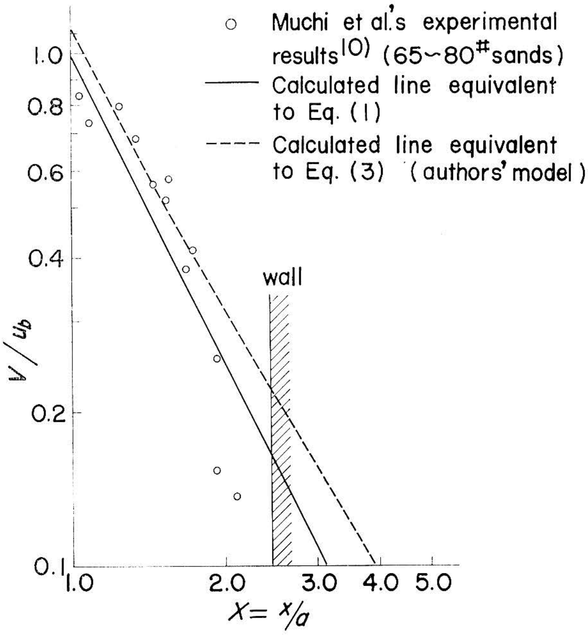


Fig. 12. Absolute velocities of particles for various x at $y=0$.

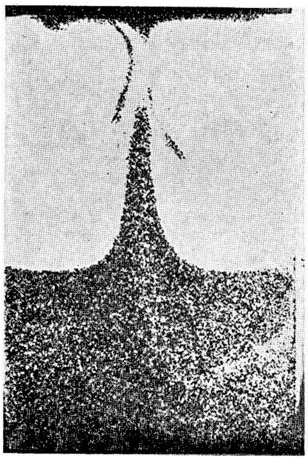


Fig. 13. Drift line after the bubble has passed through the three dimensional bed thoroughly (Row et al.[6])

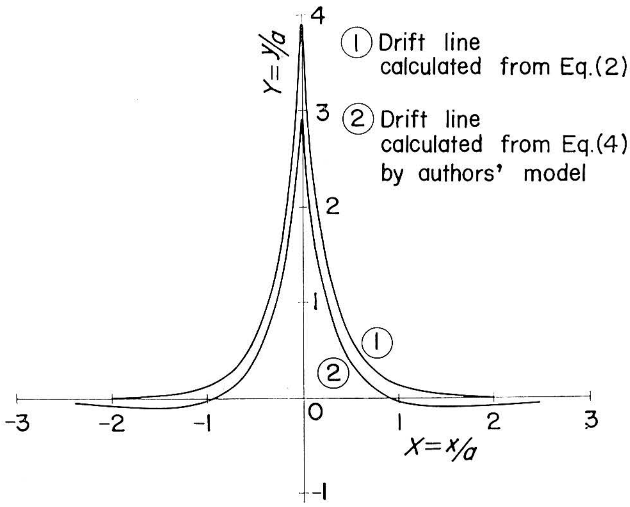


Fig. 14. Drift line after the bubble has passed through the three dimensional bed thoroughly.

flow is obtained. The absolute stream lines of Eq. (3) are shown on **Fig. 10** (**a**) (right side) and that equivalent to Eq. (1) is shown on Fig. 10 (b) (left side) respectively. **Fig. 11** shows the photograph of Muchi et al.'s experimental result[10] about the stream line of absolute flow. At the upper side of the bubble, the stream lines of Eqs. (3), (1) and experiments do not differ so much. At the lower side of the bubble, the stream lines of the model Fig. 10 (a) by Eq. (3) and that of experimental results of Fig. 11 resemble very well, but the stream lines by Eq. (1) differ very much from the experimental ones. **Fig. 12** shows the velocity of particles at various x for $y=0$.

The solid curve shows the curve calculated from Eq. (1), the dotted curve from Eq. (3) and the experimental points show the results of Muchi et al.'s[10]. The authors' model (dotted curve) coincides better with the results of Muchi et al.'s except near the wall.

Rowe et al.[6] obtained the drift line after the bubble has passed through the bed in the three dimensional fluidised bed. **Fig. 13** shows the result of it. The curve ① of **Fig. 14** shows the calculated drift line from Eq. (2) for the three dimensional case and ② shows the calculated one from the authors' model Eq. (4) for the three dimensional case taking $y_0=2a$ as for the two dimensional case. In this case, the authors' model coincided with the Rowe et al.'s experimental result taking the bubble diameter as 5.6 cm. In their paper, the bubble diameter was written as ca. 5 cm.

4. Conclusion

The motion of particles caused by a single bubble which was blown into the two dimensional fluidised bed was studied and the curve ① of Fig. 5 was obtained as the generalized drift line after the bubble had passed through.

This result could be explained fairly well by a model that there was the imaginary wall at about $2a$ below the center of the bubble and that it rose with the same velocity of the bubble. It was considered that this model also could be applied to the three dimensional case comparing with the results by Rowe et al..

Notation

A_b	:	area of bubble	[cm²]
A_l	:	drifted area of non coloured particles below from base line	[cm²]
A_u	:	drifted area of coloured particles upper from base line	[cm²]
a	:	radius of bubble	[cm]
h	:	displacement of particle to y direction	[cm]

U_b	:	rising velocity of bubble	[cm/sec]
V	:	velocity of particles	[cm/sec]
X	:	x/a	[—]
x	:	the direction perpendicular to the rising direction of bubble	[cm]
Y	:	y/a	[—]
y	:	the direction parallel to the rising direction of bubble	[cm]
η	:	dimensionless stream function	[—]
		$\frac{\Psi}{U_b a}$ for two dimensional case	
Ψ	:	stream function	
		for two dimensional case	$\left[\frac{cm^2}{sec}\right]$
		for three dimensional case	$\left[\frac{cm^3}{sec}\right]$

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