Failure Criterion of Cement Mortar Under Triaxial Compression

By

Yoshiji NIWA* and Shoichi KOBAYASHI*

(Received September 30, 1966)

In the first half of this paper, two-dimensional Griffith type fracture criteria were extended into three dimensions. Based on the result, a possible failure criterion of macroscopically homogeneous and isotropic brittle materials was proposed. The criterion is expressed in the principal stress space (σ_1, σ_2, σ_3) by a convex surface with the space diagonal of its axis. The surface consists of the following two types; one is expressed by Max. $(\sigma_1, \sigma_2, \sigma_3)=K$ (uniaxial tensile strength), and the other is such that the right sections of the surface are slightly bulged from an equilateral triangle and almost isotropically expand with an increase of hydrostatic pressure.

In the second half, experimental work for cement mortar was treated. In the compressive range examined here the failure criterion of cement mortar is found to be very similar to that described above. It is also found that the criterion is independent of the loading path and little affected by the intermediate stress in the experimental range.

1. Introduction

Knowledge of failure criteria for brittle materials such as rocks, concrete, mortar etc. subjected to multiaxial stresses is of basic importance for estimating safety factors of structures and foundations. Plenty of work has been devoted both theoretically and experimentally so as to obtain failure criteria, though most of them have some strong limitations in application owing to the complexity of fracture processes. Heterogeneity and anisotropy of materials add more difficulties on the subject.

From the general point of view the convexity condition of failure surfaces postulated by Drucker is the most fundamental. The theoretical microscopic fracture criteria based on Griffith's concept have some physical foundations on the macroscopic failure (or collapse) criteria, although the former were originally derived with an assumption of microscopically homogeneous isotropic linear elasticity.

In the first half of this paper, Griffith's and modified Griffith's criteria will be

* Department of Civil Engineering.

formally extended into three dimensions. Based on the results, a possible failure criterion of macroscopically homogeneous and isotropic brittle materials will be proposed. The criterion, of cause, satisfies the convexity condition.

In the second half, experimental work will be treated. Cement mortar specimens are used in the experiment because of their sufficient homogeneity and isotropy in macroscale, and of uniformity and easiness of fabrication. A special triaxial compression test apparatus designed for multiaxial compression test will be briefly described.

Test results will be discussed with reference to general failure criteria.

2. Some Considerations of Failure Criteria

The generalized stress or strain state of (systems of) materials at the moment of failure may be conveniently represented by a point in the stress or strain space. A set of such points forms a surface, which is called a failure surface. In other terms, the failure surface expresses all the possible combinations of stresses or strains at which failure occurs. Thus, the most general failure criteria can be fully expressed by failure surfaces of (systems of) materials. In the following, properties of failure surfaces will be discussed.

2.1 Fundamental Properties of Failure Surfaces

According to Drucker's postulate of stability[1] of (systems of) materials, failure surface of (systems of) brittle materials represented in the generalized stress space must be convex and the generalized plastic strain increment vector is normal to the surface at the stress point. These results are valid for all the brittle materials and for all the systems of brittle materials "such as composite materials, or assemblages of materials, or continua of one, two or three dimensions", whether linear, or non-linear, or elastic-plastic materials, provided time- and temperature-independent.

2.2 Possible Failure Criteria

As the convexity of failure surface and normality of strain increment vector are thus assured, the next step is to examine possible failure surfaces. We will start with Griffith's concept[2].

A) Griffith's and modified Griffith's criteria.

Based on the concept that fracture will occur at the instant when the maximum tensile stress caused by high stress concentrations at randomly distributed microcracks or flaws in the material reaches the ideal tensile strength of the material, Griffith derived the following fracture criteria[2),3] with assumptions of homogeneous and isotropic linear elastic continuum and of existence of so-called Griffith cracks (flat elliptic cracks).

In uniaxial tension the tensile strength K of the material in a direction perpendicular to the major axis of the crack is the minimum and

$$K = \sqrt{\frac{2TE}{\pi c}} \qquad \text{for plane stress} \tag{1}$$

$$= \sqrt{\frac{2TE}{\pi c(1-\nu^2)}} \qquad \text{for plane strain,} \tag{1'}$$

where E, ν, T and $2c$ are elastic modulus, Poisson's ratio, surface energy per unit area and the major axis of the elliptic crack, respectively.

In the biaxial stress state, the fracture criteria are as follows,

(1) $$\sigma_1 = K\,, \quad \text{if} \quad 3\sigma_1+\sigma_3 \leq 0\,, \tag{2a}$$

(2) $$(\sigma_1-\sigma_3)^2+8K(\sigma_1+\sigma_3) = 0\,, \quad \text{if} \quad 3\sigma_1+\sigma_3 \leq 0\,, \tag{2b}$$

where σ_1 and σ_3 are the principal stresses with $\sigma_1 > \sigma_3$. K is given by Eq. (1) or (1)'.

The angles φ of the major axis of the most dangerous crack measured from σ_3 direction are (1) $\varphi=0$, and (2) $\varphi=\cos^{-1}\{(\sigma_3-\sigma_1)/2(\sigma_1+\sigma_3)\}/2$, respectively. In uniaxial compression, it is easily shown that $\sigma_1=0$, $\sigma_3=-8K$ and $\varphi=30°$.

The geometrical configuration is assumed unaltered in the analysis of Griffith's, though the Griffith cracks might easily close when the confining pressure increases. Considering the effect of normal and frictional stresses transmitted across the closed surfaces of Griffith cracks, McClintock and Walsh[4] modified Griffith's criterion as follows,

$$\mu(\sigma_1+\sigma_3-2\sigma_c)+(\sigma_1-\sigma_3)\sqrt{1+\mu^2} = 4\sqrt{1-\frac{\sigma_c}{K}}\,, \tag{3}$$

where μ is the coefficient of friction and σ_c is normal stress to the crack axis required to close it. When cracks are very flat, then $\sigma_c \approx 0$ and simply

$$\mu(\sigma_1+\sigma_3)+(\sigma_1-\sigma_3)\sqrt{1+\mu^2} = 4K\,. \tag{4}$$

These expressions are approximately valid in the compression range.

B) Relation between Griffith's and Mohr's criteria.

As pointed out by Murrell[5], Griffith's criterion is a special case of Mohr's quadratic criteria, which is expressed by

$$(\sigma_1-\sigma_3)^2+2\alpha(\sigma_1+\sigma_3) = 4t_1-\alpha^2\,, \tag{5}$$

where α and t_1 are material constants to be determined experimentally. The substitution of $\alpha=4K$ and $t_1=4K^2$ into the above expression directly leads to Griffith's criterion (2b). If $3\sigma_1+\sigma_3 \geq 0$, all the stress states correspond to the apex of Mohr's criterion.

The modified Griffith's criterion (4) is the same as Mohr-Coulomb criterion

$$\sigma_3 = \frac{\sqrt{1+\mu^2}+\mu}{\sqrt{1+\mu^2}-\mu}\sigma_1 - \frac{2t_0}{\sqrt{1+\mu^2}-\mu} \qquad (6)$$

with $t_0=2K$.[6]

The microscopic fracture criteria seem to provide sound physical foundations for the macroscopic ones. The former, however, is a necessary and sufficient condition for only local fracture initiation[7] from microcracks and does not include any propagation processes preceding failure (or collapse), so the former does not necessarily correspond to the latter. There exist extremely complicated propagation processes from the initiation of fracture to failure or collapse.

The modified Griffith's criterion is often said to well explain experimental results, but the reason may be found in such that the coefficient of friction μ can be selected as a parameter to fit the test results. We do not know at all how the coefficient of friction behaves during the process of fracture. It may not remain constant.

The other parametric representation of Griffith's criterion in the compression range was derived by the authors as follows. Assuming that the maximum tensile stress at the most dangerous crack is reduced by a factor of $1/(1+k)$ under the confining pressure (although this assumption may not be based on physical phenomena) and satisfying the uniaxial compression condition, the following criterion was derived.

$$(\sigma_1-\sigma_3)^2+8K(1+k)(\sigma_1+\sigma_3)+64K^2k = 0\,, \qquad (7)$$

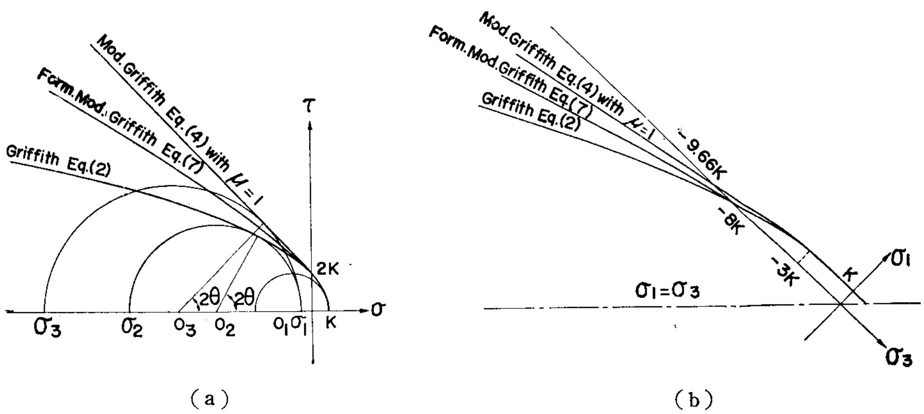


Fig. 1. Representation of Griffith's and modified Griffith's criteria in a) normal-shear stress coordinates, and b) principal stress coordinates.

where k is a parameter. The above expressions are schematically expressed in Fig. 1.

C) Extended Griffith's criterion.

According to Sack's analysis[8] of three dimensional Griffith cracks, the most dangerous is found to be a penny-shaped crack lying in the intermediate stress plane, provided that the confining pressure is not very high. Therefore, fracture may occure only with some special combinations of the maximum and the minimum principal stresses and the failure surfaces are parallel to the intermediate stress plane. Intermediate stress itself essentially adds no contribution to failure conditions.

Based on this idea, two dimensional failure criteria can be easily extended into three dimensions. The failure surface thus extended into the principal stress space must remain unaltered by the cyclic permutations of principal axes owing to the assumption of homogeneity and isotropy. Therefore, the shape of the failure surface is easily visualized by a section cut by Rendulic plane (i.e. a plane containing one of the principal axes and the space diagonal) and the right sections (i.e. a plane such that the sum of the principal stresses is constant).

Transforming a coordinate system $(\sigma_1, \sigma_2, \sigma_3)$ to a new one $(\sigma_1', \sigma_2', \sigma_3')$ according to

$$\left.\begin{aligned} \sigma_1 &= -\frac{1}{\sqrt{6}}\sigma_1' - \frac{1}{\sqrt{2}}\sigma_2' + \frac{1}{\sqrt{3}}\sigma_3', \\ \sigma_2 &= \sqrt{\frac{2}{3}}\sigma_1' \qquad\qquad\quad + \frac{1}{\sqrt{3}}\sigma_3', \\ \sigma_3 &= -\frac{1}{\sqrt{6}}\sigma_1' + \frac{1}{\sqrt{2}}\sigma_2' + \frac{1}{\sqrt{3}}\sigma_3', \end{aligned}\right\} \quad (8)$$

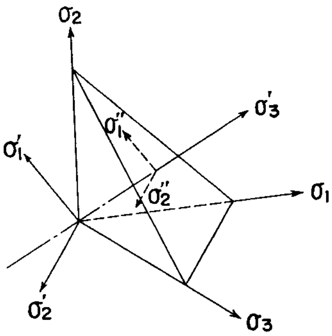


Fig. 2. Transformation of coordinates.

with condition $\sigma_1 \geq \sigma_2 \geq \sigma_3$, i.e. $\sigma_2' \leq 0$ and $\sqrt{3}\sigma_1' \leq |\sigma_2'|$, the curves on Rendulic plane and the projections of right sections are represented on (σ_1', σ_3')-plane and on π-plane ($I_1 = \sigma_1 + \sigma_2 + \sigma_3 = 3\sigma_{oct} = 0$), respectively (Fig. 2).

The length of a vector originated from the space diagonal to any points on the right section is given by

$$r = \sqrt{\sigma_1'^2 + \sigma_2'^2} = \sqrt{\frac{1}{3}\{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\}} = \sqrt{3}\,\tau_{oct} \quad (9)$$

and the direction of the vector measured from σ_2'-axis is given by

$$\theta = \tan^{-1}\left(\frac{\sigma_1'}{\sigma_2'}\right) = \tan^{-1}\left[-\frac{1}{\sqrt{3}}\cdot\frac{2\sigma_1-\sigma_2-\sigma_3}{\sigma_1-\sigma_3}\right]. \tag{10}$$

Griffith's criterion, thus, extended as follows.

(1) On π-plane;

a) $\sigma_1'+\sqrt{3}\,\sigma_2' = \sqrt{\frac{2}{3}}I_1-\sqrt{6}K$, if $2\sqrt{6}\,\sigma_1'+3\sqrt{2}\,\sigma_2'\leq 4I_1$, (11a)

b) $\sigma_1' = \frac{1}{4K}\sqrt{\frac{3}{2}}\sigma_2'^2+\sqrt{\frac{2}{3}}I_1$, if $2\sqrt{6}\,\sigma_1'+3\sqrt{2}\,\sigma_2'\geq 4I_1$, (11b)

valid for $\sigma_2'\leq 0$ and $|\theta|\leq 30°$.

(2) On Rendulic plane;

a) compression test,

$$\left.\begin{aligned} &2\sigma_1'+\sqrt{2}\,\sigma_3' = \sqrt{6}K, && \text{if } \sigma_1'+2\sqrt{2}\,\sigma_3'\geq 0, \\ &\sigma_3' = -\frac{3\sqrt{3}}{8K}\sigma_1'^2+\frac{1}{\sqrt{2}}\sigma_1', && \text{if } \sigma_1'\leq 0 \text{ and } \sigma_1'+2\sqrt{2}\,\sigma_3'\leq 0 \end{aligned}\right\} \tag{12}$$

b) extension test,

$$\left.\begin{aligned} &-4\sigma_1'+\sqrt{2}\,\sigma_3' = \sqrt{6}K, && \text{if } 5\sqrt{2}\,\sigma_1'-4\sigma_3'\geq 0 \\ &\sigma_3' = -\frac{3\sqrt{3}}{8K}\sigma_1'^2+\frac{1}{\sqrt{2}}\sigma_1', && \text{if } \sigma_1'\leq 0 \text{ and } 5\sqrt{2}\,\sigma_1'-4\sigma_3'\leq 0 \end{aligned}\right\} \tag{13}$$

The surfaces expressed by Eqs. (11a) and (11b) are smoothly connected on a line $\sigma_2'=-2\sqrt{2}K$.

The length r_m corresponding to a stress state $\{\sigma_1, \sigma_3, \sigma_2=\sigma_3+m(\sigma_1-\sigma_3), 0\leq m\leq 1\}$ is given as follows,

$$\left.\begin{aligned} r_m &= \frac{1}{\cos\left(\theta+\frac{\pi}{6}\right)}\left(\sqrt{\frac{3}{2}}K-\frac{1}{\sqrt{6}}I_1\right) && \text{corresponding to Eq. (11a)} \\ &= \frac{4\sqrt{2(1-m+m^2)}}{3\sqrt{3}}\{-(1-2m)+\sqrt{K^2(1-2m)^2-9KI_1}\} && \text{corresponding to Eq. (11b)} \end{aligned}\right\} \tag{14}$$

and

$$\left.\begin{aligned} r_1 &= 2r_0 && \text{corresponding to Eq. (11a)} \\ &= r_0+\frac{8\sqrt{6}}{9}K && \text{corresponding to Eq. (11b)} \end{aligned}\right\}, \tag{15}$$

where r_1 and r_0 correspond to compression and extension tests, respectively.

The extended Griffith's criterion is schematically represented in Figs. 3 and 4.

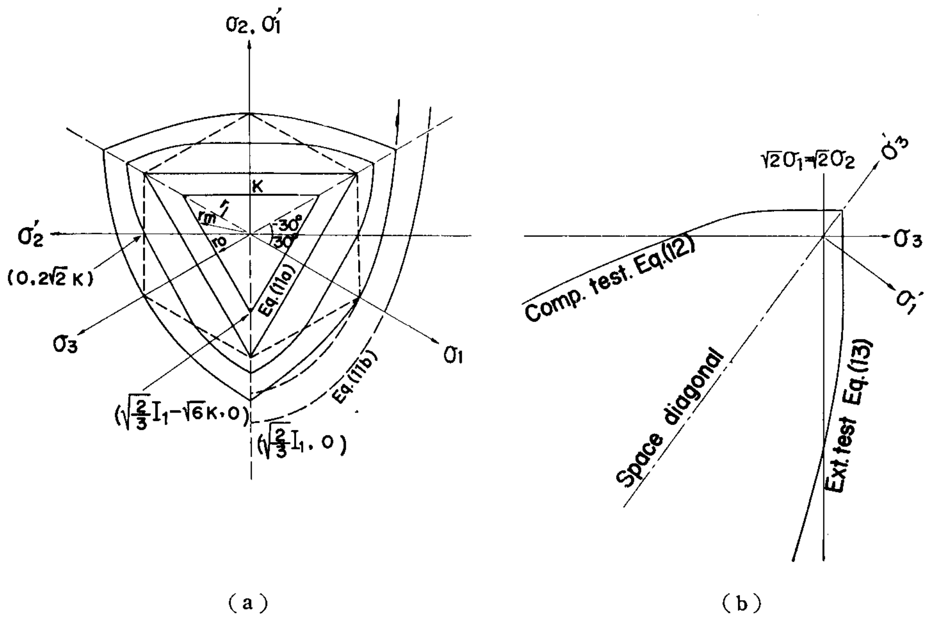


Fig. 3. Extended Griffith's criterion represented on a) π-plane, and b) Rendulic plane.

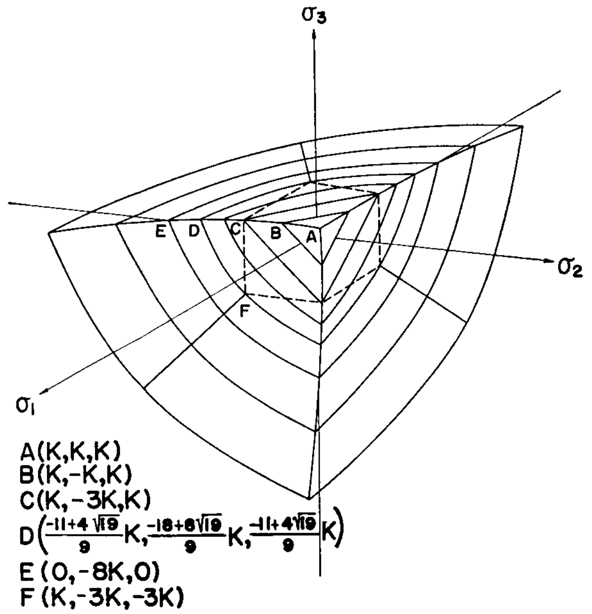


Fig. 4. A schematic view of the extended Girffith's criterion.

The extended modified Griffith's criterion is obtained in the same manner.

(1) On π-plane;

$$\sigma_2' = -\frac{1}{\sqrt{3}}\cdot\frac{\mu}{\sqrt{1+\mu^2}}\cdot\sigma_1'+\frac{\sqrt{2}}{3\sqrt{1+\mu^2}}I_1-2\sqrt{2(1+\mu^2)}K\,, \tag{16}$$

for $\sigma_2'\leq 0$ and $|\theta|\leq 30°$.

(2) On Rendulic plane;

a) compression test,

$$(3\sqrt{1+\mu^2}-\mu)\sigma_1'+\sqrt{2}\,\mu\sigma_3' = 2\sqrt{6}\,(1+\mu^2)K\,, \quad \sigma_1'\geq 0\,, \tag{17}$$

b) extension test,

$$-(3\sqrt{1+\mu^2}+\mu)\sigma_1'+\sqrt{2}\,\mu\sigma_3' = 2\sqrt{6}\,(1+\mu^2)K\,, \quad \sigma_1'\leq 0\,. \tag{18}$$

The relations

$$I_1 = \frac{2K}{\mu}-\frac{1}{6}\sqrt{\frac{3}{2}}\cdot\frac{3\sqrt{1+\mu^2}+\mu(1-2m)}{\sqrt{1-m+m^2}}\cdot\frac{r_m}{\mu}\,, \tag{19}$$

and

$$\frac{r_1}{r_0} = \frac{3\sqrt{1+\mu^2}+\mu}{3\sqrt{1+\mu^2}-\mu} \tag{20}$$

are also obtained.

The shape of the extended modified Griffith's criterion may be visualized by the aid of Figs. 3 and 4.

The extension of the parametrically expressed Griffith's criterion (7) is the same as the extended Griffith's criterion with $K(1+k)$ instead of K.

D) A possible failure criterion of macroscopically homogeneous and isotorpic brittle materials

The extended Griffith's and modified Griffith's criteria suggest a possible macroscopic failure criterion. The failure criterion is expressed by a convex surface with space diagonal of its axis. The surface consists of the following two types which are smoothly connected to each other. One is expressed by Max. $(\sigma_1, \sigma_2, \sigma_3)=K$ (uniaxial tensile strength), and the other is a surface of almost isotropically expanding with an increase of hydrostatic pressure and the right sections of it are slightly bulged from an equilateral triangle. With an increase of hydrostatic pressure, the failure surface must expand more rapidly than the extended Griffith's. The right sections of the surface may be more bulged than those of Griffith's, because the intermediate stress actually has some effects on the failure criterion. The section of the surface cut by a plane containing the space diagonal may be approximated by straight lines in the tensile range and straight lines or parabolas in the compression range.

3. Experimental Work

Data on the strength of cement mortar under multiaxial stress state are scarce. The strength under biaxial compression has been investigated by Bellamy[9], Vile and Sigvaldason[10], Iyengar, Chandrashekhara and Krishnaswamy[11]. Bellamy's experiment by the use of hollow cylinders is not a biaxial test in the true sense, since the cylinders are in a confined state by the external pressure. The failure criterion of Iyengar et al. is erroneous, because the effect of friction between the platens and surfaces of specimens is not considered. The general triaxial test for mortar has never been done so far.

Here below the authors are chiefly concerned with multiaxial compression of mortar.

A) Description of equipments

In order to obtain a general failure surface, equipment was designed for subjecting cubical specimens to combined loads. The equipment consists of a set of three Riehlé type universal hydraulic machines (Fig. 5). The load is applied by two 200 ton hydraulic jacks located in the vertical position and by four 100 ton hydraulic jacks located in a heavy horizontal frame in two orthogonal directions. These jacks except the lower vertical are equipped with a spherical-seated head. The machine is properly designed to operate each jack independently, or the three pairs of jacks, either independently or in parallel. Therefore, cubical specimens are easily set right in the center of the jacks and can be kept in the same position during the test.

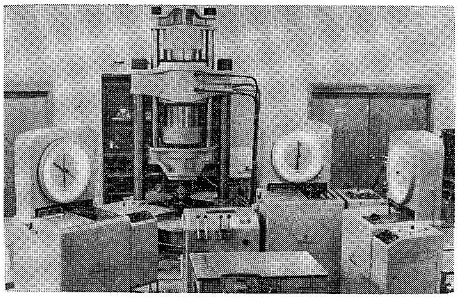


Fig. 5. General view of the equipment for triaxial test.

The applying load is arbitrarily controlled by an automatic load control apparatus either proportionally or stepwise. The load can be kept constant while deformations occur. The ratio of the proportional loading can be selected in the range from one to hundred. The maximum usable size of cubic specimen is of 20 cm.

B) Test specimens

All the tests reported herein have been carried out on 10.7 cm cubical specimens. The mix used throughout was made of ordinary portland cement (JIS, R5201–1964) and Toyoura standard sand with proportions by weight;

Cement : Water : Sand = 1 : 0.6 : 2 .

Specimens were cast in machined steel molds. The molds were stripped after 24 hr, and then cured in water (20°±1°C) for 54 days, and after which they were put in the moist room (20°±1°C, 90±5 percent relative humidity) for 1 day before testing. All specimens were thus tested at 56 days. The standard test results are shown in Table 1.

Table 1. Standard test results of cement mortar.

	3 days	7 days	28 days
Modulus of rupture (kg/cm²)	30.8	40.8	67.2
Compressive strength (kg/cm²)	118	193	343

C) Test procedures

It is widely realized that the friction between the platen and end surface of specimens plays an important role in the compression test, especially of cubical specimens. Preliminary tests were carried out in order to estimate the effect of friction. Coefficients of friction of several lubricants examined in the range of stresses about 10 to 90 percent of axial compressive strength are listed in Table 2.

Table 2. Test results of end friction.

Lubricant	Coefficient of friction μ
Without lubricant	0.46–0.65
Alminum powder	0.43–0.53
Graphite powder	0.28–0.31
Cup grease	0.15–0.24

Relations between uniaxial crushing strengths and lubricants are shown in Fig. 6. From these reults soft thin rubber sheet applied with silicon grease are very effective. The results with cup grease, however, will be chiefly discussed here. The test with the rubber sheet and silicon grease is now being carried on.

The partial loading by this type of test equipment (in this experiemt 10 cm square platens were used for 10.7 cm cubical specimens) was found to have no significant effect even on uniaxial compressive strength, provided that the faces are well lubricated.

The loads were applied in the following manner.

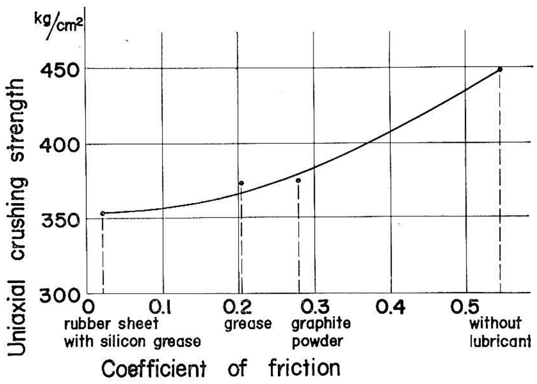


Fig. 6. Relations between uniaxial crushing strengths and coefficients of end friction of specimens.

(1) Sequent loading; one or two axial loads are kept constant while the remainder either increases or decreases,

(2) Proportional loading; the ratio of the three axial loads is kept constant while they increase,

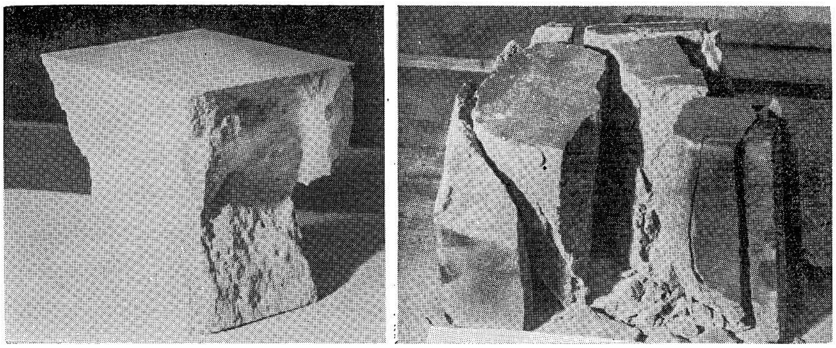
(3) Arbitrary loading; combinations of the above.

The loading rate was 100–200 kg/cm^2/min. in uniaxial tests. In biaxial and triaxial tests, the minimum loading rate was chosen 100–200 kg/cm^2/min. in the proportional loading.

D) Test results

(1) Uniaxial compressive strength

The test reuslts widely vary with different lubricants, as shown already in Fig. 6. The average uniaxial compressive strength of 26 specimens with cup grease as lubricant is 373 kg/cm^2. Typical fracture patterns are shown in Fig. 7.



(a) (b)

Fig. 7. Typical uniaxial fracture patterns tested a) without lubricant, and b) with lubricant.

The result is used as a standard as to nondimensionalize the results of biaxial and triaxial tests. Although the effect of the end friction of specimens may differ in these cases from that in the uniaxial case, the errors may be small enough if the end friction is well reduced, say by grease.

(2) Biaxial tests

About 10 specimens for each stress ratio $\sigma_1/\sigma_2=1/4$, $1/2$, $3/4$ and 1 were tested by both proportional and stepwise loadings, i.e. the smaller load was incerased at first and then kept constant while the other increased. Some of the results were obtained by applying load along an arbitrary path. No significant variation was observed in the results obtained by these loadings. The test results are shown in Fig. 8, in which the results for mortar and for concrete by Vile et al.[10] and those

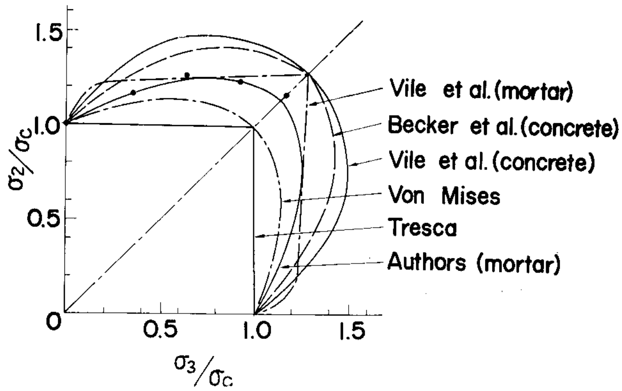


Fig. 8. Principal stresses at failure in biaxial tests.

for concrete by Becker et al.[12] were also shown for reference. Data of Vile et al. are insufficient, though their original curves are shown in the figure.

The effect of intermediate stress may be less than that evaluated from the graphs, since the end friction of specimens, however small it may be, does work to prevent fracturing.

(3) Triaxial tests

Triaxial tests were carried out on 85 specimens. The test results carried out for several specimens by proportional loading and stepwise loading (similar to the biaxial case) showed no significant variation. The results are plotted in Fig. 9. In Fig. 10 some results for concrete obtained by several investigators[13)~15)] are also shown for reference.

Using Eqs. (9) and (10) with calculated values of $(-\tau_{oct}/\sigma_c)$ corresponding to the same values of σ_{oct}/σ_c by interpolation from Fig. 9, the projections of right

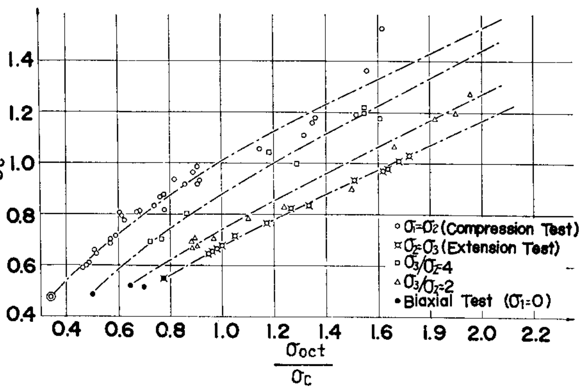


Fig. 9. Relations between octahedral stresses at failure (mortar).

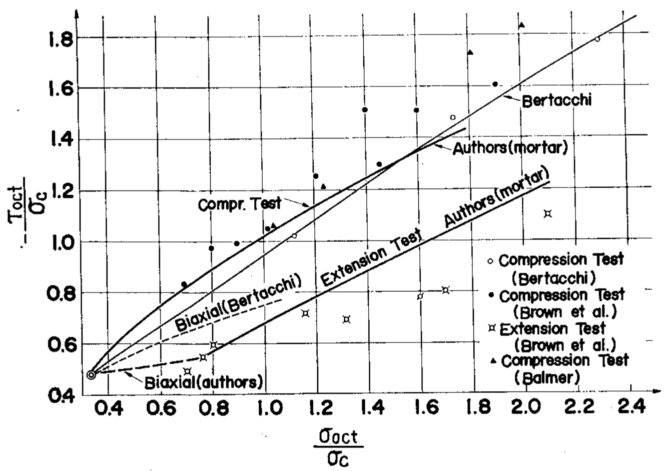


Fig. 10. Relations between octahedral stresses at failure (concrete).

sections on π-plane are obtained as shown in Fig. 11. The broken line shows the biaxial test results. The stress state corresponding to the inside of this line cannot be achieved by this experiment. The compression and extension test results are plotted on Rendulic plane as shown in Fig. 12. The failure surface thus obtained agrees well with that predicted by theoretical considerations.

In Fig. 13 test results are plotted for $(\sigma_1+\sigma_3)/2\sigma_c$ versus $-(\sigma_1-\sigma_3)/2\sigma_c$. The results may be approximated by

$$\left(\frac{\sigma_1+\sigma_3}{2\sigma_c}\right)=\frac{1}{2.10}\left\{\frac{(\sigma_1-\sigma_3)^2}{2\sigma_c^2}+\frac{1.10}{2}\right\}. \qquad (21)$$

In the same figure Griffith's and modified Griffith's criteria, i.e. Eqs. (2) and (4) are shown for reference. From Fig. 13, it may concluded that the intermediate

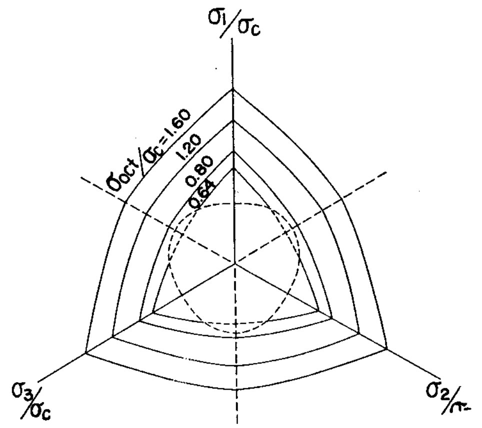


Fig. 11. Right sections projected on π-plane.

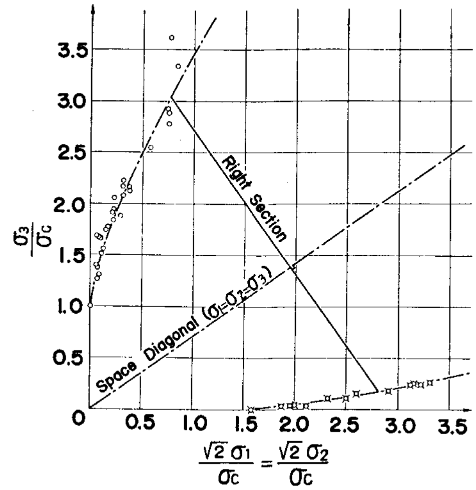


Fig. 12. Triaxial compression and estension test results.

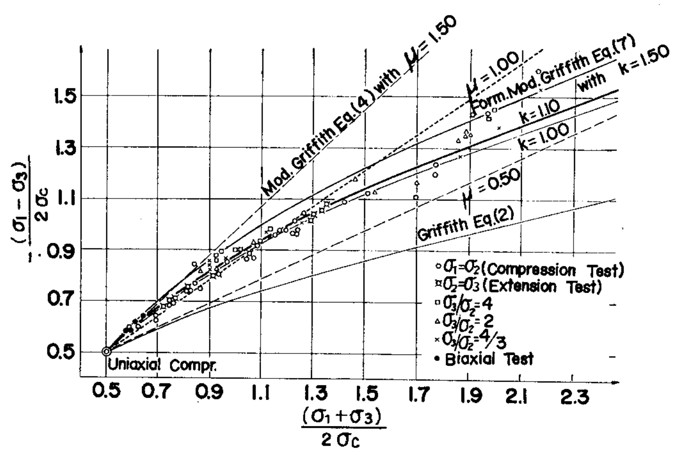


Fig. 13. Relations between $\frac{(\sigma_1+\sigma_3)}{2\sigma_c}$ versus $-\frac{(\sigma_1-\sigma_3)}{2\sigma_c}$.

stress practically has little effect on failure conditions of mortar. This fact means that a failure surface of mortar can be approximately constructed from Griffith's criterion with a parameter by the formal extension as considered in **2.2** C).

4. Conclusions

A failure criterion for macroscopically homogeneous and isotropic brittle materials was proposed. The criterion is expressed by a convex surface with space diagonal of its axis. The surface consists of the following two types of smoothly connected ones. One is expressed by Max. $(\sigma_1, \sigma_2, \sigma_3)=K$ (uniaxial tensile strength), and the other is a surface which almost isotropically expands with an increase of hydrostatic pressure, and whose right sections are slightly bulged from an equilateral triangle. The surface may expand more rapidly than the extended Griffith's criterion.

The above criterion is of cause suitable for cement mortar subjected to triaxial compression.

In order to obtain the general failure criteria an universal triaxial test equipment is necessary, though approximate failure criteria may be determined by compression and extension tests.

The intermediate stress has little effect on the failure criterion for cement mortar. The results are well fit to the formally modified Griffith's criterion with a parameter $k=1.10$ in Eq. (7).

The failure criterion obtained herein is independent of loading path.

As cement mortar has typical properties of brittle materials, the results obtained in this experiment may be applicable to similar materials.

Acknowledgment

The authors wish to express their gratitude to Mr. Ken-ichi Hirashima, graduate student, for carrying out the experimental work and for valuable assistance in the course of analysis.

References

1) Drucker, D.C.; "Structural Mechanics", ed. J.N. Goodier and N.J. Hoff, 407, Pergamon (1960), and Tech. Rep. No. 34, Div. of Eng., Brown Univ., (1963).
2) Griffith, R.A.; Phil. Trans. Roy. Soc., **A221**, 163 (1921).
3) Griffith, R.A.; Proc. 1st Intern. Congr. Appl. Mech., 55 (1924).
4) McClintock, F.A. and J.B. Walsh; 4th U.S. Nat. Congr. Appl. Mech., 1015 (1962).
5) Murrell, S.A.F.; "Mechanical Properties of Non-metallic Materials", ed. W.H. Walton, Butterworths (1958).
6) Brace, W.F.; J. Geophys. Res., **65**, 3477 (1960).
7) Orowan, E.; Weld. J. (Res. Suppl.), 157-S (1949).
8) Sack, R.A.; Proc. Phys. Soc. London, **58**, 729 (1946).
9) Bellamy, C.J.; J. ACI, **58**, 367 (1961).
10) Vile, G.W.D. and O.T. Sigvaldason; J. ACI, **62**, 1192 (1965).
11) Sundra Raja Iyengar, K.T., K. Chandrashekhara and K.T. Krishnaswamy; J. ACI, **62**, 239 (1965).
12) Weigler, H. und Becker, G.; Bauing., **10**, 390 (1961).
13) Richart, E.P., A. Brandtzaeg and R.L. Brown; Bulletin No. 185, Univ. of Ill., Eng. Exper. Station, (Nov. 1928).
14) Bertacchi, P.; 8th Intern. Congr. on Large Dams, R17, Q30, **3**, 279 (1964).
15) Balmer, G.G.; Lab. Rep. No. SP-23, Bureau of Reclamation (1964).