Measurement of Variation in Stress by a Photoelastic Stressmeter

By

Yoshio HIRAMATSU*, Yoshiji NIWA** and Yukitoshi OKA*

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Researches have been carried out to obtain a suitable apparatus and technique for field stress measurement in rock or concrete constructions by application of photoelasticity. The principle of this method is that a gage is beforehand fixed to a body whose stress is required to be measured. When the stress state of the body varies, there appears some stress in the gage. By observing the stress pattern in the gage by a polariscope, the variation in stress of the body can be determined.

In the first place a polariscope and photoelastic gages suitable for this measurement were investigated, and three types of gages, i.e. a hollow cylinder, a solid cylinder and a rectangular prism of borosilicate glass and a portable polariscope, 500 gr in weight, were obtained. This polariscope has a detachable compensator so that it can be used for two purposes, namely to measure the relative retardation at an optional point on a gage by means of the compensator, and to observe stress patterns without the compensator. The stress in gages was analysed by the theory of elasticity to obtain fundamental data for finding a proper technique to determine variation in stress in a body. Referring to the results of analysis and taking into account the results of calibration, some techniques to determine variation in stress of the body that may be recommendable were obtained.

Many experiences in practical application of this method of stress measurement have proved that the method is suitable for measurement extending over a long period of time.

1. Introduction

In order to get the fundamental data for a safe design of concrete structures or proper layout of mining underground mineral deposits, or in order to inspect the stress state in concrete structures or mine pillars, it is essential to be able to measure the stress in them. The authors have investigated, for about a decade into an instrument and techniques for this purpose based on the principle of photoelasticity.

Previously the authors published an intermediate report[1] on this research, in which the development of a portable polariscope and glass gages and the analysis

* Department of Mineral Science and Technology.
** Department of Civil Engineering.

of stress in three kinds of glass gages were described. It was concluded that this method was suited for measurement of stress extending over a long period of time, though its sensitivity is not high. Recently Roberts and others have commenced a similar study[2]. Research has yet been carried on thereafter aiming at improvement in accuracy. The present paper is the final report of this research.

2. Apparatus Obtained

The principle of a photoelastic stressmeter which the authors have originated is as follows; a photoelastic gage is fixed tightly beforehand to a optional position, where the stress is required to be measured, on or near the wall surface of a body (a concrete construction, rock, etc.). As a matter of course, there then appears no stress in the gage. When the stress in the body varies, however, some stress will occur in the gage. Measuring the stress in the gage, we can determine the stress variation occurred in the body, because there must be a definite relation between the variation in stress in the body and that in the gage.

The first thing to do is to design and produce a handy apparatus to measure stress in a gage based on the principle of photoelasticity, together with photoelastic gages which are sensible and have a constant sensitivity over a long period of time.

After a series of design and trial manufacture, three types of glass gages and a portable polariscope of reflection type were obtained,[3] which are briefly described below.

(1) Photoelastic Gages

As the material for photoelastic gages, borosilicate glass is chosen since it was found, from tests,[1] glass kept a constant photoelastic sensitivity over a long period of time.

Three types of gages were designed, a hollow cylinder, a solid cylinder and a rectangular prism. They are silvered on the back surface so that the polarized light projected into them normally to the front surface passes through the gage and is reflected back at the silvered back surface.

A hollow cylindrical gage is 30 mm long and its inner and outer diameters are 6 mm and 36 mm respectively. On the front surface of it, two fine concentric circles are drawn. See Fig. 1. This type of gage was proved to be suited for determining by a single observation the magnitudes and directions of principal stresses.

A solid cylindrical gage is of the same size as the hollow one, and has no circle on the front surface. This gage is useful for determining the difference between two principal stresses as well as the directions of them.

A rectangular gage is 30 mm long and 25 mm × 20 mm in the cross section. When this gage is fixed to a body, it is preferable to leave clearance on the two lateral surfaces by putting a sheet of paper on each surface in order that the gage is not sensible to the variation in stress in the lateral direction. This gage is useful to determine the variation in normal stress in the direction parallel to the longer side.

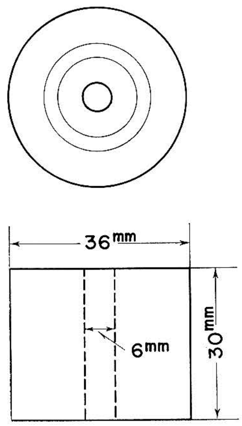


Fig. 1. Hollow cylindrical gage.

(2) Photoelastic Stressmeter

Figs. 2 and 3 show respectively the aspect and the construction of the final type of photo-

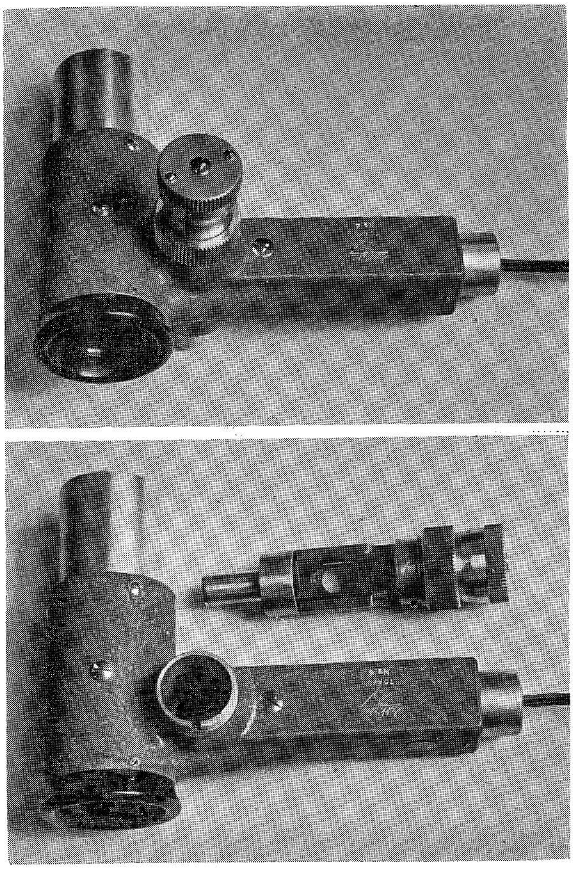


Fig. 2. Photoelastic stressmeter. (The lower photograph shows the stressmeter whose compensator is detached.

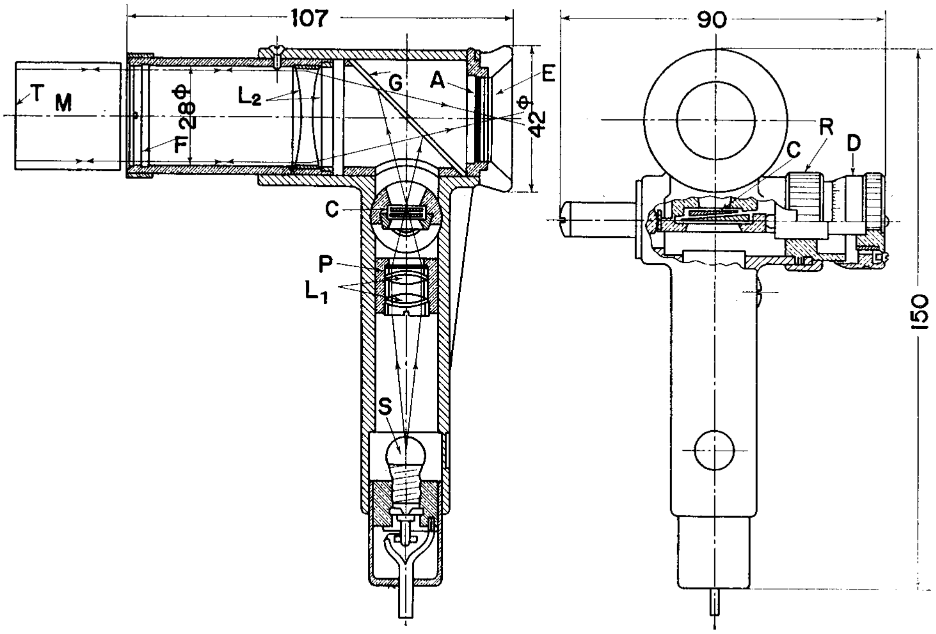


Fig. 3. Diagram showing the construction of a photoelastic stressmeter.

elastic stressmeter designed by the authors. This is a small instrument, 500 gr in weight. Natural light coming from a miniature electric bulb is employed for observation, because it yields more colored stress patterns than dark stress fringes to be seen by monochromatic light.

The light from the source S is converged by the condenser lenses L_1, and converted into polarized light through a polarizer P. Passing through the Babinet compensator C and being reflected by the glass plate G, placed at an angle of 45° to the light path, the light is projected into a photoelastic gage. After being reflected by the silvered bottom surface of a gage, the light comes back and again enters the stressmeter. This light, after passing through the analyzer A, is observed through the eyepiece E.

In the field of vision, we can see colored parallel lines arrange symmetrically on both sides of a dark line. These colored lines are shifted by turning the dial D of a compensator. When the dark line is brought just under the cross lines, the retardation occurred in the gage is completely compensated. Then the reading of the scale on the dial gives the relative retardation, a unit reading corresponding to about 500Å.

The compensator is detachable. When it is detached and a hollow cylindrical gage is observed, we can see, around the hole, colored stress patterns.

3. Apparent Principal Stresses

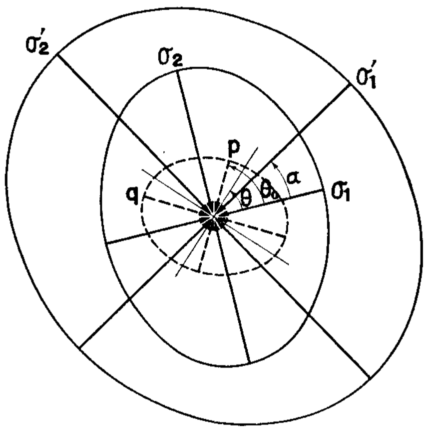


Fig. 4. Diagram showing the initial, present and apparent principal stresses.

Let us assume that principal stresses at an arbitrary point on a surface of a body was, at one time σ_1 and σ_2, and that those are now σ_1' and σ_2' respectively due to variation in stress. Denote the angle between the directions of σ_1 and σ_1' by α, as shown in Fig. 4. The variations in the stress components, $\Delta\sigma$ and $\Delta\tau$, acting on a plane making an angle of θ with the plane on which σ_1 acted are, considering the fundamental theory about stress, written as:

$$\Delta\sigma=\frac{\sigma_1'+\sigma_2'-\sigma_1-\sigma_2}{2}+\frac{\sigma_1'-\sigma_2'}{2}\cos 2(\theta-\alpha)-\frac{\sigma_1-\sigma_2}{2}\cos 2\theta\,, \tag{1}$$

$$\Delta\tau=-\frac{\sigma_1'-\sigma_2'}{2}\sin 2(\theta-\alpha)+\frac{\sigma_1-\sigma_2}{2}\sin 2\theta\,. \tag{2}$$

Differentiating Eq. (1) and considering Eq. (2), we can see that, at any point, there are two planes perpendicular to each other where $\Delta\tau$ vanishes and $\Delta\sigma$ take a maximum and a minimum values. Let them be p and q respectively. Then p and q as well as the angle θ_0 between the plane on which p acts and the plane on which σ_1 acts are given by:

$$\left.\begin{aligned}\left.\begin{matrix}p\\q\end{matrix}\right\}&=\frac{(\sigma_1'+\sigma_2')-(\sigma_1+\sigma_2)}{2}\pm\{(\sigma_1'-\sigma_2')^2+(\sigma_1-\sigma_2)^2-2(\sigma_1'-\sigma_2')(\sigma_1-\sigma_2)\cos 2\alpha\}^{\frac{1}{2}},\\ \tan\theta_0&=\frac{(\sigma_1'-\sigma_2')\sin 2\alpha}{2p-\{\sigma_1'+\sigma_2'-2\sigma_2-(\sigma_1'-\sigma_2')\cos 2\alpha\}}\,.\end{aligned}\right\} \tag{3}$$

It is demonstrated, by further calculation, that the magnitudes of $\Delta\sigma$ and $\Delta\tau$ on a plane making an angle β with the plane on which $\Delta\tau$ vanishes are given by:

$$\left.\begin{aligned}\Delta\sigma&=\frac{p+q}{2}+\frac{p-q}{2}\cos 2\beta\,,\\ \Delta\tau&=-\frac{p-q}{2}\sin 2\beta\,.\end{aligned}\right\} \tag{4}$$

The consideration above mentioned leads to a conclusion that $\Delta\sigma$ and $\Delta\tau$ can be treated as if they were a normal and a shear stresses respectively, thus p and q can be treated as if they were principal stresses. Let us call p and q apparent principal stresses. Photoelastic gages respond to the variation in normal stress. Therefore, it may be possible to determine apparent principal stresses. In a special case that both σ_1 and σ_2 are zero, p and q coincide with σ_1' and σ_2'. In short, if a gage was fixed to a body already subjected to stress, the present apparent principal stresses can be determined, but if a gage was fixed to a body free from stress, the present actual principal stresses can be determined.

If the initial principal stresses, σ_1 and σ_2, are known, the present principal stresses are able to be calculated by the following formulas:

$$\left.\begin{aligned} \left.\begin{matrix}\sigma_1'\\ \sigma_2'\end{matrix}\right\} &= \frac{\sigma_1+\sigma_2+p+q}{2} \pm \frac{1}{2}\{(\sigma_1-\sigma_2)^2+(p-q)^2+2(\sigma_1-\sigma_2)(p-q)\cos 2\theta_0\}^{\frac{1}{2}}, \\ \tan\alpha &= \frac{(p-q)\sin 2\theta_0}{2\sigma_1'-\{2\sigma_2+(p+q)-(p-q)\cos 2\theta_0\}}\ . \end{aligned}\right\} \quad (5)$$

By close examination of Eq. (5), it is seen that θ_0 is always greater than α, that is to say, apparent princiapl stresses show exaggerately the variation in direction of principal stresses especially when the original principal stresses are comparatively great.

4. Stress in Gages

(1) In Case the Original Stress Was Zero

In the first case, we shall treat of the stress in a gage which was fixed to a certain point on an elastic body when the body was free from stress. Suppose that the body is now subjected to compressive stress, the principal stresses being p and q in the vicinity of the gage where the stress is not affected by the presence of the gage. It is assumed, in this paper, that a compressive stress takes a negative sign, and that p is algebraically greater than q.

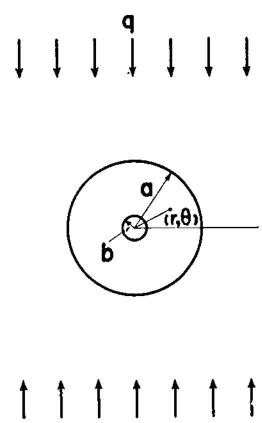


Fig. 5.

In a hollow cylindrical gage, as may be expected, there will appear complicated stresses, which depend upon p and q, the Young's modulus E and the Poisson's ratio ν of the body. When the gage is observed with a photoelastic stressmeter whose compensator is detached, we can see colored stress patterns, i.e. isochromatics, each line being a contour of the same difference between two principal stresses in the gage. Let the Young's modulus and the Poisson's ratio of glass be E' and ν' respectively, the outer and the inner radii be a, b, and define polar coordinates as shown in Fig. 5. Assuming that the gage is fixed to the body directly and tightly, without any binding agent and that $p=0$, the stress components $\overset{*}{\sigma}_r$, $\overset{*}{\sigma}_\theta$ and $\overset{*}{\tau}_{r\theta}$ at any point in the gage, according to the theory of elasticity, are represented as:[3)]

$$\left.\begin{aligned}
\overset{*}{\sigma}_r&=2A_0+B_0r^{-2}-(6B_2r^{-4}+2C_2+4D_2r^{-2})\cos 2\theta\,,\\
\overset{*}{\sigma}_\theta&=2A_0-B_0r^{-2}+(12A_2r^2+6B_2r^{-4}+2C_2)\cos 2\theta\,,\\
\overset{*}{\tau}_{r\theta}&=(6A_2r^2-6B_2r^{-4}+2C_2-2D_2r^{-2})\sin 2\theta\,,
\end{aligned}\right\}\qquad(6)$$

where

$$A_0=-\frac{1}{2b^2}B_0\,,$$

$$B_0=-\frac{aq}{E}\left\{\frac{1-\nu'}{E'}\frac{a}{b^2}+\frac{1+\nu'}{E'}\frac{1}{a}-\frac{1+\nu}{E}\left(\frac{1}{a}-\frac{a}{b^2}\right)\right\}^{-1},$$

$$A_2=-\frac{1}{3b^2}\left(2C_2+D_2\frac{1}{b^2}\right),$$

$$B_2=-\frac{b^4}{3}\left(C_2+2D_2\frac{1}{b^2}\right),$$

$$C_2=(\beta'\gamma-\beta\gamma')/(\alpha\beta'-\alpha'\beta)\,,$$

$$D_2=(\alpha'\gamma-\alpha\gamma')/(\alpha'\beta-\alpha\beta')\,,$$

$$\alpha=4\left(1-\frac{a^2}{b^2}\right)\left(\frac{3-\nu}{E}+\frac{1+\nu'}{E'}\right),$$

$$\beta=\frac{2}{a^2}\left\{\frac{3-\nu}{E}\left(1-\frac{a^4}{b^4}\right)-\frac{1+\nu'}{E'}\frac{a^4}{b^4}-\frac{3-\nu'}{E'}\right\},$$

$$\gamma=-4q/E'\,,$$

$$\alpha'=-\frac{2(1+\nu)(3-\nu)}{E}\left(\frac{b^4}{a^4}-\frac{4a^2}{b^2}+3\right)+\frac{8(3+\nu\nu')}{E'}\frac{a^2}{b^2}$$
$$+\frac{2(1+\nu')(3-\nu)}{E'}\frac{b^4}{a^4}-\frac{6(1+\nu)(1+\nu')}{E'}\,,$$

$$\beta'=-\frac{4(1+\nu)(3-\nu)}{E}\left(\frac{b^2}{a^4}-\frac{a^2}{b^4}\right)+\frac{4(3+\nu\nu')}{E'}\frac{a^2}{b^4}$$
$$+\frac{4(1+\nu')(3-\nu)}{E'}\frac{b^2}{a^4}+\frac{12(\nu-\nu')}{E'}\frac{1}{a^2}\,,$$

$$\gamma'=\frac{6q(1+\nu)}{E}\,.$$

Should the principal stress q be zero but another one p have a certain magnitude, the stress components at any point in the gage may be obtained by substituting respectively p and $\theta+\frac{\pi}{2}$ for q and θ in Eq. (6). When both p and q are not zero, the stress components in the gage are given as the algebraic sums of the corresponding components under the two assumed stress states, i.e. the one is $p=0$, $q\neq0$ and the other is $q=0$, $p\neq0$.

Once the stress components at any point in the gage are found, the difference between principal stresses $\overset{*}{\sigma}_1-\overset{*}{\sigma}_2$ is calculated by the following formula:

$$\overset{*}{\sigma}_1-\overset{*}{\sigma}_2=\{(\overset{*}{\sigma}_r-\overset{*}{\sigma}_\theta)^2+4\overset{*}{\tau}{}^2_{r\theta}\}^{\frac{1}{2}}\,. \tag{7}$$

Fig. 6 shows theoretical isochromatics, drawn under the assumptions that the Young's moduli of glass and the body are 6.3×10^5, 2.1×10^5 kg/cm^2 respectively, that the Poisson's ratios of them are both 0.2 and that the gage is fixed directly to the body. The shape of isochromatics is much affected by the ratio p/q, but a little by E and ν. When p is equal to q, all isochromatics become concentric circles.

For a solid cylindrical gage, the equations corresponding to Eq. (6) assume far simpler forms as follows:

$$\left.\begin{aligned}
\overset{*}{\sigma}_r&=q\left\{\frac{E'}{E'(1+\nu)+E(1-\nu')}+\frac{1}{2}\frac{E'(5+\nu)-E(1+\nu')}{E'(3-\nu)+E(1+\nu')}\cos 2\theta\right\}\,,\\
\overset{*}{\sigma}_\theta&=q\left\{\frac{E'}{E'(1+\nu)+E(1-\nu')}-\frac{1}{2}\frac{E'(5+\nu)-E(1+\nu')}{E'(3-\nu)+E(1+\nu')}\cos 2\theta\right\}\,,\\
\overset{*}{\tau}_{r\theta}&=-\frac{q}{2}\frac{E'(5+\nu)-E(1+\nu')}{E'(3-\nu)+E(1+\nu')}\sin 2\theta\,.
\end{aligned}\right\} \tag{8}$$

The stress components under the condition that q is zero but p takes a certain value, are able to be found in the same way as described above for a hollow cylindrical gage. Thus from simple calculation, we obtain:

$$\overset{*}{\sigma}_1-\overset{*}{\sigma}_2=\frac{E'(5+\nu)-E(1+\nu')}{E'(3-\nu)+E(1+\nu')}(p-q) \tag{9}$$

It is learned from this expression that $\overset{*}{\sigma}_1-\overset{*}{\sigma}_2$ is proportional to $p-q$ and that $\overset{*}{\sigma}_1-\overset{*}{\sigma}_2$ is uniform over the whole area of the gage. Therefore it is supposed that a uniform stress colour will be seen in a solid cylindrical gage subjected to stress when observed by a photoelastic stressmeter whose compensator is detached. This was confirmed by experiments except near the margin of a gage.

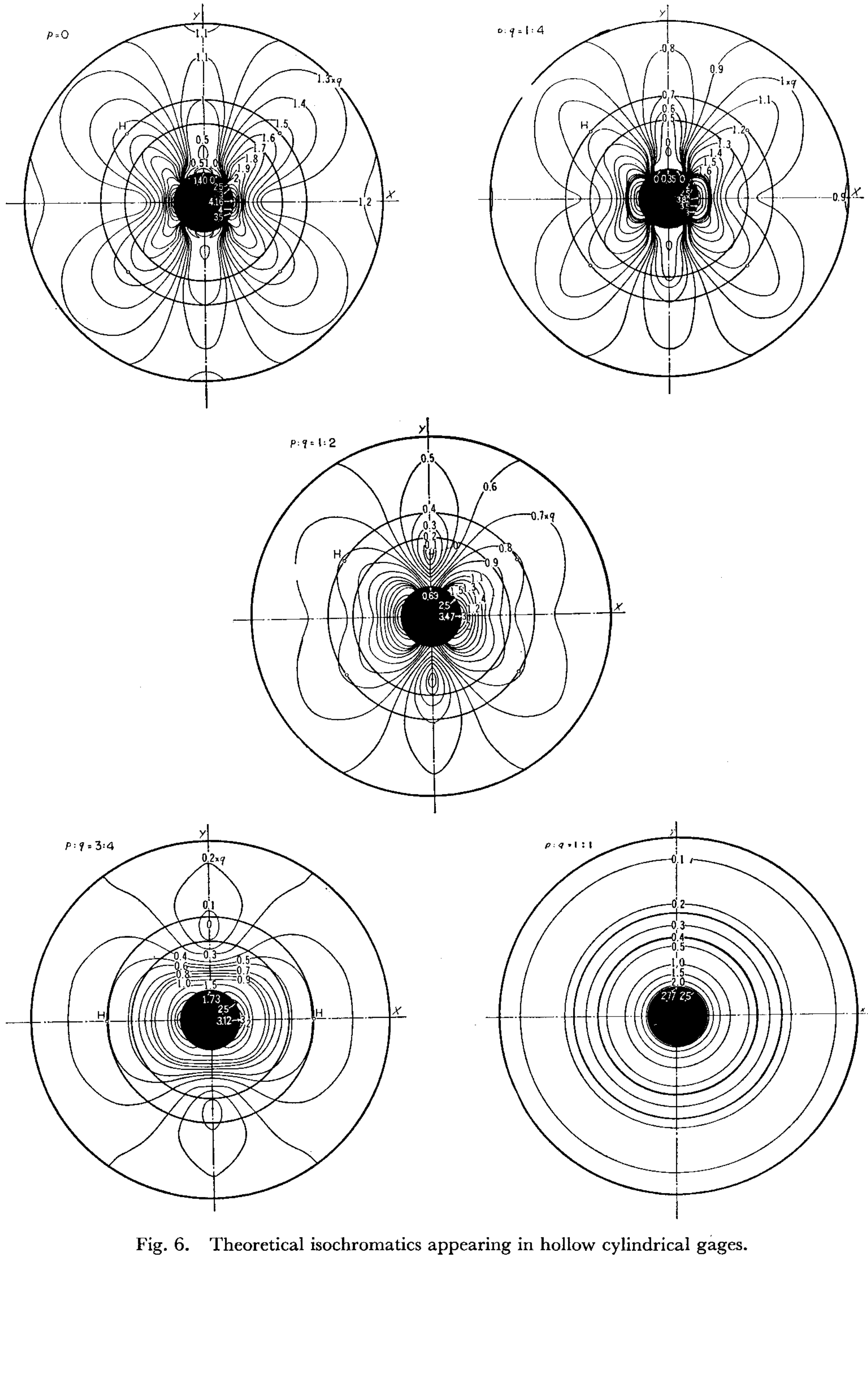


Fig. 6. Theoretical isochromatics appearing in hollow cylindrical gages.

The ratio $k=(\overset{*}{\sigma}_1-\overset{*}{\sigma}_2)/(p-q)$ depends upon E and ν as shown in Fig. 7.

In a rectangular gage of which two lateral sides are freed, one of the principal stresses $\overset{*}{\sigma}_1$ is zero and the other one, $\overset{*}{\sigma}_2$, that is parallel to the longer sides of the gage is approximately proportional to the normal stress of the body in the same direction.

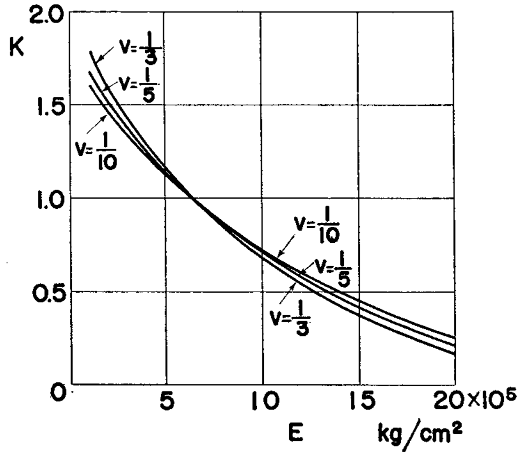


Fig. 7. Relation among the Young's modulus E and the Poisson's ratio ν of a body and the stress ratio $k=(\overset{*}{\sigma}_1-\overset{*}{\sigma}_2)/(p-q)$.

(2) In Case the Original Stress Was Not Zero

Assuming that a gage was fixed to a certain point near the surface of an elastic body which was already subjected to stress, and that the stress state is now varied, then the gage responds only to the variation in stress. From the discussion described in Section 3, everything mentioned in the first case of this section may be valid if we interpret p and q to be the apparent principal stresses.

5. Effect of Creep of a Stressed Body on the Stress Induced in a Glass Gage

When stress of a body is evaluated by measuring strain and if creep is not very small compared with elastic strain the correction due to creep is necessary. However, it was ascertained by experiments that when stress is measured by a photoelastic stressmeter and a glass gage, the error due to creep of a body is very small. Therefore, for ordinary rocks and aged concrete, the correction for creep may be omitted.

6. Determining p and q

The accuracy of determining stress depends not only on the precision of the apparatus but also on the technique of how to determine stress from the stress pattern observed or from the reading of a compensator. Much effort has been paid to find out an easy and yet accurate method for doing it. In this section some of the methods obtained that would be recommendable are explained.

Before entering into a description of the methods, a short note will be given on how to fix glass gages. Every gage should be fixed to a solid part of a body where no cracks are to be seen. A shallow bore hole, about 37 mm in diameter and 35 mm deep, is made on the surface of a body for a cylindrical gage. In this bore hole, a gage is fixed with a strong and hard binding agent, for which we are using the Araldite Type 121-S with a good result. The gap between the bore hole and a gage, that is the thickness of the binding agent, should be as small as possible.

(1) Determining p and q from Stress Patterns Observed in a Hollow Cylindrical Gage

Observe the stress patterns in a hollow cylindrical gage with the stressmeter, and turn the instrument on its optical axis to find the position where symmetrical stress patterns are to be seen. Then the axes of symmetry of the stress patterns will give the directions of the apparent principal stresses, p and q. We can easily discern which of them is the direction of p by consulting with the theoretical stress patterns shown in Fig. 6.

The stress ratio p/q can be determined from the shapes of isochromatics in the following way. Define Points A, B, C, etc. on the gage as shown in Fig. 8. At Point H, the difference between principal stresses $\overset{*}{\sigma}_1-\overset{*}{\sigma}_2$ takes a maximum value on the outer etched circle, and Point X has the same value of $\overset{*}{\sigma}_1-\overset{*}{\sigma}_2$ as Point H does. At Point Y found on the y-axis, $\overset{*}{\sigma}_1-\overset{*}{\sigma}_2$ takes a minimum value in the gage. Now from the position of Point X or Point

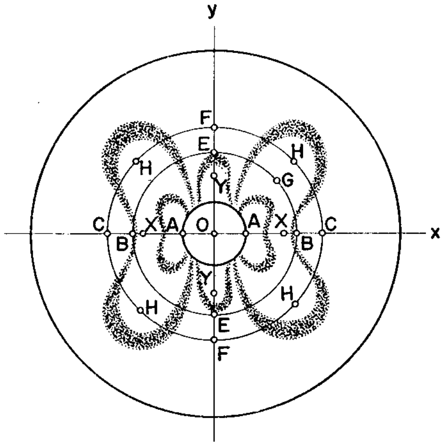


Fig. 8. Diagram illustrating the positions of several particular points on a stress pattern.

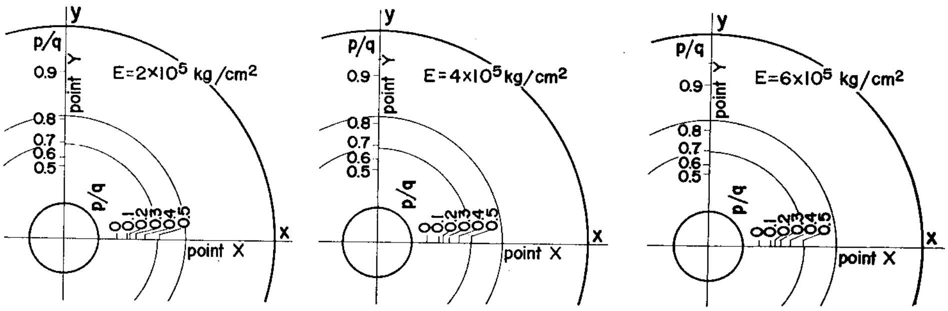


Fig. 9. Diagrams showing the relation between the positions of Point X and Y and the stress ratio p/q.

Y, we can determine the stress ratio p/q referring to the relationship shown in Fig. 9. The Young's modulus E of a body has a little influence on this relation, and the Poisson's ratio has a less influence.

The magnitude of q can be determined by the following relation:

$$q=k_1k_2q', \tag{10}$$

where k_1 and k_2 are correction factors and q' is an assumed stress. The value of q' is determined by the stress color appearing at any point of A, H and C. For

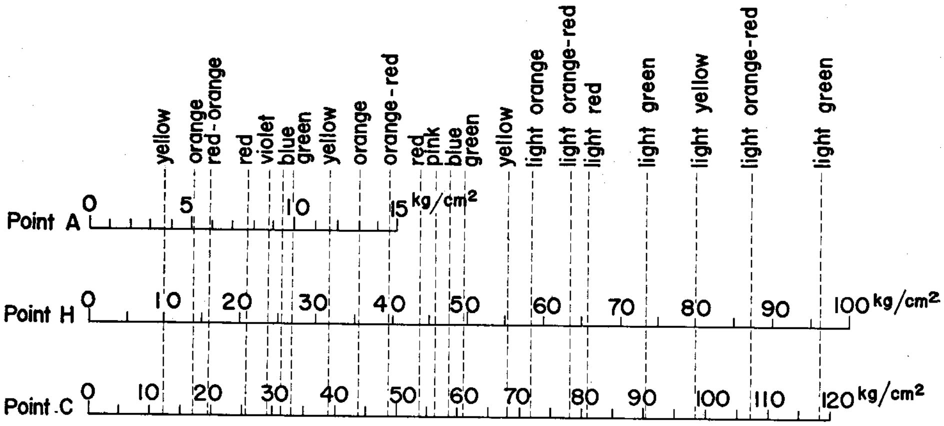


Fig. 10. Diagrams showing the relation between absolute values of q' and stress colors.

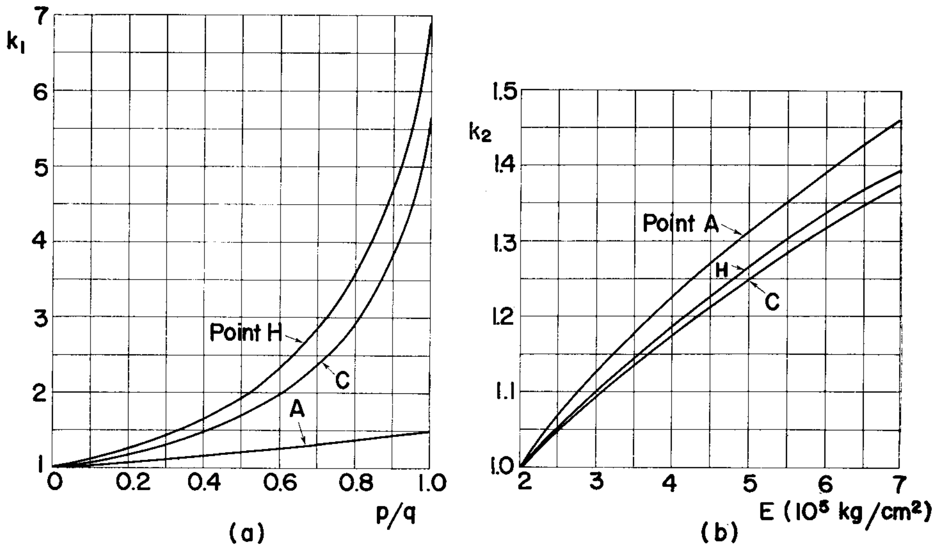
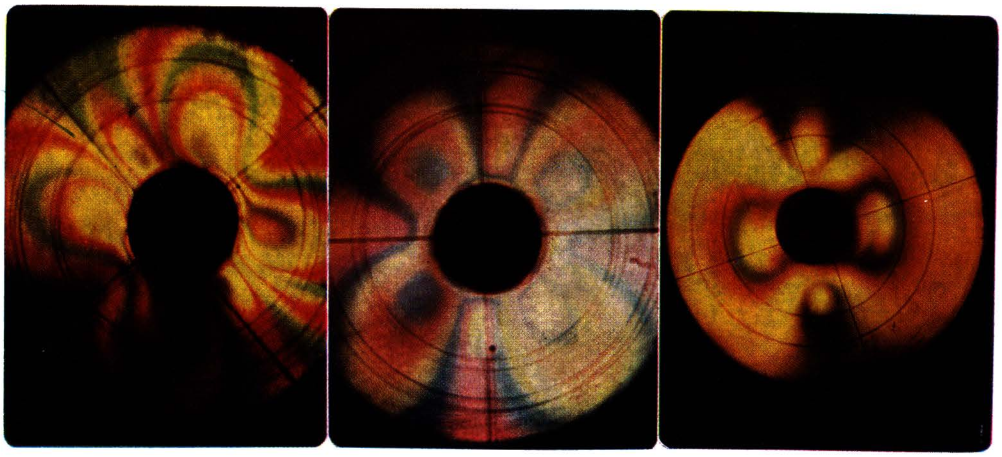


Fig. 11. (a) Relation between k_1 and p/q.
(b) Relation between k_2 and E.



(1) $p=-9.5$ kg/cm^2 $q=-95$ kg/cm^2 (2) $p=30$ kg/cm^2 $q=-25$ kg/cm^2 (3) $p=-146$ kg/cm^2 $q=-209$ kg/cm^2

Fig. 12. Some examples of stress patterns. (No. 1 and No. 2 were taken on the two gages fixed to the wall of the Fourth Lateral Road, driven 30 m below the seam, at the Sanyo-Muen anthracite mine. The former gage was fixed about 67 m in front of the face and was photographed after the face advanced about 27 m, while the latter gage was fixed 5.5 m behind the face and was photographed after the face advanced about 8 m. No. 3 was taken on a gage fixed to the wall of a drift in the Kawayama mine.)

this determination, the result of calibration, illustrated in Fig. 10, can be used provided that the layer of the binding agent is thin. When the stress in a gage is small and isochromatics are hardly seen, the color at Point A should be adopted, but when there appear a number of isochromatics it is preferable to observe the stress color either at Point H or C. The factor k_1 depends upon the stress ratio p/q as shown in Fig. 11-a, while another factor k_2 depends upon the Young's modulus of a body and is given by Fig. 11-b.

It is advisable to take color photographs of the stress patterns for the purpose of keeping records of measurement.

Example: Fig. 12 shows some photographs of stress pattern taken on hollow cylindrical gages. A brief comment will be given here to evaluation of p and q on Photograph (1). This photograph indicates, at a glance, that the direction of q inclines about 35° anticlockwise from the vertical. By examining the position of Point X, we have $p/q=0.1$ upon consulting with Fig. 9 under the assumption that $E=4\times10^5$ kg/cm^2. The color seen at Point H is green which appeared on the third time. Therefore, from Fig. 10, q' is estimated to be -73 kg/cm^2. Since we learn that $k_1=1.15$ and $k_2=1.19$ from Fig. 11, we obtain:

$$q=1.15\times 1.18\times(-73)=-95\ \text{kg/cm}^2\,,$$

and

$$p=-95\times 0.1=-9.5\ \text{kg/cm}^2\,.$$

Fig. 12 (2) is a photograph of the stress pattern taken when p is positive, and (3) is one taken when p has a considerable magnitude compared with q.

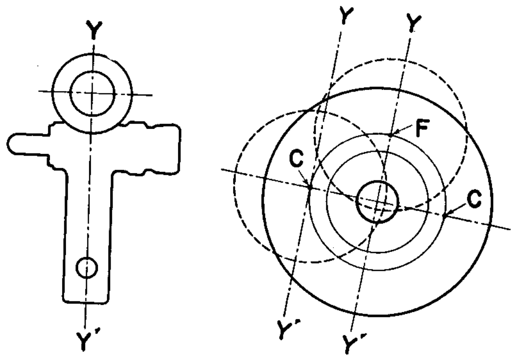


Fig. 13. Diagram showing how to hold the stressmeter against a hollow cylindrical gage.

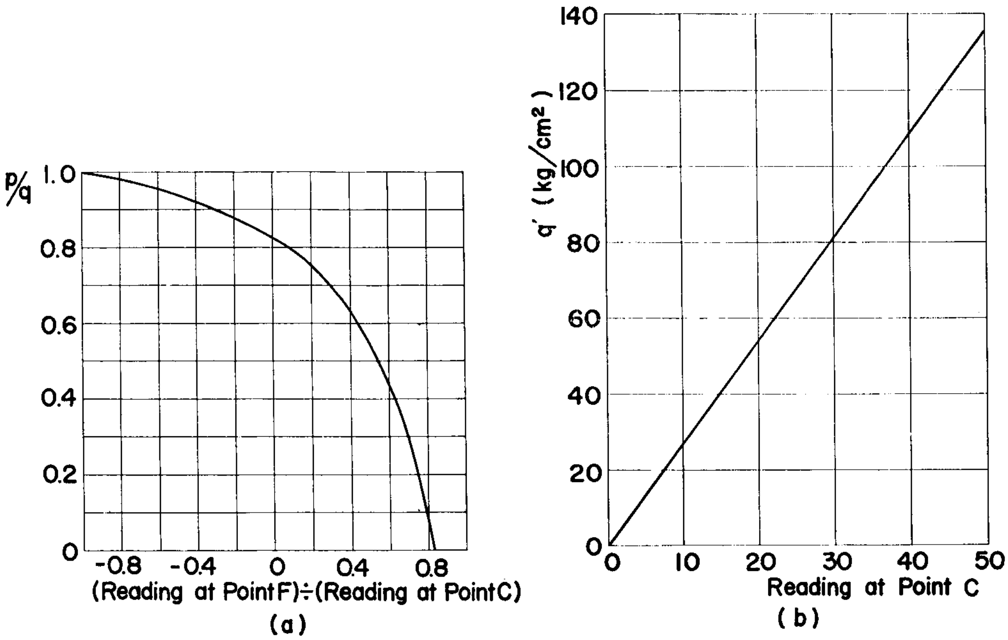


Fig. 14. (a) Relation between p/q and the ratio of readings at points F and C.
(b) Relation between q' and the reading at point C.

(2) Determining p and q from Readings of a Compensator on a Hollow Cylindrical Gage

First determine the directions of p and q in the same way as described above. Then attach the compensator to the stressmeter, and take the readings of it at Points C and F, holding the stressmeter in such a direction that the axis YY′ of the handle is parallel to the y-axis of the stress pattern. See Fig. 13. From the ratio of the readings at Point F and C, the stress ratio p/q is determined referring to Fig. 14-a. The assumed stress q' is found from the reading at Point C referring to Fig. 14-b. Then the stress q is determined by Eq. (10), where k_1 and k_2 are the same as those given by Fig. 11.

(3) Determining p and q from Readings of Compensator on Rectangular Gages

As described before, a rectangular gage can be used to determine the variation in a normal stress, $\Delta\sigma$, parallel to the longer sides of a gage. Calibration is necessary for the evaluation of $\Delta\sigma$ from the reading of the compensator. Fig. 15 shows an example of a calibration curve. When three rectangular gages are employed, we can determine, by simple calculation, p and q from the readings on them.

Consider that three rectangular gages are fixed to a body in such an arrange-

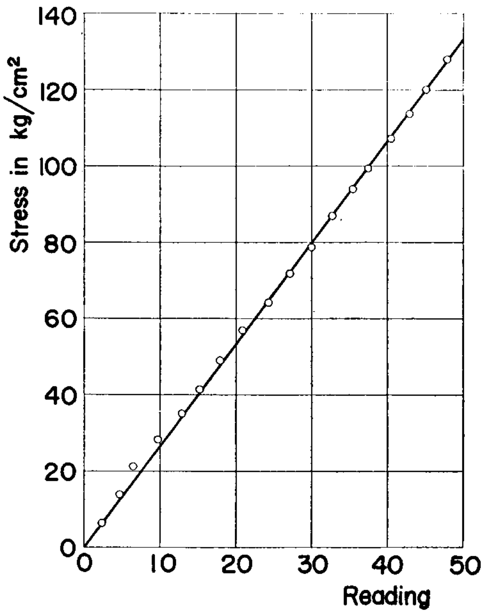


Fig. 15. An example of calibration curves for a rectangular gage fixed on a concrete block.

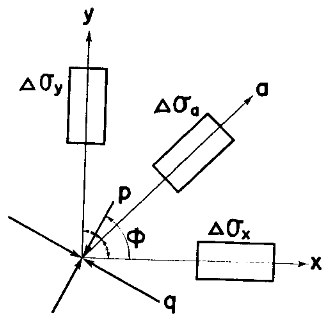


Fig. 16. Photoelastic rosette gages.

ment as shown in Fig. 16. Let the measured values by these three gages be $\Delta\sigma_x$, $\Delta\sigma_y$ and $\Delta\sigma_a$. Then we have:

$$\left.\begin{aligned} \left.\begin{matrix} p \\ q \end{matrix}\right\} &= \frac{1}{2}(\Delta\sigma_x+\Delta\sigma_y) \pm \frac{1}{2}\left\{(\Delta\sigma_x-\Delta\sigma_y)^2+(\Delta\sigma_x+\Delta\sigma_y-2\Delta\sigma_a)^2\right\}^{\frac{1}{2}}, \\ \phi &= \tan^{-1}\frac{2\Delta\sigma_a-\Delta\sigma_x-\Delta\sigma_y}{2(p-\Delta\sigma_y)}, \end{aligned}\right\} \quad (11)$$

where ϕ is the angle between the x-axis and the direction of p.

(4) Determining $p-q$ from Readings of Compensator on a Solid Cylindrical Gage

Adjust the reading of the compensator to zero, and turn the stressmeter on its optical axis holding it against the surface of a gage, and you can find out such a position that a dark line seen in the field of vision just passes the intersection of the crosslines. Then the direction of the handle (the direction of YY′-axis illustrated in Fig. 13) makes an angle of 45° with the direction of p and q.

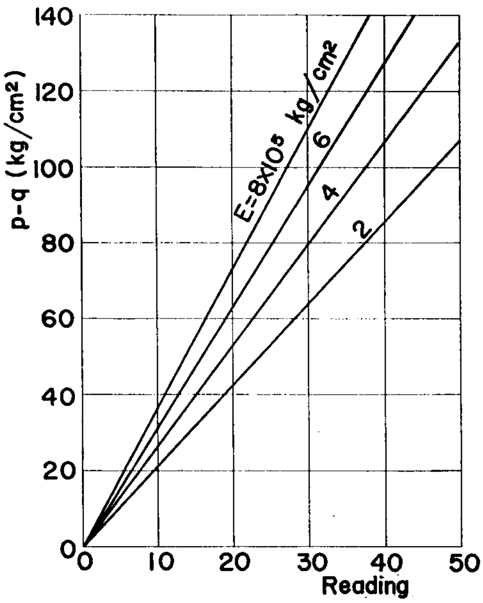


Fig. 17. Relation between $p-q$ and the reading of the compensator for solid cylindrical gages.

Direct the YY′-axis to the direetion of p or q, and take the reading of the compensator, from which $p-q$ can be determined referring to calibration curves. Fig. 17 shows an example of calibration curves. By a solid cylindrical gage, we cannot determine the individual values of p and q.

7. Field Application

The photoelastic stressmeter and glass gages developed by the authors have been used in most Japanese mines working massive ore depostis, some coal mines, and many civil engineering fields. This method of stress measurement, being a reliable means to measure stress over a long period of time, has yielded many valuable data in the field of rock mechanics. As example of application of this method the measurement of rock stress carried out at the Sanyo-Muen anthracite mine, near Mine City, will be cited here.

In this mine, two anthracite seams, lying about 65 meters apart and dipping at about 30 degrees, are mined as shown in Fig. 18. The thickness of the upper seam is from 2.5 m to 3 m, while that of the lower seam is about 2 meters. From February 1962 to February 1963, rock stress around a longwall face was investigated

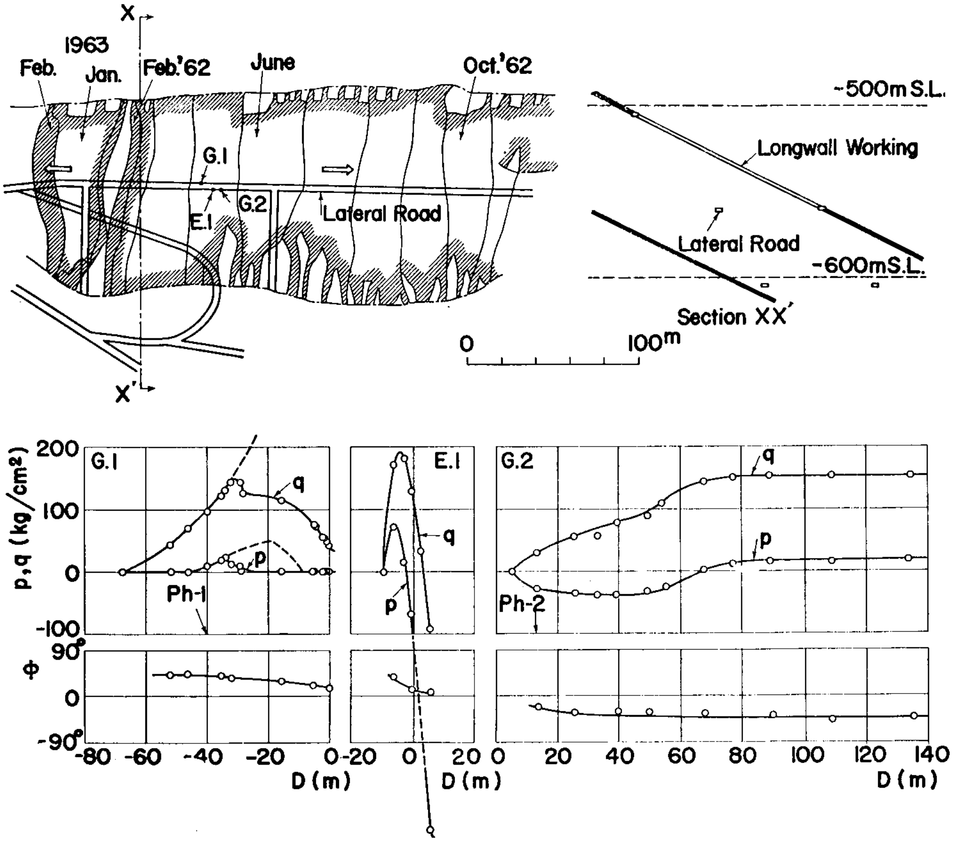


Fig. 18. Diagrams showing the district where stress in rock was measured at the Sanyo-Muen anthracite mine and the results obtained, ϕ being the angle between p and the horizontal, D the distance of the face from the gage. (G: Glass gage, E: Electrical-resistance gage)

for the purpose of obtaining data for strata control on the 4th longwall face of the upper seam about 700 m deep from the surface.

A lateral road had been driven through a hard sandstone in the direction of the strike, 30 m below the center of this longwall face. At a definite position on the road, the variation in stress with the advance of face was measured by the photo-elastic stressmeter on the wall surface. Half a dozen glass gages were fixed on the wall of the road, 1.5 m high from the floor when the face was about 70 m from the position of the gages. When the face came near to the position just above the gages, the rock around the gages failed. Then new gages were fixed to continue further measurement on a solid wall surface revealed by cutting off failed rock pieces. Determination of p and q was carried out wholly on color photographs. For the short period when the face was passing the position just above the measuring station, electrical-resistance rosette gages were used.

The lower graphs in Fig. 18 show the variation in stress expressed by the apparent principal stresses p and q during three stages, i.e. the stage the face is approaching, the stage it is passing and the stage it is going away. Photographs (1) and (2) in Fig. 12 were picked out from a number of photographs taken on this investigation and arranged somewhat rotated, the position of the face at the moment those photographs were taken being denoted respectively by Ph-1 and Ph-2 in the graphs.

8. Conclusion

Investigations have been carried out into the technique and apparatus for measuring stress in rock or concrete constructions by application of photoeleasticity. As the results, a photoelastic stressmeter and three types of glass gages have been developed and some techniques have been suggested as the easiest and most reliable method.

This instrument and techniques have been widely employed in Japanese mines and civil engineering fields, and it has been proved that this method is suitable particularly for determining variation with considerable accuracy in stress extending over a long period of time.

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