

## Criteria for the Scaling Up of Mixing Vessels

By

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(Received September 22, 1966)

There are several concepts for the scaling up of mixing vessels. They are neither consistent nor conclusive. The authors propose an idea that the selection must be made depending upon the mixing objects. The classification of the types of scaling up is as follows:

(1) Similarity for power requirement.

Power requirement is correlated by Reynolds-, Froude- and Weber-numbers. The latter two are negligible in an ordinary correlation.

(2) Similarity for mixing velocity of homogeneous liquid phase.

The authors conclude that the similarity in blending speed is obtained by equal impeller speeds.

(3) Similarity for heat and mass transfer on the fixed surface.

Rushton et al. proposed the method of scaling-up by the following equation.

$$\left(\frac{n_2}{n_1}\right) = \left(\frac{D_1}{D_2}\right)^{(2x-1)/x}$$

This criterion should be limited in the case of the heat and mass transfer on a fixed surface.

(4) Similarity for suspension of solid particles, dispersion of gas and liquid, and mass transfer on dispersed objects.

The authors support the criterion of equal power per unit volume proposed by W. Büche.

### Introduction<sup>1)</sup>

The necessary conditions for the similarity of liquid flow in mixing vessels are obtained by taking ratios of each term in the Navier-Stokes Equation:

(1) Reynolds number:  $Re = Lv\rho/\mu$

(2) Froude number:  $Fr = v^2/Lg$

(3) Pressure coefficient:  $\Delta p/\rho v^2$

Thus the following equation is obtained:

$$\frac{\Delta p}{\rho v^2} = F\left(\frac{Lv\rho}{\mu}, \frac{v^2}{Lg}\right) \quad (1)$$

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The following equation must also be taken into consideration for the similarity of mass transfer in mixing vessels under forced convection where  $D_f$  is a combined diffusivity of molecular ( $D_v$ )- and turbulent ( $D_t$ )-diffusivities:

$$\left( \underbrace{\mu \frac{\delta c}{\delta x}}_I + v \frac{\delta c}{\delta y} + w \frac{\delta c}{\delta z} \right) - D_f \left( \frac{\delta^2 c}{\delta x^2} + \frac{\delta^2 c}{\delta y^2} + \frac{\delta^2 c}{\delta z^2} \right) = - \frac{\delta c}{\delta t} \quad (2)$$

Term I shows the rate of mass transfer by convection flow, term II shows the rate by diffusion and term III shows the rate of concentration change. The dimension of each term is shown as follows:

$$\left[ \frac{vc}{L} \right] - \left[ \frac{cD_f}{L^2} \right] = - \left[ \frac{c}{t} \right] \quad (3)$$

I                  II                  III

As the terms I and III have the same quality, they are reduced to one dimensionless group. In a steady state, the following equation should be considered by taking the ratio of the terms I and II.

$$F_1 \left( \frac{vL}{D_f} \right) = \text{constant} \quad \text{or} \quad F_1 \left( \frac{vL}{D_v}, \frac{D_t}{D_v} \right) = \text{constant} \quad (4)$$

In place of  $(vL/D_v)$ , the Schmidt group which is a ratio of  $(vL/D_v)$  and  $(Lv\rho/\mu)$  may be considered. The ratio,  $D_t/D_v$  is controlled by the degree of turbulence, in other words by Reynolds number.

Therefore Eq. (4) is reduced to:

$$F_2 \left( \frac{\mu}{\rho D_v}, \frac{Lv\rho}{\mu} \right) = \text{constant} \quad (4')$$

As the term  $(\Delta p/\rho v^2)$  has nothing to do with mass transfer, we have from Eqs. (1) and (4'):

$$F' \left( \frac{Lv\rho}{\mu}, \frac{v^2}{Lg}, \frac{\mu}{\rho D_v} \right) = \text{constant} \quad (5)$$

Therefore, the Re- and Fr- numbers should be kept constant in Eq. (5) in order to obtain a perfect similarity in mixing vessels. Rushton has suggested the use of liquids having different viscosities which are related by Eq. (6) in order to keep Re- and Fr- numbers equal<sup>2)</sup>.

$$\frac{(\mu/\rho)_1}{(\mu/\rho)_2} = \frac{L_1 v_1}{L_2 v_2} = \left( \frac{L_1}{L_2} \right)^{3/2} = \left( \frac{v_1}{v_2} \right)^3 \quad (6)$$

However, the scaling up based upon Eq. (6) is actually difficult to realize.

For this reason the authors would like to give up the procedure based on the theoretically correct criteria and develop a more practical and convenient form of scaling up.

**(1) Similarity for Power Requirement.**

It is well-known that Eq. (1) reduces to the following form in the correlation of power requirement of mixing vessels.

$$\frac{Pg_c}{\rho n^3 d^5} = F'' \left( \frac{D^2 n \rho}{\mu}, \frac{Dn^2}{g}, \text{shape factor} \right) \quad (7)$$

Rushton et al<sup>3)</sup> suggest that Froude number ( $Dn^2/g$ ) is important in power correlation of mixing vessels without baffle plates.

The authors measured the power consumption of mixing impellers by using a precise dynamometer and concluded that the effect of the Froude number was relatively slight and might be neglected except for a special case<sup>4)</sup>.

To support this view, the authors measured the flow pattern of liquid in mixing vessels<sup>5)</sup>, and reached the conclusion that the power consumption inside the liquid around an impeller of about 6% of the total volume reached 60–80% of the total power. Therefore the authors conclude that the increase in power caused by the rise of liquid periphery is negligible, i.e. the Froude group can be omitted in the power correlation.

**(2) Similarity for Mixing Rate of Homogeneous Liquid Phase.**

In the case of taking the so-called gross blend time as a time of complete mixing as suggested by Godleski et al<sup>6)</sup>, the effect of diffusion, i.e. term II in Eq. (2), may sometimes be negligible compared to that of convection, i.e., term I.

Dimensionless expression may be obtained as follows: by taking the impeller diameter ( $d$ ) as the representative length, the tip velocity of an impeller ( $dn$ ) as the representative velocity and the final average concentration  $C_m$  as the standard concentration.

$$\begin{aligned} c^* &= \frac{c}{C_m}, & x^* &= \frac{x}{d}, & y^* &= \frac{y}{d}, & z^* &= \frac{z}{d} \\ t^* &= nt, & u^* &= \frac{u}{dn}, & v^* &= \frac{v}{dn}, & w^* &= \frac{w}{dn} \\ \frac{\delta c^*}{\delta (nt)} &= - \left\{ \frac{\delta}{\delta x^*} (u^* c^*) + \frac{\delta}{\delta y^*} (v^* c^*) + \frac{\delta}{\delta z^*} (w^* c^*) \right\} \end{aligned} \quad (9)$$

Each term of the right hand side of Eq. (9) is connected with the flow pattern of liquid and the distribution of concentration. Therefore, if the flow pattern of the liquid is similar, the state and degree of mixing should be fixed as a function of the

term ( $nt$ ) depending on the initial and boundary conditions of concentration distribution. Thus we have a simple relation between the gross blending time,  $T_M$  and impeller speed,  $n$  for geometrically similar mixing vessels as follows:

$$T_M \cdot n = N_{T_M} = \text{constant} \quad (10)$$

The time of mixing  $T_M$  is inversely proportional to the impeller speed notwithstanding the vessel size. However, when a dead space of a stagnant zone exists, or mixing proceeds in more than two mechanisms, the second term of Eq. (2) is no longer negligible and the simplified relation as shown by Eq. (10) does not hold.

The authors<sup>7)</sup> measured the time of mixing for a highly viscous liquid using a helical type of mixer and obtained the result shown by **Fig. 1**. This indicates the existence of Eq. (10) because the unitary circulation flow occurs in this case. On

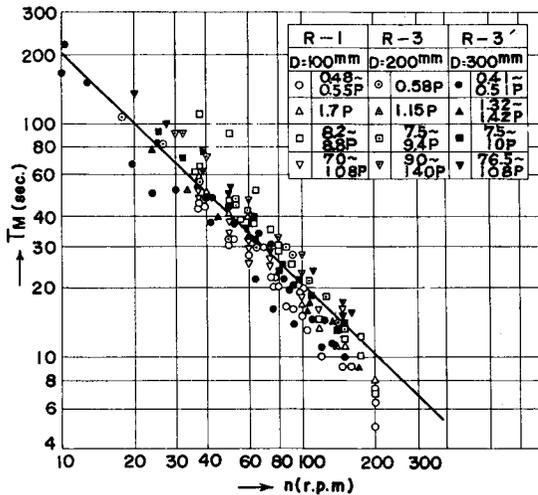


Fig. 1.  $n \sim T_M$ -Diagram of Ribbon Mixers (Effect of Vessel Diameter and Liquid Viscosity).

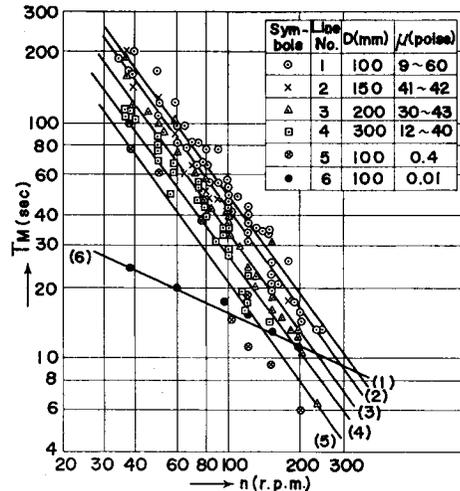


Fig. 2.  $n \sim T_M$ -Diagram of Paddle Mixer (Effect of Vessel Diameter and Liquid Viscosity).

the other hand, in case of a paddle agitator, lack of uniformity takes place caused by a stagnant zone whose size may be changed by increasing the agitator speed. **Fig. 2** shows the result of mixing time  $T_M$  versus impeller speed  $n$ . Thus, we have the following equation<sup>7)</sup> for a paddle agitator:

$$T_M \cdot n^{1.4} D^{0.56} = \text{constant} \quad (11)$$

Kramers et al<sup>8)</sup> studied the time of mixing  $T_M$  in a turbulent region in a mixing vessel with baffle plates and observed the existence of Eq. (10). Also, van de Vusse<sup>9)</sup> studied the mixing speed in unbaffled mixing vessels using the Schlieren

technique and obtained a more complicated result. For scaling up, he proposed the following relation:

$$T_M \propto n^{-1.6} d^{-0.3} \quad (12)$$

$$\text{or} \quad N_{TM} = T_M \cdot n \propto \frac{1}{(n^2 d)^{0.3}} \quad (12')$$

In the unbaffled agitation, Froude number may be involved in the mixing speed and the appearance of the so-called forced vortex may have an influence in the complication of the mixing rate. Under the condition of  $T_M = \text{constant}$ , however,  $nd^{0.2}$  is equal to a constant as shown by Eq. (12) which means that the vessel diameter has a slight influence on the mixing speed. Therefore, the scaling up may be approximately made on the basis of equal impeller speed.

Yamamoto<sup>10)</sup> studied the time of mixing by a similar procedure as that of Kramers and obtained the results as shown by Fig. 3. The concentration difference between the two distant locations in mixing vessels shows several peaks before complete mixing.

Although mixing proceeds accompanied with liquid circulation, the number of peaks which appears before complete mixing varies widely with the type of mixers.

The authors derived a general formula of the mixing speed for various impellers including the range of experiments;  $d/D=0.06-0.8$ ,  $b/d=0.125-0.33$ ,  $n_p=2-16$  and  $Re > 5 \times 10^3$ .

$$\frac{1}{T_M} = k \left\{ n \left( \frac{d}{D} \right)^3 N_{q_d} + 0.21 n \left( \frac{d}{D} \right) \left( \frac{N_p}{N_{q_d}} \right)^{1/2} \right\} \{ 1 - e^{-13(d/D)^2} \} \quad (13)$$

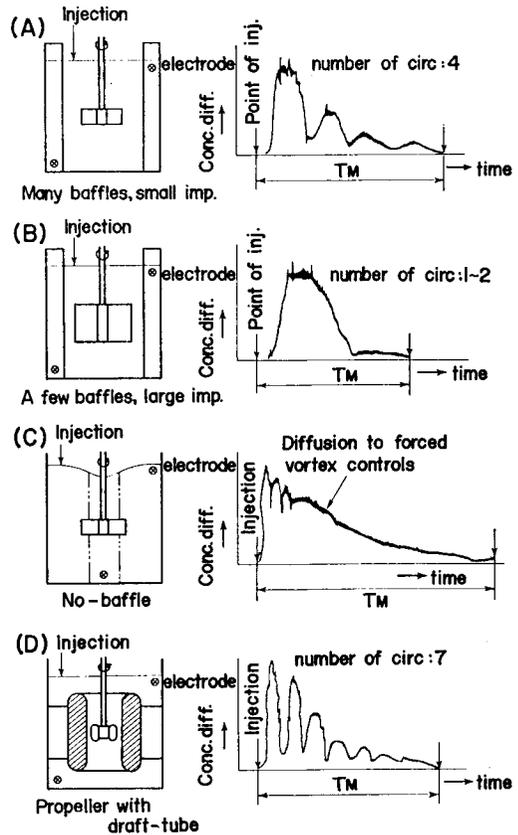


Fig. 3. Diagram showing the process of mixing by various impellers.

$$\text{or } \frac{1}{T_M \cdot n} = \frac{1}{N_{T_M}} = k \left\{ \left( \frac{d}{D} \right)^3 N_{q_d} + 0.21 \left( \frac{d}{D} \right) \left( \frac{N_P}{N_{q_d}} \right)^{1/2} \right\} \{1 - e^{-13(d/D)^2}\} \quad (13')$$

where the dimensionless discharge flow  $N_{q_d}$  is defined as follows:

$$N_{q_d} = \frac{q_d}{nd^3} \quad (14)$$

Therefore, the discharge flow rate  $q_d$  is equal to  $N_{q_d} \cdot nd^3$ . Since the total liquid volume to be circulated for mixing is proportional to  $D^3$ ,  $N_{q_d} \cdot n(d/D)^3$  is a sort of circulation flow speed.

On the other hand, power consumption per unit discharge flow is expressed as follows:

$$\frac{P}{\rho q_d} = \frac{\rho N_P n^3 d^5}{\rho N_{q_d} n d^3} = \left( \frac{N_P}{N_{q_d}} \right) n^2 d^2 \quad \left[ \frac{\text{cm}^2}{\text{sec}^2} \right] \quad (15)$$

Since  $nd(N_P/N_{q_d})^{1/2}$  corresponds to a turbulent velocity, and the distance of liquid circulation is proportional to the vessel diameter,  $D$ ,  $nd(N_P/N_{q_d})^{1/2}/D$  shows the mixing speed caused by large turbulence. Thus the mixing speed  $1/T_M$  is expressed by Eq. (13).

In Eq. (13'), the first term  $(d/D)^3 N_{q_d}$  shows the intensity of circulation and the second term  $(d/D)(N_P/N_{q_d})^{1/2}$  shows the intensity of turbulent diffusion. The third term  $\{1 - e^{-13(d/D)^2}\}$  shows the degree of the spreading of circulation flow in the whole vessel.

Eq. (13) shows that the rate of mixing is affected by both the circulation and turbulence, and the mixing time ( $T_M$ ) is inversely proportional to the impeller speed ( $n$ ). Nevertheless, the criterion for the similarity in blending speed is that the impeller speed should be equal notwithstanding the vessel diameter for geometrically similar mixing vessels without a dead space.

### (3) Similarity for Heat and Mass Transfer on the Fixed Surface.

The heat and mass transfer in an agitation vessel should be correlated by Eq. (5) using dimensionless groups, Re, Fr, and Pr(or Sc). In the case of heat transfer to a stationary solid surface as a jacket wall and a cooling coil, the Forude number may be eliminated.

Denoting the heat transfer coefficient as  $h$ , and the mass transfer coefficient on a fixed body as  $K$ , the following wellknown equations are applicable.

$$\frac{hD}{k} = c_1 \left( \frac{D^2 n \rho}{\mu} \right)^{x_h} (Pr)^y \quad (16)$$

$$\frac{KD}{D_v} = c_2 \left( \frac{D^2 n \rho}{\mu} \right)^{x_m} (Sc)^y \quad (17)$$

Rushton<sup>2)</sup> derived the scaling up criteria for the equality of  $h$  and  $K$  from Eqs. (16) and (17) as follows:

$$\frac{n_2}{n_1} = \left(\frac{D_1}{D_2}\right)^{(2x-1)/x} \quad (18)$$

Thus the impeller speed at which heat and mass transfer coefficient are equalized is a function of vessel diameter ratio ( $D_1/D_2$ ) and Reynolds-index ( $x$ ). This point of view is, of course, theoretically correct, but must be based on the assumption that heat and mass transfer is correlated thoroughly by Eqs. (16) and (17). Therefore we must restrict the range of application of Eq. (18) to the heat and mass transfer on a fixed surface.

#### (4) Similarity for Suspension of Solid Particles, Dispersion of Gas and Liquid, and Mass Transfer on Dispersed Objects.

Rushton proposed the use of Eq. (18) as a similarity criterion in getting the equal rate of dissolution of solid particles in an agitated liquid. According to the author's opinion, application of Eq. (18) should be limited to the particles fixed on a vessel wall or bottom. For suspended particles, the Froude number should never be ignored as shown by Eq. (5). The authors correlated the Re-index ( $x$ ) for the dissolution of suspended solid particles under unbaffled conditions as shown by Eq. (19)<sup>11)</sup>.

$$x = 0.0802 \left(\frac{D^3 g \rho^2}{\mu^2}\right)^{0.0772} \left\{ \log \left(\frac{\rho_s - \rho}{\rho} + 0.043\right) + 1.35 \right\} 10^{-13.5(d_p/D)} \quad (19)$$

The authors also studied the rate of dissolution of solid particles using 15 sets of solid-liquid systems and obtained a general correlation as follows:

$$\left(\frac{KD}{D_v}\right) = 3.60 \times 10^{12} \left(\frac{D^2 n \rho}{\mu}\right)^x \left(\frac{\mu}{\rho D_v}\right)^y \left(\frac{D_v^2}{d^3 g}\right)^{0.627} \left(\frac{d_p}{D}\right)^{3.08} \left(\frac{\rho_s - \rho}{\rho}\right)^{-2.82} \left(\frac{\varepsilon}{\pi}\right)^{-1} \quad (20)$$

where the Re-index ( $x$ ) should be calculated by Eq. (19), and the exponent  $y$  is calculated by Eq. (21).

$$y = -14.4 \left(\frac{d_p}{D}\right) + 1.84 \left(\frac{\rho_s - \rho}{\rho}\right)^{0.116} \quad (21)$$

The dimensionless group ( $d^3 g/D_v^2$ ) is easily transformed into the Froude number by combining with Re and  $S_c$  groups. Eq. (19) and Eq. (20) both have complicated forms, showing that the Froude group should never be neglected. Therefore the simple form such as shown by Eq. (17) is not applicable.

The critical agitator speed ( $n_f$ ) at which all the solid particles suspend, is shown by the following equation<sup>12)</sup>:

$$n_f = KD^{-2/3}d_p^{1/3}\left(\frac{\rho_s - \rho}{\rho}\right)^{2/3}\left(\frac{\mu}{\rho}\right)^{-1/9}\left(\frac{V_p'}{V_p}\right)^{0.7} \quad (22)$$

Barker and Treybal<sup>13)</sup> studied the rate of dissolution of solid particles under standard baffled conditions. According to their results, Re-index ( $x$ ) seems to be constant with change in  $(\rho_s - \rho)/\rho$ . If their result is correct, Eq. (17) holds good and the similarity equation (18) may be used. According to our experiments, Re-index ( $x$ ) varies with the change of density difference even in the case of baffled conditions as shown by **Fig. 4**. Therefore Eq. (17) is not complete and the procedures presented by Rushton may not be applicable. Presumably, the effect of the Froude number should never be neglected in dealing with suspension or settling in the field of gravity. Except for using liquid related to Eq. (6), it is generally impossible to adopt the similarity condition considering both Re- and Fr-groups. Therefore the authors would like to derive a similarity relation based solely on experimental results.

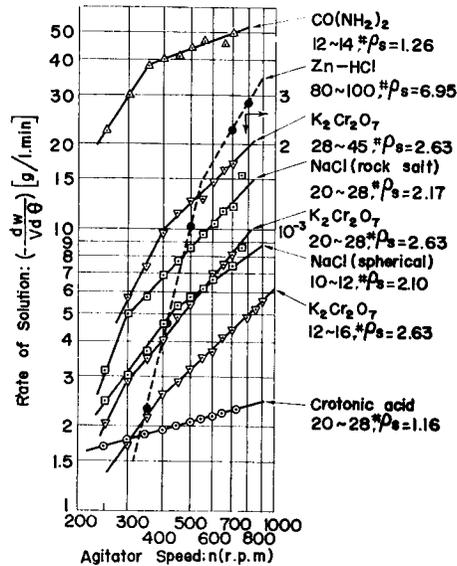


Fig. 4. Effect of Impeller Speed on the Rate of Solution of Solids on a Baffled Vessel.

In the case of agitating the same liquid in geometrically similar vessels, the critical agitator speed, at which the state of suspension of solid particles (refer to Eq. (22))<sup>12)</sup>, the rate of dissolution of solid particles<sup>14)</sup>, and the dispersion of liquid droplets and gas bubbles are equal<sup>14),15)</sup>, is found at the condition of equal power per unit volume ( $P_v$ ) which is proportional to  $n^3D^2$ . For example, **Fig. 5(a)** shows the time ( $\theta$ ) of extraction of  $I_2$  in toluene to water in geometrically similar vessels of four different diameters. Though the plotted points of  $\theta$  versus  $n$  lie on different curves, the rearranged points of  $\theta$  versus  $nD^{2/3}$  lie on a single curve as shown by **Fig. 5(b)**.  $nD^{2/3}$  is a modified agitator speed at which the power per unit volume is equal. **Fig. 6** shows the rate of dissolution ( $1/\theta_0$ ) of benzoic acid tablets in dilute alkaline solution. All the plotted points of  $(1/\theta_0)$  vs.  $nD^{2/3}$  lie on a single straight line.

According to Vermeulen<sup>15)</sup>, the diameter of dispersed droplets in liquid-liquid agitation vessels, i.e., the interfacial area per unit volume ( $a$ ), is correlated by a

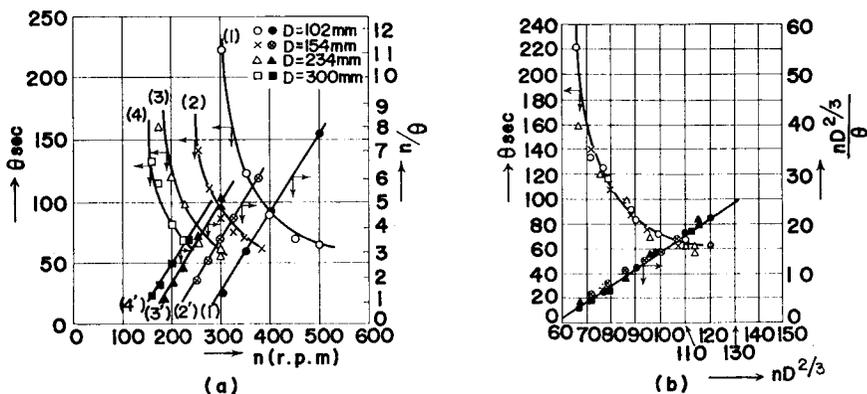


Fig. 5. Time of Extraction vs. Agitator Speed (Baffled Agitation).

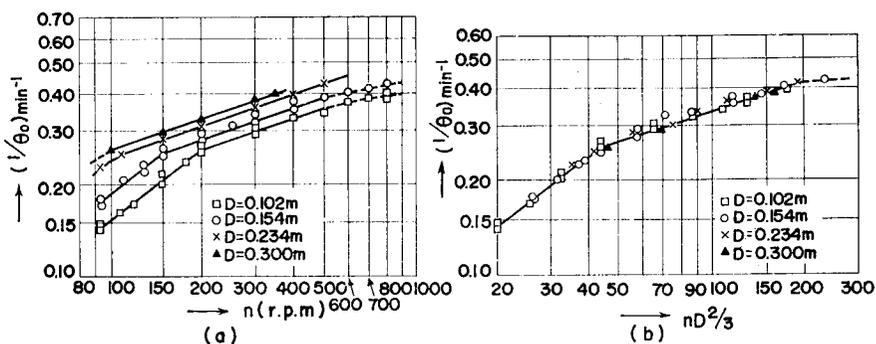


Fig. 6. Rate of Solution of Benzoic acid tablets in water vs. Agitator speed.

power function of Weber number as shown by Eq. (23) and the interfacial area per unit volume in gas-liquid agitation vessels, is correlated by a product of power functions of We- and Re- numbers as shown by Eq. (24):

$$aD \propto \left( \frac{D^3 n^2 \rho}{\sigma} \right)^{0.6} \quad (23)$$

$$aD \propto \left( \frac{D^3 n^2 \rho}{\sigma} \right) \left( \frac{D^2 n \rho}{\mu} \right)^{-0.5} \quad (24)$$

For the same liquid-liquid systems, Eq. (23) is reduced to:

$$a \propto (n^3 D^2)^{0.4} \quad (23')$$

and for the same gas-liquid systems, Eq. (24) is reduced to:

$$a \propto (n^3 D^2)^{0.5} \quad (24')$$

These results show that the interfacial area per unit volume is equal at the condition of equal power per unit volume.

Cooper et al<sup>16)</sup> and Yoshida et al<sup>17)</sup> showed experimentally that the absorption coefficient was equal at the condition of equal power per unit volume ( $P_v \propto n^3 D^2$ ). Those arguments may be on flimsy ground, but the intensity of turbulence in agitation vessels seems to be equal as a rough estimate on the basis of equal power per unit volume. This method of scaling up was formerly proposed by W. Büche<sup>18)</sup> and the authors would like to support his proposal as a simple and convenient method of scaling up for suspensions.

### Conclusion

There are many proposals for the scaling up of mixing vessels. They are neither consistent nor conclusive. The authors propose an idea to select the method of scaling up depending upon the mixing ingredients for convenient use. The classification of the types of scaling up is as follows:

- (1) Similarity for power requirement;  $N_p$  and  $Re$  are used for correlation.
- (2) Similarity for mixing speed of homogeneous liquid phase; Use equal impeller speed.
- (3) Similarity for heat and mass transfer on a fixed surface; Eq. (18) proposed by Rushton is recommended.
- (4) Similarity for suspension of solid particles, dispersion of gas and liquid, and mass transfer on dispersed ingredients; Büche's proposal of using  $P_v (\propto n^3 D^2)$  is recommended.

### Notations used

- $a$  : Interfacial area per unit volume  
 $b$  : Width of impeller blades  
 $c$  : Concentration of liquid  
 $C_m$  : Final average concentration of liquid  
 $D$  : Diameter of mixing vessels  
 $D_f$  : Combined diffusivity of  $D_v$  and  $D_t$ .  
 $D_t$  : Turbulent diffusivity  
 $D_v$  : Molecular diffusivity  
 $d$  : Impeller diameter  
 $d_p$  : Particle diameter  
 $Fr = v^2/Lg$  or  $Dn^2/g$ : Froude number  
 $g$  : Gravitational constant  
 $h$  : Heat transfer coefficient  
 $K$  : Mass transfer coefficient

- $k$  : Thermal conductivity  
 $L$  : Representative length of mixing vessel  
 $N_{TM} = T_M \cdot n$ : Dimensionless mixing time. Number of rotation of mixing impeller for complete mixing  
 $N_P = \frac{P \cdot g_c}{\rho n^3 d^5}$ : Power number  
 $N_{q_d} = \frac{q_d}{nd^3}$ : Discharge flow number  
 $n$  : Impeller speed  
 $n_f$  : Critical impeller speed for suspension of solid particles  
 $n_p$  : Number of impeller blades  
 $P$  : Power consumption of mixing impellers  
 $Pr = c_p \mu / k$ : Prandtl number  
 $\Delta p / \rho v^2$ : Pressure coefficient  
 $q_d$  : Discharge flow rate from mixing impellers  
 $Re = Lv\rho / \mu$  or  $D^2 n \rho / \mu$ : Reynolds number  
 $S_c = \mu / \rho D_v$ : Schmidt number  
 $T_M$  : Gross blending time  
 $t$  : Time  
 $V_p$  : Net volume of solid particles  
 $V_p'$  : Bulk volume of solid particles  
 $v$  : Liquid velocity  
 $x_h, x_m, (x)$ : Reynolds index for heat and mass transfer, refer to Eqs. (16), (17), (19) and (20).  
 $u, v, w$ :  $x, y, z$ -component of liquid velocity  
 Greek letters  
 $\theta$  : Time of extraction  
 $\theta_0$  : Time of dissolution of solid particles  
 $\mu$  : Viscosity of liquid  
 $\rho$  : Density of liquid  
 $\rho_s$  : Density of solid particles  
 $\sigma$  : Interfacial tension  
 $\varepsilon$  : Shape factor of solid particles

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