On the Heaving Motion of Circular Peripheral Jet Ground Effect Machines

By

Hiroshi MAEDA* and Yoshihiko KONDO*

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The heaving motion of peripheral jet ground effect machines with circular planform has been studied and the simple expressions of the equation of motion are formulated. The analysis is separated into two parts, i.e. the original peripheral jet configuration and the modified configuration equipped with the stability jets. In both cases, the behavior of the jets is quantitatively treated as the "underfed" and the "modified overfed" operation and after some simplification, second order nonlinear differential equations are derived as the equations of heaving motion. Those equations are still valid for a large amplitude heaving oscillation or for a large initial hover height. Furthermore, the frequency and damping characteristics of the small amplitude heaving oscillation can be predicted by the linearized equations of motion which are derived by the use of the approximation of small amplitude. The agreement between the analytical and experimental results is found to be comparatively good.

1. Introduction

The dynamic behaviors of a three-dimensional ground effect machine (GEM or so-called Hovercraft) have already been studied by several investigators, but since the phenomena are so complex and peculiar compared with other familiar vehicles, simple and convenient expressions of the motion have not been obtained so far.

The heaving motion of GEM has been studied by Tulin[1], Webster and Lin[2] and other workers[3][4]. However, in recent years the pitching and rolling motions of GEM are frequently discussed but the reports related to the heaving motion are rarely found. Presumably this fact depends on the following reasons, i.e. in practice the heaving motion of GEM has so small amplitude and heavy damping characteristics that no important problems occur in the usual heaving oscillation. However, since the heaving motion is one of the most fundamental characteristics of GEM dynamics, the authors have in this paper tried to analyze the heaving motion without forward velocity. In addition, the authors wish to apply the results of this analysis to GETOL planes in the future, and therefore the general

* Department of Aeronautical Engineering

characteristics of heaving oscillation including not only the small amplitude but the fairly large initial hover height will be treated here.

Furthermore, practical GEMs are generally equipped with the so-called flexible skirts and stability jets or compartment partitions underneath the GEM body. Among those accessories stability jets affect so much on the heaving motion that the discussions of the effects are also carried out in this paper.

2. Motion of a original peripheral jet GEM

For the simplification of calculation the assumptions are given as follows:

1) The plan form of peripheral jet nozzle is circular.
2) The simple momentum theory is applied to the analysis of balanced jet condition*.
3) The jet momentum is constant throughout the overall motion despite the hover height.
4) The incompressibility condition is satisfied throughout the motion.
5) The forward velocity, pitching and rolling angles are all zero.
6) The jet flow occurs with no complicating viscous effects.

Unless a GEM has the circular plan form, it can be transformed into an equivalent circular GEM taking into consideration the bottom area and the jet momentum, because well designed GEMs have nearly uniform jet distribution regardless of the configuration.

2.1 Balanced jet condition

Applying the simple momentum theory, the cushion pressure rise p for the

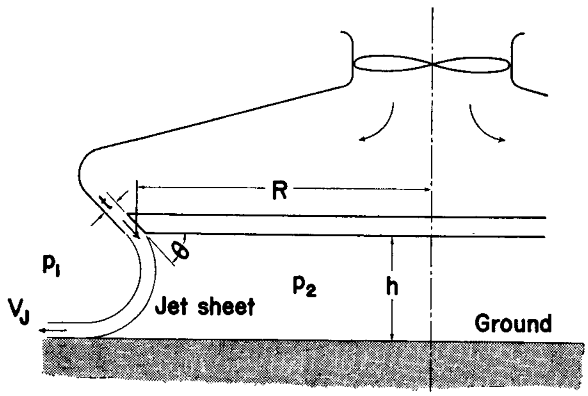

Fig. 1. Balanced jet condition of a circular peripheral jet GEM,

* See Appendix,

balanced jet condition is determined by (cf. Fig. 1)

$$p = \frac{M(1+\cos\theta)}{h \cdot 2\pi R} \tag{1}$$

where $p = p_2 - p_1$, $M = 2\pi R \cdot t \cdot \rho V_j^2$.

Therefore, the lift force L_p acted on the bottom surface of GEM by the cushion pressure p is expressed by

$$L_p = \pi R^2 \cdot p \tag{2}$$

Furthermore, when the lift component due to jet reaction is indicated by L_j, the total lift L becomes

$$L = L_p + L_j \tag{3}$$

where $L_j = M \sin\theta$.

In the case of the balanced jet condition, the total lift is equal to the total weight of GEM, W, thus

$$W = L = \frac{M(1+\cos\theta)R}{2h} + M\sin\theta \tag{4}$$

Since the jet momentum M is assumed to be constant and the second term L_j is in general much smaller than the first term L_p, the lift L is nearly proportional to the inverse of the hover height h for the balanced jet condition. This relationship

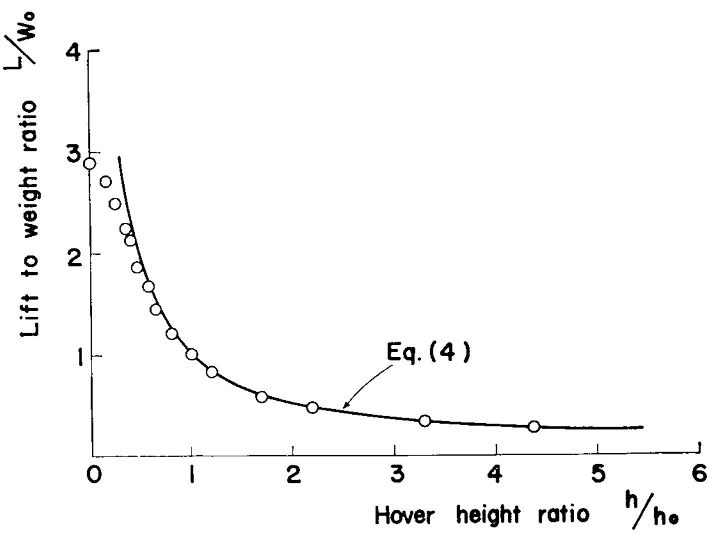

Fig. 2. Relationship between lift and hover height at balanced jet condition.

is illustrated in Fig. 2, and compared with an experimental result. In this diagram, h_0 shows the hover height at $L=W_0$, where W_0 is the normal weight of GEM.

2.2 Moving downward condition

When a GEM moves downwards, as shown in Fig. 3, the volume of pressurized air sealed between the jet sheet and the bottom surface of GEM decreases in pro-

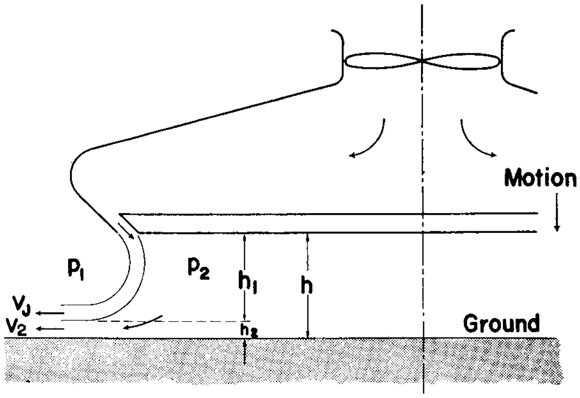

Fig. 3. Moving downwards or underfed condition.

portion to the falling velocity $\dot{h}$. Accordingly, with the assumption of incompressibility the excessive air volume has to escape along the ground to the atmosphere, i.e. the "underfed" operation must be performed. Therefore

$$h_2 \cdot V_2 \cdot 2\pi R = -\dot{h} \cdot \pi R^2 \tag{5}$$

or

$$h_2 = -\frac{\dot{h}R}{2V_2} \tag{6}$$

where $\dot{h}$ is negative because the motion is falling downward. Utilizing Bernoulli's equation, the escape velocity V_2 has to satisfy the following relation.

$$V_2 = \sqrt{\frac{2 \cdot p}{\rho}} \tag{7}$$

Substituting Eq. (7) into Eq. (6),

$$h_2 = -\frac{\dot{h}R}{2}\sqrt{\frac{\rho}{2p}} \tag{8}$$

Since the hover height h_1 shown in Fig. 3 can be treated in the same way as the balanced jet condition,

$$p=\frac{M(1+\cos\theta)}{h_1\cdot 2\pi R} \tag{9}$$

Substituting the total height $h=h_1+h_2$ into Eq. (9),

$$p=\frac{M(1+\cos\theta)}{\left\{h+\frac{hR}{2}\sqrt{\frac{\rho}{2p}}\right\}2\pi R} \tag{10}$$

Solving Eq. (10) in p,

$$p=\frac{M^2(1+\cos\theta)^2}{\pi^2R^2\left\{\dot{h}\frac{R}{2}\sqrt{\frac{\rho}{2}}+\sqrt{\dot{h}\frac{R^2\rho}{8}+\frac{4M(1+\cos\theta)h}{2\pi R}}\right\}^2} \tag{11}$$

For convenience, if the variable h in Eq. (11) is replaced by a new variable $x=h-h_0$, and also it is assumed that

$$\frac{\dot{x}}{V_J}\ll 1 \tag{12}$$

then the following approximate expression of L_p is obtained by neglecting the second order small terms of $\dot{x}/V_J$ in Eq. (11).

$$L_p\doteqdot\frac{L_{p_0}}{1+\frac{x}{h_0}}\left\{1-\frac{1}{\sqrt{1+\frac{x}{h_0}}}\cdot\frac{R}{2\sqrt{2th_0(1+\cos\theta)}}\frac{\dot{x}}{V_J}\right\} \tag{13}$$

where L_{p_0} corresponds to the value of L_p for the balanced jet condition and it is a constant value. Therefore, at the moving downward condition, the total lift is given by

$$L=\frac{L_{p_0}}{1+\frac{x}{h_0}}\left\{1-\frac{1}{\sqrt{1+\frac{x}{h_0}}}\cdot\frac{R}{2\sqrt{2th_0(1+\cos\theta)}}\cdot\frac{\dot{x}}{V_J}\right\}+L_j \tag{14}$$

2.3 Moving upward condition

In the same manner as the moving downward condition, when the GEM moves upwards, as shown in Fig. 4, the air volume sealed between the jet sheet and the bottom surface of GEM increases in proportion to the rising velocity $\dot{h}$. Therefore, if this increasing air volume is supplied with a part of peripheral jet air in order to meet the incompressibility condition, the jet momentum has to decrease in proportion to this air volume. In other words, the "overfed" condition somewhat modified should be met in this case, and the cushion pressure p becomes

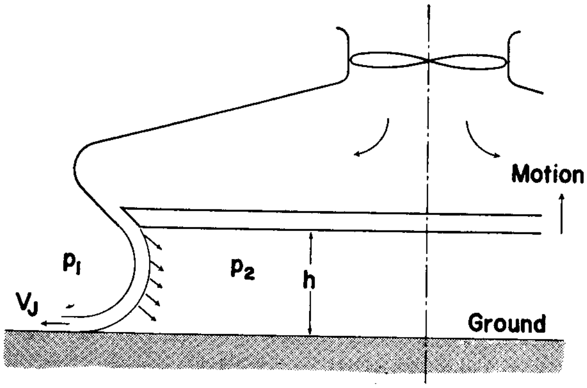

Fig. 4. Moving upwards or modified overfed condition.

$$p=\frac{(M-\Delta M)(1+\cos\theta)}{h\cdot 2\pi R} \tag{15}$$

where ΔM means the jet momentum reduction and is expressed by

$$\Delta M=2\pi R\cdot\Delta t\cdot\rho\cdot V_J^2 \tag{16}$$

By the use of the incompressibility condition, the jet thickness decrement Δt in Eq. (16) is

$$\Delta t=\frac{\dot{h}R}{2V_J} \tag{17}$$

and therefore, by Eq. (15), the cushion pressure p becomes

$$p=\frac{\left(t-\frac{\dot{h}R}{2V_J}\right)\cdot\rho V_J^2(1+\cos\theta)}{h} \tag{18}$$

In the same way as the moving downward condition, by introducing the new variable x in place of h the lift force yields, after some simplification

$$L_p\doteqdot\frac{L_{p_0}}{1+\frac{x}{h_0}}\left\{1-\frac{1}{2}\cdot\frac{R}{t}\cdot\frac{\dot{x}}{V_J}\right\}+L_j \tag{19}$$

2.4 Equations of heaving motion

The equation of heaving motion of a GEM is expressed by

$$m\ddot{h}=L-W \tag{20}$$

where m is the mass of GEM. At the balanced hover condition

$$W = L_{p_0} + L_j \tag{21}$$

therefore the equations of motion are expressed as follows:

(1) Moving downward condition: $\dot{x} < 0$

$$\frac{W}{g}\ddot{x} + \frac{L_{p_0}}{\left(1+\frac{x}{h_0}\right)^{3/2}} \cdot \frac{R}{2\sqrt{2th_0(1+\cos\theta)}} \cdot \frac{\dot{x}}{V_J} + \frac{L_{p_0}}{1+\frac{x}{h_0}} \cdot \frac{x}{h_0} = 0 \tag{22}$$

(2) Moving upward condition: $\dot{x} > 0$

$$\frac{W}{g}\ddot{x} + \frac{L_{p_0}}{1+\frac{x}{h_0}} \cdot \frac{1}{2} \cdot \frac{R}{t} \cdot \frac{\dot{x}}{V_J} + \frac{L_{p_0}}{1+\frac{x}{h_0}} \cdot \frac{x}{h_0} = 0 \tag{23}$$

Since both of those equations have the following form of nonlinear differential equation,

$$\ddot{x} + f(x)\dot{x} + g(x) = 0 \tag{24}$$

then they can be solved, for example, by the Isocline method graphically.

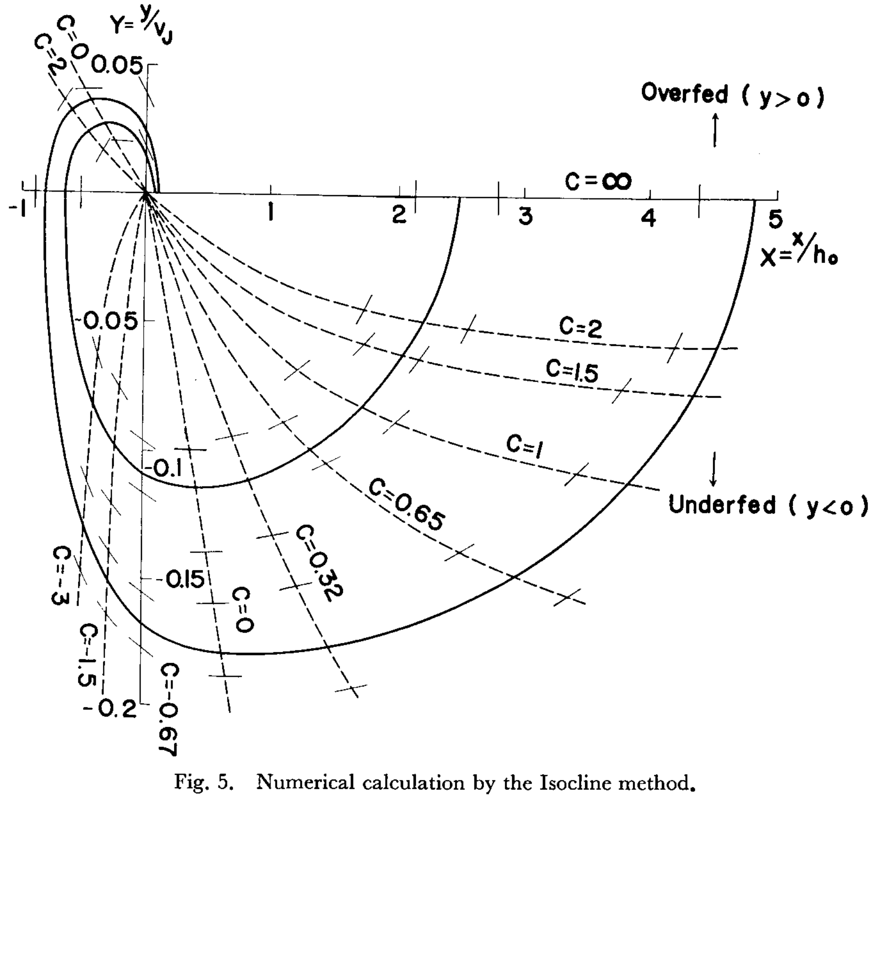

Fig. 5. Numerical calculation by the Isocline method.

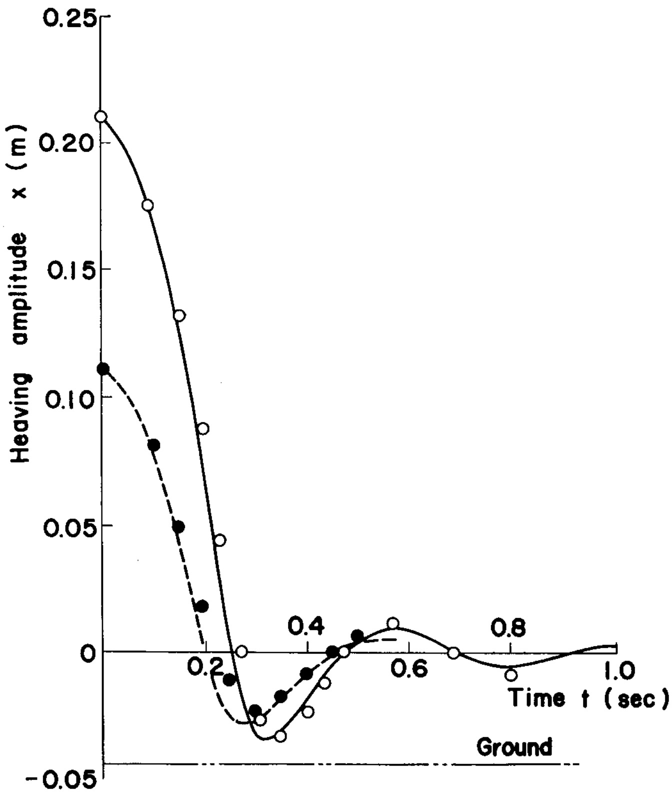

Fig. 6. Time history of heaving oscillation.

As a numerical example, the analytical calculation of a model GEM whose numerical data are listed below were performed and the results are compared with the experimental ones as shown in Fig. 5 and Fig. 6.

$$W = 4.45\ \text{kg}, \quad R_{eq} = 0.50\ \text{m}, \quad t = 0.02\ \text{m}$$
$$h_0 = 0.044\ \text{m}, \quad \theta = 45°$$

The agreement of both results looks good.

3. Motion of a peripheral jet GEM with stability jets

In general, GEMs are equipped with the so-called flexible skirt to increase the effective hover height and the stability jets or partitions to hold the pitching or rolling stability. Among those accessories the stability jets affect the heaving characteristics so much that the influences will be discussed here.

3.1 Balanced jet condition

The balanced hover condition with stability jets is illustrated in Fig. 7. In this case, the total hover height is given by

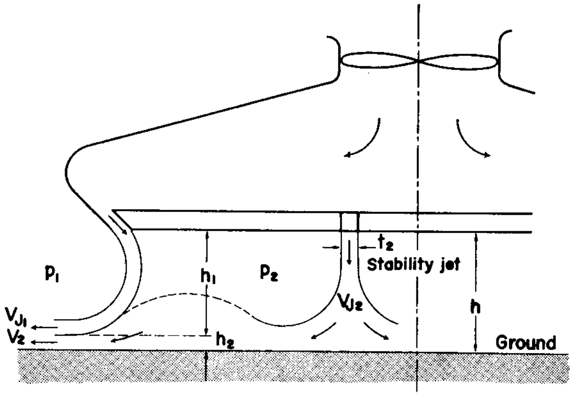

Fig. 7. Balanced jet condition of a circular GEM with stability jets.

$$h=h_1+h_2 \tag{25}$$

where h_2 is the increment of hover height due to the stability jets. If it is assumed that the total jet momentum is constant,

$$M=M_1+M_2=\text{const.} \tag{26}$$

where

$$M_1=2\pi R\cdot t\cdot \rho\cdot V_{J_1}^2$$

$$M_2=l\cdot t_2\cdot \rho\cdot V_{J_2}^2$$

M_1 and M_2 are the peripheral jet momentum and the stability jet momentum respectively.

The hover height increment h_2 in Eq. (25) is determined, by Bernoulli's theorem, as follows:

$$h_2=\frac{l}{2\pi R}\cdot t_2\cdot V_{J2}\sqrt{\frac{\rho}{2p}} \tag{27}$$

Also, h_1 is determined by the balanced jet condition, and Eq. (26) becomes

$$V_J^2=V_{J1}^2+\frac{lt_2}{2\pi Rt}\cdot V_{J2}^2 \tag{28}$$

when it is assumed that $V_{J2}=\eta\cdot V_{J1}(\eta<1)$, by Eq. (28)

$$V_{J1}=\frac{V_J}{\sqrt{1+\frac{\eta^2 lt_2}{2\pi Rt}}}, \quad V_{J2}=\frac{\eta V_J}{\sqrt{1+\frac{\eta^2 lt_2}{2\pi Rt}}} \tag{29}$$

hence the total hover height and the cushion pressure are given by the following equations respectively.

$$h=\frac{t}{p}\cdot\rho V_J^2\frac{1+\cos\theta}{1+\frac{\eta^2 lt_2}{2\pi Rt}}+\frac{lt_2}{2\pi R}\cdot\frac{\eta V_J}{\sqrt{1+\frac{\eta^2 lt_2}{2\pi Rt}}}\sqrt{\frac{\rho}{2p}} \tag{30}$$

$$p=\frac{t\rho V_J^2(1+\cos\theta)}{h}\cdot\frac{1}{1+\frac{\eta^2 lt_2}{2\pi Rt}}\cdot\frac{1}{1-\frac{\eta lt_2}{2\pi R\sqrt{2ht(1+\cos\theta)}}} \tag{31}$$

As well as the previous case, if $h=h_0$ at $L=W_0$,

$$W_0=p_{h=h_0}\cdot\pi R^2+M_1\sin\theta+M_2 \tag{32}$$

3.2 Moving downward condition

As stated previously, when a GEM moves downwards the air volume proportional to the falling velocity escapes along the ground according to the incompressibility condition. Therefore, if the velocity and thickness of this air flow are expressed by V_3 and h_3, as shown in Fig. 8, respectively

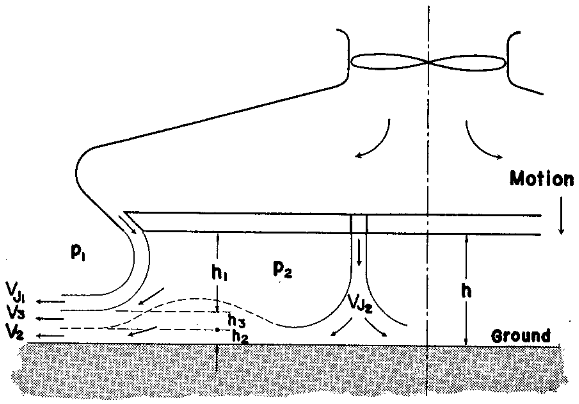

Fig. 8. Moving downwards condition with stability jets.

$$h_3=-\frac{\dot{h}R}{2}\cdot\frac{1}{V_3}=-\frac{\dot{h}R}{2}\sqrt{\frac{\rho}{2p}} \tag{33}$$

and hence the cushion pressure becomes

$$p=\frac{t\rho V_{J1}^2(1+\cos\theta)}{h-\left\{\frac{lt_2}{2\pi R}V_{J2}-\frac{\dot{h}R}{2}\right\}\sqrt{\frac{\rho}{2p}}} \tag{34}$$

where $h=h_1+h_2+h_3$.

Solving Eq. (34) in p,

$$p=\frac{t\rho V_J^2(1+\cos\theta)}{h}\cdot\frac{1}{1+\frac{\eta^2 lt_2}{2\pi Rt}}\cdot\frac{1}{1-\frac{1}{\sqrt{2ht(1+\cos\theta)}}\left\{\frac{lt_2\eta}{2\pi R}-\frac{R}{2}\cdot\frac{\dot{h}}{V_J}\sqrt{1+\frac{\eta^2 lt_2}{2\pi Rt}}\right\}} \tag{35}$$

In this expression, second order small terms of $\dot{h}/V_J$ are discarded in the same way as the previous case. Hence

$$L_p\doteqdot\frac{Lp_0}{1+\frac{x}{h_0}}\left\{1-\frac{\frac{R}{2}\sqrt{1+\frac{\eta^2 lt_2}{2\pi Rt}}}{\sqrt{2h_0t(1+\cos\theta)}\sqrt{1+\frac{x}{h_0}}-\frac{lt_2\eta}{2\pi R}}\cdot\frac{\dot{x}}{V_J}\right\} \tag{36}$$

where Lp_0 is a constant and is given in Eq. (32).

3.3 Moving upward condition

When a GEM moves upwards, the increasing air volume proportional to the rising velocity $\dot{h}$ is considered to be supplied with a part of stability jets in this case, as shown in Fig. 9. Therefore, with the constant mass flow condition the hover height increment h_2 becomes

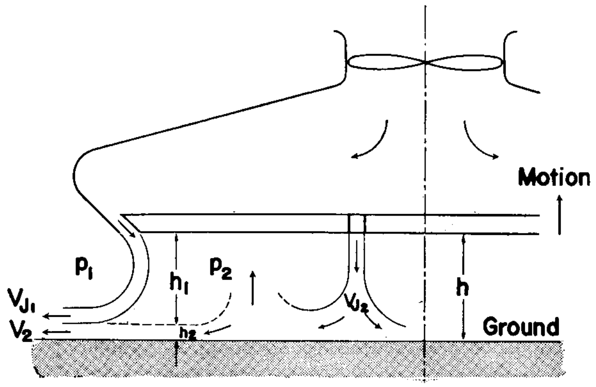

Fig. 9. Moving upwards condition with stability jets.

$$h_2=\frac{lt_2V_{J2}-\pi R^2\cdot\dot{h}}{2\pi R\cdot V_2} \tag{37}$$

and hence the cushion pressure p becomes

$$p=\frac{t\rho V_J{}^2(1+\cos\theta)}{h-\left\{\dfrac{lt_2}{2\pi R}\cdot V_{J_2}-\dfrac{\dot{h}R}{2}\right\}\sqrt{\dfrac{\rho}{2p}}} \tag{38}$$

where $h=h_1+h_2$.

Since Eq. (38) has the same form as Eq. (34), the cushion pressure p and accordingly the pressure lift L_p are given by Eq. (35) and (36), too.

3.4 Equation of heaving motion

The equation of motion of a GEM with stability jets is, therefore, expressed as follows:

$$\frac{W}{g}\ddot{x}+\frac{L_{p_0}}{1+\dfrac{x}{h_0}}\frac{\dfrac{R}{2}\sqrt{1+\dfrac{\eta^2 lt_2}{2\pi Rt}}}{\sqrt{2h_0t(1+\cos\theta)}\sqrt{1+\dfrac{x}{h_0}}-\dfrac{lt_2\eta}{2\pi R}}\cdot\frac{\dot{x}}{V_J}+\frac{L_{p_0}}{1+\dfrac{x}{h_0}}\cdot\frac{x}{h_0}=0 \tag{39}$$

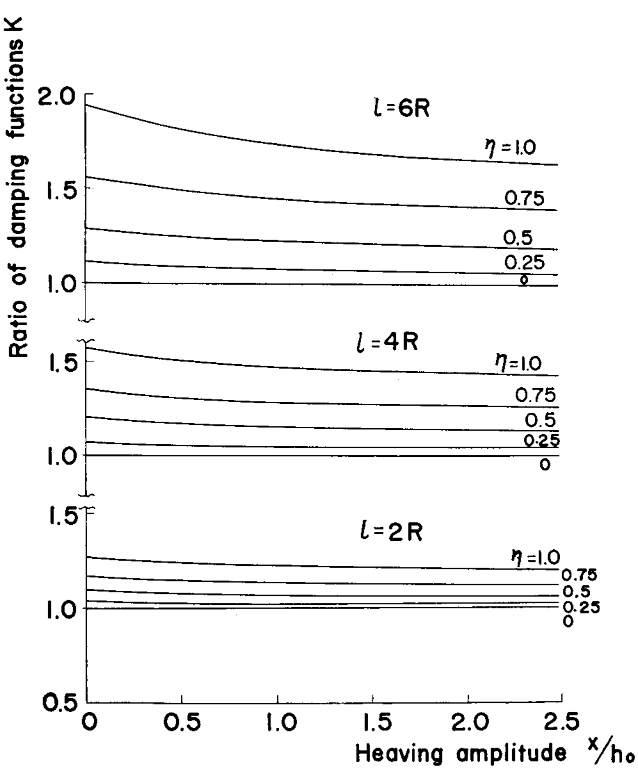

Fig. 10. Influence of velocity ratio η on the damping characteristics.

It is noticed that the equation of motion of this case is expressed by one equation, which is different from the previous case.

In Eq. (39) the velocity ratio η only is undetermined. Since the stability jet velocity is a function of cushion pressure, η varies according to the hover height. However, the influence of η on the damping characteristics of the heaving oscillation is illustrated in Fig. 10. (It is clear that η does not affect the natural frequency of the oscillation. (see Eq. (39)). In this diagram, K denotes

$$K=\frac{f(x)}{f_0(x)} \tag{40}$$

where $f(x)$ is defined in Eq. (24) and $f_0(x)$ denotes the value of $f(x)$ without stability jets, and therefore K means the variation of damping factors with and without stability jets. Refering to the diagrams, it is found that η has relatively small effect on K even in the case when the stability jet length is nearly equal to the peripheral jet length. Therefore, it will be satisfactory to assume that η is constant throughout the motion in the small amplitude heaving oscillation.

Since Eq. (39) has the same form as Eq. (24), it can be solved by the Isocline method, too. However, for the convenience of comparing with some experimental results let us derive the linearized equation of motion by assuming that the amplitude ratio x/h_0 is a small quantity. It is expressed as follows:

$$\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n{}^2x=0 \tag{41}$$

where

$$2\zeta\omega_n=\frac{Lp_0\cdot g}{W}\cdot\frac{\frac{R}{2}\sqrt{1+\frac{\eta^2lt_2}{2\pi Rt}}}{\sqrt{2h_0t(1+\cos\theta)}-\frac{lt_2\eta}{2\pi R}}\cdot\frac{1}{V_J}$$

$$\omega_n{}^2=\frac{Lp_0\cdot g}{W}\cdot\frac{1}{h_0}$$

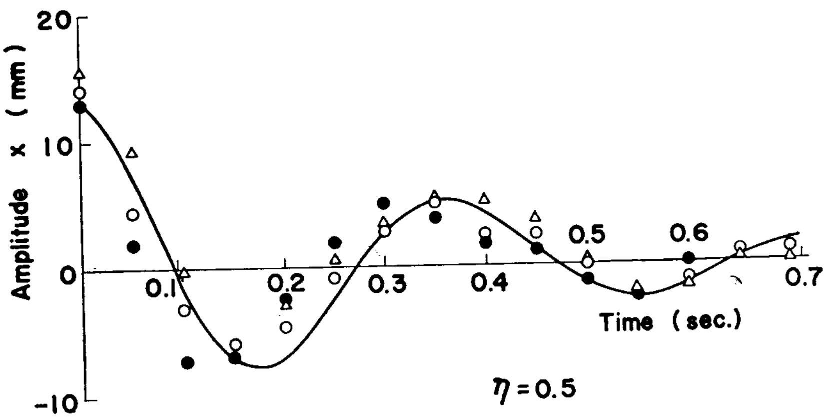

Fig. 11. Time history of heaving oscillation with stability jets.

As a numerical example, the frequency and damping ratio calculated by this linearized equation are compared with the experimental data in Fig. 11.

4. Concluding remarks

The heaving motion of circular peripheral jet ground effect machines has been studied and the expressions of equation of motion are formulated in this report. The analysis is separated into two parts, i.e. the motion of an original peripheral jet GEM and the motion of a GEM equipped with stability jets respectively. In both cases, the heaving behavior of GEMs has been treated analytically and by assuming that the heaving velocity is much smaller than the peripheral jet velocity, the second order nonlinear differential equations are derived as the equations of motion.

Furthermore, the linearized equations of motion can easily be derived from the original equations by assuming the small amplitude, and accordingly the frequency and the damping characteristics of heaving oscillation are easily predicted. The numerical calculation of this analysis is carried out for the original GEM and the modified GEM with stability jets separately. Those results have been compared with the experimental ones and the agreement between them is found to be comparatively good.

Nomenclature

g	:	acceleration of gravity
h	:	hover height
h_0	:	hover height at balanced hover condition
h_1, h_2, h_3	:	hover height components at moving condition
K	:	ratio of damping functions
l	:	total length of stability jets
L	:	total lift force
L_j	:	lift force component due to jet momentum
L_p	:	lift force component due to cushion pressure
L_{p_0}	:	pressure lift at balanced hover condition
m	:	mass of GEM
M	:	jet momentum
M_1	:	peripheral jet momentum
M_2	:	stability jet momentum
ΔM	:	jet momentum decrement
p_1	:	base pressure

p_2 : atmospheric pressure
p : cushion pressure rise or the pressure difference due to jet ground effect
R : radius of circular jet slot
t : jet thickness
Δt : jet thickness decrement
t_2 : stability jet thickness
V_J : jet velocity
V_{J_1} : peripheral jet velocity
V_{J_2} : stability jet velocity
V_2, V_3 : escape velocities of pressurized air
W : weight of GEM
W_0 : normal weight of GEM
x : heaving amplitude
η : velocity ratio of stability jet and peripheral jet
θ : peripheral jet inclination angle
ρ : air density
ζ : damping factor
ω_n : natural frequency

dot denotes the differentiation with respect to time

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Appendix

In this report, the cushion pressure rise at the balanced hover condition has been calculated with the application of the simple momentum theory. Apparently the cushion pressure estimated with this method disagrees with the experimental results in the vicinity of the small hover height, or in other words when the ratio of balanced hover height and jet thickness is small. (see Fig. 1)

From this point of view, it is considered that the exponential theory is more suitable to the analysis of balanced hover condition, i.e. the cushion pressure is expressed by

$$\frac{p}{H_j}=1-e^{-2x} \tag{A-1}$$

where
$$x=\frac{t(1+\cos\theta)}{h^*} \tag{A-2}$$

and h^* is the balanced hover height. The total pressure of peripheral jet H_j becomes

$$H_j=\frac{1}{2}\rho V_J^2+p_j\doteqdot\frac{1}{2}\rho V_J^2 \tag{A-3}$$

where the jet static pressure p_j is supposed to be about one half of the cushion pressure, but it is so small compared with the jet dynamic pressure that it can be neglected approximately. Therefore, at the balanced hove rcondition,

$$p\doteqdot\frac{1}{2}\rho V_J^2(1-e^{-2x}) \tag{A-4}$$

when $x\ll 1$ or $h^*\gg t$, e^{-2x} is expanded as follows:

$$e^{-2x}=1-2x+\frac{4x^2}{2!}\cdots \tag{A-5}$$

or
$$1-e^{-2x}\doteqdot 2x \tag{A-6}$$

Hence the cusion pressure becomes

$$p=\frac{\rho V_J^2 t(1+\cos\theta)}{h^*} \tag{A-7}$$

This expression is the same as that given in Eq. (1).

Since in the majority of regions of the heaving motion the condition given in Eq. (A-6) is met, then it is considered to be satisfactory to utilize the simple momentum theory in place of the exponential theory in the analysis.