A Study on Optimal Transmission System Planning

By

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In this paper, the author deals with the optimal transmission system planning based on topological considerations.

The development of mathematical technique, particularly, the network flow theory has enabled the system planning taking account of topological situations. The construction cost characteristics for the actual transmission system are frequently expressed by a staircase function.

The paper describes the method for solving these transmission system planning by means of integer linear programming with zero-one variables.

1. Introduction

In recent years, the development of mathematical technique and the digital computer has enabled more difficult transmission system planning considering load growth[1]. In particular, the network flow theory has been available for this purpose[2]; first, we can represent a transmission system as a directed network and intend to solve the network problem by means of a method based on topological considerations. In these studies, they assume that the cost characteristics for a construction of system elements are expressed by a linear function. However, in case of a transmission system, the cost characteristic is frequently expressed by a staircase function.

Thus, in the present paper, from the supply-demand theorm of the network flow theory, the author introduces the necessary and sufficient condition to satisfy the demands in the transmission system. The optimal plan for a given load pattern has a minimal cost, so that the optimal system planning can be written as the problem of an integer linear programming with zero-one variables. There are several methods available for solving an integer linear program. We can make use of R. E. Gomory's method[3] and E. Balas's additive algorithm[4].

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2. Directed Network for the Transmission System

Consider the transmission system which consists of power plants, substations, transmission lines and loads as shown in Fig. 2.1. This transmission system is represented by the directed network in which transmission lines correspond to arcs and power plants, substations and loads correspond to nodes, as indicated in Fig. 2.2. A number $g(x)$ may be thought of as the supply of power at source x, $l(x)$ the demand of power at sink x. We can suppose that each arc and some nodes have capacities; it may be thought of intutively as representing the maximal amount of transmitted electrical power. Therefore, in this network, there are multiple sources and sinks, and several nodes have capacities.

However, the addition of two nodes and several arcs to the multiple source, multiple sink network reduces the problem to the case of a single source and sink. Furthermore, by a simple device the network with both arc and node capacities can be reduced to the extended network with only arc capacities. If the given network is that of Fig. 2.2, the extended network with only arc capacities is shown in Fig. 2.3, where $g(x)$, $d(x)$, $l(x)$ and $t(x)$ are respectively capacities of power plants, substations, loads and transmission lines.

Consider the flow from source s_0 to sink t_0 on the network shown in Fig. 2.3,

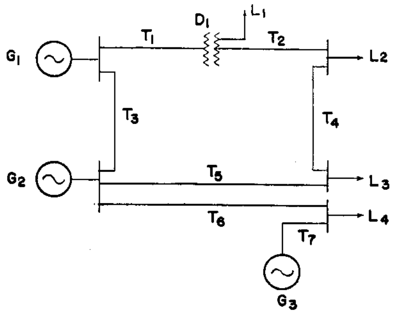

Fig. 2.1. Transmission system.

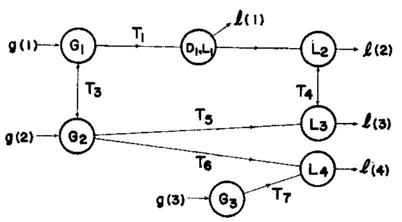

Fig. 2.2. Network for the transmission system shown in Fig. 2.1.

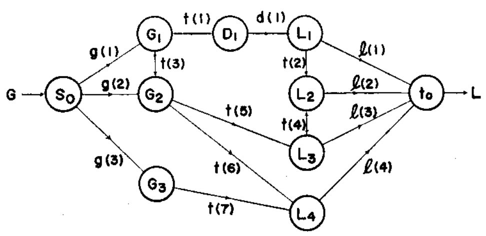

Fig. 2.3. Extended network for the transmission system shown in Fig. 2.1.

the pattern of this flow is interpreted as the power flow distribution of the transmission system shown in Fig. 2.1.

Therefore, in order to investigate whether the demand can be fulfilled from the supplies in the transmission system, calculate the maximal flow from source s_0 to sink t_0 on the extended network shown in Fig. 2.3. Let F_m be the maximal power flow obtained from the calculation and if we put the overall supply power and load power as G and L, these are written as

$$\left.\begin{aligned} G &= \sum_{x\in S} g(x) \\ L &= \sum_{x\in T} l(x) \end{aligned}\right\} \qquad (2.1)$$

where S and T are sets of all sources and all sinks, respectively.

Hence the demand-supply conditions of the transmission system will be classified as follows;

(1) If $F_m=L\leq G$, the demands can be fulfilled from the supplies.

(2) If $F_m=G<L$, the system has a lack of supply power.

(3) If $F_m<L\leq G$, the capacity of the transmission line is insufficient.

(4) If $F_m<G<L$, the system has a lack of supply power and the capacity of the transmission line is insufficient.

When we intend to design the transmission system to satisfy the increasing demand, the extended network corresponding to the transmission system involved plants, substations and transmission lines to be constructed in the future must satisfy the condition (1), i.e., $F_m=L\leq G$.

If we assume that the construction cost is proportional to the capacity for an increase, the optimal (minimal cost) expansion planning of the transmission system can be reduced to the problem of constructing network flows that minimize cost. The minimal cost flow problem has been treated by many authors and the effective algorithms for digital computer are now well known.

However, for example, suppose an increase of plants or transmission lines being already constructed, a generator capacity of plants or a transmission capacity of lines for an increase will be limited by the capacity of the existing generators or lines. Therefore, the cost characteristics are expressed by a staircase function, since we must study the discrete programming problem instead of linear programming problems.

3. Cost Characteristic and Expression of the Capacity of Arc

In this paper, we shall treat such a case that the cost for a construction can be shown in Fig. 3.1, that is, the relation between capacity and cost is expressed by a staircase function. In this case, each arc can be expressed by a parallel connection of arcs having a constant capacity as indicated in Fig. 3.2 where $C^{(0)}(x,y)$ is an arc being already constructed and $\Delta C^{(1)}(x,y)$, $\Delta C^{(2)}(x,y)$, $\cdots$ and $\Delta C^{(m)}(x,y)$ are capacities of increasing arcs.

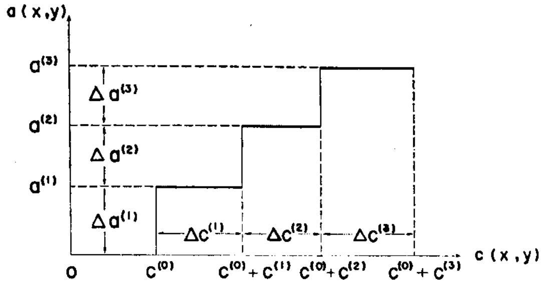

Fig. 3.1. Construction cost characteristic.

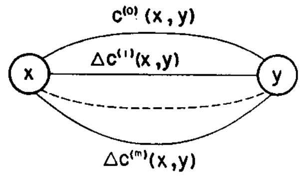

Fig. 3.2. Parallel connection of arc.

Now, introduce the variables $f^{(i)}(x,y)$ and $\Delta f^{(i)}(x,y)$ constrained to take only one of the values 0 or 1. Hence the capacity of arc (x,y), $C(x,y)$, may be written in two ways as follows:

Case 1:

$$C(x,y) = C^{(0)}(x,y) + \sum_{i=1}^{m(x,y)} C^{(i)}(x,y) f^{(i)}(x,y) \tag{3.1}$$

where

$$C^{(i)}(x,y) = \sum_{j=1}^{i} \Delta C^{(j)}(x,y) \tag{3.2}$$

and

$$f^{(i)}(x,y) = \begin{cases} 1, & \text{if } C(x,y) = C^{(0)}(x,y) + C^{(i)}(x,y) \\ 0, & \text{if } C(x,y) \neq C^{(0)}(x,y) + C^{(i)}(x,y) \end{cases} \tag{3.3}$$

In this case, the following constrained relation is required for the variables $f^{(i)}(x,y)$, $(i=0, 1, \cdots, m(x,y))$, namely;

$$\sum_{i=0}^{m(x,y)} f^{(i)}(x,y) = 1 \tag{3.4}$$

where

$$f^{(0)}(x,y) = \begin{cases} 1, & \text{if } C(x,y) = C^{(0)}(x,y) \\ 0, & \text{if } C(x,y) \neq C^{(0)}(x,y) \end{cases} \tag{3.5}$$

Case 2:

$$C(x,y) = C^{(0)}(x,y) + \sum_{i=1}^{m(x,y)} \Delta C^{(i)}(x,y) \Delta f^{(i)}(x,y) \tag{3.6}$$

where the variable $\Delta f^{(i)}(x,y)$ can be taken the value 1 in case of

$$\Delta f^{(1)}(x,y) = \Delta f^{(2)}(x,y) = \cdots \qquad = \Delta f^{(i-1)}(x,y) = 1 \tag{3.7}$$

Introducing the variables $\Delta f_0^{(i)}(x,y)$, $(i=1, 2, \cdots, m(x,y))$ constrained to take only one of the values 0 or 1, we can also express the constraints described above as the constrained equations, that is,

$$\sum_{j=1}^{i} \Delta f_0^{(j)}(x, y)+\Delta f^{(i)}(x, y) = 1 \qquad (i=1, 2, \cdots, m(x,y)) \tag{3.8}$$

4. Formulation as Integer Linear Programs

If any cut separating source s_0 and sink t_0 is expressed by $(X, \bar{X})$ and its capacity by $C(X, \bar{X})$, the necessary and sufficient condition under which the demands can be fulfilled from supplies in the extended network shown in Fig. 2.3 can be written that

$$C(X, \bar{X}) \geq L \qquad (s_0 \in X,\ t_0 \in \bar{X}) \tag{4.1}$$

holds for every subset $X \subseteq N$. (See Appendix)

Using the expression of Eq. (3.1) as an arc capacity, $C(x,y)$, for any cut P_j we obtain the constraint as follows;

$$\sum_{(x,y)\in P_j} \left[\sum_{i=1}^{m(x,y)} C^{(i)}(x,y) f^{(i)}(x,y)\right] + \sum_{(x,y)\in P_j} C^{(0)}(x,y) \geq L \quad (j=1, 2, \cdots, n) \tag{4.2}$$

or

$$\sum_{(x,y)\in P_j} \left[\sum_{i=1}^{m(x,y)} C^{(i)}(x,y) f^{(i)}(x,y)\right] \geq L - \sum_{(x,y)\in P_j} C^{(0)}(x,y) \quad (j=1, 2, \cdots, n) \tag{4.3}$$

where n is a number of cut separating source s_0 and sink t_0. In the case of using these expressions, another constraints is required for the variables $f^{(i)}(x, y)$, $(i=0, 1, \cdots, m(x, y))$ as

$$\sum_{i=0}^{m(x,y)} f^{(i)}(x, y) = 1 \tag{4.4}$$

In Eq. (4.2) or (4.3), if the inequality

$$\sum_{(x,y)\in P_j} C^{(0)}(x, y) \geq L \tag{4.5}$$

will be satisfied for any cut P_j, that is, the cut capacity being already constructed exceeds an overall load power L, we take away the inequality corresponding to cut P_j from Eq. (4.3).

The optimal transmission planning for a given load growth pattern is to determine the number $f^{(i)}(x,y)$ in order that the overall construction cost

$$\sum_{(x,y)\in A} \left[\sum_{i=1}^{m(x,y)} a^{(i)}(x, y) f^{(i)}(x, y)\right] \tag{4.7}$$

remains a minimum, where A is a set of all arcs and

$$a^{(i)}(x, y) = \sum_{j=1}^{i} a^{(j)}(x, y) \tag{4.8}$$

Consequently, we have to find the values of $f^{(i)}(x, y)$ which reduce the function Eq. (4.7) to a minimum, knowing that the variables are subject to the constrained conditions Eqs. (4.3) and (4.4).

Futhermore, when we use the expression of Eq. (3.6) as an arc capacity $C(x, y)$, another linear program will be obtained, namely;

The optimal system planning is to find the number $\Delta f^{(i)}(x, y)$ constrained to take only one of the values 0 or 1 which minimizes the linear form

$$\sum_{(x,y)\in A}\left[\sum_{i=1}^{m(x,y)} \Delta a^{(i)}(x, y)\, \Delta f^{(i)}(x, y)\right] \tag{4.9}$$

subject to the linear constraints

$$\sum_{(x,y)\in P_j}\left[\sum_{i=1}^{m(x,y)} \Delta C^{(i)}(x, y)\, \Delta f^{(i)}(x, y)\right] \geq L - \sum_{(x,y)\in P_j} C^{(0)}(x, y) \qquad (j=1, 2, \cdots, n) \tag{4.10}$$

and

$$\sum_{j=1}^{i} \Delta f_0^{(j)}(x, y) + \Delta f^{(i)}(x, y) = 1 \qquad (j=1, 2, \cdots, m(x, y)),\ (x, y)\in A \tag{4.11}$$

Therefore, the problem of the optimal transmission planning is to solve a special type of integer linear programming problem in which the integer variable has to be either 0 or 1, depending on whether or not some increase of the system element is used.

At present, several methods are available for solving integer linear programs. The best known among them is R.E. Gomory's algorithm. They use the dual simplex method and impose the integer condition by adding new constraints to the original constraint set. Another effective algorithm which is proposed by B. Ealas represents a combinatorial approach to the problem of solving discrete-variable linear programs with zero-one variables and it seems to work very efficiently.

5. Extension to the Long Term Design

The optimal long term planning for load growth should have the lowest cost of accumulation of annual requirments. Therefore the planning problem is to find the economically optimal sequence.

If we adopt n years as the period considered for the planning, the capacity of arc (x, y) may be written as

$$\begin{aligned} C(x, y) = C^{(0)}(x, y) &+ \Delta C^{(1)}(x, y)\{\Delta f_1^{(1)}(x, y) + \Delta f_2^{(1)}(x, y) + \cdots + \Delta f_n^{(1)}(x, y)\} \\ &+ \Delta C^{(2)}(x, y)\{\Delta f_1^{(2)}(x, y) + \Delta f_2^{(2)}(x, y) + \cdots + \Delta f_n^{(2)}(x, y)\} \\ &+ \cdots\cdots \\ &+ \Delta C^{(m)}(x, y)\{\Delta f_1^{(m)}(x, y) + \Delta f_2^{(m)}(x, y) + \cdots + \Delta f_n^{(m)}(x, y)\} \end{aligned} \tag{5.1}$$

where $\Delta f_j^{(i)}(x,y)$ is a zero-one variable having to be either 1 or 0, depending upon whether or not the i-th increasing element of arc (x,y) is constructed at the j-th year. In this case, the following constrained relations are required for the variables $\Delta f_j^{(i)}(x,y)$ and newly introduced variables $\Delta f_{0j}^{(i)}(x,y)$, namely;

$$\sum_{j=0}^{n} \Delta f_j^{(i)}(x,y) = 1 \qquad (i=1,2,\cdots,m(x,y)) \tag{5.2}$$

and

$$\left.\begin{aligned} &\Delta f_{0j}^{(1)}(x,y)+\sum_{i=1}^{j}\Delta f_i^{(1)}(x,y) = 1 \\ &\Delta f_{0j}^{(1)}(x,y)+\Delta f_{0j}^{(2)}(x,y)+\sum_{i=1}^{j}\Delta f_i^{(2)}(x,y) = 1 \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &\Delta f_{0j}^{(1)}(x,y)+\cdots+\Delta f_{0j}^{(2)}(x,y)+\sum_{i=1}^{j}\Delta f_i^{(n)}(x,y) = 1 \\ &\qquad\qquad (j=1,2,\cdots,n) \end{aligned}\right\} \tag{5.3}$$

By using Eq. (5.1) as the expression of the arc capacity, $C(x,y)$, we can obtain the similar integer linear program as obtained in the proceeding chapter. In the present case a criterion function (linear form having to minimize) may be expressed by

$$\sum_{(x,y)\in A}\left[\sum_{i=1}^{m(x,y)}\sum_{j=1}^{n}\Delta a_j^{(i)}(x,y)\,\Delta f_j^{(i)}(x,y)\right] \tag{5.4}$$

The linear program obtained in this fashion has an enormous number of constrained equations such as Eqs. (5.2) and (5.3). However, using the algorithm obtained by the principle of a combinatorial approach for solving an integer linear programming problem, we can deal with these constraints as a criterion for selecting the basic feasible solutions.

References

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3) R.E. Gomory; "An Algorithm for Integer Solutions to Linear Programs", in R.L. Graves and P.H. Wolfe (eds.), "Recent Advances in Mathematical Programming", pp. 269~302, McGraw-Hill, New York (1963)
4) E. Balas; Op. Res. vol. 12, pp. 517~546 (1965)

Appendix

Let $[N; A]$ be an arbitrary network with capacity function C, and suppose that N (set of all nodes) is partitioned into sources S, intermediate nodes R and sinks T. Associate with each $x \in S$ a non-negative number $a(x)$, to be thought of as the supply of some commodity at x, and with each $x \in T$ a non-negative number $b(x)$, the demand for the commodity at x. Under what conditions can the demands at the sinks be fulfilled from the supplies at the sources? The following theorem gives the answer for the above problem.

Theorem (Supply-Demand Theorem)

$$\left.\begin{array}{ll} f(x, N)-f(N, x) \leq a(x) & x \in S \\ f(x, N)-f(N, x) = 0 & x \in R \\ f(x, N)-f(N, x) \geq b(x) & x \in T \\ 0 \leq f(x, y) \leq C(x, y) & (x, y) \in A \end{array}\right\} \quad \text{(A,1)}$$

where $a(x) \geq 0$, $b(x) \geq 0$, are feasible if and only if

$$b(T \cap X) - a(S \cap \bar{X}) \leq C(X, \bar{X}) \quad \text{(A,2)}$$

holds for every subset $X \subseteq N$.

Where the commodity flow from x to y and the capacity of the cut $(X, \bar{X})$ are respectively expressed by $f(x, y)$ and $C(X, \bar{X})$, and

$$f(X, Y) = \sum_{(x, y) \in (X, Y)} f(x, y) \quad \text{(A,3)}$$

Extend the network $[N; A]$ to a new network $[N^*; A^*]$ by adjoining a fictious source s_0, sink t_0 and arcs (s_0, S), (T, t_0). The capacity function on A^* is defined by

$$\left.\begin{array}{ll} C^*(s_0, x) = a(x) & x \in S \\ C^*(x, t_0) = b(x) & x \in T \\ C^*(x, y) = C(x, y) & (x, y) \in A \end{array}\right\} \quad \text{(A,4)}$$

On the new network $[N^*; A^*]$, the cut separating s_0 and t_0 and its capacity are respectively expressed by $(Y, \bar{Y})$ and $C'(Y, \bar{Y})$. Since $Y = X \cup s_0$, $\bar{Y} = \bar{X} \cup t_0$, we have

$$C(X, \bar{X}) = C'(Y, \bar{Y}) - a(S \cap \bar{X}) - b(T \cap X) \quad \text{(A,5)}$$

By using Eq. (A,5), Eq. (A,2) yields

$$C'(Y, \bar{Y}) \geq b(T \cap X) + b(T \cap \bar{X}) = \sum_{x \in T} b(x) = L \quad \text{(A,6)}$$

Eq. (A, 6) is the necessary and sufficient condition under which the demand at the sink can be fulfilled from the supply at the source on the extended network $[N^*; A^*]$.