A Theoretical Model of Rapid Transit System Planning Within a Metropolitan Area

By

KOZO AMANO* and Yoshitaka AOYAMA*

(Received December 28, 1967)

The increasing process of the total population in a metropolitan area is based on economic and sociologic problems. But the variation of the population distribution within each zone of the metropolitan area is, much more effected by the transportation system. The zonal population is the occurrence source of the commuter transportation demand, so the interaction between the commuter transportation system and the zonal population distribution should be analyzed first, in order to establish the transportation system planning. In this theses we have analyzed the interaction by applying the information theory.

Consequently, it could be said that the variation of the zonal population distribution maximizes entropy per unit characteristic value.

Then we showed that distributed and diverted transportation volume of commutation can be presumed in making the transportation system the endogenous variable and making the employee population and the transit fares policy the exogenous variable. And then we investigated how to evaluate the transportation system planning with measurement. Evidently economic and sociologic research is needed for this problem, but at first we tried to approach it in a physical respect.

At the end, utilizing these analyses, we proposed a practical means for transportation system planning in a metropolis. The effect of any projected transportation system, total population in the future, and transit fares policy in the metropolitan area will be measured by computing through this means. We showed the applying process in the flow diagram.

1. Introduction

Rapid transit system planning in a metropolitan area is a partial field in comprehensive urban planning. But first it should be taken up, when urban planning is studied in the physical aspect. The reason is that commuter transportation facilities are not only important for practical traffic demand, but also have much effect on further development of metropolitan area. In this thesis we will take up first the interaction between commuter transportation system which has been mentioned above and growth of urbanized area, population, traffic volume and so on. Then we will analyse them by applying the information theory and

---

* Department of Transportation Engineering

the regression analysis. Finally we will show a practical process of transportation system planning which produces a good effect upon the metropolitan area in the future.

2. The Mechanism of Suburbanization Influenced by Rapid Transit Network

Urban households will survey residential conditions of suburban areas such as accessibility to C.B.D., land price, natural and social environment, before they make a decision about their house's location.

In order to analyze the residential informations processing, the notations will be given in the following model of metropolitan urbanization.

1) A metropolis has been divided to λ zones which have equal area s.
2) t_i is urban life prevention coefficient in zone i.
3) Residential districts are located in suburban areas and they are distinguished from central districts.
4) The probability that one population falls into zone i would be $1/\lambda$, if all of households can't get any residential information about each zone.
5) But the probability that one population falls into zone i after getting residential information is not yet known.

We have to find the probability p_i in this problem. Then we have

$$n_i = p_i \cdot N \tag{1}$$

$$\therefore \sum_{i=1}^{\lambda} p_i = \sum_{i=1}^{\lambda} \frac{n_i}{N} = 1 \tag{2}$$

in which

n_i : population in zone i
N : total population in the metropolitan area.

The probability U realizing the micro-condition that population n_i is residing in zone i, will be written as

$$U = \prod_{i=1}^{\lambda} \left(\frac{1}{\lambda}\right)^{n_i} = \lambda^{-N} \tag{3}$$

The same micro-condition may also be realized with population interchanging of inhabitants within each zone which is considered having no effect. Therefore the number of micro-conditions V may be expressed as

$$V = \frac{N!}{\prod_{i=1}^{\lambda} n_i!} \tag{4}$$

Hence the combined probability $\boldsymbol{W}$ realizing a pattern of population distribution, that assumed for macro-condition, will be a product of $\boldsymbol{U}$ and $\boldsymbol{V}$ as follows

$$\boldsymbol{W} = \frac{N!}{\prod_{i=1}^{\lambda} n_i!} \cdot \lambda^{-N} \tag{5}$$

In a condition in which a population has not any residential information, a pattern of population distribution will be realized as the combined probability reaching maximum. And naturally the maximum of $\boldsymbol{W}$ will be reached with that of $\boldsymbol{V}$ at the same time, because $\boldsymbol{U}$ is constant, as mentioned in Eq. 3. Logarithmic equation for $\boldsymbol{V}$ will be written in the form

$$\log \boldsymbol{V} = \log N! - \sum_{i=1}^{\lambda} \log n_i! \tag{6}$$

by using Stirling Formula

$$\begin{aligned} \log \boldsymbol{V} &= N \log N - N - \sum_{i=1}^{\lambda} (n_i \log n_i - n_i) \\ &= N \log N - \sum_{i=1}^{\lambda} n_i \log n_i \end{aligned} \tag{7}$$

Substituting Eq. 1 to Eq. 7

$$\begin{aligned} \log \boldsymbol{V} &= N \log N - \sum_{i=1}^{\lambda} p_i N \log (p_i N) \\ &= -N \cdot \sum_{i=1}^{\lambda} p_i \log p_i \\ &= N \cdot \boldsymbol{H} \end{aligned} \tag{8}$$

in which

$$\boldsymbol{H} = -\sum_{i=1}^{\lambda} p_i \log p_i$$

We shall call $\boldsymbol{H}$ entropy of population distribution. Now we can get residential information and life prevention coefficient t_i, which is the characteristic value in zone i. The total life prevention volume in Metropolitan area is

$$\sum_{i=1}^{\lambda} n_i t_i = N \cdot \sum_{i=1}^{\lambda} p_i t_i \tag{9}$$

We shall call it loss energy. Urban inhabitants have a strong will to decrease the loss energy. Next we can get the following formula showing entropy per unit loss energy according to the information theory.

$$\frac{-N \cdot \sum_{i=1}^{\lambda} p_i \log p_i}{\sum_{i=1}^{\lambda} n_i t_i} = \frac{-\sum_{i=1}^{\lambda} p_i \log p_i}{\sum_{i=1}^{\lambda} p_i t_i} = \frac{\boldsymbol{H}}{\bar{t}} \tag{10}$$

We assume that the most realizable pattern of distribution is considered to be attained when the entropy per loss energy becomes maximum. The extreme value of p_i, when the value of Eq. 10 reaches its maximum according to the conditional Eq. 2, may be determined by means of Lagrange's mathematical formula. In solving this problem, let us apply Lagrange's function with constant π as follows

$$F(p_i, \pi) = \frac{\boldsymbol{H}}{\bar{t}} + \pi \Bigl(\sum_{i=1}^{\lambda} p_i - 1\Bigr) \tag{11}$$

Differentiating Eq. 11 with respect to p_i and π, we have the simultaneous equations

$$\frac{\partial F}{\partial p_i} = \frac{(-1-\log p_i)\sum_{i=1}^{\lambda} p_i t_i - \bigl(-\sum^{\lambda} p_i \log p_i\bigr)\cdot t_i}{\bigl(\sum_{i=1}^{\lambda} p_i t_i\bigr)^2} + \pi = 0 \tag{12}$$

$$\frac{\partial F}{\partial \pi} = \sum_{i=1}^{\lambda} p_i - 1 = 0 \tag{13}$$

by multiplying p_i to Eq. 12 and summalizing as follows

$$\frac{\bigl(-1-\sum_{i=1}^{\lambda} p_i \log p_i\bigr)\cdot\sum_{i=1}^{\lambda} p_i t_i - \bigl(-\sum_{i=1}^{\lambda} p_i \log p_i\bigr)\cdot\sum_{i=1}^{\lambda} p_i t_i}{\bigl(\sum_{i=1}^{\lambda} p_i t_i\bigr)^2} + \pi = 0 \tag{14}$$

where

$$-\sum_{i=1}^{\lambda} p_i \log p_i = \boldsymbol{H}, \qquad \sum_{i=1}^{\lambda} p_i t_i = \bar{t} \tag{15}$$

From Eq. 14

$$-1 + \boldsymbol{H} - \boldsymbol{H} + \pi\cdot\bar{t} = 0, \qquad \therefore\ \pi = 1/\bar{t} \tag{16}$$

From Eq. 14 and Eq. 16, we obtain

$$\log p_i = -\frac{\boldsymbol{H}}{\bar{t}}\cdot t_i, \qquad \therefore\ p_i = e^{-(H/\bar{t})\cdot t_i} \tag{17}$$

Eq. 2 becomes

$$\sum_{i=1}^{\lambda} e^{-(H/\bar{t})\cdot t_i} = 1 \tag{18}$$

By substituting $e^{-H/\bar{t}} = X$, Eq. 18 becomes

$$\sum_{i=1}^{\lambda} X^{t_i} = 1 \qquad (0 < X < 1) \tag{19}$$

Thus, solution p_i making Eq. 11 maximum is

$$p_i = X_0^{t_i} \qquad (i = 1, 2, \cdots, \lambda) \tag{20}$$

where X_0 is a positive real root of Eq. 19, which can be determined by numerical analysis. The specific ratio of population density distribution for each zone is equal to that of population, because areas of respective zones are uniformly equal to s. Hence for a general urban model, which consists of several zones being different in areas, distribution ratios of population density may be applied to contervail the influence due to their difference in areas.

Now we have Eq. 21, where ρ_i is a population density and s_i is area in zone i.

$$\rho_i = K \cdot p_i \qquad (i = 1, 2, \cdots, \lambda) \tag{21}$$

$$\therefore \quad n_i = \rho_i \cdot s_i = K \cdot p_i \cdot s_i \tag{22}$$

in which K is constant.

Table 1. Population in each zone (Tokyo)

* : evaluated value

		1950		1955		1960		1965	
zone	t_i (min.)	n_i	n_i*	n_i	n_i*	n_i	n_i*	n_i	n_i*
1	35	745	814	934	981	1,139	1,169	1,384	1,518
2	40	682	817	859	985	1,038	1,174	1,337	1,524
3	45	655	632	824	761	1,006	907	1,213	1,178
4	50	534	664	662	800	877	954	1,105	1,239
5	55	353	614	462	739	610	881	884	1,145
6	60	386	551	479	665	676	792	992	1,029
7	65	383	510	461	615	602	732	858	951
8	70	366	461	422	555	526	662	756	860
9	75	339	326	398	392	467	468	691	607
10	80	405	447	501	541	586	645	839	837
11	85	155	171	190	206	231	246	322	319
12	90	413	330	482	397	506	474	663	615
13	95	288	199	322	240	340	286	406	372
14	100	159	127	183	153	201	183	295	238
15	105	134	77	139	93	135	111	138	144
16	110	235	103	250	124	269	148	301	193
17	115	164	69	188	83	201	99	224	129
18	120	164	47	173	57	176	68	181	88
19	125	124	36	144	43	145	52	168	67
20	130	161	43	174	52	172	61	173	80
21	135	89	21	91	25	90	29	91	38
22	140	57	14	59	16	54	20	58	26
23	145	61	12	61	14	59	17	60	22
24	150	45	11	47	13	44	16	52	21
25	155	57	11	60	13	59	15	62	20
Total		7,107	7,107	8.565	8,565	10,209	10,209	13,258	13,258

(unit: 1000 persons)

We have

$$\sum_{i=1}^{\lambda} n_i = N \tag{23}$$

$$\therefore \quad K = \frac{N}{\sum_{i=1}^{\lambda} p_i s_i} \tag{24}$$

so that

$$\rho_i = \frac{p_i}{\sum_{i=1}^{\lambda} p_i s_i} \cdot N \tag{25}$$

$$n_i = \frac{p_i s_i}{\sum_{i=1}^{\lambda} p_i s_i} \cdot N \tag{26}$$

Table 2. Population Density in Each Zone (Tokyo)

* : evaluated value

		1950		1955		1960		1965	
zone	t_i (min.)	ρ_i	ρ_i*	ρ_i	ρ_i*	ρ_i	ρ_i*	ρ_i	ρ_i*
1	35	6,324	6,909	7,929	8,326	9,669	9,924	11,749	12,888
2	40	4,596	5,505	5,788	6,635	6,995	7,908	9,009	10,270
3	45	4,549	4,386	5,722	5,286	6,986	6,301	8,458	8,183
4	50	2,811	3,495	3,484	4,212	4,616	5,020	5,816	6,519
5	55	1,602	2,785	2,097	3,356	2,769	4,000	4,012	5,195
6	60	1,541	2,219	1,927	2,674	2,720	3,188	3,992	4,140
7	65	1,269	1,763	1,599	2,131	2,087	2,540	2,975	3,298
8	70	1,035	1,409	1,290	1,698	1,608	2,024	2,311	2,628
9	75	1,167	1,123	1,372	1,353	1,610	1,613	2,383	2,095
10	80	807	895	999	1,078	1,168	1,285	1,673	1,669
11	85	606	713	792	859	963	1,024	1,342	1,329
12	90	711	568	830	684	871	316	1,142	1,059
13	95	654	452	731	545	772	650	921	844
14	100	450	361	518	435	569	518	835	763
15	105	499	287	518	346	503	413	514	536
16	110	521	229	554	276	596	329	667	427
17	115	432	182	496	220	530	262	591	340
18	120	505	145	532	175	542	209	557	271
19	125	397	116	461	139	465	166	538	216
20	130	347	92	375	111	371	132	373	172
21	135	319	74	326	89	323	106	326	137
22	140	244	59	253	71	232	84	249	109
23	145	248	47	248	56	239	67	243	87
24	150	152	37	158	45	148	54	175	70
25	155	156	30	164	36	161	42	170	55

(unit: person/km^2)

Thus, by substituting p_i of Eq. 20 to Eq. 25 and Eq. 26, we can estimate the population density ρ_i and population n_i in each zone. However we can not yet explain the characteristic value t_i of zone i in the term of measurement. Therefore, at the present, the approximation of the characteristic value may be used for the evidential investigation. The most evident measure among factors of the characteristic value is the accessibility or time-distance from each zone to C.B.D.. Actual and estimated values of population, population density and population density ratio in suburban zones within Tokyo Metropolis are shown in Table 1, 2, 3.

Table 3. Population Density Ratio in Each Zone (Tokyo)

* : evaluated value

zone	t_i (min.)	p_i				p_i*
		1950 year	1955	1960	1965	
1	35	0.19773	0.20245	0.20350	0.19253	0.20390
2	40	0.14368	0.14780	0.14722	0.14764	0.16247
3	45	0.14221	0.14610	0.14704	0.13861	0.12945
4	50	0.08787	0.08896	0.09715	0.09531	0.10314
5	55	0.05010	0.05355	0.05828	0.06576	0.08219
6	60	0.04819	0.04922	0.05725	0.06542	0.06549
7	65	0.03968	0.04081	0.04393	0.04875	0.05218
8	70	0.03240	0.03294	0.03385	0.03789	0.04158
9	75	0.03655	0.03504	0.03389	0.03905	0.03314
10	80	0.02524	0.02550	0.02459	0.02741	0.02640
11	85	0.02020	0.02022	0.02027	0.02200	0.02103
12	90	0.02223	0.02119	0.01834	0.01871	0.01076
13	95	0.02043	0.01866	0.01624	0.01510	0.01335
14	100	0.01407	0.01323	0.01198	0.01369	0.01064
15	105	0.01561	0.01323	0.01059	0.00843	0.00848
16	110	0.01628	0.01414	0.01254	0.01093	0.00675
17	115	0.01352	0.01266	0.01115	0.00967	0.00538
18	120	0.01578	0.01359	0.01140	0.00913	0.00429
19	125	0.01242	0.01178	0.00978	0.00882	0.00341
20	130	0.01085	0.00958	0.00780	0.00671	0.00272
21	135	0.00997	0.00833	0.00679	0.00535	0.00217
22	140	0.00764	0.00646	0.00487	0.00407	0.00173
23	145	0.00774	0.00632	0.00504	0.00399	0.00138
24	150	0.00474	0.00404	0.00312	0.00287	0.00110
25	155	0.00487	0.00419	0.00339	0.00278	0.00087
Total		1.00000	1.00000	1.00000	1.00000	1.00000
H		1.16116	1.11426	1.12253	1.12382	1.06895
$\bar{t}$		60.62616	59.21663	57.67546	57.41117	54.17671
H $/\bar{t}$		0.01915	0.01929	0.01946	0.01957	0.01973

The difference between actual and estimated values will be caused by the other factors of characteristic value, natural and cosial conditions, etc.. Therefore, in macro-scope-analysis, the population distribution in metropolitan area can be estimated by time-distances as the characteristic value of respective zones, but it will be better when natural and social conditions can be measured additionaly.

3. Eastimation of Volume of Commuting Passengers

At first, it is necessary for us to estimate the generating volume of commuting passengers. In the macro-condition such as Tokyo Metropolis or Osaka Metropolis, several destinations may be coordinated in one region in which many commercial organizations concentrated, for example, inner area within the railway loop-lines of Tokyo and Osaka Metropolis.

In chapter 2 we have proposed an estimation method for population distribution within a metropolitan area. So in this chapter we will consider the ratio of generating commuter per population in each zone. We called commuter generating ratio α_i, which is shown in Eq. 27.

$$\alpha_i = a \cdot k_i^b \cdot e^{-ct_i} \tag{27}$$

where

α_i: commuter generating ratio in zone i
k_i: inflow commuters/outflow commuters
t_i: time-distance from zone i to C.B.D. (min.)
a, b, c: constants

The conditional equation is

$$\sum_{i=1}^{\lambda} \alpha_i n_i = \sum_{i=1}^{\lambda} m_i = M \tag{28}$$

in which

n_i: population in zone i
m_i: generating commuters in zone i
M: total commuters into C.B.D.

We can obtain the following equations from Eq. 27 and Eq. 28.

$$a = \frac{M}{\sum_{i=1}^{\lambda} n_i k_i^b e^{-ct_i}} \tag{29}$$

$$\therefore \quad m_i = \frac{n_i k_i^b e^{-ct_i}}{\sum_{i=1}^{\lambda} n_i k_i^b e^{-ct_i}} \cdot M \tag{30}$$

We have calculated constants, a, b and c as shown in Table 4 by multiple regression analysis for Osaka Metropolis (1967).

Table 4. Regression Coefficients.

	for OSAKA	*for* KYOTO	*for* KOBE
a	0.2794	0.1961	0.0964
b	−0.9463	−0.8737	−0.2979
c	0.0344	0.0314	0.0202
m.r.	0.8906	0.8655	0.7473

It is possible to forecast future generating commuter volume m_i^* by substituting these constants and projected values of n_i^*, k_i^*, t_i^* and M^* in the future to Eq. 30.

There are multi-traffic-routes generally, for each origin and destination of unban commuters. Each commuter selects one of them respectively comparing with some criterias. And also the commuting passenger is a daily mass, regular flow, and the statistical criteria for selection seems to be settled pretty consciously. So this regular flow can be analyzed statistically. The relation between characteristics of the route and diverted traffic volume in each route have been analyzed in various ways. As it has been defined by Dr. K. Amano that a diverted ratio is got by dividing traffic volume in the route by OD traffic volume, and that the diverted ratio is statistically espressed as

$$p_{ij} = \alpha\left(t_{ij} - \frac{\sum_{j=1}^{r_i} t_{ij}}{r_i}\right) + \beta\left(c_{ij} - \frac{\sum_{j=1}^{r_i} c_{ij}}{r_i}\right) + \delta \tag{31}$$

where

i : a certain origin-destination
p_{ij}: diverted ratio from zone i to route j
t_{ij}: time-distance from zone i to C.B.D. passing route j
c_{ij}: passenger fares from zone i to C.B.D. passing route j
r_i : number of routes from zone i to C.B.D.
α, β, δ: contants

The diverted ratio p_{ij} must satisfy the following equation

$$\sum_{j=1}^{r_i} p_{ij} = 1 \tag{32}$$

substituting Eq. 31 to Eq. 32

Table 5. Commuter Diverted Traffic Ratio in Tokyo Metropolis.

* : evaluated value

i	j	p_{ij}	t_{ij}	c_{ij}	p_{ij}^*
1	1	0.220	33.00	660	0.086
	2	0.780	24.00	630	0.914
2	1	0.240	29.00	660	0.331
	2	0.760	25.00	750	0.669
3	1	0.152	31.00	690	0.327
	2	0.848	27.00	750	0.673
4	1	0.880	33.00	710	0.825
	2	0.120	40.00	750	0.175
5	1	0.120	29.00	660	0.103
	2	0.880	20.00	750	0.897
6	1	0.827	23.00	490	1.130
	2	0.173	36.00	750	0.130
7	1	0.871	27.00	590	0.742
	2	0.129	32.00	690	0.258
8	1	0.111	29.00	660	0.149
	2	0.889	21.00	750	0.851
9	1	0.152	31.00	690	0.327
	2	0.848	27.00	750	0.673
10	1	0.765	33.00	710	0.642
	2	0.235	36.00	750	0.358
11	1	0.879	22.00	540	0.795
	2	0.121	28.00	690	0.205
12	1	0.181	30.00	590	0.071
	2	0.819	20.00	770	0.929
13	1	0.981	19.00	420	0.958
	2	0.019	28.00	750	0.042
14	1	0.989	22.00	490	0.993
	2	0.011	32.00	750	0.007
15	1	0.216	32.00	710	0.293
	2	0.784	28.00	420	0.707
16	1	0.997	19.00	420	1.000
	2	0.003	30.00	750	0.000
17	1	0.985	22.00	490	1.000
	2	0.015	34.00	750	0.000
18	1	0.273	29.00	630	0.244
	2	0.727	23.00	750	0.756
19	1	0.705	25.00	630	0.791
	2	0.295	31.00	750	0.209
20	1	0.157	23.00	660	0.377
	2	0.843	20.00	750	0.623
21	1	0.429	36.00	770	0.452
	2	0.571	35.00	750	0.548
22	1	0.804	25.00	630	0.745
	2	0.196	30.00	750	0.255
23	1	0.250	26.00	690	0.372
	2	0.750	23.00	750	0.628
24	1	0.428	28.00	690	0.372
	2	0.572	25.00	750	0.628
25	1	0.141	24.00	660	0.331
	2	0.859	20.00	750	0.669
26	1	0.695	28.00	710	0.543
	2	0.305	29.00	690	0.457
27	1	0.911	29.00	660	0.738
	2	0.089	34.00	730	0.262
28	1	0.468	30.00	730	0.353
	2	0.532	27.00	660	0.647
29	1	0.070	33.00	710	0.133
	2	0.930	25.00	690	0.867
30	1	0.127	33.00	660	0.280
	2	0.873	28.00	710	0.720

i : a certain OD
j : a certain route
p_{ij} : Commuter Diverted Traffic Ratio
t_{ij} : time-distance (unit: minute)
c_{ij} : passenger fare (unit: yen)

$$\alpha\left(\sum_{j=1}^{\gamma_i} t_{ij}-\frac{r_i\cdot\sum_{j=1}^{\gamma_i} t_{ij}}{r_i}\right)+\beta\left(\sum_{j=1}^{\gamma_i} c_{ij}-\frac{r_i\cdot\sum_{j=1}^{\gamma_i} c_{ij}}{r_i}\right)+r_i\cdot\delta = 1 \tag{33}$$

$$\therefore\quad \delta=\frac{1}{r_i} \tag{34}$$

and substituting Eq. 34 to Eq. 31

$$p_{ij}-\frac{1}{r_i}=\alpha\left(t_{ij}-\frac{\sum_{j=1}^{\gamma_i} t_{ij}}{r_i}\right)+\beta\left(c_{ij}-\frac{\sum_{j=1}^{\gamma_i} c_{ij}}{r_i}\right) \tag{35}$$

Diverted ratio function is obtained by computing regression coefficients by multiple regression analysis using sampling data p_{ij}, t_{ij} and c_{ij}. Assuming that the stations along suburban lines are origins and the stations on a loop line are destinations, sampling data from diverted commuting passengers in Tokyo Metroplis are shown in Table 5.

The regression model is

$$p_{ij}-\frac{1}{r_i} = -5.46368\left(t_{ij}-\frac{\sum_{j=1}^{\gamma_i} t_{ij}}{r_i}\right)-0.29281\left(c_{ij}-\frac{\sum_{j=1}^{\gamma_i} c_{ij}}{r_i}\right) \tag{36}$$

The correlation coefficient is 0.93086.

And the unit are

t_{ij}: hour
c_{ij}: 1,000 yen

In this way, the diverted ratio function is decided, including time-distance and passenger fares as parameter.

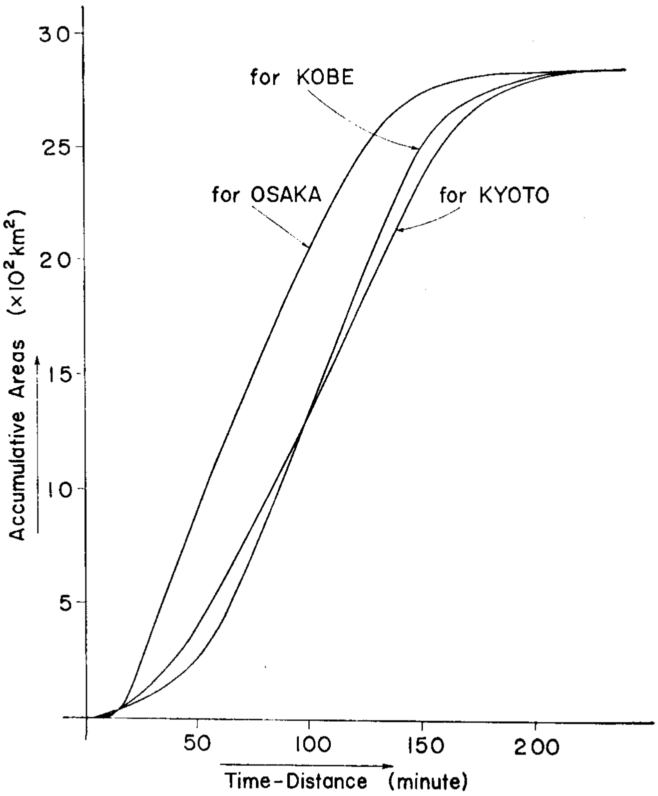
Diverted traffic volume m_{ij} to route j can be calculated by m_i from Eq. 30 and p_{ij} from Eq. 36. as follows

$$m_{ij}=m_i\cdot p_{ij} \tag{37}$$

4. The Criteria for Planning Evaluation

A) Time-Distance and Accumulative Areas

The location of cities and the progress of urbanization in a metropolitan area have been heavily influenced by available means of transit network. And the metropolitan area has been growing in accordance with the development of the urban transportation system. The Sprawling of urbanized areas is not always desirable, but the accumulative area of regions from which commuters can flow into C.B.D. within a short time, is a concrete effect of the urban transportation system.



(available regions as residential districts in Osaka Metropolis)

Fig. 1

The accumulative areas in Osaka Metropolis are shown in Fig. 1. Assuming that some proposed transportation systems are completed, the accumulative curve will grow up in comparison with present conditions. And we can calculate the evident effects of the additional transportation systems by the differences of two accumulative curves.

B) Saving the Time-Distance

Principal requirements for urban commuter transportation system are rapid, safe and comfort. In this section we paid attention to the item concerning the scheduled speed of transit. All commuters have been expecting to save their time-distances by means of transportation planning. Now we consider the loss energy which is enforced on all commuters for their travel, as shown in Eq. 38.

$$I = \sum_j \sum_i m_i p_{ij} t_{ij} \tag{38}$$

in which

I : total loss energy for all commuters in metropolitan area
m_{ij}: commuting passengers of OD i
p_{ij} : diverted ratio of generated commuter to route j within OD i
t_{ij} : time-distance passing route j within OD i

And the loss energy per unit commuter is

$$J = \frac{I}{\sum_{i=1}^{\lambda} m_i} \tag{39}$$

Saving of I or J will be a principal measure in evaluating for transportation planning.

5. A Process of Commuter Transportation System Planning

By systematization of the above mentioned analysis, we have proposed a

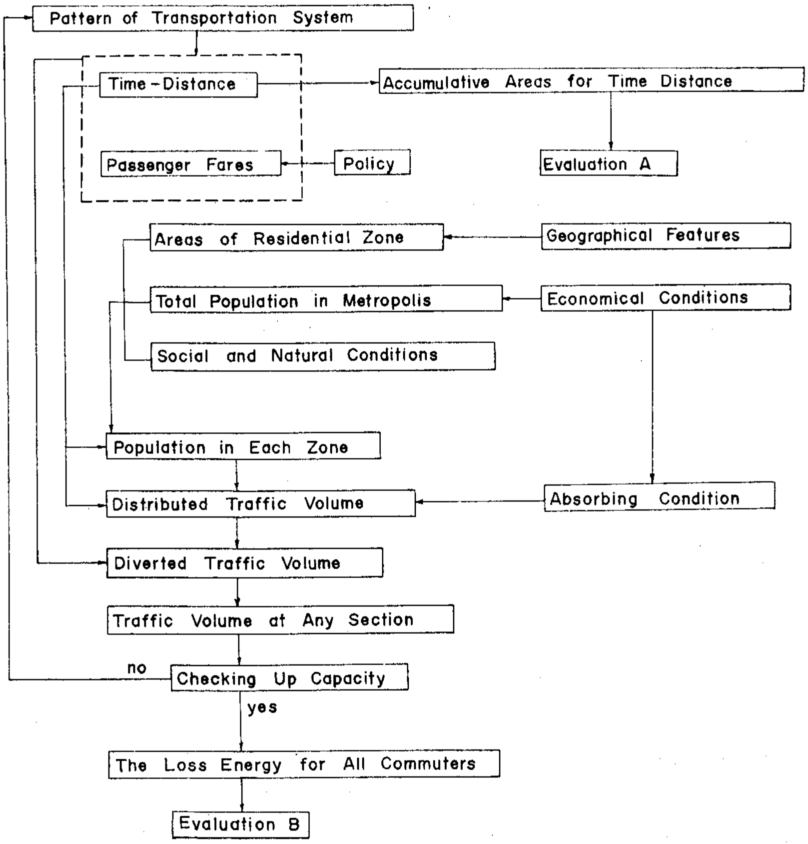


Fig. 2

mathematical process for planning of commuter transportation system as shown in Fig. 2. The steps of forecasting process are as follows.

1) At first, we suppose a certain pattern of the transportation systems which we want to compare respectively.

2) According to the supposed system, time-distances and passenger fares from each residential zone to C.B.D. can be surveyed.

3) "Accumulated Areas——Time-Distance Curve" can be drawn up by time-distance and areas in each zone.

4) Population in each zone is estimated by Eq. 26, in which time-distances are taken as endogenous variables and total population, social and natural conditions are taken as exogenous variables. Total population in a metropolitan area will be given principally by the economic conditions of the metropolis.

5) Distribution traffic volumes are estimated by Eq. 30, in which time-distances and population in each zone have been given already and absorbing conditions can be given in according to the commercial activities in C.B.D..

6) Diverted ratios are calculated by Eq. 36, in which time-distances and passenger fares are used as parameters. And then diverted traffic volume can be estimated by diverting the distribution traffic volume which is mentioned above.

7) The traffic volume at any section is given by the sum of the diverted traffic volume passing there. And we can check this pattern in comparison with capacity at each section.

8) If the capacities cover the traffic demands at all sections, the total loss energy for all commuters can be calculated by substituting time-distances and diverted traffic volumes to Eq. 38.

9) This process has to be repeated for all patterns which we want to compare with each other.

10) As the result, we can find the most desirable pattern according to evaluation A and B.

6. Conclusion

We think that this process of transportation system planning has two specific features. One is considering the transportation system as the inducing means of the population distribution and transportation requirements, and the other is expressing the planning process in the term of measurements.

In contrary by reason of these features themselves, it has some weak points. It is due to the fact that the transportation system is not always perfect as the inducing means and all elements of planning are not always measurable. It will be the principal subject in the future to investigate how to amend these weak points.

References

1) K. Amano, Y. Aoyama and M. Fujita; "The Information Theory applied to A Study on the Distribution Pattern of Urban Population."
Transactions of the Japan Society of Civil Engineerings, No. 142 p. 31–36 (1968)
2) K. Amano; "A Study on the Mechanism Constituting Traffic Share", Transportation & Economy, Vol. 27, No. 12, p. 10–18 (1967)
3) E. Kometani and Y. Aoyama; "A Study on Rapid Transit System Planning in the Aspect of Urban Pattern", Paper Presented to the 22nd Annual Conference of Japan Soc. Civil Engrs. Part IV p. 180 (1967)