Simple Low Energy Gamma Ray Spectral Analysis Using Large NaI(Tl) Scintilation Spectrometers*

BY

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A simple method of conversion of pulse-height distributions measured with a large NaI(Tl) scintillator to energy spectra of gamma rays is described. This method is quite adequate for a low energy region where the peak-to-total ratio is large and for the application of approximate analysis in the outside field of the strict gamma ray spectroscopy. In practical application of this method one needs to know only the peak-to-total ratio and efficiency of the scintillation counter as a function of photon energy. Simulation by a calculation was carried out for the purpose of demonstrating the utility of this method.

Introduction

The need for a simple method of a coversion of the pulse height distribution measured by a NaI(Tl) scintillator to an energy spectrum of gamma rays become evident during the course of an investigation on the penetration and scattering of gammma rays.

The inverse matrix method[1,2], strip-off method[1] and least-squares method[3] has been used for unfolding a scintillation pulse height distribution to the gamma ray energy spectrum. The strip-off method consists of successively subtracting the photopeaks with the corresponding tails step by step. The analysis using the least-spuares method has been applied as the progress of electronic computers inspite of the graphical strip-off. In most applications of the strip-off method and the least squares method, the response functions obtained by experiment or calculation with monochromatic gamma rays were used, and in a few works the photopeaks were approximated by delta functions or triangles having similar width of photopeaks [1,3-6]. A large number of response functions of the scintillator for monochromatic gamma rays are necessary as input data to compose a response function matrix.

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Experimental determination of the response functions with desired monochromatic gamma rays is not easy, because a limited number of monochromatic gamma source is available and scattered gamma rays are incident to the scintillator. Calculations of response functions carried out with ideal conditions often disagreed with experimental values in low energy region.

Recently, large NaI(Tl) scintillators having very good characteristics are easily obtainable. The scintillators such as 8-in. diameter by 8-in. long have very large photofraction. The determination of the photofraction and pulse-height distribution for desired energy gamma rays is not easy; however, interpolation is easier for the photofraction than for the pulse-height distribution, if we can get them at several energies of gamma rays. Taking advantage of these facts, the simpler method of converting the pulse-height distributions to the gamma ray energy spectra, has been developed. It promises to be useful for the approximate spectral analysis of gamma rays in the outside field of strict gamma ray spectroscopy.

This paper describes a simple unfolding mehtod* and a simulation by computation which demonstrates the utility of the method and several examples of application with a 8-in. diameter by 4-in. long NaI(Tl) scintillator.

Simplification of Unfolding

The simple unfolding method described in this paper is illustrated schematically in Fig. 1. Each unfolding strip has two parts. Part A, namely"peak", and part B, "tail", correspond to the photopeak and the Compton electron distribution in the usual strip-off method, respectively. The ratio, ("peak" area)/("peak" area+ "tail" area), is made equal to the photofraction at the energy corresponding to the "peak" for the first approximation. The width of the "peak" is decide somewhat arbitrarily, keeping free from the pulse-height distribution or widths of photopeaks. It has to be determined considering the energy of the gamma rays, type of measuring instrument, and capacity of caluclating machine. In most cases, it is smaller than that of photopeak and has a constant value in all energy regions. In the usual strip-off method, on the contrary, the width of the photopeak is not arbitrary; it depends on the gamma ray energy and, worse, on the characteristics of the individual instrument.

A pulse-height distribution $P(\varepsilon)$ is sliced up first into pulse-hieght intervals which are made equal to the width of the "peak" of the unfolding strip and get the one-column matrix $\langle P(\varepsilon)\rangle$, where ε denotes pulse height. Starting from the

* The similar method was used independently by Nakata and described shortly in: M. Nakata, T. Fuse and K. Takeuchi, Unken Hokoku **11**, 561–568 (1961) (in Japanese)

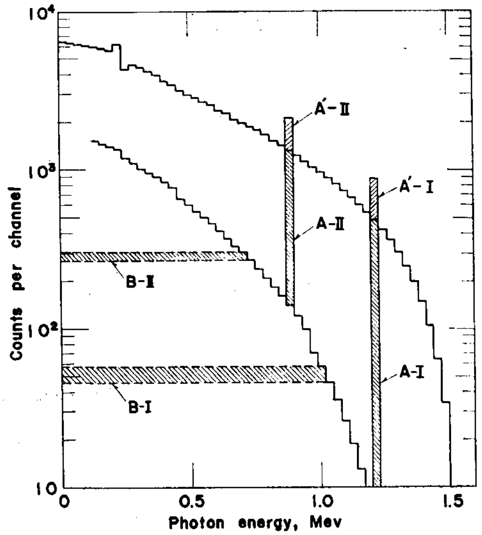

Fig. 1. A schematic illustration of the simple unfolding method. A pulse-height distribution $P(\varepsilon)$ is shown by the dark line. The areas A-I and A-II show examples of the "peaks". The areas B-I are B-II are the "tails" corresponding to A-I and A-II, respectively. The tails are subtracted from the pulse-height distribution. The area [(A-I)+(A'-I)] and that of [(A-II)+(A'-II)] are elements of $Q(E)$, where the area of A'-I and A'-II are equal to that of B-I and B-II, respectively.

highest pulse-height element of $\langle P(\varepsilon)\rangle$, the "tails" are subtarcted from the pulse-height distribution and added to the corresponding "peak". This process is carried out in successive steps and a spectrum $\langle Q(E)\rangle$ is obtained as shown schematically in Fig. 1, where E denotes the energy of gamma rays.

For the purpose of mathematical explanation of this method, we have an array numbers a_{ij} in a matrix form. Each column of the matrix represents the unfolding strip, and its "peak" is located on a diagonal element of the matrix. Thus the elements of the $n\times n$ matrix A, a_{ij}, are given in the first approximation as,

$$\begin{aligned} a_{ij} &= \gamma(E_j) && \text{for} \quad i=j \\ &= [1-\gamma(E_j)]/f(E_j) && \text{for} \quad i\leq f(E_j) \\ &= 0 && \text{for} \quad i>f(E_j) \text{ and } i\neq j \end{aligned} \tag{1}$$

where $f(E_j)$ refers to the location of the pulse height interval containing the edge of the Compton electron distribution in the usual meaning, and $\gamma(E_j)$, the photofraction of the scintillator.

The application of the unfolding method to the convertion of a pulse height distribution $P(\varepsilon)$ to a energy spectrum $Q(E)$ is written as,

$$a_{ij}q_i = p_i - \sum_{k=i+1}^{n} a_{ik}q_k. \qquad (2)$$

where p_i and q_i represent the elements of $P(\varepsilon)$ and $Q(E)$, respectively. Considering the matrix A is triangular as shown in Eqs. (1),

$$\langle P(\varepsilon)\rangle = A\langle Q(E)\rangle \qquad (3)$$

then

$$\langle Q(E)\rangle = A^{-1}\langle P(\varepsilon)\rangle \qquad (4)$$

Thus we know that this simple unfolding method is also the simplification of inverse matrix method.

Simulation of Utility by Calculation

A series of numerical calculations has been carried out for the purpose of demonstrating the utility of this method.

Figure 2a shows the "ideal" case of measurement of gamma rays by a scintillation spectrometer and an "ideal" conversion method is applied to pulse-height distribution and resultant spectrum may be identical to the spectrum of the incident gamma rays, $N(E)$. It is difficult to know that the incident gamma ray specturm strictly, because there is no "ideal" conversion method. A very similar spectrum to that of the incident gamma rays may be obtainable only when geometrical arrangement is free from scattered radiation and incident gamma ray spectrum are very simple. Figure 2b shows the case of the usual unfolding method.

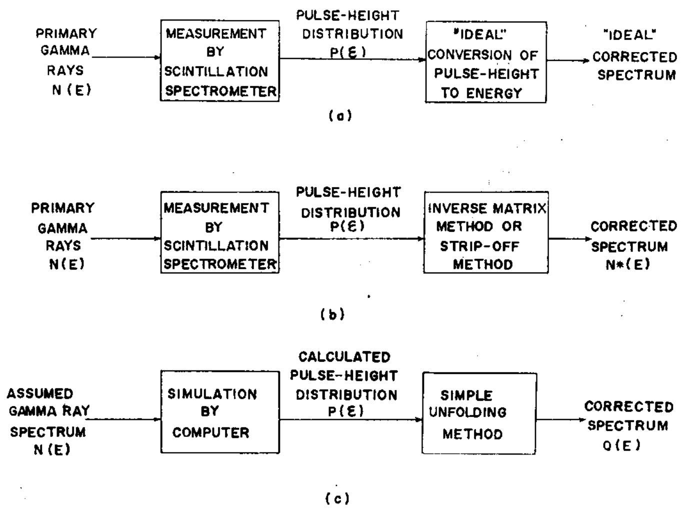

Fig. 2. Schematic illustrations of (a) measurement of gamma rays and "ideal unfolding"; (b) me measurement of gamma rays and unfolding; (c) simulating calculation and application of the simple unfolding method.

The fact mentioned above is the reason that the demonstration of the utility of a new method is not easy. For the purpose of taking away this difficulty, the measurement in Fig. 2b is replaced by the simulating calculation.

The response functions of a scintillator are postulated first for many monochromatic gamma rays. These functions compose a response function matrix M. Multiplicating this matrix to the postulated gamma ray spectrum $N(E)$, and the calculated pulse-height distribution $P(\varepsilon)$ is obtained. Then

$$\langle P(\varepsilon)\rangle = M\langle N(E)\rangle \tag{5}$$

where $\langle P(\varepsilon)\rangle$ and $\langle N(E)\rangle$ are one-column matrices.

This specturm $\langle P(\varepsilon)\rangle$ is converted to the corrected gamma ray specturm $\langle Q(E)\rangle$ by the method described previous paragraph. The utility of this method may be understood by comparing $N(E)$ with $Q(E)$.

The postulated response function matrix M is shown in Table I. It is composed of fifty pulse-height distributions of monochromatec gamma rays with energies from 15 keV to 1485 keV. Each pulse-height distribution has a photopeak being assumed as the Gaussian distribution and a flat Compton electron distribution. The photofractions used in this simulating calculating are shown in Fig. 3 as a function of energy. Curve I shows the photofraction for 8-in. diameter by 4-in. long NaI(Tl) scintillator obtained by experiment and Curve II shows photofraction of scintillator as large as or larger than 8-in. diameter by 8-in. long posturated from the data obtained by Kreger and Brown[6] and Love and Chapman[7]. The half-width of the scintillators are shown in Fig. 4. For the sake of simplicity, the

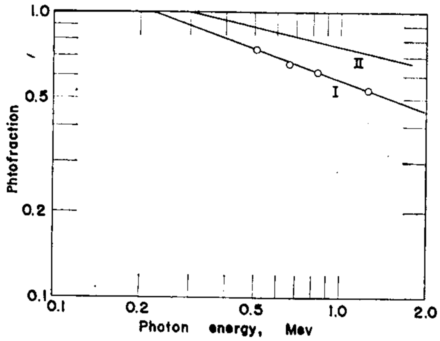

Fig. 3. The photofractions used in the simulating calculations. The Curve I is the photofraction obtained with 8-in. diameter by 4-in. long NaI(Tl) scintillator and the Curve II is the postulated photofraction for a NaI (Tl) scintillator larger than 8-in. diameter by 8-in. long.

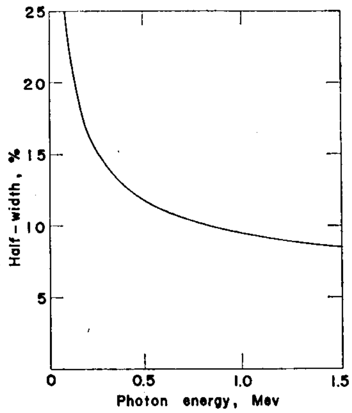

Fig. 4. The postulated half-width used in this calculation.

efficiency of the scintillator was postulated to be equal to unity throughout the region of gamma ray energies involved. The matrix A which is used in this calculation is shown in Table II. The photofraction shown in Fig. 3 was also used for the calculation of the element of Matrix A.

Several typical gamma ray spectra $N(E)$ were assumed. The pluse-hight distributions $P(\varepsilon)$ and the corrected spectra $Q(E)$ corresponding to the assumed spectra $N(E)$ were calculated and given in Table III a to c.

These spectra are not corrected for the Gaussian broadening of the photopeak. The correction for this broadening is not simple. For the application to analysis of discrete gamma rays, the determination of incident gamma ray energy and intensity is not as difficult the same usual strip off method.

Example of Application to the Analysis of Gamma Rays

This unfolding method was applied to the analysis of pulse height distributions obtained by an 8-in. in diameter by 4-in. long NaI(Tl) scintillator placed in a

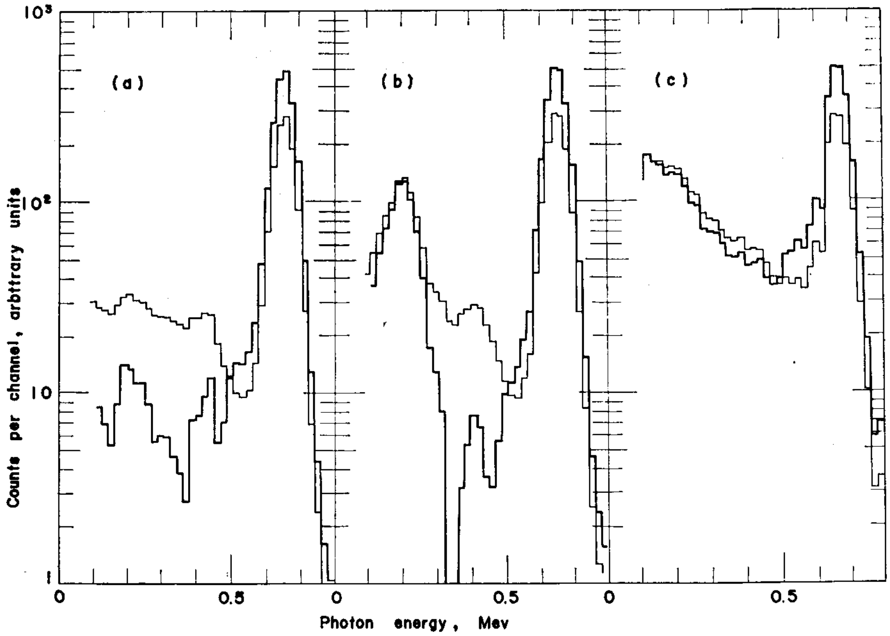

Fig. 5. Shows Examples of application of the simple unfolding method for scintillation spectra obtained with 8-in. diameter by 4-in. long NaI(Tl) scintillator. Gamma ray spectra from a ^{137}Cs point isotropic source are shown: (a) from a point source placed 4 cm from the surface of the scintillator; (b) from a point source placed on a 1 cm thick iron plate placed at a distance 50 cm from the surface of the scintillator; (c) from ^{137}Cs solution in a phantom with the capacity of 10.86 l.

Table I. The postulated response function matrix, M. The efficiency of the scintillators for gamma-rays was postulated to be equal to unity. The width of each interval is equal to 30 keV. In the table E_j represents the center of the energy interval (keV), and ε_i represents the center of pulse-height interval (keV), to yield the matrix elements the number in the table should be multiplied by 10^{-4}.

ε_j \ E_j	15	45	75	105	135	165	195	225	255	285
15	100000									74
45		100000		2						74
75			100000	8547	9					74
105				82902	10738	33				74
135				8547	78505	12805	93			74
165					10738	74323	14870	196		30
195					9	12805	70075	16606	360	1
225						33	14870	66396	18152	579
255							93	16606	62975	19357
285								196	18152	59725
315									360	19357
345										579
375										1
405										
435										
465										
495										
525										
555										
585										
615										
645										
675										
705										
735										
765										
795										
825										
855										
885										
915										
945										
975										
1005										
1035										
1065										
1095										
1125										
1155										
1185										
1215										
1245										
1275										
1305										
1335										
1365										
1395										
1425										
1455										
1485										

ε_j \ E_j	315	345	375	405	435	465	495	525	555	585
15	291	455	560	653	699	746	777	818	838	853
45	291	455	560	653	699	746	777	818	838	853
75	291	455	560	653	699	746	777	818	838	853
105	291	455	560	653	699	746	777	818	838	853
135	291	455	560	653	699	746	777	818	838	853
165	291	455	560	653	699	746	777	818	838	853
195	54	455	560	653	699	746	777	818	838	853
225	3	15	480	653	699	746	777	818	838	853
255	850	8		476	699	746	777	818	838	853
285	20151	1185	18		409	746	777	818	838	853
315	56192	20774	1534	33		340	777	818	838	853
345	20151	52866	21195	1946	55	1	253	818	838	853
375	850	20774	50108	21492	2350	92	1	184	838	853
405	3	1185	21195	47359	21683	2846	139	4	106	853
435		8	1534	21492	45124	21798	3313	197	4	58
465			18	1946	21683	42726	21803	3751	269	9
495				33	2350	21798	40687	21670	4184	346
525					55	2846	21803	38756	21518	4566
555						92	3313	21670	37050	21309
585						1	139	3751	21518	35540
615							1	197	4184	21309
645								4	269	4566
675									4	346
705										9
735										
765										
795										
825										
855										
885										
915										
945										
975										
1005										
1035										
1065										
1095										
1125										
1155										
1185										
1215										
1245										
1275										
1305										
1335										
1365										
1395										
1425										
1455										
1485										

ε_j \ E_j	615	645	675	705	735	765	795	825	855	885
15	871	876	876	869	874	868	867	868	865	865
45	871	876	876	869	874	868	867	868	865	865
75	871	876	876	869	874	868	867	868	865	865
105	871	876	876	869	874	868	867	868	865	865
135	871	876	876	869	874	868	867	868	865	865
165	871	876	876	869	874	868	867	868	865	865
195	871	876	876	869	874	868	867	868	865	865
225	871	876	876	869	874	868	867	868	865	865
255	871	876	876	869	874	868	867	868	865	865
285	871	876	876	869	874	868	867	868	865	865
315	871	876	876	869	874	868	867	868	865	865
345	871	876	876	869	874	868	867	868	865	865
375	871	876	876	869	874	868	867	868	865	865
405	871	876	876	869	874	868	867	868	865	865
435	806	876	876	869	874	868	867	868	865	865
465		760	876	869	874	868	867	868	865	865
495	14		684	869	874	868	867	868	865	865
525	429	23	1	627	874	868	867	868	865	865
555	4910	552	30	1	568	868	867	868	865	865
585	21063	5346	641	41	2	508	867	868	865	865
615	34167	20797	5611	743	56	5	460	868	865	865
645	21063	32665	20549	5884	877	76	4	393	865	865
675	4910	20797	31636	20305	6195	1013	101	6	370	865
705	429	5346	20549	30654	19976	6496	1176	128	8	305
735	14	552	5611	20305	29488	19680	6774	1321	157	12
765		23	641	5884	19976	28464	19337	6996	1462	192
795			30	743	6195	19680	27415	19001	7185	1612
825			1	41	877	6496	19337	26495	18665	7366
855				1	56	1013	6774	19001	25645	18316
885					2	76	1176	6996	18665	24804
915						5	101	1321	7185	18316
945							4	128	1462	7366
975								6	157	1612
1005									8	192
1035										12
1065										
1095										
1125										
1155										
1185										
1215										
1245										
1275										
1305										
1335										
1365										
1395										
1425										
1455										
1485										

ε_j \ E_j	915	945	975	1005	1035	1065	1095	1125	1155	1185
15	855	854	850	846	842	834	828	822	813	806
45	855	854	850	846	842	834	828	822	813	806
75	855	854	850	846	842	834	828	822	813	806
105	855	854	850	846	842	834	828	822	813	806
135	855	854	850	846	842	834	828	822	813	806
165	855	854	850	846	842	834	828	822	813	806
195	855	854	850	846	842	834	828	822	813	806
225	855	854	850	846	842	834	828	822	813	806
255	855	854	850	846	842	834	828	822	813	806
285	855	854	850	846	842	834	828	822	813	806
315	855	854	850	846	842	834	828	822	813	806
345	855	854	850	846	842	834	828	822	813	806
375	855	854	850	846	842	834	828	822	813	806
405	855	854	850	846	842	834	828	822	813	806
435	855	854	850	846	842	834	828	822	813	806
465	855	854	850	846	842	834	828	822	813	806
495	855	854	850	846	842	834	828	822	813	806
525	855	854	850	846	842	834	828	822	813	806
555	855	854	850	846	842	834	828	822	813	806
585	855	854	850	846	842	834	828	822	813	806
615	855	854	850	846	842	834	828	822	813	806
645	855	854	850	846	842	834	828	822	813	806
675	855	854	850	846	842	834	828	822	813	806
705	855	854	850	846	842	834	828	822	813	806
735	280	854	850	846	842	834	828	822	813	806
765	16	250	850	846	842	834	828	822	813	806
795	229	22	200	846	842	834	828	822	813	806
825	1758	266	28	158	842	834	828	822	813	806
855	7531	1888	309	35	134	834	828	822	813	806
885	18020	7645	2025	350	42	116	828	822	813	806
915	24092	17687	7761	2147	395	50	65	822	813	806
945	18020	23384	17370	7846	2271	438	64	25	797	806
975	7531	17687	22714	17068	7925	2382	511	76	8	208
1005	1758	7645	17370	22108	16760	7985	2560	563	87	11
1035	229	1888	7761	17068	21508	16493	8097	2674	609	102
1065	16	266	2025	7846	16760	20999	16150	8142	2771	669
1095		21	309	2147	7925	16493	20326	15874	8175	2887
1125		1	27	350	2271	7985	16150	19827	15642	8208
1155			1	34	395	2382	8097	15874	19415	15360
1185				1	42	438	2560	8142	15642	18925
1215					3	50	511	2674	8175	15360
1245						2	64	563	2771	8208
1275							5	76	609	2887
1305								7	87	669
1335									8	102
1365										11
1395										
1425										
1455										
1485										

ε_j \ E_j	1215	1245	1275	1305	1335	1365	1395	1425	1455	1485
15	801	793	789	781	777	773	765	758	752	744
45	801	793	789	781	777	773	765	758	752	744
75	801	793	789	781	777	773	765	758	752	744
105	801	793	789	781	777	773	765	758	752	744
135	801	793	789	781	777	773	765	758	752	744
165	801	793	789	781	777	773	765	758	752	744
195	801	793	789	781	777	773	765	758	752	744
225	801	793	789	781	777	773	765	758	752	744
255	801	793	789	781	777	773	765	758	752	744
285	801	793	789	781	777	773	765	758	752	744
315	801	793	789	781	777	773	765	758	752	744
345	801	793	789	781	777	773	765	758	752	744
375	801	793	789	781	777	773	765	758	752	744
405	801	793	789	781	777	773	765	758	752	744
435	801	793	789	781	777	773	765	758	752	744
465	801	793	789	781	777	773	765	758	752	744
495	801	793	789	781	777	773	765	758	752	744
525	801	793	789	781	777	773	765	758	752	744
555	801	793	789	781	777	773	765	758	752	744
585	801	793	789	781	777	773	765	758	752	744
615	801	793	789	781	777	773	765	758	752	744
645	801	793	789	781	777	773	765	758	752	744
675	801	793	789	781	777	773	765	758	752	744
705	801	793	789	781	777	773	765	758	752	744
735	801	793	789	781	777	773	765	758	752	744
765	801	793	789	781	777	773	765	758	752	744
795	801	793	789	781	777	773	765	758	752	744
825	801	793	789	781	777	773	765	758	752	744
855	801	793	789	781	777	773	765	758	752	744
885	801	793	789	781	777	773	765	758	752	744
915	801	793	789	781	777	773	765	758	752	744
945	801	793	789	781	777	773	765	758	752	744
975	801	793	789	781	777	773	765	758	752	744
1005	767	793	789	781	777	773	765	758	752	744
1035	14	738	789	781	777	773	765	758	752	744
1065	123	16	686	781	777	773	765	758	752	744
1095	744	136	21	636	777	773	765	758	752	744
1125	3028	789	162	24	658	773	765	758	752	744
1155	8250	3101	873	178	27	629	765	758	752	744
1185	15049	8251	3242	923	211	34	570	758	752	744
1215	18385	14837	8277	3315	1017	226	43	587	752	744
1245	15049	18040	14519	8269	3454	1054	260	52	575	744
1275	8250	14837	17508	14304	8278	3496	1146	298	58	567
1305	3028	8251	14519	17170	13981	8239	3629	1234	317	68
1335	744	3101	8277	14303	16649	13774	8271	3734	1277	354
1365	123	789	3242	8269	13981	16347	13555	8237	3777	1358
1395	14	136	873	3315	8278	13774	15981	13236	8199	3871
1425		16	162	923	3454	8239	13555	15503	13048	8182
1455			21	178	1017	3496	8271	13236	15233	12815
1485			2	24	211	1054	3629	8237	13048	14883

Table II. Matrix A. The efficiency of the scintillator for gamma rays is postulated to be equal to unity. The width of each energy and pulse-height interval is equal to 30 keV. In the table E_j represents the center of the energy interval (keV), and ε_i represents the center of pulse-height interval (keV), to yield the matrix elements the number in the table should be multiplied by 10^{-4}.

ε_j \ E_j	15	45	75	105	135	165	195	225	255	285
15	100000									74
45		100000								74
75			100000							74
105				100000						74
135					100000					74
165						100000				30
195							100000			
225								100000		
255									100000	
285										100000
315										
345										
375										
405										
435										
465										
495										
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675										
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885										
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1095										
1125										
1155										
1185										
1215										
1245										
1275										
1305										
1335										
1365										
1395										
1425										
1455										
1485										

ε_j \ E_j	315	345	375	405	435	465	495	525	555	585
15	296	470	586	692	749	809	852	909	942	969
45	296	470	586	692	749	809	852	909	942	969
75	296	470	586	692	749	809	852	909	942	969
105	296	470	586	692	749	809	852	909	942	969
135	296	470	586	692	749	809	852	909	942	969
165	296	470	586	692	749	809	852	909	942	969
195	55	470	586	692	749	809	852	909	942	969
225		15	502	692	749	809	852	909	942	969
255				505	749	809	852	909	942	969
285					438	809	852	909	942	969
315	100000					369	852	909	942	969
345		100000					277	909	942	969
375			100000					204	942	969
405				100000					119	969
435					100000					66
465						100000				
495							100000			
525								100000		
555									100000	
585										100000
615										
645										
675										
705										
735										
765										
795										
825										
855										
885										
915										
945										
975										
1005										
1035										
1065										
1095										
1125										
1155										
1185										
1215										
1245										
1275										
1305										
1335										
1365										
1395										
1425										
1455										
1485										

ε_j \ E_j	615	645	678	705	735	765	795	825	855	885
15	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
45	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
75	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
105	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
135	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
165	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
195	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
225	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
255	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
285	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
315	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
345	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
375	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
405	1001	1017	1027	1027	1044	1046	1055	1066	1073	1080
435	926	1017	1027	1027	1044	1046	1055	1066	1073	1080
465		883	1027	1027	1044	1046	1055	1066	1073	1080
495			802	1027	1044	1046	1055	1066	1073	1080
525				741	1044	1046	1055	1066	1073	1080
555					679	1046	1055	1066	1073	1080
585						612	1055	1066	1073	1080
615	100000						560	1066	1073	1080
645		100000						483	1073	1080
675			100000						459	1080
705				100000						382
735					100000					
765						100000				
795							100000			
825								100000		
855									100000	
885										100000
915										
945										
975										
1005										
1035										
1065										
1095										
1125										
1155										
1185										
1215										
1245										
1275										
1305										
1335										
1365										
1395										
1425										
1455										
1485										

ε_j \ E_j	915	945	975	1005	1035	1065	1095	1125	1155	1185
15	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
45	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
75	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
105	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
135	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
165	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
195	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
225	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
255	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
285	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
315	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
345	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
375	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
405	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
435	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
465	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
495	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
525	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
555	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
585	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
615	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
645	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
675	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
705	1084	1089	1094	1099	1104	1102	1103	1103	1099	1098
735	354	1089	1094	1099	1104	1102	1103	1103	1099	1098
765		319	1094	1099	1104	1102	1103	1103	1099	1098
795			257	1099	1104	1102	1103	1103	1099	1098
825				205	1104	1102	1103	1103	1099	1098
855					170	1102	1103	1103	1099	1098
885						151	1103	1103	1099	1098
915	100000						80	1103	1099	1098
945		100000						24	1077	1098
975			100000							283
1005				100000						
1035					100000					
1065						100000				
1095							100000			
1125								100000		
1155									100000	
1185										100000
1215										
1245										
1275										
1305										
1335										
1365										
1395										
1425										
1455										
1485										

ε_j \ E_j	1215	1245	1275	1305	1335	1365	1395	1425	1455	1485
15	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
45	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
75	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
105	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
135	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
165	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
195	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
225	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
255	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
285	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
315	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
345	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
375	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
405	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
435	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
465	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
495	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
525	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
555	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
585	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
615	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
645	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
675	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
705	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
735	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
765	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
795	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
825	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
855	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
885	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
915	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
945	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
975	1100	1097	1100	1097	1101	1100	1099	1097	1096	1091
1005	1054	1097	1100	1097	1101	1100	1099	1097	1096	1091
1035		1021	1100	1097	1101	1100	1099	1097	1096	1091
1065			955	1097	1101	1100	1099	1097	1096	1091
1095				890	1101	1100	1099	1097	1096	1091
1125					922	1100	1099	1097	1096	1091
1155						89	1099	1097	1096	1091
1185							812	1097	1096	1091
1215	100000							839	1096	1091
1245		100000							828	1091
1275			100000							809
1305				100000						
1335					100000					
1365						100000				
1395							100000			
1425								100000		
1455									100000	
1485										100000

Table III (a). The results of a series of simulating calculations. $N(E)$ are assumed to be incident gamma ray spectra, $P(\epsilon)$ are calculated pulse height distribution, $Q(E)$ are response corrected spe ctra.

Medium energy of interval (keV)	$N(E)$	8″φ×4″		Larger than 8″φ×8″	
		$P(\epsilon)$	$Q(E)$	$P(\epsilon)$	$Q(E)$
15	0	68	0	40	0
45	0	68	0	40	0
75	0	68	0	40	0
105	0	68	0	40	0
135	0	68	0	40	0
165	0	68	0	40	0
195	0	68	0	40	0
225	0	68	0	40	0
255	0	68	0	40	0
285	0	68	0	40	0
315	0	68	0	40	0
345	0	68	0	40	0
375	0	68	0	40	0
405	0	68	0	40	0
435	0	68	0	40	0
465	0	68	0	40	0
495	0	68	0	40	0
525	0	68	0	40	0
555	0	68	0	40	0
585	0	68	0	40	0
615	0	68	0	40	0
645	0	68	0	40	0
675	0	68	0	40	0
705	0	68	0	40	0
735	0	68	0	40	0
765	0	68	0	40	0
y95	0	68	0	40	0
825	0	68	1	40	0
855	0	68	0	40	0
885	0	68	0	40	0
915	0	68	0	40	2
945	0	68	5	40	8
975	0	68	43	40	20
1005	0	0	−43	0	2
1035	0	4	−12	5	−7
1065	0	25	38	33	4
1095	0	108	192	144	190
1125	0	308	549	492	546
1155	0	575	1054	768	1031
1185	5000	709	1288	946	1279
1215	0	575	1065	768	1046
1245	0	308	569	492	564
1275	0	108	204	144	200
1305	0	25	47	33	48
1335	0	4	8	5	7
1365	0	0	0	0	0
1395	0	0	0	0	0
1425	0	0	0	0	0
1455	0	0	0	0	0
1485	0	0	0	0	0

Table III (b)

Medium energy of interval (keV)	$N(E)$	$8''\phi\times4''$		Larger than $8''\phi\times8''$	
		$P(\varepsilon)$	$Q(E)$	$P(\varepsilon)$	$Q(E)$
15	5000	8402	5074	6577	5028
45	5000	8402	5074	6577	5028
75	5000	8402	5074	7005	5456
105	5000	8402	5074	6261	4720
135	5000	8402	5074	6575	5026
165	5000	8351	5065	6582	5032
195	5000	8276	5049	6555	5050
225	5000	9079	5937	6535	5064
255	5000	6905	4171	6494	5062
285	5000	6432	3887	6433	5062
315	5000	7179	4987	6333	5077
315	5000	7179	4987	6333	5077
345	5000	6932	4995	6217	5076
375	5000	6722	4988	6116	5083
405	5000	6516	4983	6008	5086
435	5000	6322	4992	5912	5045
465	5000	6014	4983	5800	5033
495	5000	5813	4982	5696	5027
525	5000	5632	5006	5596	5034
555	5000	5740	5028	5496	5031
585	5000	5318	5015	5405	5038
615	5000	5158	5060	5311	5041
645	5000	4999	5031	5208	5035
675	5000	4866	5039	5132	5037
705	5000	4744	5055	5058	5047
735	5000	4610	5053	4971	5038
765	5000	4475	5048	4881	5025
795	5000	4347	5052	4792	5020
825	5000	4233	5074	4708	5022
855	5000	4125	5023	4627	5025
885	5000	4013	5040	4546	5028
915	4000	3903	5062	4465	5020
945	5000	3794	5086	4386	5024
975	5000	3621	5033	4281	4989
1005	5000	3526	5075	4237	5034
1035	5000	3434	5034	4117	5046
1065	5000	3338	5069	4091	5042
1095	5000	3239	5101	4013	5034
1125	5000	3145	5048	3946	5035
1155	5000	3064	5106	3882	5047
1185	5000	2975	5055	3814	5041
1215	5000	2881	5082	3748	5040
1245	5000	2805	5043	3681	5030
1275	5000	2721	5067	3616	5020
1305	5000	2640	4980	3650	5000
1335	5000	2600	4999	3530	5000
1365	5000	2584	4969	3485	4978
1395	5000	2511	4828	3382	4860
1425	5000	2304	4589	3154	4563
1455	5000	2011	3942	2713	3950
1485	5000	1521	3042	2054	3012

Table III (c)

Medium energy of interval (keV)	$N(E)$	$8''\phi \times 4''$		Larger than $9''\phi \times 8''$	
		$P(\varepsilon)$	$Q(E)$	$P(\varepsilon)$	$Q(E)$
15	4900	6418	4894	5499	4910
45	4800	6318	4794	5399	4801
75	4700	6218	4694	5693	4501
105	4600	6118	4594	4897	4301
135	4500	6018	4494	5092	4498
165	4400	5817	4327	4997	4413
195	4300	5717	4286	4889	4327
225	4200	6279	4914	4762	4224
255	4100	4391	3352	4632	4121
285	4000	4686	3911	4486	4020
315	3900	4478	3897	4313	3930
345	3800	4212	3805	4130	3827
375	3700	3980	3699	3963	3731
405	3600	3757	3597	3794	3631
435	3500	3548	3502	3637	3506
465	3400	3277	3395	3472	3397
495	3300	3078	3280	3316	3293
525	3200	2897	3208	3166	3197
555	3100	2733	3092	3019	3094
585	3000	2579	2984	2880	2998
615	2900	2425	2909	2743	2899
645	2800	2275	2792	2604	2795
675	2700	2144	2694	2481	2696
705	2600	2021	2602	2363	2601
735	2500	1896	2500	2239	2501
765	2400	1774	2396	2117	2394
795	2300	1659	2296	1998	2292
825	2200	1554	2204	1884	2192
855	2100	1454	2097	1774	2093
885	2000	1355	1989	1666	1994
915	1900	1259	1896	1561	1891
945	1800	1167	1803	1459	1793
975	1700	1065	1683	1354	1694
1005	1600	984	1595	1263	1595
1035	1500	906	1482	1170	1497
1065	1400	829	1392	1078	1396
1095	1300	755	1300	987	1293
1125	1200	683	1185	900	1193
1155	1100	617	1098	816	1096
1185	1000	547	979	734	994
1215	900	487	893	653	894
1245	800	429	788	574	793
1275	700	369	695	497	693
1305	600	313	590	422	593
1335	600	258	496	348	493
1365	400	205	394	276	394
1395	300	153	294	206	296
1425	200	105	206	142	205
1455	100	64	99	87	126
1485	0	34	68	46	67

steel iron box lined by 3-mm thick lead plate*. The photofraction of the scintillator is shown in Fig. 3 as Curve I. Figure 5 shows spectra of gamma rays from ^{137}Cs point source placed at a distance 4-cm from the face of the scintillator without scatterer and from the source placed on a 1-cm thick iron plate placed at a distance 50-cm from the face of the scintillator, and ^{137}Cs solution in a phantom[8] with the capacity of 10.86*l*. They might be considered response corrected spectra.

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