Studies on the Responses of Multi-degrees of Freedom Systems Subjected to Random Excitation with Applications to the Tower and Pier Systems of Long Span Suspension Bridges

By

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In this paper the responses of multi-degrees of freedom systems with applications to the pier and tower systems of long span suspension bridges due to earthquake motions are studied by the stochastic process theory and the results are compared with the direct-integration.

For the present system, foundation conditon coupled with the pier dimensions has significant effects on the structural dynamic characteristics. The response behaviors and then their evaluation become complex for some foundation ranges. Here, on the assumption that the normal mode analysis can be applied, the direct-effects of individual modes and their cross-effects to the dynamic response characteristics are investigated by simulating earthquake motions to a suitable form.

1. Introduction

The asismic design of a tall structure must take both the earthquake characteristics and the system dynamic characteristics, including those of the foundation layer on which the structure is constructed, into consideration. They are related by the so-called input-output formula in the spectral analysis[1]; therefore, the response through the linear system is easily calculated.

In this sense, the system dyanmics for a specified model with variation of the foundation layer are first found out. The response analysis due to the stationary excitation is made on typical cases on the basis of the results obtained in section 2; i.e. system natural frequencies are in proximity and sufficiently separated. In the former case the closeness of natural frequencies and the value of damping factor play an essential part in the system behavior and make the complexity of the response analysis. That is, even in the system to be analyzed in the normal mode

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co-ordinates, the coupling terms between them make a great contribution to the system response against the proposal by Jennings, R.L. and Newmark, N.M[2]., and the larger the damping factor gets, the more their effects increase. The shape of distributed error caused by dropping out these terms differs greatly according to where the proximity occurs. On the other hand, they are negligibly small in the latter case.

After getting the informations on response evaluation in connection with the system natural frequencies, the response characteristics which permit the estimation of safety of the system in random excitation are of interest. Hence, in section 4, the investigations on the stochastic quantities and deterministic problems of earthquake motions are made, bringing out the data on their mathematical formulation as non-stationary random excitations. The input characteristics thus constructed analytically have the properties that they are varied to some extent in the parameter regions. Then, the time-evolving response characteristics due to them are evaluated provided that the system characteristics are time-invariant.

In the last section, for corroboration of the spectral analysis, comparison is made between the results obtained by it and the ones direct-integrated by β-method for the actual earthquake motions.

2. Dynamic Characteristics of the System

As for the system considered, a tower and pier system of long span suspension bridges is continued to adopt[3,4] in this paper, the dimensions of which are listed on Table 1. The numerical computations are carried out on its idealized system with ten degrees of freedom as shown in Fig. 1. The free vibration motion without damping is governed by

$$[M]\{\ddot{X}\}+[K]\{X\}=0 \tag{2.1}$$

where $[M]$ and $[K]$ are square symmetric matrices. Eq. (2.1) can be written in terms of the normal mode co-ordinates by putting

$$\{X\}=[V]\{Y\} \tag{2.2}$$

where the square matrix $[V]$ is defined such that

$$[V]^T[K][V],\ [V]^T[M][V] \quad \text{are diagonal matrices} \tag{2.3}$$

$$[V]^T[M][V] \quad \text{is unit matrix} \tag{2.4}$$

In Eq. (2.1), it may be more difined as

$$[V]^T[K][V]=[\diagdown\omega^2\diagdown] \tag{2.5}$$

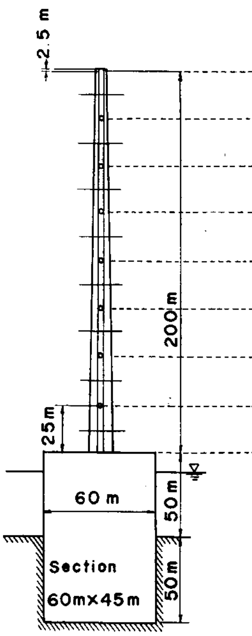


Table. 1

Values per one shaft						Total
B (m)	H (m)	A (m^2)	I (m^4)	C (m)	C/I (m^{-3})	W (ton)
3.00	5.00	1.62	3.75	2.50	0.667	177
3.50	5.75	2.25	6.65	2.88	0.433	455
4.00	6.50	2.70	10.99	3.25	0.296	585
4.50	7.25	3.15	17.15	3.63	0.212	731
5.00	8.00	3.60	25.60	4.00	0.156	893
5.50	8.75	4.07	36.85	4.38	0.119	1071
6.00	9.50	4.50	51.44	4.75	0.0923	1266
6.50	10.25	4.95	70.00	5.13	0.0733	1476
7.00	11.00	5.40	93.17	5.50	0.0590	852
						7506

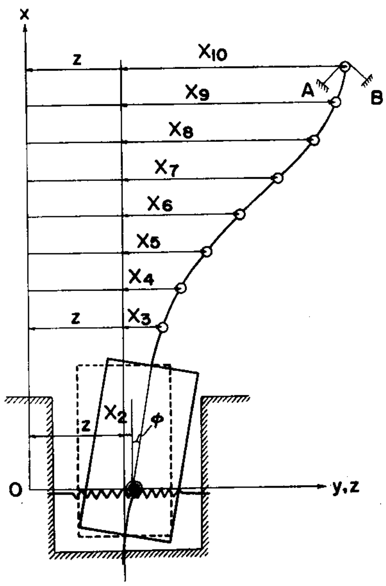


Fig. 1 System considered

The diagonalization of Eq. (2. 1) is evident and the normalization of co-ordinates as specified in Eq. (2.4) ensures that all generalized mass is unity. Eq. (2.1) then becomes

$$\{\ddot{Y}\} + [\diagdown\omega^2\diagdown]\{Y\} = 0 \tag{2.6}$$

The system dynamic characteristics in connection with foundation conditions are detected by use of this equation. Fig. 2 shows the variation of the lower natural frequencies against the foundation constant and Fig 3. illustrates the aspects of normal mode shapes modified by it. It is recognized from the latter that if the natural frequencies get closed each other, the corresponding normal mode shapes come to

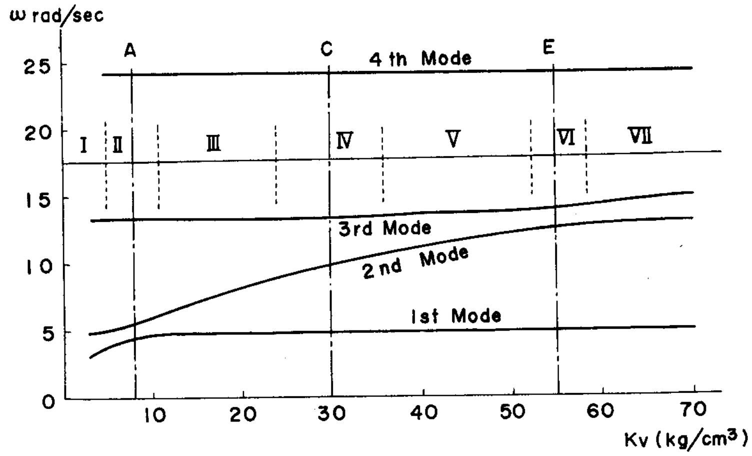


Fig. 2 Natural Frequencies of the System

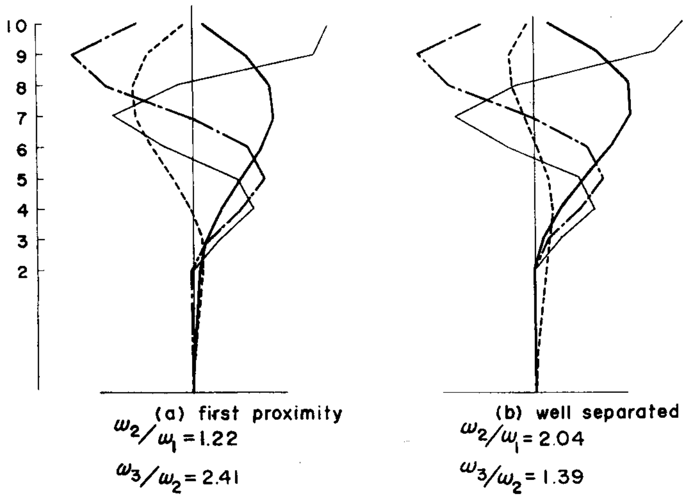


Fig. 3

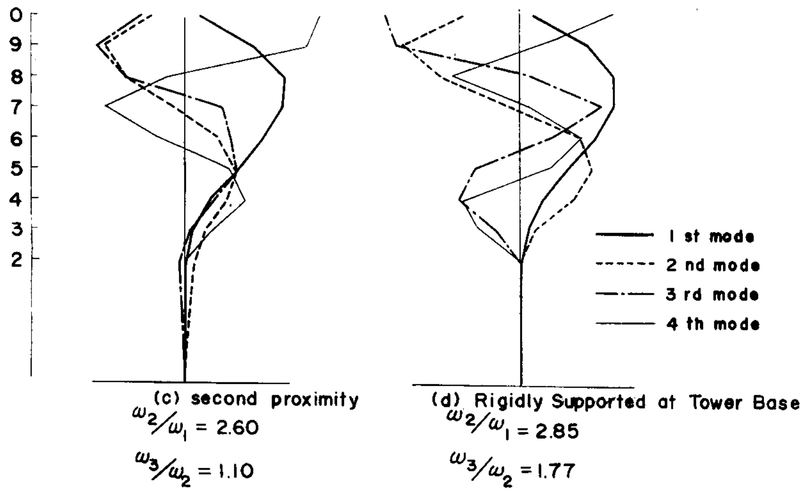


Fig. 3 Mode Shapes modified by Foundation Layer

be alike. Form Fig. 2 the proximity of natural frequencies occurs only between two adjacent ones due to the inertia force of the pier and the reaction of the foundation layer, practically between the fundamental and the second, and between the second and the third. In this state the vibration phenomenon becomes complex as a matter of course.

Hence, the investigations have to be made in the following cases. One is in the proximity of the natural frequencies in the foundation constant region II, VI in Fig. 2 and the other is in the condition of thier being well separated in region IV. Each case corresponds to the foundation KV=KH=KHV=8 kg/cm^3, KV=55 kg/cm^3 and KH=KHV=9.5 kg/cm^3, and KV=30 kg/cm^3 KH=KHV=10 kg/cm^3, respectively.

3. Response of the System due to Stationary Random Excitation

It is not possible to extend the normal mode method of analysis to the general case of a damped system with multi-degrees of freedom without considerable complications, because the damping force induces a coupling motion between the normal modes, especially in the system with nearly equal natural frequencies[5]. No coupling phenomenon between them arises, however, if the damping force is distributed in such a form as proportional to the inertia force, the elastic force or their special combination[6]. These different damping types come from the different treatment of the damping factor. There, the damping term in the normal mode coordinates is presented as

$$[V]^T[C][V] = \beta'[I] \tag{3.1}$$

$$= \beta''[\diagdown\omega^2\diagdown] \tag{3.2}$$

$$= \beta[\diagdown\omega\diagdown] \tag{3.3}$$

which are denoted here by the damping type 1, damping type 2 and damping type 3, respectively. The damping effects of these types are set to equal at the fundamental mode.

In the above cases the analysis can be extended fairly easily to the normal mode method, including the effects of damping. The governing equation in terms of normal mode co-ordinates is expressed as

$$\{\ddot{Y}\} + [V]^T[C][V]\{\dot{Y}\} + [V]^T[K][V]\{Y\} = \{Q\} \tag{3.4}$$

$$\{Q\} = [V]^T\{F\} \tag{3.5}$$

where the external force $\{F\}$ can only be explained in the stochastic meaning. The Eq. (3.4) is solved as

$$\{Y\} = [\diagdown h\diagdown] \circledast \{Q\} \tag{3.6}$$

or

$$\mathcal{F}\{Y\} = [\diagdown H\diagdown]\cdot\mathcal{F}\{Q\} \tag{3.7}$$

where the asterisk $\circledast$ means the convolution integral and $\mathcal{F}\{\cdot\}$ signifies the Fourier transformation of $\{\cdot\}$. $[\diagdown h\diagdown]$ is the diagonal matrix of impulsive response function. Its Fourier transformation $[\diagdown H\diagdown]$ is a frequency response function matrix, of which element $H_j(\omega)$ is in general complex and has the form according to the type of damping term;

damping type 1 $$H_j(\omega) = \frac{1}{\omega_j^2}\cdot\frac{1}{\left\{1-\left(\frac{\omega}{\omega_j}\right)^2\right\}+i\left\{2\beta'\frac{\omega}{\omega_j^2}\right\}} \tag{3.8-a}$$

damping type 2 $$H_j(\omega) = \frac{1}{\omega_j^2}\cdot\frac{1}{\left\{1-\left(\frac{\omega}{\omega_j}\right)^2\right\}+i\{2\beta''\omega\}} \tag{3.8-b}$$

damping type 3 $$H_j(\omega) = \frac{1}{\omega_j^2}\cdot\frac{1}{\left\{1-\left(\frac{\omega}{\omega_j}\right)^2\right\}+i\left\{2\beta\frac{\omega}{\omega_j}\right\}} \tag{3.8-c}$$

The response in terms of the original co-ordinates is then

$$\mathcal{F}\{X\} = [V]\mathcal{F}\{Y\} = [V][\diagdown H\diagdown]\mathcal{F}\{Q\} = [V][\diagdown H\diagdown][V]^T\mathcal{F}\{F\} \tag{3.9}$$

In Eq. (3.9) the frequency response function matrix $[V][\diagdown H\diagdown][V]^T$ is not of course diagonal but symmetric. Here, one may write

$$[\mathcal{A}] = [V][\diagdown H\diagdown][V]^T \tag{3.10}$$

Then,

$$\mathcal{F}\{X\} = [\mathcal{A}]\mathcal{F}\{F\} \tag{3.11}$$

In these equations $[\mathcal{A}]$ makes the frequency response function matrix in the response co-ordinates and is called the receptance matrix. The relationship of Eq. (3.9) or Eq. (3.11) forms the basis connecting the spectral density of random excitation and that of the system response.

The symbol $[S^F]$ and $[S^X]$ may be used to denote the spectral density for the excitation and the system response. If $[\mathcal{A}^*]$ is the matrix of which elements are the complex conjugate of those of Eq. (3.10), then it leads to

$$[S^X] = [\mathcal{A}][S^F][\mathcal{A}^*]^T \tag{3.12}$$

The following technique, however, will be preferable for the system to which the normal mode method can be applied, clarifing the correlation between the normal modes. It follows that from Eq. (3.11)

$$[S^Y] = [\diagdown H \diagdown][S^Q][\diagdown H \diagdown^*]^T \tag{3.13}$$

where $[S^Q]$ and $[S^Y]$ represent the input and output spectral density matrices in the normal mode co-ordinates. This is particularly a simple relationship, because the frequency response function matrix is diagonal. Then, the response power spectral density of the original coordinates is from Eq. (2.2) as

$$[S^X] = [V][S^Y][V]^T \tag{3.14}$$

From Eq. (3.12) or Eq. (3.13) the response power spectral density depends on two factors; one is the characteristics of the system itself and the other those of the external forces. The former is expressed only by the natural frequencies and the damping factors. Considering Eq. (3.13) can, therefore, reveal something essential to the specified system behavior in random exictation in terms of their fundamental characteristics.

If it can be assumed that the excitation is "white" (it means the elements of $[S^F]$ are independent of frequency), which may be sufficient to investigate the contributions of direct- and cross- spectral density of normal modes to the system response[7], then the mean-square-response depends on n^2 integrals defined by

$$\tilde{I}_{jk} = \int_0^\infty H_j H^*_k \, d\omega \tag{3.15}$$

and the respose in the original co-ordinates is obtained as

$$\{X\}_{R.M.S.} = \text{diag.} \left\{\int_0^\infty [S^X] \, d\omega\right\}^{1/2} \tag{3.16}$$

The integral is carried out according to each type of damping term as follows;

for the damping type 3

$$\tilde{I}_{jk} = \frac{2\pi(\beta_j\omega_j+\beta_k\omega_k)}{(\omega_k^2-\omega_j^2)^2+4(\beta_j\omega_j+\beta_k\omega_k)(\beta_j\omega_k+\beta_k\omega_j)\omega_j\omega_k} \tag{3.17}$$

The corresponding integral arising from the direct spectral density is a special case and it is found as

$$\tilde{I}_{jj} = \frac{\pi}{4\beta_j\omega_j^3} \tag{3.18}$$

Similarly,

for the damping type 1

$$\tilde{I}_{jk} = \frac{2\pi(\beta_j'+\beta_k')}{(\omega_k^2-\omega_j^2)^2+4(\beta_j'+\beta_k')(\beta_j'\omega_k^2+\beta_k'\omega_j^2)} \tag{3.19}$$

$$\tilde{I}_{jj} = \frac{\pi}{4\beta_j'\omega_j^2} \tag{3.20}$$

for the damping type 2

$$\tilde{I}_{jk} = \frac{2\pi(\beta_j''\omega_j^2+\beta_k''\omega_k^2)}{(\omega_k^2-\omega_j^2)^2+4(\beta_j''\omega_j^2+\beta_k''\omega_k^2)(\beta_j''+\beta_k'')\omega_j^2\omega_k^2} \tag{3.21}$$

$$\tilde{I}_{jj} = \frac{\pi}{4\beta_j''\omega_j^4} \tag{3.22}$$

Comparing the integrated values for each different type of damping term, it is understood that the contributions of the direct- and cross-spectral densities to the total spectral density are increased in magnitude in ascending order of the damping type 1, damping type 2, damping type 3. (This tendency can be observed directly in the mean-square-response in Fig. 6.)

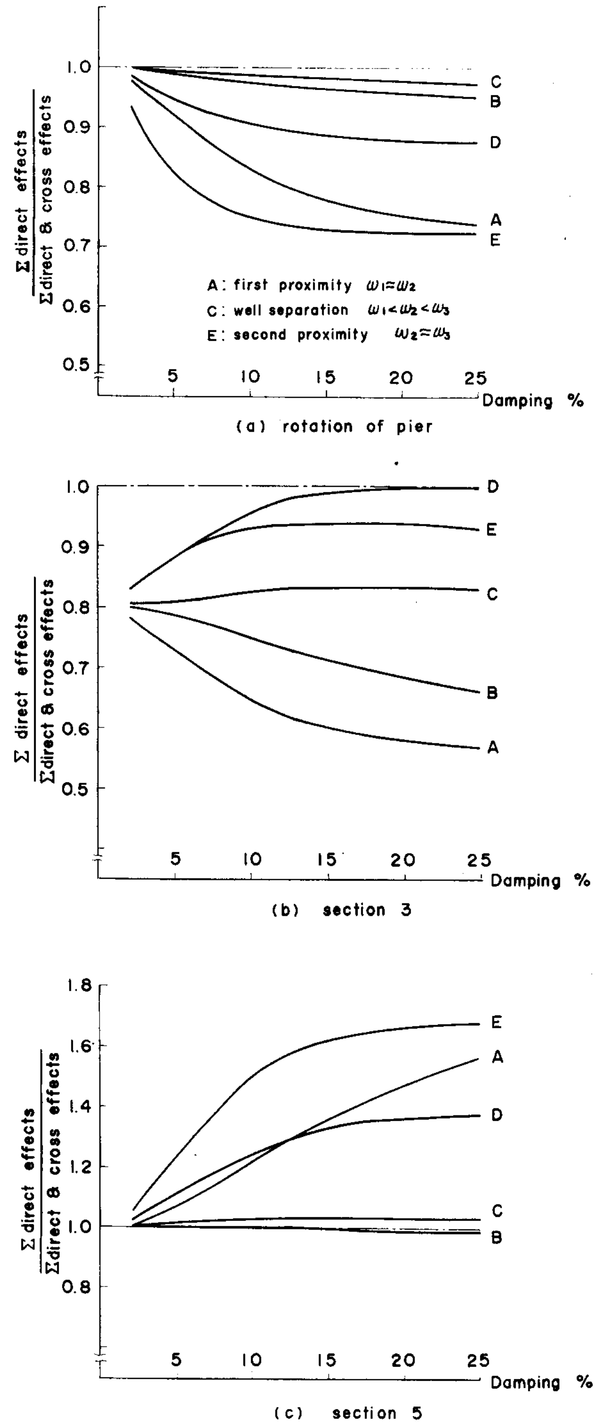
In the case of two lower natural frequencies becoming close to each other, the order of the cross-spectral densities (the non-diagonal elements of Eq. (3.13)) is increased in comparison with the direct ones (the diagonal elements of Eq. (3.13)). The conditions where the order of the cross-spectral density is negligibly small to the direct one are found. They are according to the type of damping term;

damping type 1 $\left\{1-\left(\frac{\omega_k}{\omega_j}\right)^2\right\}^2 \gg \frac{8\beta'^2_j}{\omega_j^2}$; $\omega_j > \omega_k$ (3.23-a)

damping type 2 $1-\left(\frac{\omega_k}{\omega_j}\right)^2 \gg 4\beta''^2_j\omega_j^2\left\{\left(\frac{\omega_k}{\omega_j}\right)^2+1\right\}$; $\omega_j > \omega_k$ (3.23-b)

damping type 3 $1-\left(\frac{\omega_k}{\omega_j}\right)^2 \gg 4\beta_j^2\left\{\left(\frac{\omega_k}{\omega_j}\right)+2\right\}$; $\omega_j > \omega_k$ (3.23-c)

They indicate that the larger the damping factor is, the more sufficiently the natural



(c) section 5

Fig. 4

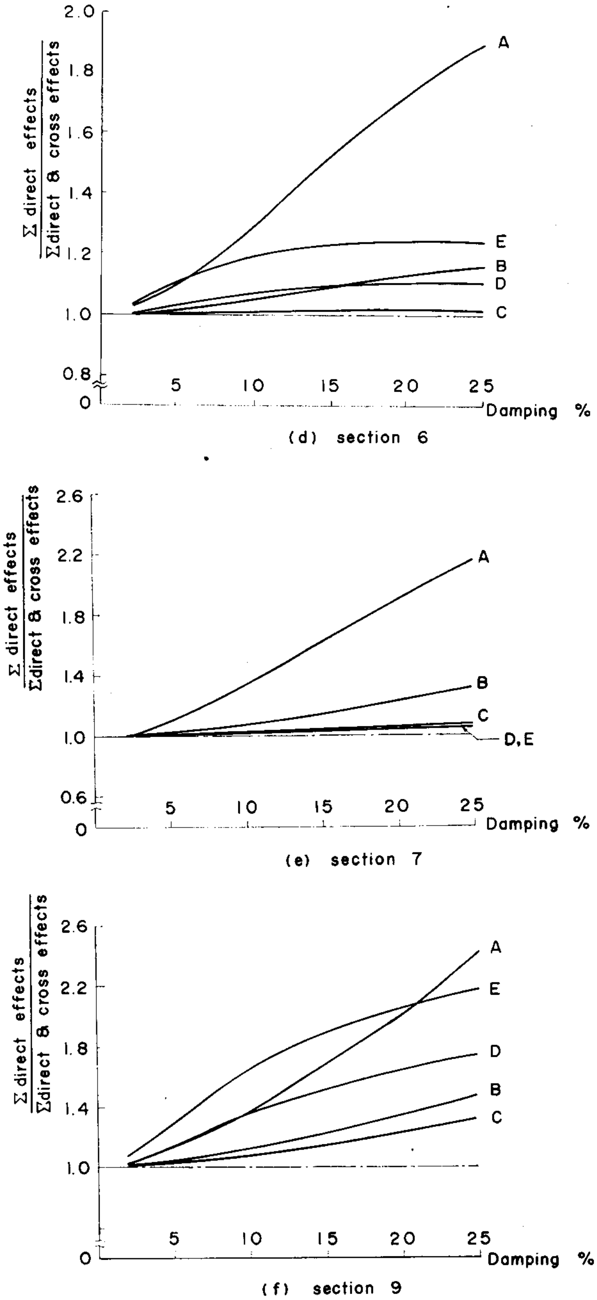
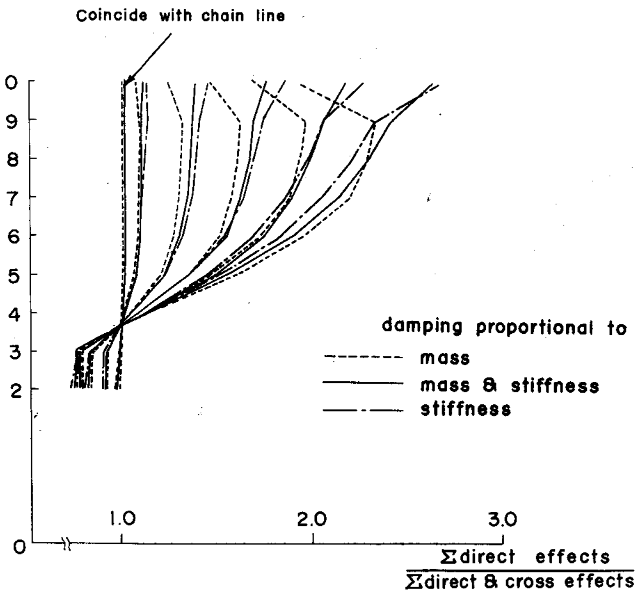


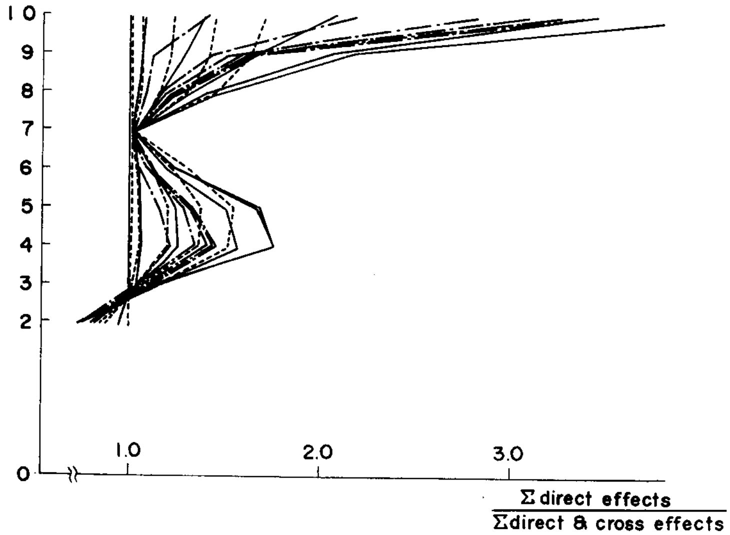
Fig. 4 Cross-relation between normal modes (Ratio of the response totaled only by the direct effects of individual normal modes to the one by both their direct and cross effects is preasented.)

frequencies concerned have to be separated. This condition becomes severe in ascending order of the damping type 1, damping type 3, damping type 2.

It is quite evident that the lower vibration modes play a dominant part in the system response. Moreover, it is the lower modes that can be affected by the circumstances surrounding the system. (see Fig. 2 and Fig 3) Hence, the ratio of the mean-square response totaled only by the direct effects of individual normal modes to the one by both their direct and cross effects is calculated at each section of the system against the damping factor with ω_2/ω_1, ω_3/ω_2 as parameter. They are plotted in Fig. 4, where the curve A corresponds to the case of the first mode and the second in proximity, and the curve E the second and the third in the same condition while the curve C of thier well separation. The ratio is distributed along the whole system at the first proximity as in Fig. 5-(a) and at the second as in Fig. 5-(b) with damping factor as parameter. In the former case the error caused by dropping out the cross terms between the normal modes cannot be negligible in proportion to the damping factor and the height of the system. In the latter case a fairly large extent of error is also noticed at the upper part and lower but not at the central part. On the other hand, in case of the natural frequencies sufficiently separated, this error becomes negligibly small over the whole system. In all cases the ratio less than 1.0 appears at the pier top, which is due to the orthogonality condition of vibration mode. These results coupled with the mode shapes in Fig.

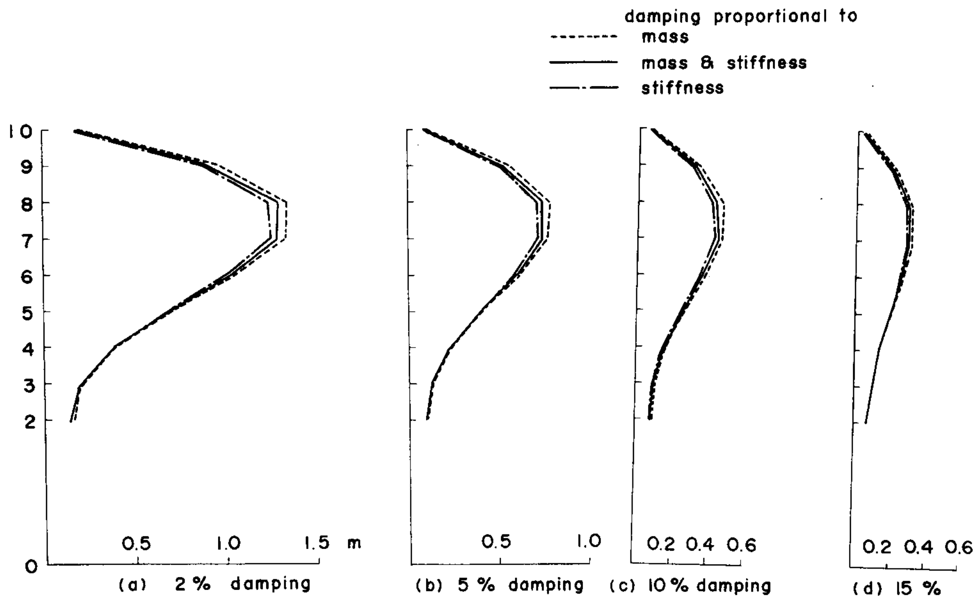


(a) first proximity
damping factor 2%, 5%, 10%, 15%, 20%, 25%
Fig. 5



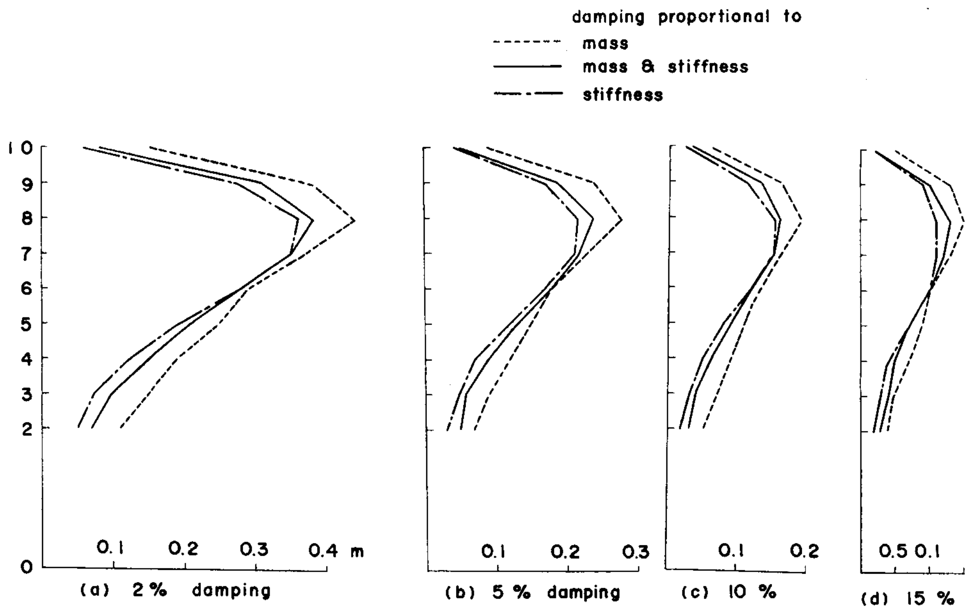
(b) second proximity
damping factor 2%, 5%, 10%, 15%, 20%, 25%

Fig. 5 Cross-relation between normal modes distributed along the system

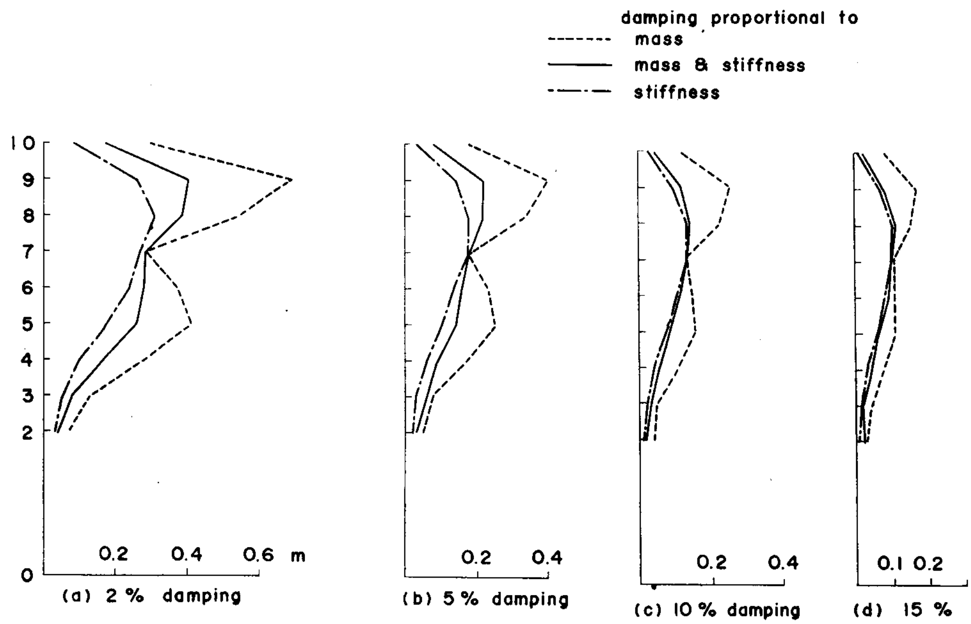


(1) first proximity

Fig. 6



(2) well separation



(3) second proximity

Fig. 6 Response of the system due to white noise (Level 800 cm^2/sec^5)

3 substantiate the fact that the normal modes in proximity should occur in phase and that the ones sufficiently separated in quadrature.

The responses obtained from Eq. (3.16) are illustrated in Fig. 6 with various damping factors, corresponding to the above three cases. In view of these figures it is recognized that the assumption of vibration damping together with the disposition of natural frequencies has significant effects on the response.

4. Simulation of Earthquake Motion as Non-Stationary Random Excitation

The earthquake motions evidently have a non-stationary stochastic process by nature. It is essential for the later investigation of the precise system response to present these motions in a mathematical expression.

Let the generalized earthquake acceleration be synthesized in a form[8]

$$\ddot{z} = f(t) = \sum_{\nu} \psi_\nu(\alpha_1, \alpha_2, \cdots, \alpha_r; t) g_\nu(\alpha_{r+1}, \cdots, \alpha_n; t) \tag{4.1}$$

where $\alpha_1, \cdots, \alpha_n$ are the random parameters, or the integral criteria of the seismic forces, such as the nature of the earthquake itself, the distance from the epicenter and the characteristics of foundation layer. $\psi(\alpha_1, \cdots, \alpha_r; t)$ is the deterministic function of time (which gives the envelope of accelerogram) and $g(\alpha_{r+1}, \cdots, \alpha_n; t)$ is a stationary random function of time and is further assumed to be subject to the Gaussian process.

For a rough approximation, the following expression is adopted in this paper such as

$$f(t) = \psi(\alpha_1, \alpha_2; t) g(\alpha_3, \alpha_4, \alpha_5; t) \tag{4.2}$$

and its derivative in a form

$$\dot{f}(t) = \dot{\psi}(\alpha_1, \alpha_2; t) g(\alpha_3, \alpha_4, \alpha_5; t) + \psi(\alpha_1, \alpha_2; t) \dot{g}(\alpha_3, \alpha_4, \alpha_5; t) \tag{4.3}$$

The observation of the actual accelerograms gives the exponential form to the deterministic function.

$$\psi(t) = (e^{-\alpha_1 t} - e^{-\alpha_2 t}) H(t) \tag{4.4}$$

where $H(t)$ is the Heaviside unit step function, and α_1 and α_2 are the positive constants through which the rapidity of buildup and decay of the seismic force intensity can be controlled. The stationary Gaussian process $g(t)$ is assumed to have the zero mean and the following cross-variance[9].

$$E\{g(t_1) g(t_2)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(\omega) e^{i\omega(t_1 - t_2)} d\omega \tag{4.5}$$

$$E\{\dot{g}(t_1)g(t_2)\} = -E\{g(t_1)\dot{g}(t_2)\} = \frac{i}{2\pi}\int_{-\infty}^{\infty}\omega\Phi(\omega)e^{i\omega(t_1-t_2)}d\omega \tag{4.6}$$

$$E\{\dot{g}(t_1)\dot{g}(t_2)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\omega^2\Phi(\omega)e^{i\omega(t_1-t_2)}d\omega \tag{4.7}$$

where $\Phi(\omega)$ is the power spectral density of a stationary part only. Since $g(t)$ is the Gaussian process with mean zero, $\dot{g}(t)$ is also the Gaussian process with mean zero. It then follows that

$$E\{f(t)\} = E\{\dot{f}(t)\} = 0 \tag{4.8}$$

The cross-variance between $f(t)$ and $\dot{f}(t)$, and the variance of $\dot{f}(t)$ are

$$E\{f(t_1)\dot{f}(t_2)\} = \psi(t_1)\dot{\psi}(t_2)E\{g(t_1)g(t_2)\} + \psi(t_1)\psi(t_2)E\{g(t_1)\dot{g}(t_2)\} \tag{4.9}$$

$$\begin{aligned}E\{\dot{f}(t_1)\dot{f}(t_2)\} = {} & \dot{\psi}(t_1)\dot{\psi}(t_2)E\{g(t_1)g(t_2)\} + \{\psi(t_1)\dot{\psi}(t_2) \\ & -\dot{\psi}(t_1)\psi(t_2)\}E\{\dot{g}(t_1)g(t_2)\} + \psi(t_1)\psi(t_2)E\{\dot{g}(t_1)\dot{g}(t_2)\}\end{aligned} \tag{4.10}$$

Now the stationary Gaussian process $g(t)$ may be replaced as the response of the system with a second order differential equation to which the Gaussian white noise $n(t)$ is input.

$$\ddot{g}(t) + 2\mu_0\dot{g}(t) + (\omega_0^2 + \mu_0^2)g(t) = n(t) \tag{4.11}$$

Let the mean value of $n(t)$ be zero and the correlation be

$$E\{n(t_1)n(t_2)\} = D\delta(t_1 - t_2) \tag{4.12}$$

The frequency response function of Eq. (4.11) is then

$$H_0(\omega) = \frac{(\omega_0^2 + \mu_0^2 - \omega^2) - 2i\mu_0\omega}{(\omega_0^2 + \mu_0^2 - \omega^2)^2 + 4\mu_0^2\omega^2} \tag{4.13}$$

in which the parameter ω_0, μ_0 are chosen to represent the characteristics of the input power spectral density, in other words, the conditions of the foundation layer in addition to the seismicity. The Eq. (4.13) is related to the power spectral density $\Phi(\omega)$ in such a form as

$$\Phi(\omega) = D|H_0(\omega)|^2 \tag{4.14}$$

Hence, the cross-variances of the external forces with non-stationrity in the form of Eq. (4.2) are calculated by use of Eq. (4.5) to Eq. (4.7) and Eq. (4.14), and the power spectral densities by Eq. (4.9) to Eq. (4.10) and Eq. (4.14). The latter have the expression of

$$S^f(t,\omega) = DJ^{(0)}(t)\{L_1^{(0)}(t)K_1(\omega) - L_2^{(0)}(t)K_2(\omega)\}/4W_1 \tag{4.15}$$

$$\begin{aligned}S^{\dot{f}f}(t,\omega) = {} & DJ^{(0)}(t)\{L_1^{(1)}(t)K_1(\omega) - L_2^{(1)}(t)K_2(\omega)\}/4W_1 \\ & + DJ^{(1)}(t)\{L_1^{(0)}(t)K_3(\omega) - L_2^{(0)}(t)K_4(\omega)\}/4W_2\end{aligned} \tag{4.16}$$

$$S^{f}(t,\omega)=DJ^{(1)}(t)\{L_1^{(1)}(t)K_1(\omega)-L_2^{(1)}(t)K_2(\omega)\}/4W_1$$
$$+D[J^{(1)}(t)\{L_1^{(0)}(t)K_3(\omega)-L_2^{(0)}(t)K_4(\omega)\}$$
$$-J^{(0)}(t)\{L_1^{(1)}(t)K_3(\omega)-L_2^{(1)}(t)K_4(\omega)\}$$
$$+J^{(0)}(t)\{L_1^{(1)}(t)K_5(\omega)-L_2^{(1)}(t)K_6(\omega)\}]/4W_2 \quad (4.17)$$

$$J^{(0)}(t)=(e^{-\alpha_1 t}-e^{-\alpha_2 t}),\quad J^{(1)}(t)=\frac{d}{dt}J^{(0)}(t)$$

$$L_i^{(0)}(t)=e^{-\alpha_i t},\quad L_i^{(1)}(t)=\frac{d}{dt}L_i^{(0)}(t)$$

$$K_i(\omega)=\frac{(\alpha_i+\mu_0)\omega_0+\mu_0(\omega+\omega_0)}{(\alpha_i+\mu_0)^2+(\omega+\omega_0)^2}+\frac{(\alpha_i+\mu_0)\omega_0-\mu_0(\omega-\omega_0)}{(\alpha_i+\mu_0)^2+(\omega-\omega_0)^2}$$

$$K_{i+2}(\omega)=\frac{\omega+\omega_0}{(\alpha_i+\mu_0)^2+(\omega+\omega_0)^2}-\frac{\omega-\omega_0}{(\alpha_i+\mu_0)^2+(\omega-\omega_0)^2}$$

$$K_{i+4}(\omega)=\frac{(\alpha_i+\mu_0)\omega_0+\mu_0(\omega-\omega_0)}{(\alpha_i+\mu_0)^2+(\omega-\omega_0)^2}+\frac{(\alpha_i+\mu_0)\omega_0-\mu_0(\omega+\omega_0)}{(\alpha_i+\mu_0)^2+(\omega+\omega_0)^2}$$

$$W_1=\mu_0\omega_0(\mu_0^2+\omega_0^2)$$

$$W_2=\mu_0\omega_0$$

$(i=1$, or $2)$

The abstraction of the stochastic characteristics, i.e. the auto-correlation function and the power spectral density is made on several earthquake accelerograms for description of eatrhquake motions in a mathematical way as in Eq. (4.2). One of them is presented in Fig. 8–a, Fig. 8–b for the EL CENTRO earthquake, 1940 NS component in Fig. 7. On inspection of these spectral densities, they generally have one conspicuous peak at about 1 to 2.5 c/sec (6.3 to 16 rad/sec) with the maximum value of 200 cm²/sec⁵ to 600 cm²/sec⁵ and a very marked decrease about 3

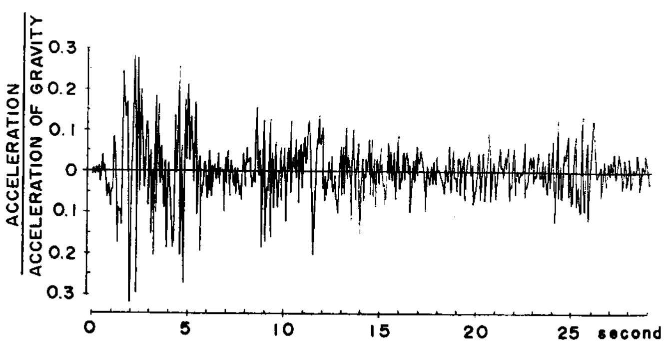


Fig. 7 EL CENTRO earthquake 1940, NS component

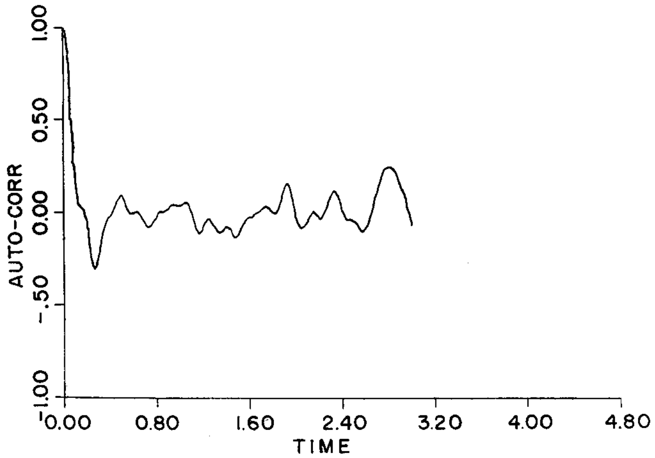


Fig. 8-a Auto-correlation of EL CENTRO earthquake 1940, NS component (plotted by KDC-II computer)

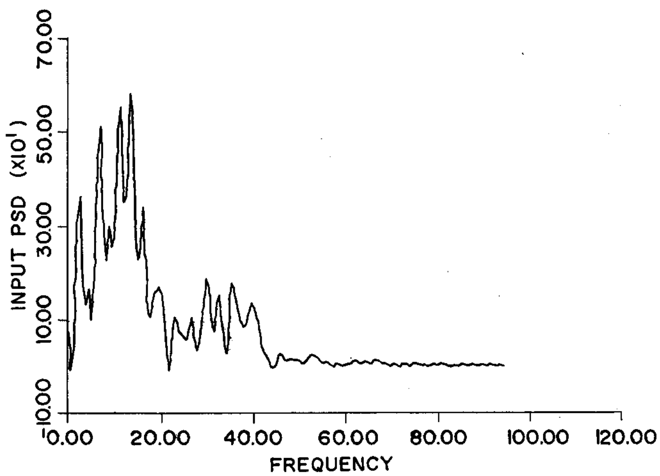


Fig. 8-b Power Spectral Density of EL CENTRO earthquake 1940, NS component (plotted by KDC-II computer)

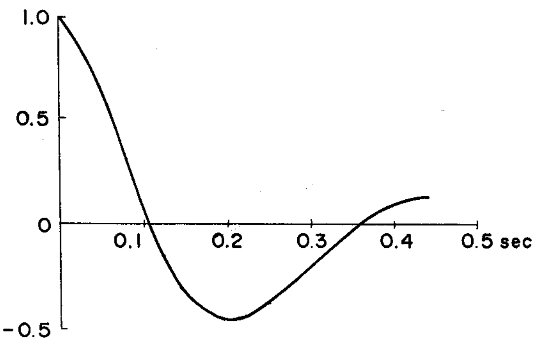


Fig. 9-a Auto-correlation Function of $g(t)$ in Eq. (4.2)

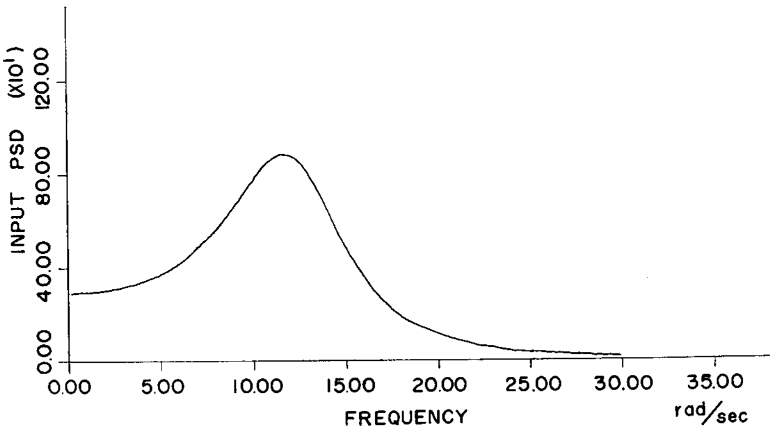


Fig. 9-b Power Spectral Density of $g(t)$ in Eq. (4.2)

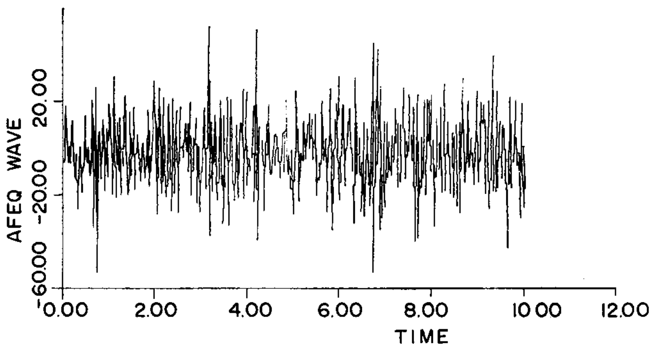


Fig. 10 Artificial Earthquake generated
(plotted by KDC-II computer)

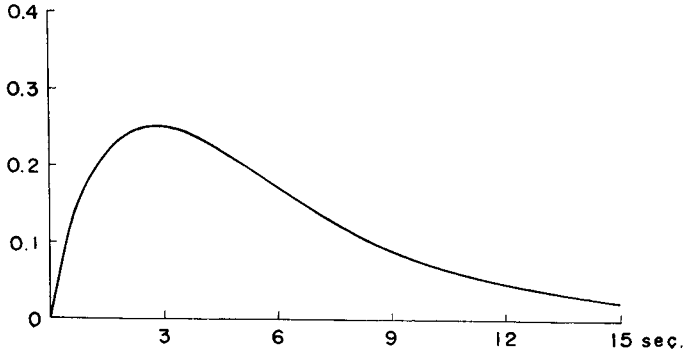


Fig. 11 Function $\psi(t)$ in Eq. (4.2)

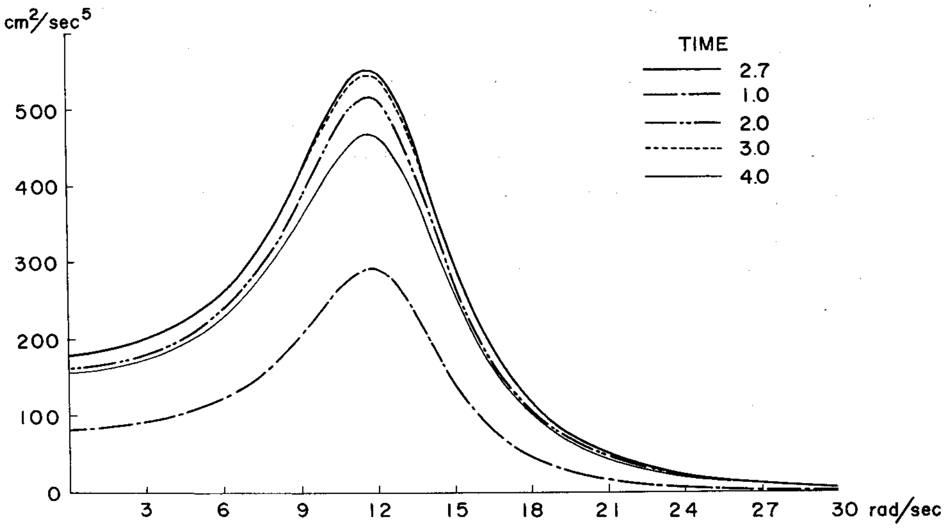


Fig. 12 Power Spectral Density of the Non-stationary Excitation formulated in Eq. (4.2)

c/sec (25 rad/sec), and then they vanish about more than 10 c/sec (60 rad/sec). It might be emphasized that the input power spectral density, being taken into consideration of the natural frequencies of the structure, is practically confined within the frequency band, zero to 5 c/sec (zero to 30 rad/sec).

The formulation of the spectral density of the stochastic function $g(t)$, which is replaced in this paper by the output through the second-order differential equation of the white noise $n(t)$ input, makes the system response analysis more analytically. Its assumed function $\Phi(\omega)$ is plotted in Fig. 9–b with the parameter of $D=8.0\times10^6$ cm²/sec⁵ $\omega_0=12.6$ rad/sec $\mu_0=3.86$ rad/sec. The artificial earthquake generated by its power spectral density[10] is presented in Fig. 10 and its auto-correlation function is in Fig. 9–a. As the coefficients of the deterministic function $\psi(t)$, the values of $\alpha_1=0.25$ and $\alpha_2=0.5$ are taken as Fig. 11

The external forces constructed from the foregoings, which have the time-evolving stochastic process, will approximate to the first several seconds of the EL CENTRO earthquake 1940 NS component. The variation of the former power spectral density is shown in Fig. 12 with time as parameter.

5. Response of the System Due to Non-Stationary Random Excitation

As has been mentioned, the earthquake motions are the excitations of which stochastic quantities are time-evolving. The response through the linear system is of course subject to the same type distribution if the external forces have the nonstationarity with Gaussian. It is, therefore, desired for the precise evaluation of the system response to obtain the response characteristics with time t as parameter.

On the other hand, the application of the stochastic process theory to the structural response analysis leads successfully to the estimation of the system safety against random excitations. It is necessary for that purpose to obtain the variance of displacement, velocity and the cross-variance between them with time-evolving. Now in this paper, the structural responses due to the external forces synthesized in the preceeding sectoin are evaluated by use of the following relationships.

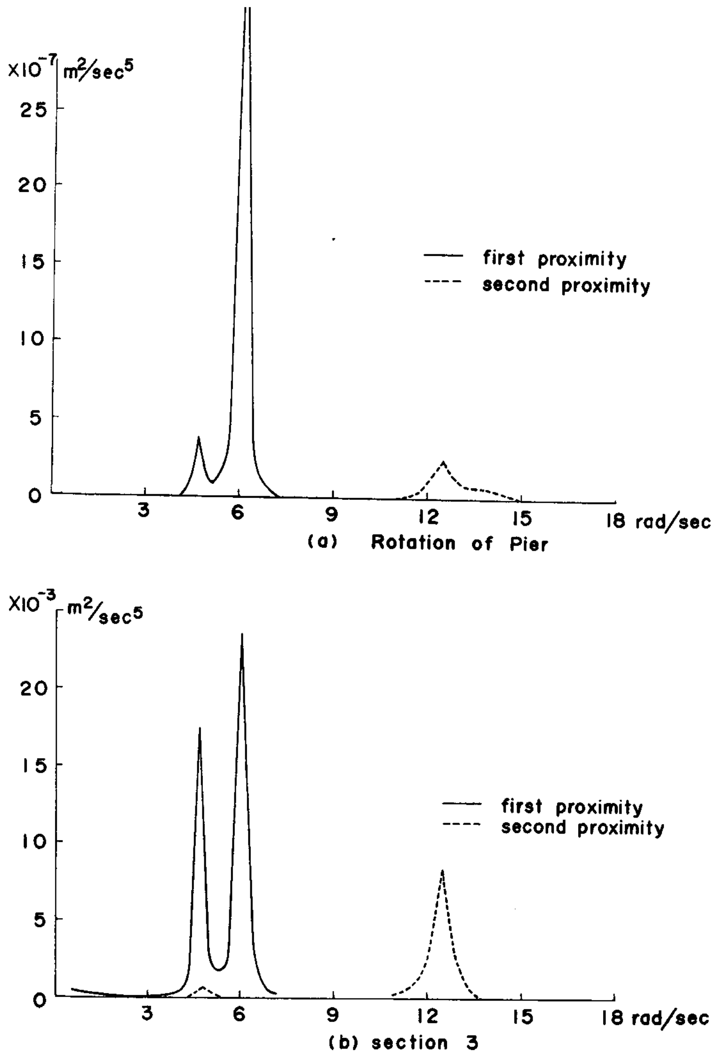
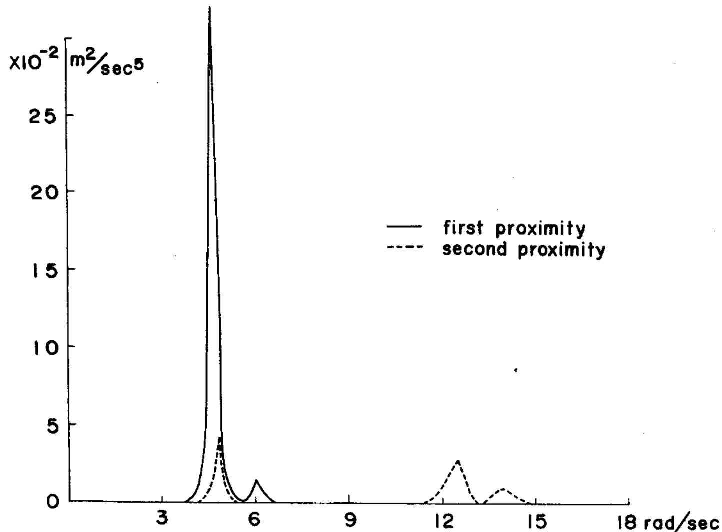
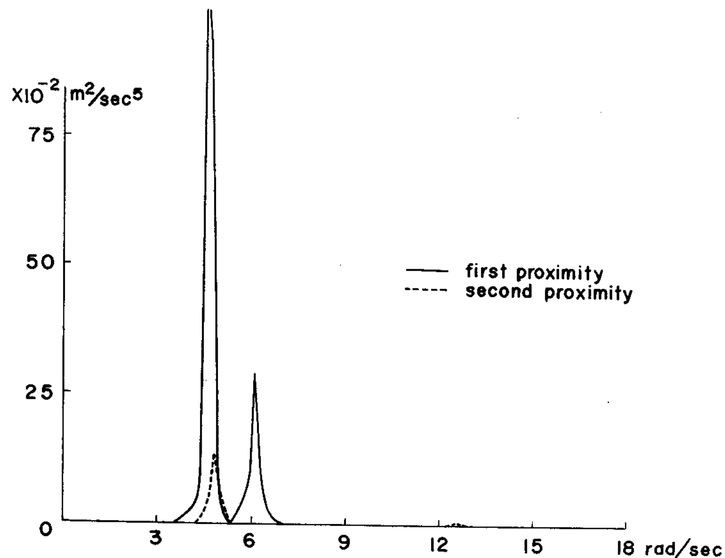


Fig. 13



(c) section 5



(d) section 7

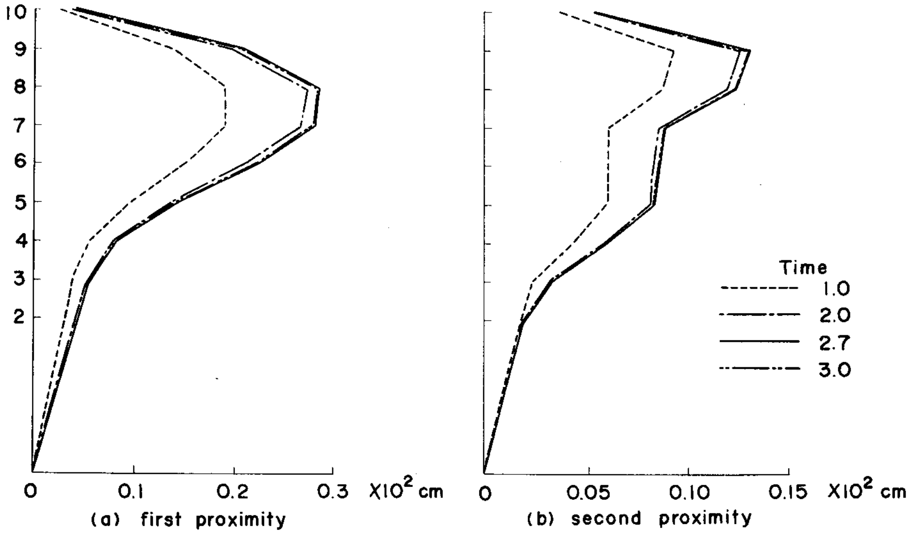
Fig. 13 Response power Spectral Density due to the Non-stationary Excitation

$$[S^X] = [V][H][V]^T[S^F][V][H^*]^T[V]^T \tag{5.1}$$

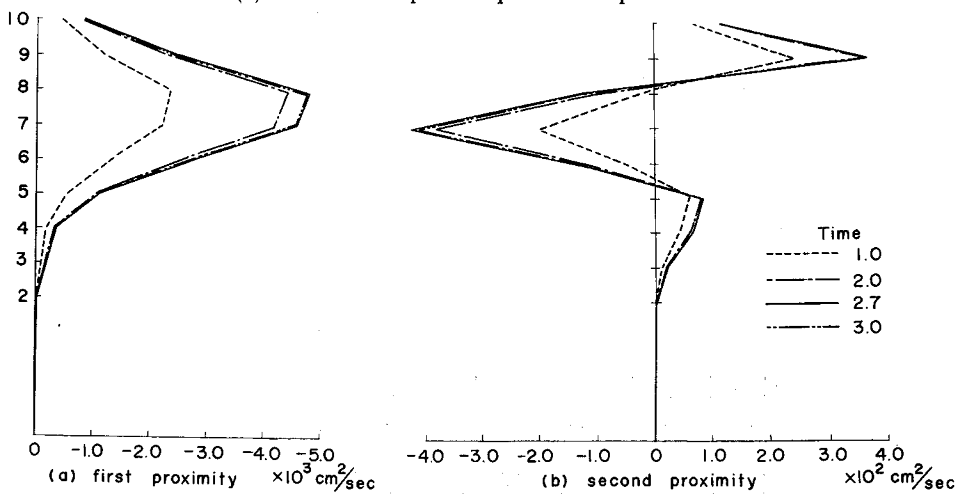
$$[S^{\dot{X}X}] = [V][H][V]^T[S^{\dot{F}F}][V][H^*]^T[V]^T \tag{5.2}$$

$$[S^{\dot{X}}] = [V][H][V]^T[S^{\dot{F}}][V][H^*]^T[V]^T \tag{5.3}$$

The response power spectral densities calculated are presented in Fig. 13, which correspond to those at time 2.78 sec. It is understood that the contribution of the pier part to the second mode is great in view of these figures. At the lower part of the tower the contribution of the second mode to the response is primary, at the central part the first mode and near the tower top the second. As the relults of the inverse Fourier Transformation of the power spectral densities, three kinds of stochastic quantities are obtained. The root-mean-square responses in Eq. (3.16) are shown in Fig. 14–1, those of the first derivative in Fig. 14–3 and the cross-variances between them in Fig. 14–2. It is recognized that the closeness of the natural fre-

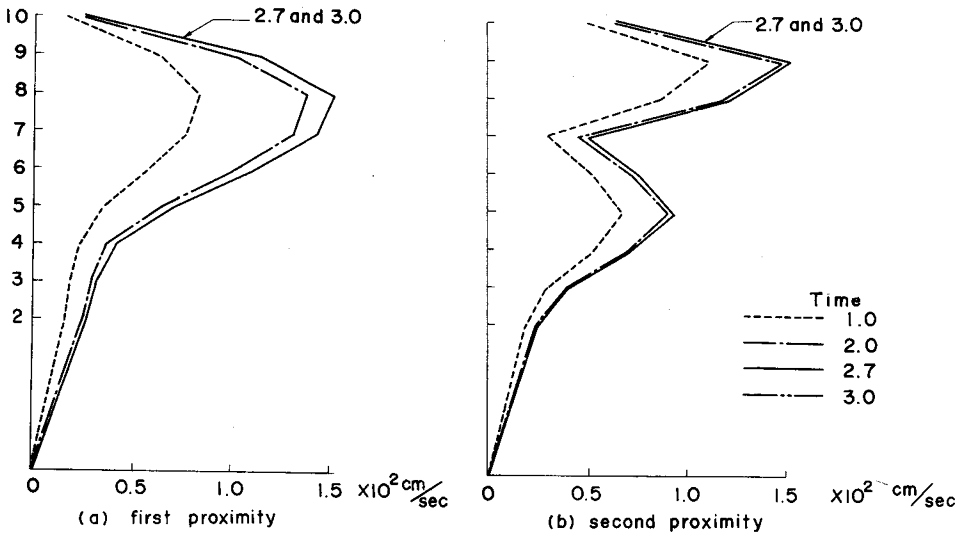


(1) Root-mean-square response of displacement



(2) Cross-variance between displacement and velocity

Fig. 14



(3) Root-mean-square response of velocity

Fig. 14 Response Characteristics of the System due to the Non-stationary Excitation

quencies exerts an influence on the order of cross-variance. In the first proximity it is greater than in the second. They will be conveniently utilized to discuss the system safety for the critical response level as mentioned in reference 11).

6. Comparison Between the Response Obtained by the Stochastic Analysis and the one by Direct-integration

The direct-integrated response gives the exact solution to the system with a general type of damping. Here, in this paper the damping effect of Type 3 is considered and the β-method of numerical caluculation[12] is adopted. As for the external force, the EL CENTRO earthquake, 1940 NS component (Fig. 7) is utilized.

The responses direct-integrated corresponding to three typical foundation conditions in Fig. 2 are shown with various damping factor in Fig. 15, where three kinds of response evaluations are taken; i.e. direct-integrated response, root-mean-square response of individual normal modes and their absolute sum.

Comparison between the results obtained by the stochastic analysis in section 3 and the ones direct-integrated gives the same informations but the maximum response values have to be made about three times in the former case. Moreover, the responses due to the non-stationary excitation obtained in section 5 not only present the same vibration aspects as the ones by direct-integration, but also give the necessary stochastic characteristics for the evaluation of the system safety.

From the facts above metnioned the spectral analysis will be well proposed to the aseismic structural design, including the earthquake parameter $\alpha_1, \cdots, \alpha_5$ in Eq. (4.2), while the response analysis by direct-integration for a certain earthquake motions represents a special one.

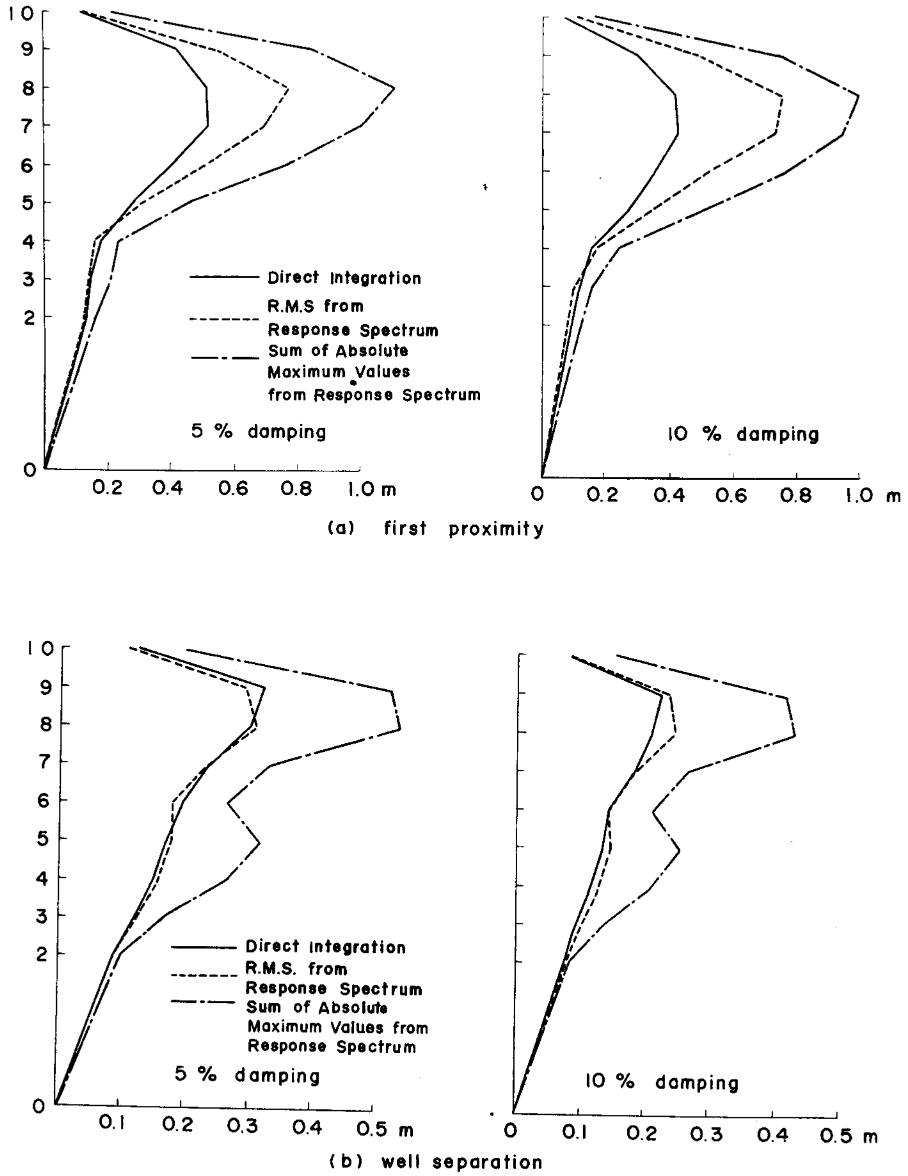


Fig. 15

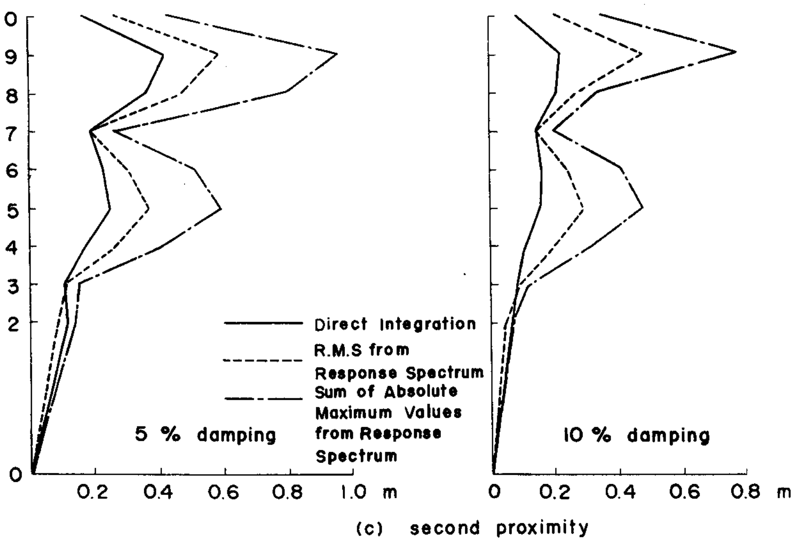


Fig. 15 Response direct-integrated for EL CENTRO earthquake 1940, NS component

7. Conclusions

From the results of the analyses in this study, the following conclusions may be derived.

1) Inertia moment of the pier coupled with the foundation constant has significant effects on the structural dynamic characteristics to such an extent that the normal modes have a good cross-relation. It is the function of the disposition of natural frequencies and the value of damping factor. Especially, the system with nearly equal natural frequencies presents, due to it, much of different response aspects from the ones obtained by the well-known response spectrum. However, in the system of their being well separated the cross terms become negligibly small. As for the damping effects, they act for the increase of the corss-relation in both cases.

2) Assumptions of vibration damping term affect the response at the relation of the system natural frequencies. Investigation on the system with general damping term has to be made for this reason.

3) The formulated mathematical expression of non-stationary random excitation is effective with some parameters for its application to the system response analysis.

4) The stochastic process theory guaranteed by the direct-integration is well proposed for the analysis of the structural integrity of a product.

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