Determination of the Stress in Rock Unaffected by Boreholes or Drifts from Measured Strains or Deformations

By

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The present paper treats the problems regarding determination of the stress in the rock unaffected by boreholes or drifts from measurements made in them. As the items of measurement, we can mention variations in borehole diameter, strains on the bottom surface of boreholes, strains of the wall surface of boreholes, variations in oblique dimension of boreholes and combinations of these as well as strains on the wall surface of drifts or shafts. The formulae to be used in practice to determine the stress in the rock from several measurements have been presented. On deducing them, the elastic constants of the rock, the rigidity of the measuring instruments used and the irregular distribution of strain within the range of each strain gauge were taken into account. The least number of boreholes or drifts necessary for each stress determination and the accuracy in the results obtained have also been discussed.

1. Introduction

In recent years, when investigating the problem of strata pressure, much effort has been made in attempt to determine the stress in the rock unaffected by boreholes or drifts from strains or deformations measured in those openings. Generally the ground may be in a three-dimensional stress state, the directions of the principal stresses inclining from the vertical and horizontal. Therefore it is supposed that the determination of the stress in the rock will need much complicated calculation.

Everling[1] and Hoek[2] showed how to determine the stress in the rock from variations in borehole diameter under the condition that the direction of the borehole axis coincided with that of one of the principal stresses. Hiramatsu and Oka[3,4] analysed the stress in the rock around a drift or shaft with a circular or various shaped cross section driven in the ground being in a general stress state. The results obtained provided the basis of determining the stress in the rock from measurements made in boreholes or drifts. Then they suggested methods of determining the stress in the rock either from strains measured at a minimum of three points on

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the wall surface of a drift or from the deformation of a borehole[5]. Leeman[6] proposed previously to determine the stress in the rock from strains on the bottom surface of a borehole under the condition that the axis of the borehole coincided with that of one of the principal stresses. Panek[7] described a statistical method of determining the mean stress in the rock from variations in borehole diameter. Recently Leeman[8] suggested the calculation of the stress in the rock from variations in borehole diameter under the condition that the ground was in a general stress state.

The authors have noticed, however, that there was still ample room for investigation into the calculation of the stress in the rock from the results of measurement, and have attempted further study. They have deduced the formulae to be used in practice to determine the stress in the rock from all kinds of measurement that they can currently think of. This study has been carried out on the assumption that the ground is perfectly elastic and is in a general state of stress and that the directions of the boreholes or drifts in which measurement is made may be optional. The irregular strain distribution within the range of each strain gauge has been taken into account. The counter-measure for employing rigid types of measuring instruments has also been discussed.

This paper treats the in situ stress determination by the stress-relief technique, but all the formulae obtained can be used in determining the variation in stress provided that the difference between the measured values at two different times be used in place of each absolute value of measurement.

2. Determining Stress from Variations in Borehole Diameter

Any stress state in the rock can be represented either by three principal stresses or by six stress components referred to a certain co-ordinates system. Now we assume a standard co-ordinates system (X, Y, Z) and auxiliary co-ordinates systems (x, y, z), (x', y', z') and (x'', y'', z''), y-, y'- and y''-axes being all horizontal, as shown in Fig. 1. Moreover let us take cylindrical co-ordinates systems (r, θ, x), (r', θ', x') and (r'', θ'', x'').

We designate the original stress components at a certain point P situated within short distances from all boreholes by $\mathring{\sigma}_x$, $\mathring{\sigma}_y$, $\mathring{\sigma}_z$, $\mathring{\tau}_{yz}$, $\mathring{\tau}_{zx}$ and $\mathring{\tau}_{xy}$ referring to the co-ordinates (x, y, z). The authors analysed previously the stress and displacement around a circular drift driven in the ground being in a three-dimensional stress state[3,4]. According to the analysis, the components of displacement, u, v and w, around the first borehole in the directions of r, θ and x are given by:

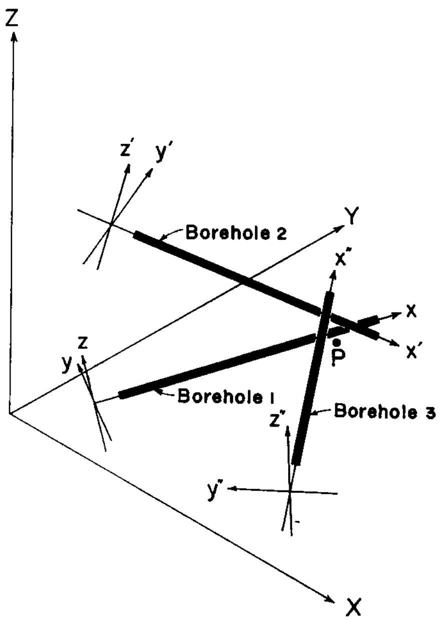

Fig. 1 Boreholes and co-ordinates systems.

$$\left.\begin{aligned}
u &= \frac{1+\nu}{E}\left[\left(\frac{1-\nu}{1+\nu}\,\frac{\overset{*}{\sigma}_y+\overset{*}{\sigma}_z}{2}-\frac{\nu}{1+\nu}\,\overset{*}{\sigma}_x\right)r+\frac{\overset{*}{\sigma}_y+\overset{*}{\sigma}_z}{2}\,\frac{a^2}{r}\right.\\
&\quad\left.+\left\{r-\frac{a^4}{r^3}+4(1-\nu)\frac{a^2}{r}\right\}\left(\frac{\overset{*}{\sigma}_y-\overset{*}{\sigma}_z}{2}\cos 2\theta+\overset{*}{\tau}_{yz}\sin 2\theta\right)\right],\\
v &= \frac{1+\nu}{E}\left\{-r-\frac{a^4}{r^3}-2(1-2\nu)\frac{a^2}{r}\right\}\left(\frac{\overset{*}{\sigma}_y-\overset{*}{\sigma}_z}{2}\sin 2\theta-\overset{*}{\tau}_{yz}\cos 2\theta\right),\\
w &= \frac{1+\nu}{E}\left\{2\left(r+\frac{a^2}{r}\right)(\overset{*}{\tau}_{zx}\sin\theta+\overset{*}{\tau}_{xy}\cos\theta)\right.\\
&\quad\left.+\frac{x}{1+\nu}\overset{*}{\sigma}_x-\frac{x\nu}{1+\nu}(\overset{*}{\sigma}_y+\overset{*}{\sigma}_z)\right\},
\end{aligned}\right\}\qquad (1)$$

where a is the radius of the borehole, E and ν are respectively the Young's modulus and the Poisson's ratio of the rock. The components of displacement, u_a, v_a, w_a at a point on the wall surface are expressed by:

$$\left.\begin{aligned}
u_a &= a\{-\nu\overset{*}{\sigma}_x+\overset{*}{\sigma}_y+\overset{*}{\sigma}_z+2(1-\nu^2)(\overset{*}{\sigma}_y-\overset{*}{\sigma}_z)\cos 2\theta\\
&\quad+4(1-\nu^2)\overset{*}{\tau}_{yz}\sin 2\theta\}/E,\\
v_a &= a\{-2(1-\nu^2)(\overset{*}{\sigma}_y-\overset{*}{\sigma}_z)\sin 2\theta+4(1-\nu^2)\overset{*}{\tau}_{yz}\cos 2\theta\}/E,\\
w_a &= a\left\{\frac{x}{a}(\overset{*}{\sigma}_x-\nu\overset{*}{\sigma}_y-\nu\overset{*}{\sigma}_z)+4(1+\nu)\overset{*}{\tau}_{zx}\sin\theta+4(1+\nu)\overset{*}{\tau}_{xy}\cos\theta\right\}/E.
\end{aligned}\right\}\qquad (2)$$

Suppose that variations in diameter, Δd_1, Δd_2 and Δd_3, are measured in a certain cross section near point P by means of the stress-relief technique. (See Fig. 2.) Point P must not be near the mouth or bottom of the borehole. The measured values, Δd_1, Δd_2 and Δd_3, ought to be approximately twice the values of u_a which are obtained by putting $\theta=\theta_1$, θ_2, θ_3 in the first equation of (2). Thus we have:

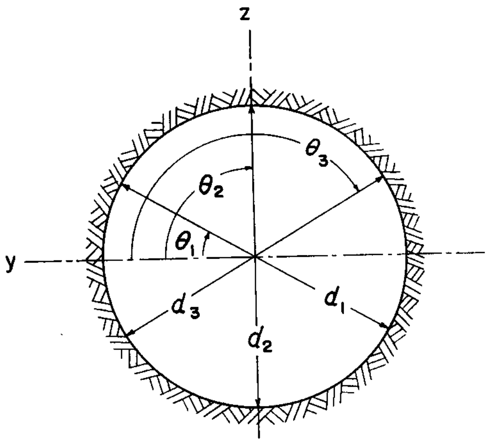

Fig. 2 Borehole diameters to be measured.

$$\Delta d_n/d = \alpha+\beta\cos 2\theta_n+\gamma\sin 2\theta_n, \quad (n=1, 2, 3, \cdots\cdots) \tag{3}$$

where

$$\alpha = (1/E)(-\nu\overset{*}{\sigma}_x+\overset{*}{\sigma}_y+\overset{*}{\sigma}_z),$$
$$\beta = (2/E)(1-\nu^2)(\overset{*}{\sigma}_y-\overset{*}{\sigma}_z),$$
$$\gamma = (4/E)(1-\nu^2)\overset{*}{\tau}_{yz}.$$

From equation (3) it is understood that we can indeed determine α, β and γ from variations in three optional diameters but the stress components σ_x, σ_y, σ_z and τ_{yz} cannot be determined, and that even from the variations in four or more diameters we can only determine more accurately the values of α, β and γ.

Before proceeding further, it may be preferable to transform equation (3) into the expressions referring to the standard co-ordinates system (X, Y, Z). Let the direction cosines of the axes x, y and z with respect to the X-, Y- and Z-axes be respectively (l_1, m_1, n_1), (l_2, m_2, n_2) and (l_3, m_3, n_3). Then from the equilibrium of force, the following relations exist:

$$\left.\begin{aligned}
\overset{*}{\sigma}_x &= l_1^2\overset{*}{\sigma}_X+m_1^2\overset{*}{\sigma}_Y+n_1^2\overset{*}{\sigma}_Z+2m_1n_1\overset{*}{\tau}_{YZ}+2n_1l_1\overset{*}{\tau}_{ZX}+2l_1m_1\overset{*}{\tau}_{XY},\\
\overset{*}{\sigma}_y &= l_2^2\overset{*}{\sigma}_X+m_2^2\overset{*}{\sigma}_Y+n_2^2\overset{*}{\sigma}_Z+2m_2n_2\overset{*}{\tau}_{YZ}+2n_2l_2\overset{*}{\tau}_{ZX}+2l_2m_2\overset{*}{\tau}_{XY},\\
\overset{*}{\sigma}_z &= l_3^2\overset{*}{\sigma}_X+m_3^2\overset{*}{\sigma}_Y+n_3^2\overset{*}{\sigma}_Z+2m_3n_3\overset{*}{\tau}_{YZ}+2n_3l_3\overset{*}{\tau}_{ZX}+2l_3m_3\overset{*}{\tau}_{XY},\\
\overset{*}{\tau}_{yz} &= l_2l_3\overset{*}{\sigma}_X+m_2m_3\overset{*}{\sigma}_Y+n_2n_3\overset{*}{\sigma}_Z+(m_2n_3+n_2m_3)\overset{*}{\tau}_{YZ}\\
&\quad+(l_2n_3+n_2l_3)\overset{*}{\tau}_{ZX}+(l_2m_3+m_2l_3)\overset{*}{\tau}_{XY},
\end{aligned}\right\} \tag{4}$$

$$\begin{aligned}\overset{*}{\tau}_{zx} &= l_3 l_1 \overset{*}{\sigma}_X + m_3 m_1 \overset{*}{\sigma}_Y + n_3 n_1 \overset{*}{\sigma}_Z + (m_3 n_1 + n_3 m_1) \overset{*}{\tau}_{YZ} \\ &\quad + (l_3 n_1 + n_3 l_1) \overset{*}{\tau}_{ZX} + (l_3 m_1 + m_3 l_1) \overset{*}{\tau}_{XY}, \\ \overset{*}{\tau}_{xy} &= l_1 l_2 \overset{*}{\sigma}_X + m_1 m_2 \overset{*}{\sigma}_Y + n_1 n_2 \overset{*}{\sigma}_Z + (m_1 n_2 + n_1 m_2) \overset{*}{\tau}_{YZ} \\ &\quad + (l_1 n_2 + n_1 l_2) \overset{*}{\tau}_{ZX} + (l_1 m_2 + m_1 l_2) \overset{*}{\tau}_{XY}. \end{aligned}$$

Substituting equations (4) in equation (3), we obtain:

$$\begin{aligned}\Delta d/d = [&\{1 - l_1^2 - \nu l_1^2 + 2(1-\nu^2)(l_2^2 - l_3^2)\cos 2\theta + 4(1-\nu^2) l_2 l_3 \sin 2\theta\} \overset{*}{\sigma}_X \\ &+ \{1 - m_1^2 - \nu m_1^2 + 2(1-\nu^2)(m_2^2 - m_3^2)\cos 2\theta + 4(1-\nu^2) m_2 m_3 \sin 2\theta\} \overset{*}{\sigma}_Y \\ &+ \{1 - n_1^2 - \nu n_1^2 + 2(1-\nu^2)(n_2^2 - n_3^2)\cos 2\theta + 4(1-\nu^2) n_2 n_3 \sin 2\theta\} \overset{*}{\sigma}_Z \\ &+ \{-2(1+\nu) m_1 n_1 + 4(1-\nu^2)(m_2 n_2 - m_3 n_3) \cos 2\theta \\ &\quad + 4(1-\nu^2)(m_2 n_3 + n_2 m_3) \sin 2\theta\} \overset{*}{\tau}_{YZ} \\ &+ \{-2(1+\nu) n_1 l_1 + 4(1-\nu^2)(n_2 l_2 - n_3 l_3) \cos 2\theta \\ &\quad + 4(1-\nu^2)(n_2 l_3 + l_2 n_3) \sin 2\theta\} \overset{*}{\tau}_{ZX} \\ &+ \{-2(1+\nu) l_1 m_1 + 4(1-\nu^2)(l_2 m_2 - l_3 m_3) \cos 2\theta \\ &\quad + 4(1-\nu^2)(l_2 m_3 + m_2 l_3) \sin 2\theta\} \overset{*}{\tau}_{XY}]/E. \end{aligned} \tag{5}$$

Equation (5) is the general formula to be used in practice to determine the stress components in the rock from variations in borehole diameter measured by a soft type of measuring instrument. Now let us discuss how many measurements should be done in how many boreholes. Assume that the variations in three diameters are measured at a cross section near point P in each borehole. By substituting the measured values, the angles that represent the directions of diameters measured and the direction cosines of the axis of each borehole in equation (5), we obtain three observation equations with six unknowns $\overset{*}{\sigma}_X$, $\overset{*}{\sigma}_Y$, $\overset{*}{\sigma}_Z$, $\overset{*}{\tau}_{YZ}$, $\overset{*}{\tau}_{ZX}$, $\overset{*}{\tau}_{XY}$ for each borehole. If we set up six simultaneous observation equations from the results of measurements made in two boreholes, it will readily be found that the rank of the determinant made up of the coefficients of unknowns is less than six. It follows that the unknowns cannot be determined from these equations. But if we set up nine observation equations from the results of measurement made in three boreholes, it will be found that the rank of the matrix of their coefficients becomes six or greater than six. It is concluded therefore that in order to determine the stress in the rock from variations in borehole diameter, it is necessary to carry out measurement in at least three boreholes, as Gray and Toews[9] pointed out. To determine six unknowns from equations greater than six in number, it is advisable to use the method of least squares.

The boreholes shown in Fig. 1 are in a random arrangement. If they are in a regular arrangement, equation (5) will become simpler. An example of simple

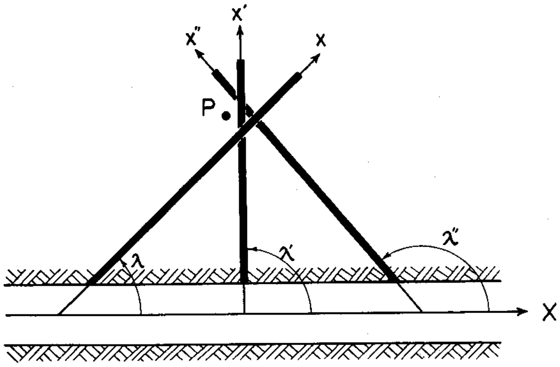

Fig. 3 An example of the arrangement of three boreholes.

arrangements of boreholes will be shown in the following. Assume that three boreholes are drilled horizontally from a drift as shown in Fig. 3. Let the angles between the axes of boreholes, x, x', x'', and the axis of the drift X be λ, λ' and λ'' respectively. Then for the first borehole

$$
\begin{aligned}
l_1 &= \cos\lambda, & m_1 &= \sin\lambda, & n_1 &= 0, \\
l_2 &= -\sin\lambda, & m_2 &= \cos\lambda, & n_2 &= 0, \\
l_3 &= 0, & m_3 &= 0, & n_3 &= 1.
\end{aligned}
$$

Thus equation (5) is transformed as:

$$
\begin{aligned}
\Delta d/d = [&\{1-(1+\nu)\cos^2\lambda+2(1-\nu^2)\sin^2\lambda\cos 2\theta\}\mathring{\sigma}_X \\
&+\{1-(1+\nu)\sin^2\lambda+2(1-\nu^2)\cos^2\lambda\cos 2\theta\}\mathring{\sigma}_Y \\
&\quad+\{1-2(1-\nu^2)\cos 2\theta\}\mathring{\sigma}_Z \\
&+\{4(1-\nu^2)\cos\lambda\sin 2\theta\}\mathring{\tau}_{YZ}+\{-4(1-\nu^2)\sin\lambda\sin 2\theta\}\mathring{\tau}_{ZX} \\
&+\{-2(1+\nu)\cos\lambda\sin\lambda-4(1-\nu^2)\cos\lambda\sin\lambda\cos 2\theta\}\mathring{\tau}_{XY}]/E. \qquad (6)
\end{aligned}
$$

By substituting λ' or λ'' for λ in equation (6), we have the equations for the second or third borehole.

3. Determining Stress from Strains on Bottoms of Boreholes

To obtain the formulae for determining the stress in the rock from strains on the bottom surfaces of boreholes, it is essential to know the stress state on the bottom surfaces. This stress state was once studied by photoelasticity by Galle[10]. But as it affords an important material for determining the stress in the rock, the authors have recently carried out investigations into the same problem, the results of which will be reported in a separate paper. However the necessary relations for the present discussion will be mentioned in the following.

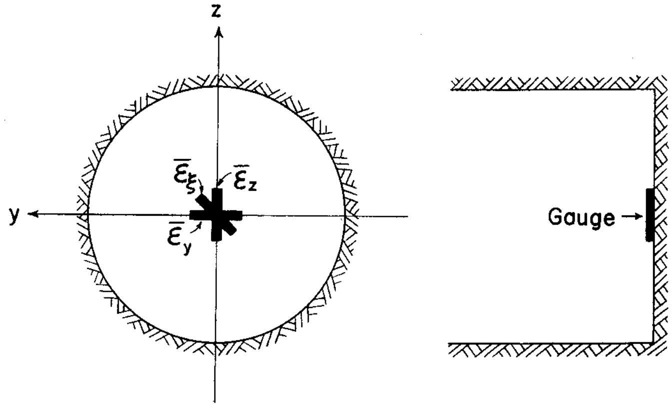

Fig. 4 Strains to be measured on the bottom surface of a borehole.

By means of strain gauges we can measure average longitudinal strains within the range of the gauges. The average strains $\bar{\varepsilon}_y$, $\bar{\varepsilon}_z$, $\bar{\varepsilon}_\xi$ in the middle of the bottom surface of each borehole in the directions of y, z and ξ, ξ being the direction making an angle of 45° with both the y- and z- axes as shown in Fig. 4, are given by the following equations:

$$\left.\begin{aligned} \bar{\varepsilon}_y &= (L\overset{*}{\sigma}_x+M\overset{*}{\sigma}_y+N\overset{*}{\sigma}_z)/E, \\ \bar{\varepsilon}_z &= (L\overset{*}{\sigma}_x+N\overset{*}{\sigma}_y+M\overset{*}{\sigma}_z)/E, \\ \bar{\varepsilon}_\xi &= \left\{L\overset{*}{\sigma}_x+\frac{M+N}{2}\overset{*}{\sigma}_y+\frac{M+N}{2}\overset{*}{\sigma}_z+(M-N)\overset{*}{\tau}_{yz}\right\}/E, \end{aligned}\right\} \quad (7)$$

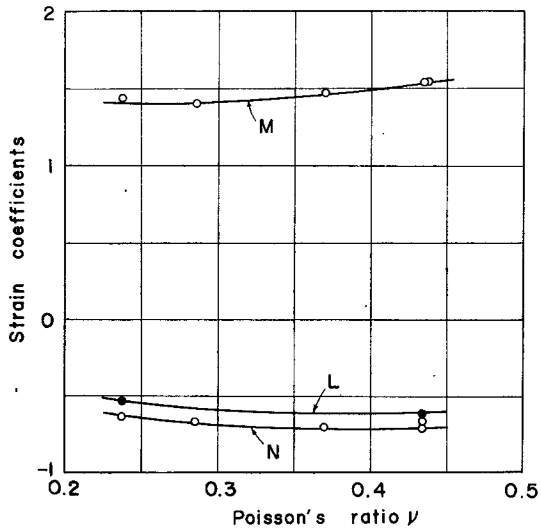

Fig. 5 Relation between strain coefficients and Poisson's ratio.

where L, M and N are coefficients which can be determined by photoelasticity. These coefficients depend a little upon the Poisson's ratio of the rock. Under the condition that strain gauges are not long, compared with the diameter of boreholes, say less than one fifth the diameter, the coefficients take such values as shown in Fig. 5.

In this case also, it can be proved that measurement must be carried out at least in three boreholes as Gray and Toews[9] pointed out. Now let us rewrite equations (7) in terms of the stress components referring to the co-ordinates system (X, Y, Z).

$$\left.\begin{aligned}
\varepsilon_y = {} & \{(Ll_1^2+Ml_2^2+Nl_3^2)\overset{*}{\sigma}_X+(Lm_1^2+Mm_2^2+Nm_3^2)\overset{*}{\sigma}_Y \\
& +(Ln_1^2+Mn_2^2+Nn_3^2)\overset{*}{\sigma}_Z+2(Lm_1n_1+Mm_2n_2+Nm_3n_3)\overset{*}{\tau}_{YZ} \\
& +2(Ln_1l_1+Mn_2l_2+Nn_3l_3)\overset{*}{\tau}_{ZX} \\
& \quad +2(Ll_1m_1+Ml_2m_2+Nl_3m_3)\overset{*}{\tau}_{XY}\}/E, \\
\varepsilon_z = {} & \{(Ll_1^2+Nl_2^2+Ml_3^2)\overset{*}{\sigma}_X+(Lm_1^2+Nm_2^2+Mm_3^2)\overset{*}{\sigma}_Y \\
& +(Ln_1^2+Nn_2^2+Mn_3^2)\overset{*}{\sigma}_Z+2(Lm_1n_1+Nm_2n_2+Mm_3n_3)\overset{*}{\tau}_{YZ} \\
& +2(Ln_1l_1+Nn_2l_2+Mn_3l_3)\overset{*}{\tau}_{ZX} \\
& \quad +2(Ll_1m_1+Nl_2m_2+Ml_3m_3)\overset{*}{\tau}_{XY}\}/E, \\
\varepsilon_\xi = {} & \Big[\Big\{Ll_1^2+\frac{M+N}{2}(l_2^2+l_3^2)+(M-N)l_2l_3\Big\}\overset{*}{\sigma}_X \\
& +\Big\{Lm_1^2+\frac{M+N}{2}(m_2^2+m_3^2)+(M-N)m_2m_3\Big\}\overset{*}{\sigma}_Y \\
& +\Big\{Ln_1^2+\frac{M+N}{2}(n_2^2+n_3^2)+(M-N)n_2n_3\Big\}\overset{*}{\sigma}_Z \\
& +\{2Lm_1n_1+(M+N)(m_2n_2+m_3n_3) \\
& \quad +(M-N)(m_2n_3+n_2m_3)\}\overset{*}{\tau}_{YZ} \\
& +\{2Ln_1l_1+(M+N)(n_2l_2+n_3l_3) \\
& \quad +(M-N)(n_2l_3+l_2n_3)\}\overset{*}{\tau}_{ZX} \\
& +\{2Ll_1m_1+(M+N)(l_2m_2+l_3m_3) \\
& \quad +(M-N)(l_2m_3+m_2l_3)\}\overset{*}{\tau}_{XY}\Big]/E.
\end{aligned}\right\} \quad (8)$$

Equations (8) are the general formulae to be used to determine the stress in the rock from the strains on the bottom surface of boreholes measured by a soft type of measuring instrument.

In the case that three boreholes are arranged as shown in Fig. 3, equations (8) for the first borehole are transformed as:

$$\begin{aligned}
\varepsilon_y = {} & \{(L\cos^2\lambda+M\sin^2\lambda)\overset{*}{\sigma}_X+(L\sin^2\lambda+M\cos^2\lambda)\overset{*}{\sigma}_Y+N\overset{*}{\sigma}_Z \\
& +2(L-M)\sin\lambda\cos\lambda\cdot\overset{*}{\tau}_{XY}\}/E,
\end{aligned}$$

$$\left.\begin{aligned}
\varepsilon_x &= \{(L\cos^2\lambda+N\sin^2\lambda)\overset{*}{\sigma}_X+(L\sin^2\lambda+N\cos^2\lambda)\overset{*}{\sigma}_Y+M\overset{*}{\sigma}_Z\\
&\quad +2(L-N)\sin\lambda\cos\lambda\overset{*}{\tau}_{XY}\}/E,\\
\varepsilon_\xi &= \Big\{\Big(L\cos^2\lambda+\frac{M+N}{2}\sin^2\lambda\Big)\overset{*}{\sigma}_X\\
&\quad +\Big(L\sin^2\lambda+\frac{M+N}{2}\cos^2\lambda\Big)\overset{*}{\sigma}_Y\\
&\quad +\frac{M+N}{2}\overset{*}{\sigma}_Z+(M-N)\cos\lambda\cdot\overset{*}{\tau}_{YZ}-(M-N)\sin\lambda\cdot\overset{*}{\tau}_{ZX}\\
&\quad +(2L-M-N)\cos\lambda\sin\lambda\cdot\overset{*}{\tau}_{XY}\Big]/E.
\end{aligned}\right\}\quad (9)$$

To obtain the formula for the second or third borehole, we have only to substitute λ' or λ'' for λ in equations (9).

4. Determining Stress from Strains on the Wall of a Borehole

The results of analysis of the stress around a circular opening obtained previously by the authors[3,4] are as follows:

$$\left.\begin{aligned}
\sigma_r &= \frac{1}{2}(\overset{*}{\sigma}_y+\overset{*}{\sigma}_z)\Big(1-\frac{a^2}{r^2}\Big)+\frac{1}{2}(\overset{*}{\sigma}_y-\overset{*}{\sigma}_z)\Big(1-4\frac{a^2}{r^2}+3\frac{a^4}{r^4}\Big)\cos 2\theta\\
&\quad +\overset{*}{\tau}_{yz}\Big(1-4\frac{a^2}{r^2}+3\frac{a^4}{r^4}\Big)\sin 2\theta,\\
\sigma_\theta &= \frac{1}{2}(\overset{*}{\sigma}_y+\overset{*}{\sigma}_z)\Big(1+\frac{a^2}{r^2}\Big)-\frac{1}{2}(\overset{*}{\sigma}_y-\overset{*}{\sigma}_z)\Big(1+3\frac{a^4}{r^4}\Big)\cos 2\theta\\
&\quad -\overset{*}{\tau}_{yz}\Big(1+3\frac{a^4}{r^4}\Big)\sin 2\theta,\\
\sigma_x &= \overset{*}{\sigma}_x-2\nu(\overset{*}{\sigma}_y-\overset{*}{\sigma}_z)\frac{a^2}{r^2}\cos 2\theta-4\nu\overset{*}{\tau}_{yz}\frac{a^2}{r^2}\sin 2\theta,\\
\tau_{\theta x} &= \overset{*}{\tau}_{zx}\Big(1+\frac{a^2}{r^2}\Big)\cos\theta-\overset{*}{\tau}_{xy}\Big(1+\frac{a^2}{r^2}\Big)\sin\theta,\\
\tau_{xr} &= \overset{*}{\tau}_{zx}\Big(1-\frac{a^2}{r^2}\Big)\sin\theta+\overset{*}{\tau}_{xy}\Big(1-\frac{a^2}{r^2}\Big)\cos\theta,\\
\tau_{r\theta} &= \frac{1}{2}(\overset{*}{\sigma}_y-\overset{*}{\sigma}_z)\Big(-1-2\frac{a^2}{r^2}+3\frac{a^4}{r^4}\Big)\sin 2\theta\\
&\quad -\overset{*}{\tau}_{yz}\Big(-1-2\frac{a^2}{r^2}+3\frac{a^4}{r^4}\Big)\cos 2\theta.
\end{aligned}\right\}\quad (10)$$

The stress components at any point $(a,\ \theta,\ x)$ on the wall of a borehole are given by:

$$\left.\begin{aligned}(\sigma_\theta)_a &= (\overset{*}{\sigma}_y+\overset{*}{\sigma}_z)-2(\overset{*}{\sigma}_y-\overset{*}{\sigma}_z)\cos 2\theta-4\overset{*}{\tau}_{yz}\sin 2\theta,\\(\sigma_x)_a &= \overset{*}{\sigma}_x-2\nu(\overset{*}{\sigma}_y-\overset{*}{\sigma}_z)\cos 2\theta-4\nu\overset{*}{\tau}_{yz}\sin 2\theta,\\(\tau_{\theta x})_a &= 2\overset{*}{\tau}_{zx}\cos\theta-2\overset{*}{\tau}_{xy}\sin\theta\,.\end{aligned}\right\}\quad(11)$$

Now suppose that longitudinal strains in three directions are measured by the stress relief technique at several points on the wall of a borehole as shown in Fig. 6. We can readily evaluate the stress components $(\sigma_\theta)_a$, $(\sigma_x)_a$, and $(\tau_{\theta x})_a$ at the same points from the measured strains if the elastic constants of the rock are known.

Assume that we have determined the three stress components at the two points where $\theta=\theta_1$, and $\theta=\theta_2$. By substituting these values as well as the angles θ_1 and θ_2 in equations (11), six observation equations are set up, the unknowns being $\overset{*}{\sigma}_x$, $\overset{*}{\sigma}_y+\overset{*}{\sigma}_z$, $\overset{*}{\sigma}_y-\overset{*}{\sigma}_z$, $\overset{*}{\tau}_{yz}$, $\overset{*}{\tau}_{zx}$, $\overset{*}{\tau}_{xy}$. However it is proved that the value of the determinant constructed by the coefficients in these simultaneous equations is always zero. It follows that not all the unknowns can be determined from the strain measurement carried out at two points on the wall of a borehole. Now assume that we have further determined the three stress components at the third point where

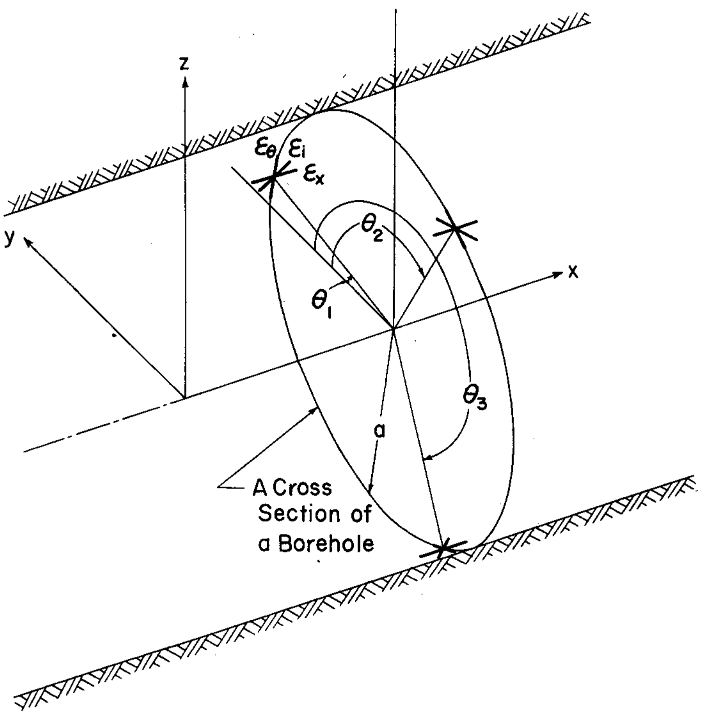

Fig. 6 Strain components to be measured on the wall of a borehole.

$\theta=\theta_3$ and three more observation equations are added. Care must be taken that the difference between any two of the three angles θ_1, θ_2 and θ_3 must not be π. Then the rank of the matrix made up of the coefficients of these observation equations become six or greater than six. Therefore all the unknowns can be determined. It follows that we can determine the stress in the rock from strains on the wall measured at least in one borehole.

Let us deduce the relation between the measured average strains and the stress in the rock. We denote the average strain components in the directions of x, θ and i by ε_x, ε_θ and ε_i respectively, the i-direction making an angle of 45° with both the x- and θ-directions. As for average stresses, let us use such a notation that the average values of σ_θ in the directions of x and θ are denoted respectively by $(\bar{\sigma}_\theta)_x$ and $(\bar{\sigma}_\theta)_\theta$. Then we have:

$$\left.\begin{aligned}\varepsilon_\theta &= \{(\bar{\sigma}_\theta)_\theta-\nu(\bar{\sigma}_x)_\theta\}/E,\\ \varepsilon_x &= \{(\bar{\sigma}_x)_x-\nu(\bar{\sigma}_\theta)_x\}/E,\\ \varepsilon_i &= [(1-\nu)\{(\bar{\sigma}_x)_i+(\bar{\sigma}_\theta)_i\}+2(1+\nu)(\bar{\tau}_{\theta x})_i]/2E.\end{aligned}\right\} \tag{12}$$

By putting $\omega=l/a$, where l is the length of strain gauges, and calculating average stresses in equation (12) by making use of equations (11), we obtain:

$$\left.\begin{aligned}\varepsilon_\theta =& \Big[-\nu\overset{*}{\sigma}_x+\Big\{1-\frac{\sin\omega}{\omega}\,2(1-\nu^2)\cos 2\theta\Big\}\overset{*}{\sigma}_y\\ &+\Big\{1+\frac{\sin\omega}{\omega}\,2(1-\nu^2)\cos 2\theta\Big\}\overset{*}{\sigma}_z\\ &-\frac{\sin\omega}{\omega}\,4(1-\nu^2)\sin 2\theta\cdot\overset{*}{\tau}_{yz}\Big]/E,\\ \varepsilon_x =& (\overset{*}{\sigma}_x-\nu\overset{*}{\sigma}_y-\nu\overset{*}{\sigma}_z)/E,\\ \varepsilon_i =& \Big[\frac{1-\nu}{2}\,\overset{*}{\sigma}_x+\Big\{\frac{1-\nu}{2}-\frac{\sin(\omega/\sqrt{2})}{\omega/\sqrt{2}}(1-\nu^2)\cos 2\theta\Big\}\overset{*}{\sigma}_y\\ &+\Big\{\frac{1-\nu}{2}+\frac{\sin(\omega/\sqrt{2})}{\omega/\sqrt{2}}(1-\nu^2)\cos 2\theta\Big\}\overset{*}{\sigma}_z\\ &-\frac{\sin(\omega/\sqrt{2})}{\omega/\sqrt{2}}\,2(1-\nu^2)\sin 2\theta\cdot\overset{*}{\tau}_{yz}\\ &+\frac{\sin(\omega/2\sqrt{2})}{\omega/2\sqrt{2}}\,2(1+\nu)\cos\theta\cdot\overset{*}{\tau}_{zx}\\ &-\frac{\sin(\omega/2\sqrt{2})}{\omega/2\sqrt{2}}\,2(1+\nu)\sin\theta\cdot\overset{*}{\tau}_{xy}\Big]/E.\end{aligned}\right\} \tag{13}$$

These are the equations which can be used for determining the stress in the rock from strains on the wall of a borehole measured by a soft type of measuring instrument.

5. Determining Stress from Deformation of a Borehole

Suppose that not only variations in diameter but also variations in length of oblique dimensions, such as RQ in Fig. 7, are measured by the stress-relief technique. We denote the length RQ by s, and assume the distance b and the angle θ as shown in Fig. 7. Then we have:

$$\begin{aligned}\Delta s/2 &= \{(a+u_a)^2+v_a^2+(b+w_a)^2\}^{1/2}-(a^2+b^2)^{1/2}\\ &\fallingdotseq (au_a+bw_a)(a^2+b^2)^{-1/2}.\end{aligned} \tag{14}$$

Substituting equation (2) in (14), we obtain:

$$\begin{aligned}\Delta s/d = 2\Big[&\Big(-\nu a+\frac{b^2}{a}\Big)\overset{*}{\sigma}_x+\Big\{a-\nu\frac{b^2}{a}+2(1-\nu^2)a\cos 2\theta\Big\}\overset{*}{\sigma}_y\\ &+\Big\{a-\nu\frac{b^2}{a}-2(1-\nu^2)a\cos 2\theta\Big\}\overset{*}{\sigma}_z+4(1-\nu^2)a\sin 2\theta\cdot\overset{*}{\tau}_{yz}\\ &+4(1+\nu)b\sin\theta\cdot\overset{*}{\tau}_{zx}+4(1+\nu)b\cos\theta\cdot\overset{*}{\tau}_{xy}\Big]/E\ s.\end{aligned} \tag{15}$$

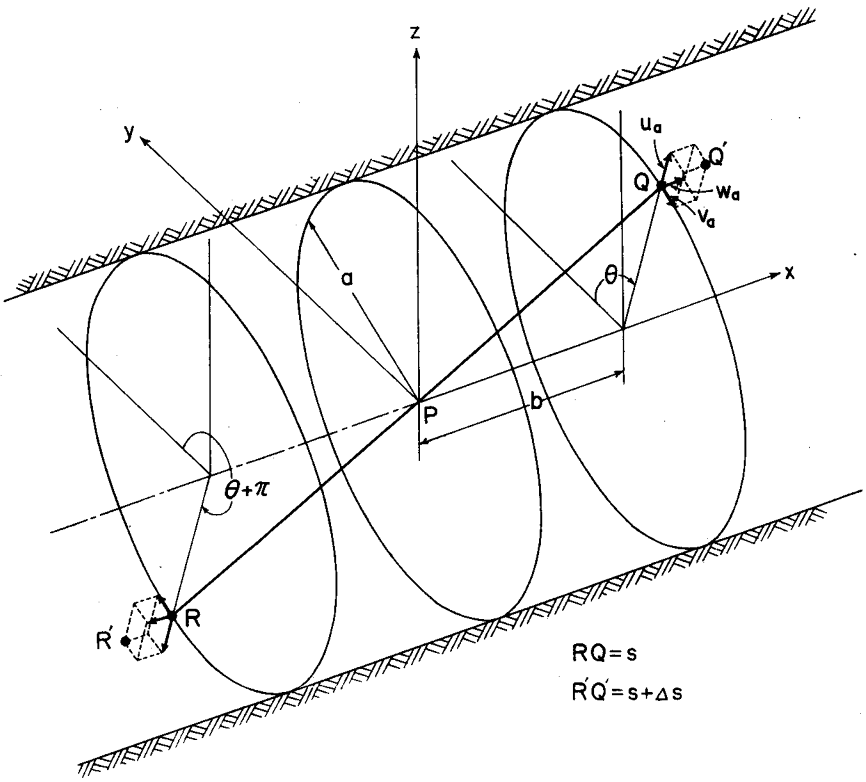

Fig. 7 A diagonal dimension in a borehole to be measured.

When $b=0$, RQ becomes a diameter and Δs is reduced to a variation in diameter Δd. Consequently, we have:

$$\Delta d/d = [-\nu\mathring{\sigma}_x + \{1+2(1-\nu^2)\cos 2\theta\}\mathring{\sigma}_y + \{1-2(1-\nu^2)\cos 2\theta\}\mathring{\sigma}_z + 4(1-\nu^2)\sin 2\theta\cdot\mathring{\tau}_{yz}]/E. \tag{16}$$

From the values of Δs and Δd measured in several directions (at least six directions in total) by a soft type of measuring instrument, we can determine the stress in the rock by substituting the measured values in equations (15) and (16), because all the observation equations obtained are independent. It must be noted here that the observation equations containing Δs up to five in number are independent while those containing Δd up to three in number are independent.

It is considered that the method of stress determination above described would indeed be hard to use in a borehole, but in a circular drift excavated by a tunnel boring machine, the method could easily be applied.

6. Determining Stress from Both Variations in Diameter and Strains on Bottoms of Boreholes

In expectation of reducing the number of boreholes in which measurements are to be done, let us discuss determining the stress in the rock from the combined measurements. The stress determination from the strains on the wall of or from the deformation of a borehole needs no consideration. Therefore the only case to be considered is that the stress in the rock is determined from both variations in diameter and strains on the bottom surfaces of boreholes.

By considering equations (3) and (7) it may be found that we can determine the four stress components, $\mathring{\sigma}_x$, $\mathring{\sigma}_y$, $\mathring{\sigma}_z$ and $\mathring{\tau}_{yz}$ from variations in diameter and strains on the bottom surface measured in one borehole. Furthermore, if the similar measurements are made in another borehole, we can construct twelve observation equations by putting the measured values in equations (5) and (8). By considering the rank of the matrix made up of the coefficients of these equations it may be seen that we can determine, in theory, the stress in the rock from the combined measurements made in two boreholes. But it is apprehended that the observation equations set up from variations in diameter and those set up from strains on the bottom surface of each borehole are close to each other. Therefore, it is supposed that the accuracy in the result obtained will be rather low.

7. Determining Stress by Making Use of Calibration Tests

By the calibration tests conducted with test pieces of the same rock as that found around the boreholes and with the measuring apparatus used in practice, all the coefficients which are necessary in calculating the stress in the rock can be determined, and the tests of the elastic constants can be omitted. When a rigid type of measuring instrument is used, we cannot help carrying out calibration tests.

Let us denote the reading of the instrument, inserted in a borehole to measure the variation in diameter, the strain on the bottom surface or the variation in any other quantity due to the deformation of the borehole, by R. The instrument may be either of a rigid type or of a soft type. To designate the three directions of measurement made in the borehole, we add the subscripts y, z, ξ to R, the direction ξ being that making an angle of 45° both with the y- and z-directions. Let these three readings in the test as illustrated by Fig. 8 (a) divided by the intensity of loading be H, I and J respectively. It is proved theoretically that J is a half the sum of H and I. In the test as shown in Fig. 8(b), the readings ought to be equal in all directions. But in practice they will more or less differ with one another. Let us denote the mean value of the three readings divided by the intensity of loading by G. Then from the similar contemplation as that made on deriving equation (7), we get

$$\left.\begin{aligned} R_y &= G\overset{*}{\sigma}_x+I\overset{*}{\sigma}_y+H\overset{*}{\sigma}_z, \\ R_z &= G\overset{*}{\sigma}_x+H\overset{*}{\sigma}_y+I\overset{*}{\sigma}_z, \\ R_\xi &= G\overset{*}{\sigma}_x+\frac{I+H}{2}\overset{*}{\sigma}_y+\frac{I+H}{2}\overset{*}{\sigma}_z+(I-H)\overset{*}{\tau}_{yz}. \end{aligned}\right\} \quad (17)$$

In the same way we can obtain the eqations that correspond to equations (8) and (9) by substituting R_y, R_z, R_ξ for ϵ_y, ϵ_z, ϵ_ξ respectively and by substituting G, I, H for L/E, M/E, N/E respectively in these equations. The equations obtained by these substitutions are to be used for determining the stress in the rock not only from strains on the bottom surfaces of boreholes but also from any other measured quantity.

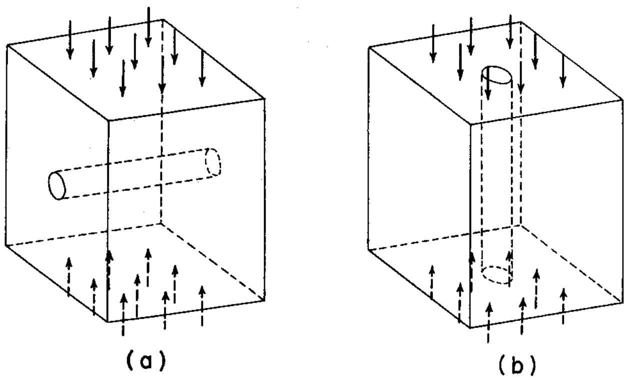

Fig. 8 Two kinds of calibration test.

8. Determining Stress from Strains on the Wall of a Drift

According to the authors' analysis of the stress on the wall of a drift[4], the stress components can be represented by:

$$\left.\begin{aligned}\sigma_t &= A_y\overset{*}{\sigma}_y+A_z\overset{*}{\sigma}_z+(2A_a-A_y-A_z)\overset{*}{\tau}_{yz},\\ \sigma_{x'} &= \overset{*}{\sigma}_x+\nu(A_y-1)\overset{*}{\sigma}_y+\nu(A_z-1)\overset{*}{\sigma}_z+\nu(2A_a-A_y-A_z)\overset{*}{\tau}_{yz},\\ \tau_{tx'} &= 2F_b\overset{*}{\tau}_{zx}+2F_c\overset{*}{\tau}_{xy},\end{aligned}\right\} \qquad (18)$$

where the x'-axis is parallel to the axis of the drift, the t-axis is another axis, perpendicular to the x'-axis, tangent to the wall surface, each being drawn from the point where strains are measured. The coefficients A_y, A_z, A_a, F_b and F_c depend upon both the shape of and the position on the contour line of the cross section of the drift. Of these coefficients, A_y, A_z and A_a are defined as the values of σ_t which will take place when the ground is loaded in the y-, z- and a-directions by turns respectively divided by the intensity of loading, the a-direction making an angle of 45° with both the y- and z-axes. The coefficients F_b and F_c are defined as the values of $\tau_{tx'}$ which will take place when the ground is loaded only in the b- and c-directions by turns respectively divided by the intensity of loading, the b-direction making 45° with the z- and x'-axes, and the c-direction making 45° with the x'- and y-axes.

It can be proved that if strains at least at three points on the wall of a drift are measured by the stress-relief technique, the stress in the rock is able to be determined. Let us find the relation between the measured strains and the stress in the rock. We denote the direction making an angle of 45° with both the x'- and t-directions by i. Then we have:

$$\left.\begin{aligned}\epsilon_t &= \{-\nu\overset{*}{\sigma}_x+(A_y-\nu^2A_y+\nu^2)\overset{*}{\sigma}_y+(A_z-\nu^2A_z+\nu^2)\overset{*}{\sigma}_z\\ &\quad+(1-\nu^2)(2A_a-A_y-A_z)\overset{*}{\tau}_{yz}\}/E,\\ \epsilon_l &= \{\overset{*}{\sigma}_x-\nu\overset{*}{\sigma}_y-\nu\overset{*}{\sigma}_z\}/E,\\ \epsilon_i &= \left\{\frac{1-\nu}{2}\overset{*}{\sigma}_x+\frac{1-\nu}{2}(A_y+\nu A_y-\nu)\overset{*}{\sigma}_y+\frac{1-\nu}{2}(A_z+\nu A_z-\nu)\overset{*}{\sigma}_z\right.\\ &\quad+\frac{1-\nu^2}{2}(2A_a-A_y-A_z)\overset{*}{\tau}_{yz}+2(1+\nu)F_b\overset{*}{\tau}_{zx}\\ &\quad\left.+2(1+\nu)F_c\overset{*}{\tau}_{xy}\right\}/E.\end{aligned}\right\} \qquad (19)$$

These are the formulae to be used for detemining the stress in the rock from strains on the wall surface of a drift.

9. Numerical Examples and Discussion on the Accuracy of the First Four Methods

Assuming that the Young's modulus of the rock be 3×10^5 kg/cm^2, the Poisson's ratio 0.25, the numerical examples of determining the stress in the rock will be shown for the four cases, from variations in borehole diameter (Case 1), from strains on the bottom surfaces of boreholes (Case 2), from strains on the wall of a borehole (Case 3), and from deformation of a borehole (Case 4). In Case 1 and 2, the three boreholes are assumed to arragne as shown in Fig. 3.

The measured values assumed and the coefficients of the observation equations which have been calculated from them are shown in Tables 1–4. The stress states in the rock expressed by six stress components which have been calculated from these observation equations are shown in the column I in Table 5. The stress states in the rock are alike in these four examples, but are not the same.

The errors in the calculated stress components caused by an error of $\pm10^{-5}$ in every measured value are shown in the column II in Table 5. The errors in the results obtained due to the error in the Young's modulus of the rock are proportional to the percentage of the error in the modulus. The influence of an error in the Poisson's ratio is irregular. The column III in Table 5 shows the error in the results obtained due to an error of -10% in the Poisson's ratio. If there are errors in the coefficients which are to be evaluated by experiments, the stress in the rock determined will of course have some error.

These sources of errors cause errors in the coefficients and constants of the normal equations obtained. The less is the value of the determinant made up of the coefficients p_{fg} of the normalized form of normal equations, the greater is the error in the results obtained. The values of the determinants made up of p_{fg} are shown in the column IV in Table 5. It seems however that the error in measured values may cause the greatest error in the results of calculation among all the factors.

From the numerical examples described above, it may be found, under the assumption that the accuracy in measurement is the same for all cases, that the accuracy in the results obtained is highest in Case 1, and it comes next in Case 3, and it is lowest in Case 2 or 4. However it is supposed that a part of the reason for the low accuracy in Case 4 which is presumed from the numerical example may lie in that in this case alone the stress in the rock has been determined from a minimum number of data.

Table 1. An Example of Calculation in Case 1.

(Arrangement of Borenholes Being as Shown in Fig. 3)

Measured values	λ	$\lambda=45°$			$\lambda'=90°$			$\lambda''=135°$		
	θ	30°	90°	150°	30°	90°	150°	30°	90°	150°
	$\frac{\Delta d}{d}(10^{-6})$	−64	−832	−265	−52	−826	−320	−128	−816	−308

Observation equations after equation (6)	Coefficients	of the stress components (10^{-6})						Constants (10^{-6})
		$\overset{*}{\sigma}_X$	$\overset{*}{\sigma}_Y$	$\overset{*}{\sigma}_Z$	$\overset{*}{\tau}_{YZ}$	$\overset{*}{\tau}_{ZX}$	$\overset{*}{\tau}_{XY}$	
	i	2.813	2.813	0.208	7.655	−7.655	−7.292	−64
	ii	−1.875	−1.875	9.583			2.083	−832
	iii	2.813	2.813	0.208	−7.655	7.655	−7.292	−265
	iv	6.458	−0.833	0.208		−10.825		−52
	v	−2.917	−0.833	9.583				−826
	vi	6.458	−0.833	0.208		10.825		−320
	vii	2.813	2.813	0.208	−7.655	−7.655	7.292	−128
	viii	−1.875	−1.875	9.583			−2.083	−816
	ix	2.813	2.813	0.208	7.655	7.655	7.292	−308

Table 2. An Example of Calculation in Case 2.

($d=75$ mm, $l=8$ mm, Arrangement of Boreholes Being as Shown in Fig. 3)

Measured values	λ	$\lambda=45°$			$\lambda'=90°$			$\lambda''=135°$		
	Directions	y	z	ξ	y'	z'	ξ'	y''	z''	ξ''
	$\bar{\varepsilon}(10^{-6})$	153	−348	−29	131	−335	−17	103	−336	−60

Observation equations after equation (9)	Coefficients	of the stress components (10^{-6})						Constants (10^{-6})
		$\overset{*}{\sigma}_X$	$\overset{*}{\sigma}_Y$	$\overset{*}{\sigma}_Z$	$\overset{*}{\tau}_{YZ}$	$\overset{*}{\tau}_{ZX}$	$\overset{*}{\tau}_{XY}$	
	i	1.417	1.417	−2.167			−6.500	153
	ii	−2.000	−2.000	4.667			0.333	−348
	iii	−0.292	−0.292	1.250	4.833	−4.833	−3.083	−29
	iv	4.667	−1.833	−2.167				131
	v	−2.167	−1.833	4.667				−335
	vi	1.250	−1.833	1.250		−6.833		−17
	vii	1.417	1.417	−2.167			6.500	103
	viii	−2.000	−2.000	4.667			−0.333	−336
	ix	−0.292	−0.292	1.250	−4.833	−4.833	3.083	−60

Table 3. An Example of Calculation in Case 3.

(d=29 mm, l=8 mm, ω= 0.552 rad)

Measured values	$\theta_1=0°$			$\theta_2=120°$			$\theta_3=240°$		
Directions	θ_1	x	i_1	θ_2	x	i_2	θ_3	x	i_3
$\bar{\varepsilon}(10^{-6})$	−821	−10	−486	−74	−9	1	−268	−9	−109

Observation equations after equation (13): Coefficients	of the stress components (10^{-6}): $\overset{*}{\sigma}_x$	$\overset{*}{\sigma}_y$	$\overset{*}{\sigma}_z$	$\overset{*}{\tau}_{yz}$	$\overset{*}{\tau}_{zx}$	$\overset{*}{\tau}_{xy}$	Constants (10^{-6})
i	−0.833	−2.724	9.390				−821
ii	3.333	−0.833	−0.833				−10
iii	1.250	−1.828	4.328		8.300		−486
iv	−0.833	6.361	0.305	10.490			−74
v	3.333	−0.833	−0.833				−9
vi	1.250	2.789	−0.289	5.332	−4.150	−7.188	1
vii	−0.833	6.361	0.305	−10.490			−268
viii	3.333	−0.833	−0.833				−9
ix	1.250	2.789	−0.289	−5.332	−4.150	7.188	−109

Table 4. An Example of Calculation in Case 4.

($b=a$)

Measured values	$\theta_1=30°$	$\theta_2=90°$	$\theta_3=150°$
$\frac{\Delta s}{d}(10^{-6})$	−108	−693	−235
$\frac{\Delta d}{d}(10^{-6})$	−63	−833	−265

Observation equations after equations (15) and (16): Coefficients	of the stress components (10^{-6}): $\overset{*}{\sigma}_x$	$\overset{*}{\sigma}_y$	$\overset{*}{\sigma}_z$	$\overset{*}{\tau}_{yz}$	$\overset{*}{\tau}_{zx}$	$\overset{*}{\tau}_{xy}$	Constants (10^{-6})
i	−0.833	6.458	0.208	10.825			−63
ii	1.768	3.977	−0.422	7.655	5.893	10.206	−108
iii	−0.833	−2.917	9.583				−833
iv	1.768	−2.652	6.187		11.785		−693
v	−0.833	6.458	0.208	−10.825			−265
vi	1.768	3.977	−0.442	−7.655	5.893	−10.206	−235

Table 5. Calculated Stresses in the Rock and Their Errors

		I Calculated stresses (kg/cm^2)	II Errors due to errors of $\pm 10^{-5}$ in every measured value (kg/cm^2)	III Errors due to -10% error in Poisson's ratio (kg/cm^2)	IV The value of the determinant of p_{fg}
Case 1	$\overset{*}{\sigma}_X$	−29.6	±1.0	−0.1	0.361
	$\overset{*}{\sigma}_Y$	−31.0	±1.8	−0.1	
	$\overset{*}{\sigma}_Z$	−97.9	±0.7	−0.8	
	$\overset{*}{\tau}_{YZ}$	0.7	±0.7	0	
	$\overset{*}{\tau}_{ZX}$	−12.4	±0.5	0.1	
	$\overset{*}{\tau}_{XY}$	−3.6	±0.7	−0.1	
Case 2	$\overset{*}{\sigma}_X$	−30.1	±6.7	0.1	0.004
	$\overset{*}{\sigma}_Y$	−31.0	±8.2	−0.2	
	$\overset{*}{\sigma}_Z$	−98.9	±6.8	0	
	$\overset{*}{\tau}_{YZ}$	0.6	±1.6	0.2	
	$\overset{*}{\tau}_{ZX}$	−12.7	±1.3	0.3	
	$\overset{*}{\tau}_{XY}$	−3.9	±1.1	0	
Case 3	$\overset{*}{\sigma}_x$	−33.9	±1.7	3.1	0.280
	$\overset{*}{\sigma}_y$	−26.6	±1.1	0.3	
	$\overset{*}{\sigma}_z$	−98.1	±1.1	1.2	
	$\overset{*}{\tau}_{yz}$	9.3	±0.7	−0.1	
	$\overset{*}{\tau}_{zx}$	−8.1	±1.1	−0.2	
	$\overset{*}{\tau}_{xy}$	−0.8	±1.1	0	
Case 4	$\overset{*}{\sigma}_x$	−32.8	±9.5	2.3	0.006
	$\overset{*}{\sigma}_y$	−26.5	±1.2	0.2	
	$\overset{*}{\sigma}_z$	−97.8	±1.8	1.1	
	$\overset{*}{\tau}_{yz}$	9.3	±0.7	−0.1	
	$\overset{*}{\tau}_{zx}$	−8.5	±2.5	0	
	$\overset{*}{\tau}_{xz}$	−0.6	±0.8	−0.2	

10. Conclusion

For all cases of determining the stress in the rock that can be considered today, the formulae to be used in practice have been presented. If the rigid types of measuring instruments are used, calibration tests are necessary, the details of which and the formulae to be used have been described. The least number of boreholes or drifts necessary for the stress determination has been pointed out. The methods of calculation have been more concretely shown by numerical examples. The sources of the error in the results obtained have been discussed. It has been suggested that if measurements of the same accuracy be carried out by all the methods, the accuracy in the stress in the rock obtained may be highest when it is determined from variations in borehole diameter, and may come next when it is determined from strains on the wall of a borehole.

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