An Experimental Study on the Diffusion Characteristics of Turbulent Flow in a Low Speed Wind Tunnel

By

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This paper describes the results of a study on diffusion characteristics of turbulent flow in a wind tunnel, in which the turbulent flow was obtained by a grid. The lateral turbulent velocity deviation $\sqrt{\overline{v^2}}$ ranged from 1.01 to 1.14 times the longitudinal turbulent velocity deviation $\sqrt{\overline{u^2}}$ for mean wind velocities from 2 to 8 m/sec and mesh sizes of grid from 4.5 to 13.5 cm, and the isotropic condition was almost satisfied. The decay of turbulence was expressed with the equation $\sqrt{\overline{u^2}}/U=m(x/M)^{-\alpha}$, where $\overline{u^2}$: longitudinal turbulent velocity variance, U: mean wind velocity, x: distance from the grid, M: mesh size of the grid, and α, m are constants. From 11 tracer gas experiments, we obtained the Lagrangian correlation coefficient and found the Lagrangian and Eulerian correlation coefficients had similar shapes within the range of time-lag from 0 to 400 milliseconds. The ratios of the Lagrangian to Eulerian time scale, β, were between 4 and 11.

1. Introduction

One of the important aspects of turbulent motion in fluid is its dispersive property. The first mathematical analysis of this problem was the work on diffusion by continuous movement by Taylor[1] in 1921. Not only did his paper lay a basis for the study of turbulent diffusion but it proposed the statistical theory of turbulence. Taylor[2] showed the Lagrangian formulation of transport problem in a homogeneous field of an isotropic and decaying turbulence. Later, a few theoretical or experimental works were done to find the relation between the Lagrangian statistical measures of a turbulent field and its Eulerian measures[3-6]. Since turbulence dynamics can be easily treated with the Eulerian terms, and turbulent diffusion with the Lagrangian, it is important to find the relation between the two terms. The object of this research on turbulent diffusion is to seek the relation between the diffusive and dynamical variables. The experiment was carried out to find the preliminary

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relationship between the Lagrangian correlation coefficient R_L and the Eulerian correlation coefficient R_E under various mean wind speeds and mesh sizes of turbulence-producing grid. The results obtained permit R_L to be specified when R_E is known or predicted, and are useful for the model experiments which are efficient to solve the problems of atmospheric pollution.

2. Equipment and Procedure

2.1 Wind Tunnel

The experiment was conducted in an Eiffel type wind tunnel with test section of 2.5 m wide, 1.5 m height, and 5.6 m long, in which flow has a turbulence level

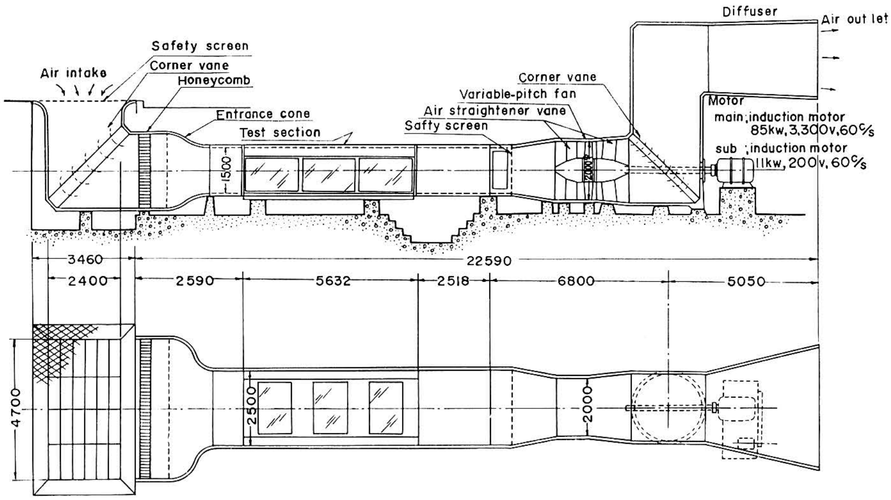

Fig. 1. Wind tunnel for the study of atmospheric diffusion.

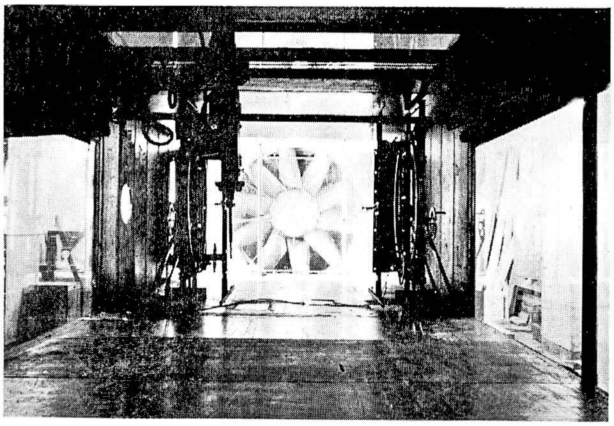

Fig. 2. Interior of wind tunnel viewing from the windward,

of $\sqrt{\overline{u^2}}/U=0.005$ at mean velocity of 15 m/sec and the fluctuation of mean velocity is always within $\pm 1\%$ (Fig. 1). It has the sampling apparatus which can be moved in three directions. Blower is of variable pitch type and consists of 10 rotating blades, diameter of which is 2000 mm. The efficiencies of the system are as follows; revolution: 590 rpm, total effective head: 65 mm Aq, maximum flow rate: 93.7 m^3/sec. The interior of the wind tunnel is shown in Fig. 2.

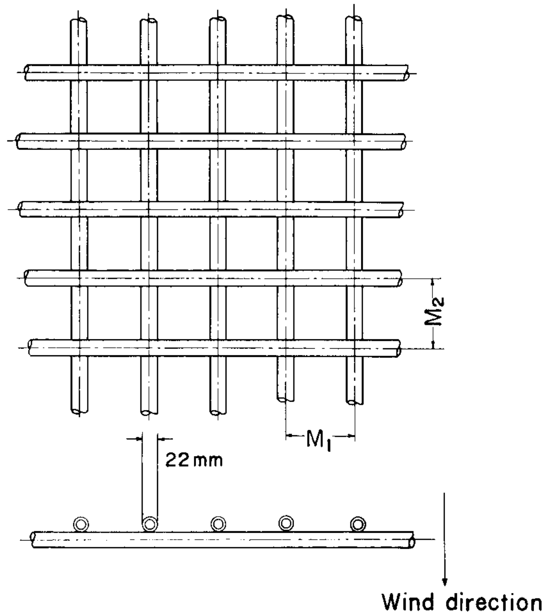

Fig. 3. Details of turbulence-producing mesh.

The turbulence-producing grid consisted of round bars of 22 mm diameter arranged in square mesh, and is shown in Fig. 3. This turbulence-producing grid was set at the windward position of the test section.

2.2 Equipment and Procedure

The mean velocity was measured with a constant current type hot wire anemometer (TYPE 19–2311, NIHON KAGAKU KOGYO CO., Japan) and a pitot tube. The turbulent velocities were measured with two constant temperature type hot wire anemometers (TYPE 28–1111, NIHON KAGAKU KOGYO CO., Japan), two function linearizers (TYPE 28–7111, NIHON KAGAKU KOGYO CO., Japan), and a calculator (TYPE 28–6211, NIHON KAGAKU KOGYO CO., Japan). The hot wire measurement system is schematically shown in Fig. 4.

The gas feeding and sampling system used is schematically shown in Fig. 5. Ethane gas was used as the diffusing gas. The gas was fed into the tunnel at a constant rate through a stainless steel pipe. The mixture of the diffusing gas was drawn through the sampling system with a vacuum pump. The sampling rate and the feed rate were measured by the flowmeters. The sampling gas was measured using an infrared gas analyzer (URAS TYPE 1, HARTMAN-BRAUN CO., Germany and SHIMADZU CO., Japan) which was previously calibrated by a standard gas. The gas analyzer has a time constant of about 2 seconds.

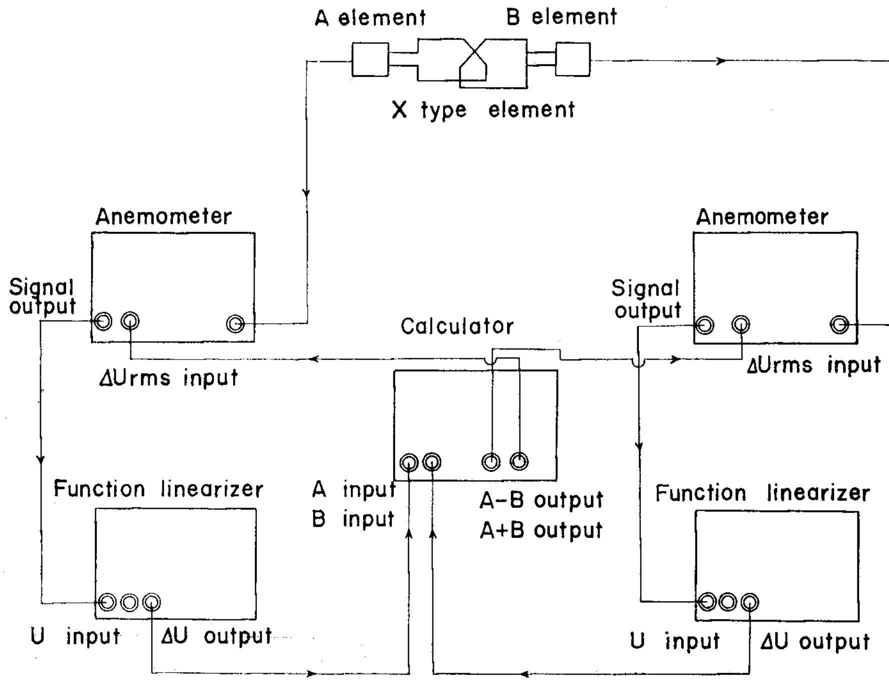

Fig. 4. Arrangement of apparatus for measuring fluctuation of wind velocity.

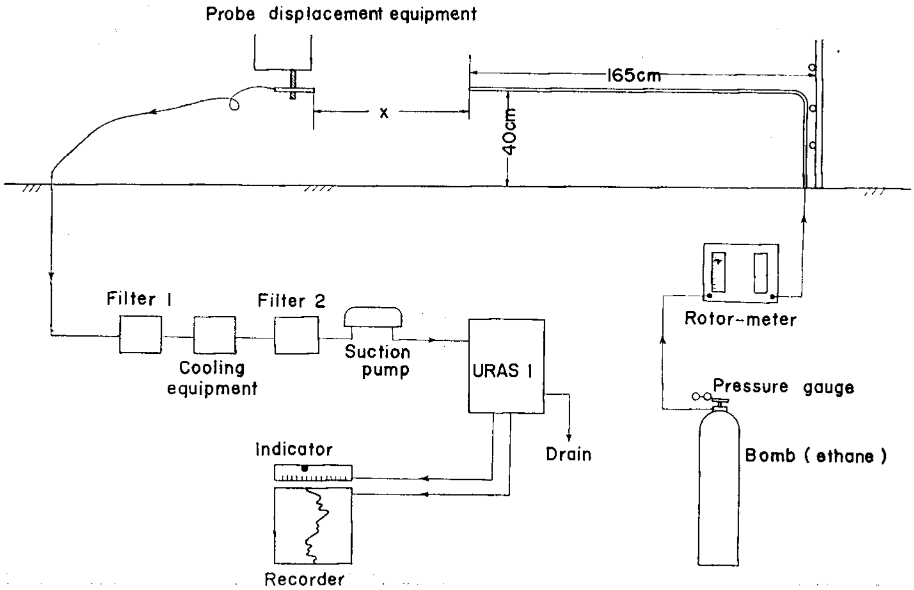

Fig. 5. Gas feeding and sampling system for concentration measurement.

Experimental conditions are as in Table 1.

Table 1. Experimental conditions.

Mean wind velocity, U: 2.0–8.0 m/sec
Mesh size of grid, M: 4.5, 9.0 and 13.5 cm
Rate of emission of tracer gas, Q: 30–65 cc/min
Ethane concentration: 99 vol %

3. Results

3.1 Isotropy of Turbulent Flow

The lateral intensity of turbulent flow $\sqrt{\overline{v^2}}/U$ and longitudinal intensity $\sqrt{\overline{u^2}}/U$ were calculated from root-mean-square readings of constant temperature type hot wire anemometers placed with declination angles (45, 135 deg.) to the flow. The ratios of $\sqrt{\overline{v^2}}/\sqrt{\overline{u^2}}$ were about 1.01 to 1.14 at leeward position from the square mesh grid ($M=9$, 13.5 cm) at various wind speeds (2–6 m/sec). The result of previous research[3] showed that the ratios of $\sqrt{\overline{v^2}}/\sqrt{\overline{u^2}}$ varied from 0.7 to 0.9 along pipe center line of turbulent flow.

3.2 Decay of Turbulence

The decay of turbulence was measured at various wind speeds ranging from 2 to 8 m/sec. Sets of results were plotted with x/M as abscissa and intensity of turbulence as ordinate both in logarithmic scale in Fig.6, where x: distance from the grid, M: mesh size of grid. The intensity of turbulence was decreased with an increase of the distance x from the grid and the following relation was obtained.

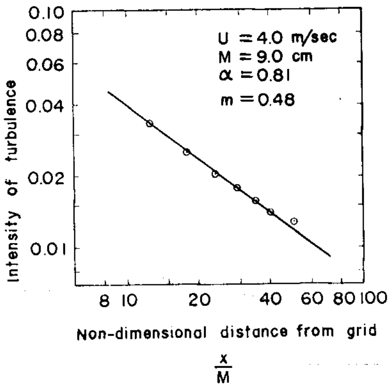

Fig. 6. Decay of turbulence behind a grid.

$$\frac{\sqrt{\overline{u^2}}}{U} = m\left(\frac{x}{M}\right)^{-\alpha} \qquad (1)$$

The parameters α and m were decided by the least square method and are shown in Table 2.

3.3 Eulerian Autocorrelation Coefficient

From the analysis of the hot wire anemometer signal readings, the autocorrelation coefficient was calculated. The measurements were made at four points 90 to 457 cm downward from the grid. An example of these data is presented in Fig. 7.

Table 2. Estimation of the parameters α and m in Eq. (1), and the ratio of the Lagrangian to the Eulerian time scale, β, in Eq.(7).

U (m/sec)	M_1 (cm)	M_2 (cm)	α (−)	m (−)	Direction of σ measurement	β (−)
2.0	13.5	13.5	0.78	0.37	y	—
2.4	9.0	9.0	0.74	0.31	y	8.8
3.0	9.0	9.0	0.79	0.28	y	—
3.0	13.5	13.5	0.85	0.45	y	—
4.0	4.5	4.5	0.79	0.44	y	18.9
4.0	9.0	9.0	0.81	0.48	y	5.9
4.0	13.5	13.5	0.87	0.45	y	4.8
6.0	9.0	9.0	0.78	0.30	y	4.4
6.0	13.5	13.5	0.80	0.31	y	—
8.0	9.0	9.0	0.74	0.27	y	10.8
4.0	9.0	18.0	0.72	0.25	y	10.1
4.0	9.0	18.0	0.72	0.25	z	6.7
4.0	9.0	*	0.85	0.38	y	11.0
4.0	9.0	*	0.85	0.38	z	6.3
4.0	**	**	—	—	y	3.1

*: Lateral bars were omitted.
**: Without grid.

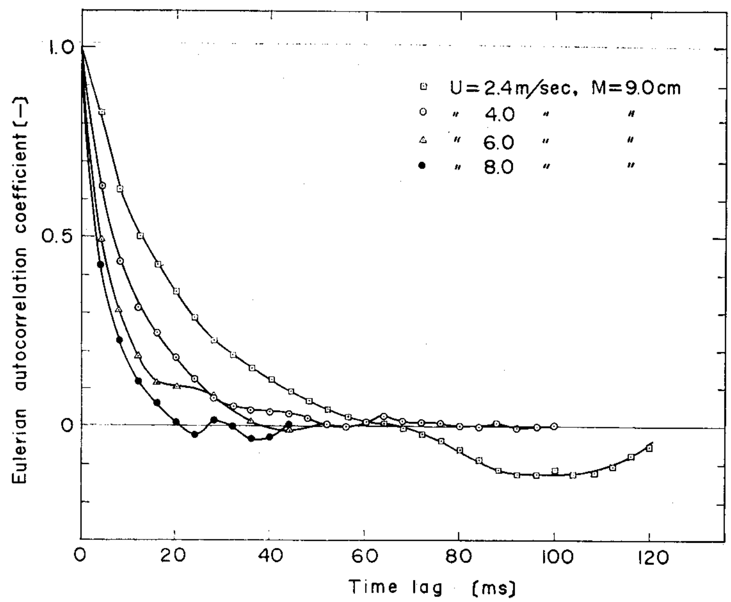

Fig. 7. Eulerian autocorrelation curves of turbulence behind a grid.

3.4 Power Spectra

The spectral functions of turbulence energy $F(n)$ were obtained indirectly by Fourier transformation of the autocorrelation curves, where n is frequency. We applied Tukey's method to this transformation which was applied previously by Panofsky, Cramer and others[7]. The method is based on the theorem that the cosine transformation of autocorrelation function of a time series is essentially equal to the spectrum of the original series.

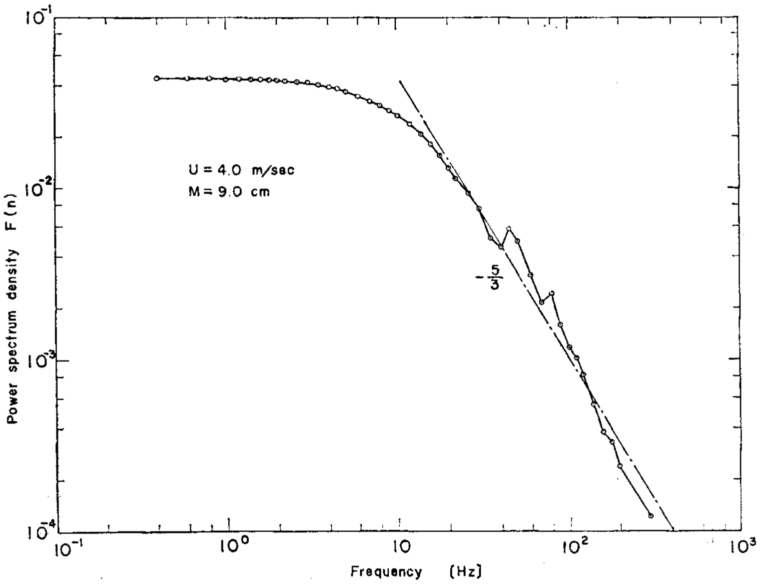

Fig. 8. Power spectral curve of turbulence behind a grid.

Some examples of the power spectra are illustrated in Fig. 8. In the turbulence produced by the grid, the evolution energy having a frequency greater than 10 Hz was slight; for the lower frequencies, the spectral curves showed that energy increases as wind speed increases. The behavior of these power spectra did not change with the distance from turbulence-producing grid.

According to Kolmogoroff[8], the inertial subrange exists in the range of high frequency of energy spectrum in turbulent flow when Reynolds number becomes large. In this range, turbulence is isotropic, and its statistical characters depend only on the rate of energy dissipation ε. The one dimensional spectrum in the inertial subrange is given by the so-called "$-5/3$" power law, as follows:

$$F(n) \propto \varepsilon^{2/3} n^{-5/3} \qquad (2)$$

where n is frequency along wind direction, $F(n)$ is power spectral function. The spectral curves in agreement with the "$-5/3$" power law were observed in the turbulence produced by the grid.

3.5 Concentration Measurement

After the photographic qualitative observation of plume behavior as shown in Fig. 9, the measurement was carried out in vertical cross sections of the test section at various positions from the source. Lateral profiles of concentration of tracer

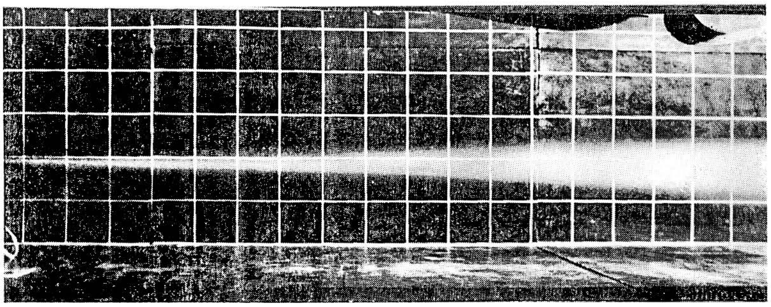

Fig. 9. Qualitative observation of plume behavior in the test section. $U=2.0$ m/sec, $M=9.0$ cm.

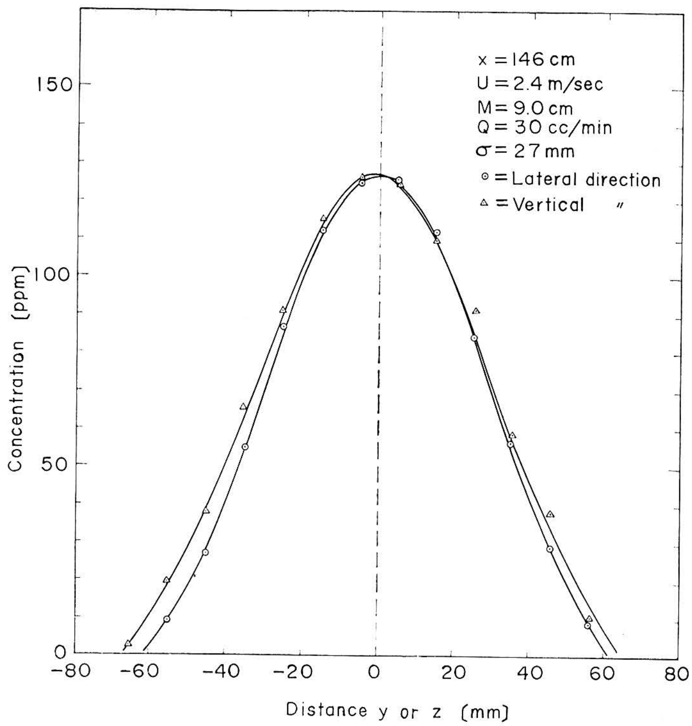

Fig. 10. Concentration distribution due to turbulence behind a grid,

gas plume were measured by traversing the sampling probe parallel to the y-axis at a fixed height of 40 cm. Also vertical profiles of concentration were obtained similarly.

Fig. 10 shows the concentration distribution in respect to y- and z- direction at the same section. In any section, the concentration distribution was fitted to the normal distribution curves as shown in Fig. 11.

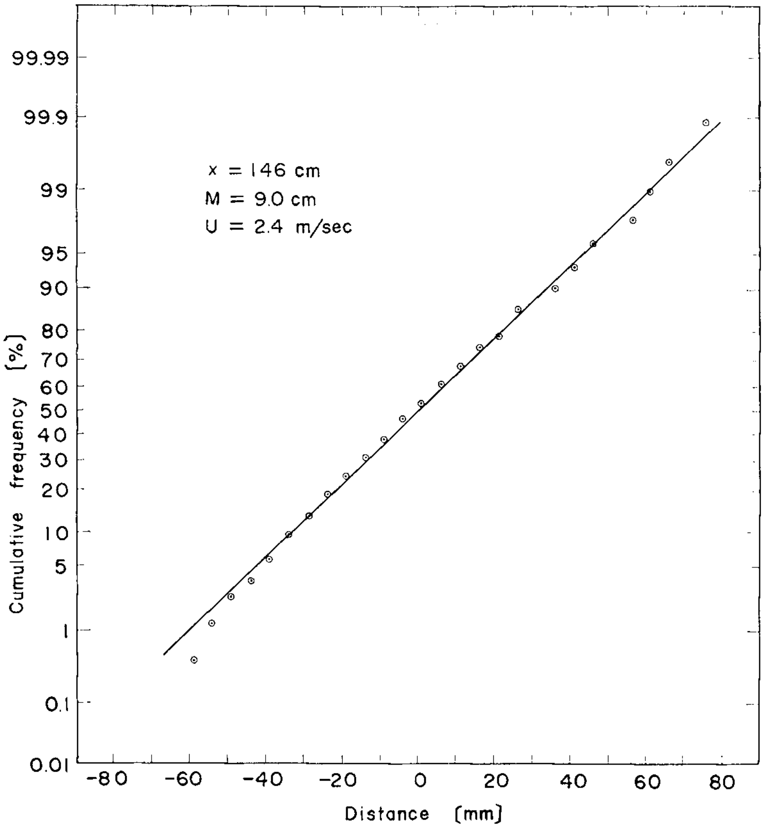

Fig. 11. Goodness of fit between experimental data and normal distribution.

4. Discussion

4.1 Decay of Turbulence

Taylor[2] showed that the relation between the value of $U/\sqrt{\overline{u^2}}$ and the distance x from the grid was linear. Dryden, Schubauer, Mock, and Skramstad[9] derived Eq. 3 from the Eulerian space correlation function $R(y)$, where a, b and B are constant, and M is mesh size of the grid.

$$\frac{U}{\sqrt{\overline{u^2}}} = \frac{5}{B^2 b}\log\left(a+b\frac{x}{M}\right) \tag{3}$$

Also, Stewart and Townsend[10] showed that $U/\sqrt{\overline{u^2}}$ is proportional to the distance x from the grid. We obtained the following equation using Fig. 6.

$$\frac{\sqrt{\overline{u^2}}}{U} = m\left(\frac{x}{M}\right)^{-\alpha} \tag{4}$$

The mean values of α and m are about 0.8 and 1.5, respectively.

4.2 Lagrangian Correlation Coefficient

Taylor[2] represented an equation of the Lagrangian correlation coefficient taking account of the decay of turbulence intensity using Schubauer's results of the dissipation of heat from a hot wire in a wind tunnel.

$$R_L = \frac{U^2}{\overline{u^2}}\frac{d}{dx}\left(\overline{Y}\frac{d\overline{Y}}{dx}\right) \tag{5}$$

Barad[6] derived the Lagrangian correlation coefficient from Eq. 5 using the results of the diffusive experiment at plane grass land near O'Neil. Mickelsen[11] compared the Eulerian correlation coefficient with the Lagrangian correlation coefficient of the homogeneous turbulent flow and obtained,

$$R_L(\xi) = R_E\left(\frac{x}{B\sqrt{\overline{u^2}}}\right) \tag{6}$$

where B is constant.

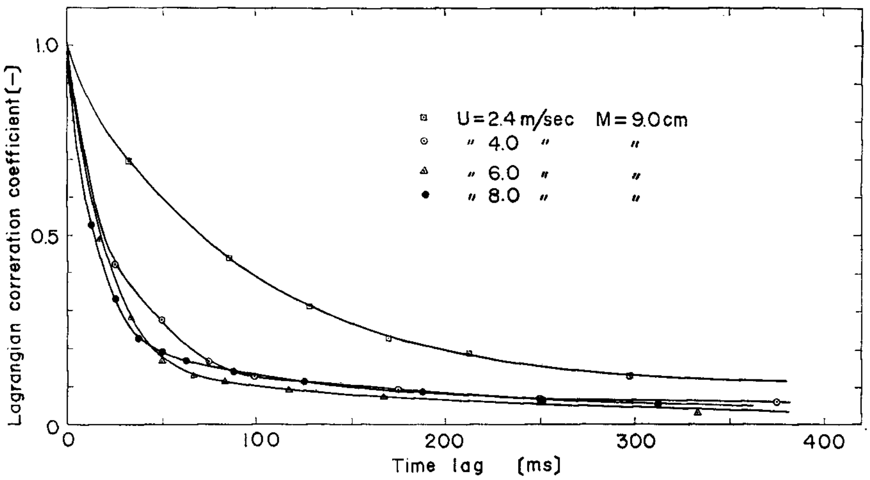

Fig. 12. Lagrangian correlation curves of turbulence behind a grid.

In the present study, the Lagrangian correlation coefficient was estimated by Taylor's method using the data on concentration distribution. An example of data is presented in Fig. 12. Applying the simple hypothesis, Hay and Pasquill[5] proposed the following relation.

$$R_L(\xi) = R_E(t) \quad \text{when} \quad \xi = \beta t \tag{7}$$

Using the above relation, the Lagrangian correlation coefficient in Fig. 12 was compared with the Eulerian autocorrelation coefficient from 0 to 400 milliseconds. Table 2 shows the ratio of the Lagrangian to the Eulerian time scale, β.

4.3 Diffusion Coefficient

A basic diffusion equation is,

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z \frac{\partial c}{\partial z}\right) - \left(U\frac{\partial c}{\partial x} + V\frac{\partial c}{\partial y} + W\frac{\partial c}{\partial z}\right) \tag{8}$$

when a source is steady and continuous,

$$D_x = D_y = D_z = \text{constant}, \quad V = W = 0$$

$$U\frac{\partial c}{\partial x} = D\left(\frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2}\right) \tag{9}$$

and the solution is,

$$C = \frac{Q}{4\pi x D} \exp\left[-\frac{U}{4xD}(y^2 + z^2)\right] \tag{10}$$

This equation shows the Gaussian distribution in y- and z- direction at the distance x from the grid.

Diffusion coefficient D can be estimated from the standard deviation σ of concentration distribution curve in y- direction. When $z=0$, σ can be estimated from the following equation.

$$\sigma^2 = \frac{\int_{-\infty}^{\infty} c y^2 dy}{\int_{-\infty}^{\infty} c\, dy} = \frac{2xD}{U} \tag{11}$$

From this equation diffusion coefficient D_1 becomes as follows:

$$D_1 = \frac{\sigma^2 U}{2x} \tag{12}$$

Also, Frenkiel[12] derived the diffusion coefficient D_2 from the Lagrangian correlation coefficient as follows:

$$D_2 = \overline{v^2} \int_0^t R_L(\xi)\, d\xi \tag{13}$$

Using this equation, D_2 can be estimated from substituting R_E into $R_L(\xi)$. Both diffusion coefficients were compared and discussed.

The intensity of turbulence at the test section in the tracer gas experiment is about 2%. From these data, D_1 is 6.2 cm²/sec, D_2 is 4.1 cm²/sec at U=2.4 m/sec, M=9.0 cm. The amounts of D_1 and D_2 were same in decimal order.

5. Conclusion

The character of turbulence produced by the square mesh grid was examined in a low speed wind tunnel. The results obtained were as follows:

1. The lateral turbulent velocity deviation $\sqrt{\overline{v^2}}$ ranged from 1.01 to 1.14 times of the longitudinal turbulent velocity deviation $\sqrt{\overline{u^2}}$ for mean wind velocities from 2 to 8 m/sec and mesh sizes of grid from 4.5 to 13.5 cm, therefore the isotropic condition was almost satisfied.

2. The decay of turbulence intensity was expressed as follows:

$$\frac{\sqrt{\overline{u^2}}}{U} = m\left(\frac{x}{M}\right)^{-\alpha}$$

where $\overline{u^2}$: longitudinal turbulent velocity variance, U: mean wind velocity, x: distance from the grid, M: mesh size of the grid. α and m are constants and in this experiment about 0.8 and 1.5, respectively.

3. The concentration distribution of tracer gas had the normal distribution curve on the plane of y and z as shown in Fig. 10.

4. The ratios of the Lagrangian to Eulerian time scale, β, were between 4 and 11.

Notation

a	:	Constant.	
B	:	Constant.	
b	:	Constant.	
C	:	Concentration of diffusing gas.	(ppm)
D	:	Diffusion coefficient.	(cm²/sec)
D_1	:	Diffusion coefficient defined in Eq. (12).	(cm²/sec)
D_2	:	Diffusion coefficient defined in Eq. (13).	(cm²/sec)
$F(n)$	:	Power spectral function.	
M	:	Mesh size of grid.	(cm)
m	:	Constant.	
n	:	Frequency.	(Hz)
Q	:	Rate of emission of diffusing gas.	(cc/min)

R_L	:	Lagrangian correlation coefficient.	
R_E	:	Eulerian correlation coefficient.	
$R(y)$	:	Eulerian space correlation function.	
t	:	Time lag, Time.	(sec)
U	:	Longitudinal mean wind velocity.	(m/sec)
u	:	Longitudinal turbulent velocity.	(cm/sec)
V	:	Lateral mean wind velocity.	(m/sec)
v	:	Lateral turbulent velocity.	(cm/sec)
W	:	Vertical mean wind velocity.	(m/sec)
w	:	Vertical turbulent velocity.	(cm/sec)
x	:	Longitudinal coordinate.	(cm)
$\overline{Y}^2$	:	Lateral displacement variance.	(cm^2)
y	:	Lateral coordinate.	(cm)
z	:	Vertical coordinate.	(cm)
Greek			
α	:	Constant.	
β	:	The ratio of the Lagrangian to the Eulerian time scale.	
ε	:	The rate of energy dissipation.	(erg/g. sec)
ξ	:	Time lag.	(sec)
σ^2	:	Lateral displacement variance.	(cm^2)

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