Correlation between Idling Vibration of Machine Tool and Machining Chatter

By

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Mechanism of the *machining chatter* caused by the forced vibration on the machine tool is presented with the metal cutting test results. From the presently conceived theory of *self-excited chatter*, a theory of *forced chatter* has been derived. According to the theory, the amplification factor of the chatter amplitude to the amplitude of the idle time forced vibration is predicted from the dynamic characteristic of the machine structure, the cutting condition, and the frequency of the forced vibration. The result of turning tests performed with the forced vibration added by a magnetic exciter proves that the amplification factor varies according to the forcing frequency in the way as predicted by the theory. From the theory and the experiment of this study, it is understood that there exist some particular sets of conditions where very high amplification factors are attained although the ratio is not greater than unity except at these conditions. The forcing frequency above the resonance frequency but in the resonance range is particularly unfavorable due to the generation of the forced chatter.

1. Introduction

The continuous vibration between the work piece and the cutting tool is conventionally termed as the *chatter* in metal cutting, and it is detrimental to the quality of the finished work and to the life of the tool. Thus the chatter is counted as one of the most critical obstructions in increasing the productivity of metal cutting operations. Owing to intensive efforts of many researchers, the fundamental mechanism of the chatter has recently come to light; especially, the mechanism of the *self-excited chatter* is well understood by the theory originally proposed by Tlusty [1]** or Tobias [2], and many useful achievements are reported. By the word self-excited, it is meant that even when no forcing vibration exists which might cause the chatter, the chatter arises by itself, when a certain set of conditions is satisfied by the combinations of the dynamic characteristics of the cutting process and the machine structure.

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** Number in brackets designate References at end of paper.

Another type of chatter occurs due to the excitation from the vibration sources such as electric motors, pumps, gears etc. equipped to the machine tool, or from the external vibration which is transmitted to the machine through its foundation. If those vibrations, which already exist when the machine tool is running idle, bring about the chatter when the work is cut, the phenomenon is called *forced chatter*.

This paper studies the mechanism through which the relative vibration between the tool and the work forced at idle running time by the sources as listed above, appears at the machining time in the form of forced chatter.

Theoretical model of the forced chatter is developed as an expansion of the presently conceived theory of the self-excited chatter. The theory predicts the response of the amplitude of the forced chatter to a change of the forcing frequency, and thereafter it is demonstrated in a metal cutting test having a forced vibration from the frequency-variable magnetic exciter.

2. Theoretical analysis on the correlations between the vibration at idle running time and the machining chatter.

2.1. Purpose of the analysis and formulation of the model

2.1.1. Purpose

Suppose a vibration Y_0 (whose amplitude being y_0) of a particular freqency dominates among the vibrations between the tool and the work when the machine is running idle with the tool being apart from the work, the amplitude of the vibration of the identical frequency is to be predicted when it appears at the machining time. The vibration Y_0 is defined in the direction of the depth of cut which is normal to the cutting direction and its sign is positive when the tool is away from the work.

2.1.2 Dynamic response of the machine structure

Since the forced vibration Y_0 occurs when the machine is running idle, and the forcing source such as the motor or the pump supposedly receives no influence from the start of the cutting, the forced vibration Y_0 is also intact in cutting.

However, the cutting brings the tool to a contact with the work through the cutting process; thus, if the cutting force fluctuates at the same frequency Z as the vibration Y_0 it acts as a vibratory external force on the mechanical system consisting of the tool, the machine, and the work, and induces a relative vibratory displacement between the tool and the work due to the elastic deformation of the system, in addition to the above-mentioned Y_0. The directional component of the vibratory deformation, caused by the vibration of the cutting force in the direction of Y_0, is denoted by X_0 (whose frequency and amplitude is Z and x_0 respectively). Provided that the resultant vibration which appears during the cutting in the direction of Y_0 is denoted by X (frequency Z and amplitude x), it is equal to the idle time vibration Y_0 superimposed by the vibration X_0 due to the cutting force, formulated as:

$$X = X_0 + Y_0 \tag{1}$$

The vibratory deformation X_0 of the machine structure is expressed by the equation

$$X_0 = F \cdot R(Z) \tag{2}$$

provided that the vibratory component of the cutting force is denoted by F(frequency Z, amplitude f), and the dynamic characteristic of the machine structure, being assumed to be linear, is expressed by a function $R(Z)$.

The dynamic characteristic $R(Z)$ is a function which describes, with the frequency Z as the varying parameter, the amplitude ratio and the phase difference of the sinusoidal vibratory displacement X_0 to the sinusoidal vibratory force F both at the frequency Z, the latter being applied between the tool and the work in the direction of the resultant cutting force. Since the machine tool is a system with the distributed mass and spring, theoretically it has an infinite number of frequency ranges at which the vibratory displacement X_0 is resonant with the exciting force. Plotting the variation of $R(Z)$ versus the frequency Z on a polar coordinate, the so-called *harmonic response loci* is obtained like the one schematicaly shown in Fig.2. The radial vector to any particular point on the loci shows the amplitude ratio of the displacement X_0 to the force F ($x_0/f = |R(Z)|$, referred to as the *compliance*), and the right hand rotated angle of the vector indicates the phase lag of X_0 to $F(-\underline{/X_0/F})$, both at the

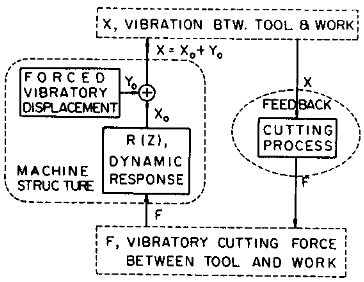

Fig. 1. Metal cutting vibration system having forced vibration.

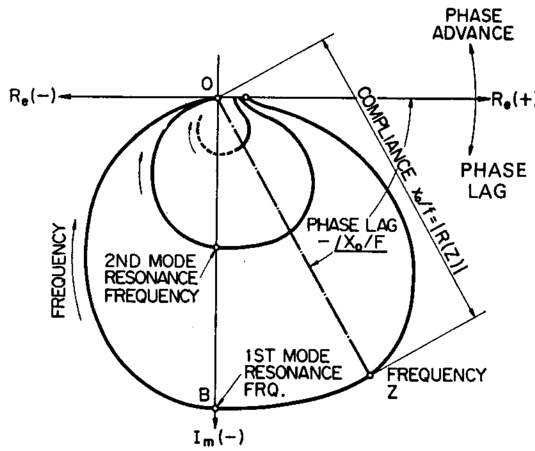

Fig. 2. Harmonic response loci of a machine structure.

frequency Z which corresponds to the point. Unless two or more resonance ranges overlap each other, a single closed loop stands for one resonance range tracking the loci clockwise with the increase of the frequency.

2.1.3. Dynamic characteristic of the cutting process

Generation of the cutting force composes the feedback pass, X to F, in the vibration system depicted in Fig. 1. For the convenience of analysis, the cutting process is modeled upon a comparatively rough approximation: namely, as frequently taken for the theory of self-excited chatter, that the cutting force is proportional to the cross section area of the metal to be removed measured at the tool tip point. Let U be the fluctuation of the thickness to be removed, the vibratory cutting force F is expressed by:

$$F = r \cdot b \cdot U$$

where b is the width of cut in orthogonal cutting, or the depth of cut in conventional cutting, and r is the ratio of the cutting force variation to the variation of the cross section area of the metal to be removed.

The fluctuation of the uncut chip thickness U is given from the apparent vibration X between the tool and the work, taking the so-called *regenerative effect*

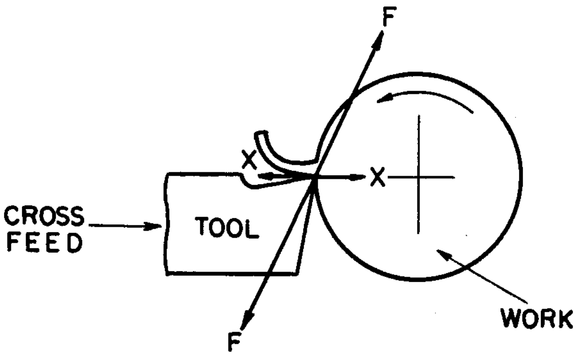

Fig. 3. Parting, an example of a lathe operation.

into account; that is, a surface wave generated by a previous cutting adds to the fluctuation of U when the wave is cut at the next tool path. Let $X(t)$ denote the relative displacement between the tool and the work at time t in a parting operation on a lathe as shown in Fig. 3, the displacement at the previous rotation is given by $X(t-T)$, where T is the time for a rotation of the work. The vibratory component of the uncut chip thickness is formulated as:

$$U(t) = \mu X(t-T) - X(t)$$

in which μ is the overlap factor that represents the effect of the regeneration. As often explained in the theory of the self-excited chatter [3], μ generally takes a value in the range of $0 \leq \mu \leq 1$; however, many actual operations including the one shown in Fig. 3 can be approximated by $\mu = 1$. In this case, it is convenient that the equation between U and X is reduced to a relatively simple form, from which the equation of the vibratory cutting force F and the vibratory displacement X is deduced as follows [4]:

$$\left.\begin{aligned} &\text{amplitude ratio} \quad \frac{|X|}{|F|} = \frac{1}{2rb \sin \frac{\varphi}{2}} \\ &\text{phase difference} \quad \angle\frac{X}{F} = -\left(\frac{\varphi}{2} + \frac{\pi}{2}\right) \end{aligned}\right\} \tag{3}$$

The angle φ in the above equation is the phase difference of the surface wave left by the previous cut to that to be generated in the succeeding cut, and it is computed by:

$$\varphi = 2\pi(1-\varepsilon) \tag{4}$$

where ε is the fraction of the number of chatter marks per revolution of the work, and this is calculated by:

$$n = \frac{60\ Z}{R_{pm}} = N\ (\text{integer}) + \varepsilon(\text{fraction}) \tag{5}$$

(R_{pm}: number of work revolutions per minute)

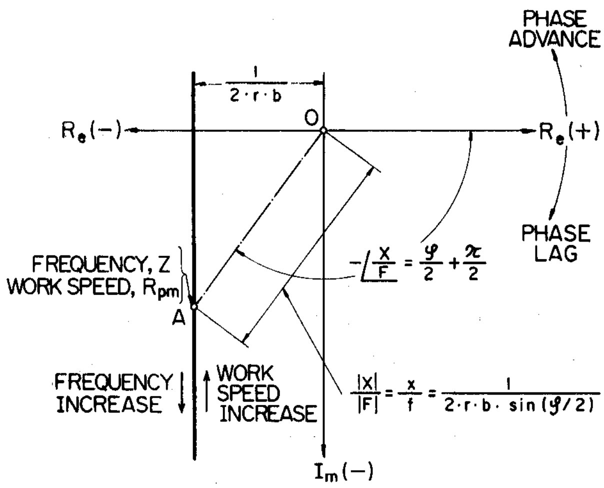

Fig. 4. Polar plot of the dynamic response of the metal cutting process [Eq. (3)].

The cutting dynamics X/F expressed by the equations (3), (4), and (5) are expressed on the polar coordinate as a straight line parallel to the Im-axis as shown in Fig. 4. A point A is found on the straight line corresponding to any particular set of a work speed R_{pm} and a frequency Z, and the vector $\overrightarrow{OA}$ represents the metal cutting dynamics. With an increase of the frequency Z or with a decrease of the work speed R_{pm}, the angle φ decreases, thus the point A moves downward. The distance between the line and the Im-axis is given as $1/(2 \cdot r \cdot b)$, which reduces, for example, with the increase in the depth of cut b in a conventional cutting; thus, the line makes a right side parallel shift.

2.2. Derivation of the theory of forced chatter from the conventional theory of self-excited chatter.

2.2.1. Conditions for the self-excited chatter without forced vibration

Existing theories of the chatter concern the occurrence of the self-exicted chatter when no forced vibration is present, that is $Y_0=0$, hence $X=X_0$ in the system shown in Fig. 1. According to those theories, the stability problem is reduced to a discussion on a polar coordinate, on which the harmonic response loci α of the machine structure and the straight line β expressing the cutting dynamics are drawn. Only when the two lines α and β intersect each other as shown in Fig. 5(a), is it possible for the self-excited chatter to occur, but impossible when they are apart as shown in Fig. 5(b). In the former case, the chatter occurs continuously at a constant amplitude when the curve α and the line β have a common frequency at either

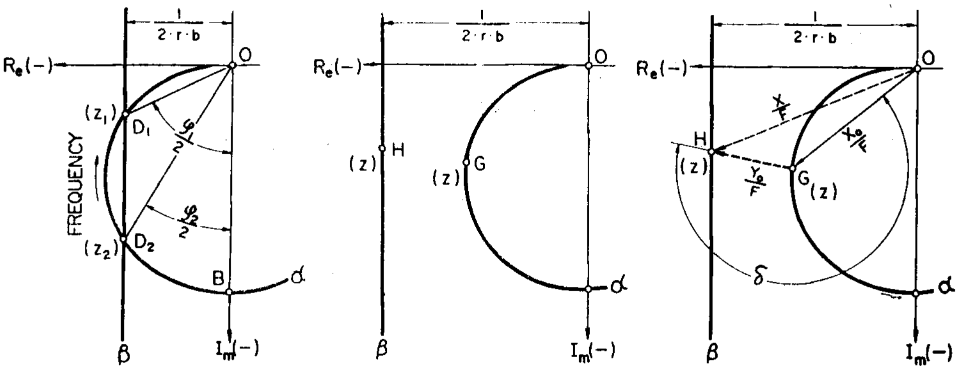

Fig. 5. Illustration of the cases where the chatter occurs or subsides depending on the combinations of the response curve α of the mechanical structure and the response line β of the cutting process.
(a) Condition of possible occurrence of the self-excited chatter.
(b) Condition where the self-excited chatter cannot occur.
(c) Occurrence of chatter due to the forced vibration Y_0.

of the two intersections D_1 and D_2; otherwise, the vibration either damps or diverges. If it damps, the chatter eventually does not occur, while if it diverges, either or both of the curve α and the lines β varies due to the nonlinear character which appears with the increase in the amplitude, until the two satisfy the above condition that allows a chatter of a constant amplitude to continue.

When a sufficiently large depth of cut is selected such that the line β intersects with the curve α as shown in Fig. 5(a), the intersection D_1 determines the upper limits R_{pm1}'s of the work speed ranges in which the self-excited chatter arises. The R_{pm1}'s are equal to the work speed R_{pm}'s which satisfy the equations (4) and (5) for the frequency $Z=Z_1$ and the phase $\varphi=\varphi_1$, where Z_1 is the frequency of the point D_1 marked on the cusve α, and $\varphi_1/2$ is the rotated angle of the vector $\overrightarrow{OD_1}$ from the $I_m(-)$ axis. Similar calculation about the intersection D_2 gives the lower limits R_{pm2}'s of the work speed ranges. If the work speed lies between the R_{pm1}'s and R_{pm2}'s, the onset of the self-excited chatter occurs irrespective of the presence of the forced chatter; therefore, in such a case, it it useless to minimize the forced vibration which exists at an idle running time in expectation of chatter free operation.

2.2.2. The mechanism through which the forced vibration stimulates the chatter

If the forced chatter is to be discussed, such conditions must be considered that the cutting is free from the self-excited chatter, for instance a small enough depth of cut to have the line β apart from the curve α. Considering a vibration of an arbitrary frequency Z for this case, the points G and H corresponding to the frequency Z on α and β respectively, are apart as seen in Fig. 5(b), and it is impossible for a vibration of the frequency to arise continuously.

However, if the forced vibration Y_0 at the identical frequency superimposes with a proper phase difference and an amplitude ratio to the vibration X_0, it is possible for a steady vibration to occur. Namely, substituting the two equations:

$$X = \overrightarrow{OH}\cdot F \quad \text{and} \quad X_0 = \overrightarrow{OG}\cdot F$$

into the foregoing equation (1) gives:

$$\overrightarrow{OH}\cdot F = \overrightarrow{OG}\cdot F + Y_0$$

Dividing this by F, the resulted equation reads:

$$\overrightarrow{OH} = \overrightarrow{OG} + Y_0/F$$

and this indicates that a steady vibration is able to continue if term Y_0/F equals to the vector GH, because in this case the equation between the three vectors:

$$\overrightarrow{OH} = \overrightarrow{OG} + \overrightarrow{GH}$$

is satisfied as seen in Fig. 5(c)

In the figure, the angle δ made by the two vectors $\overrightarrow{GH}$ and $\overrightarrow{OG}$ is the proper phase difference of the forced vibration Y_0 to X_0, and the length of the vector $\overrightarrow{GH}$ is the proper amplitude ratio of the forced vibration Y_0 to the resulted fluctuation of the cutting force F, namely y_0/f.

The theory described indicates that, when a cutting starts under the condition invulnerable to the self-excited chatter but with a forced vibration Y_0 of a frequency Z, it is possible that a fluctuation in cutting force F and the associated vibratory deformation of the machine structure X_0 arises at the same frequency, with the force amplitude f being equal to y_0/(length of the vector $\overrightarrow{GH}$), and keeping a phase difference δ between X_0 and Y_0, resulting in an apparent vibration $X = X_0 + Y_0$, between the tool and the work.

2.2.3. Amplification factor of the forced vibration by the cutting

Although, from the theoretical point, any small forced vibration Y_0 can bridge the points G and H of the Fig. 5(b) and cause the forced chatter, practically it is not recognized as a chatter if the apparent amplitude is small. To evaluate the forced vibration amplified by the cutting process, an amplification factor M is defined by $M = x/y_0$, namely the amplitude ratio of the resulted vibration X to the source vibra-

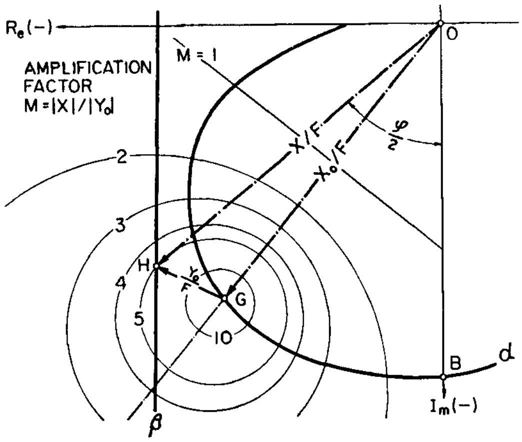

Fig. 6. Given a response curve α of a machine structure and the frequency Z of the forced vibration, the amplification factor M distributes depending on the position of the point H on the line β which represents the cutting dynamics. In the example illustrated, $M = 5$.

tion Y_0. On the harmonic response loci of Fig. 6, the amplification factor is the ratio of the magnitude of the vector $\overrightarrow{OH}$ to that of the vector $\overrightarrow{GH}$, and therefore, if the loci α of the machine structure is given and the point G is thereupon fixed corresponding to the forcing frequency Z, the value of M depends on the metal cutting dynamics represented by the position of H and it is distributed as shown in Fig. 6 when H moves in the plane of the polar coordinate. The contour lines of M are the Apollonius' circles for the two fixed points O and G, and a greater M is attained as the point H approaches G.

Starting from a condition having a self-excited chatter, gradual decrements of the depth of cut lead to a condition free from the chatter, in which the point H is slightly remote from G. When a forced oscillation is added to this condition at a frequency close to that of the previous chatter, the chatter is supposed to occur again as the result of a high amplification.

3. Verification by the cutting experiments

3.1 Purpose of the experiment

From the theory described above, the following phenomena are expected, the occurence of which is to be ascertained experimentally:

(1) It is probable that a forced vibration at idle running time is amplified by the cutting.

(2) As the frequency of the forced vibration changes, given a particular work speed R_{pm}, the point H moves along the line β, and therefore the amplification factor M varies. Consulting with the equations (4) and (5), when the forcing frequency Z changes by $R_{pm}/60$ c/s, the angle φ takes an identical value so that the point H resumes an identical position on the line. This specifies that for every $R_{pm}/60$ c/s of the forcing frequency, the distance between the points H and G is minimized resulting in a maximum amplification factor M, although a little discrepancy arises to the above interval of the frequency because the point G also moves slowly along the curve α. If a cutting test is conducted at a constant work speed of $R_{pm}=80$ rpm, this interval is supposedly close to $80/60=1.3$ c/s.

(3) When a high amplification occurs, a reduction in the depth of cut removes the line β to its left and reduce amplification factor M.

3.2. Test procedure

A long arbor having a work disc at an end is mounted between the three-jaw chuck and the center of an engine lathe, and the periphery of the disc is turned by a tool with 45 deg side cutting edge angle, as illustrated in Fig. 7. Though the setup

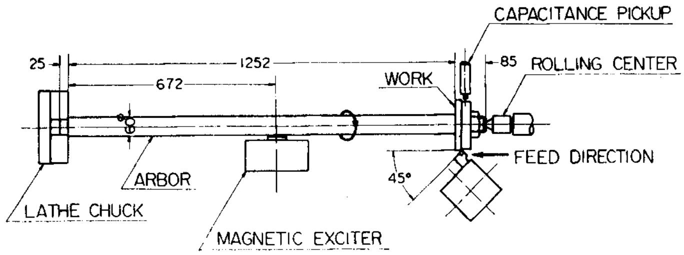

Fig. 7. Illustration of the cutting test setup on an engine lathe with a forced vibration added by a magnetic exciter.

is liable to the self-excited chatter due to the bending vibration of the arbor, a critical depth of cut free from the chatter is obtained by repeating the turning tests with gradually reduced depths of cut. The critical depth of cut thus obtained and a depth of cut smaller than that are employed as the test conditions. At those conditions, the cutting is free from the chatter unless a forced vibration is artificially added. To provide the system with a forced vibration of various frequencies, a magnetic exciter is mounted on the lathe bed which excites the middle of the arbor in the horizontal direction without making a contact with the arbor.

By conducting preliminary cutting test, the principal frequency of the chatter due to the spindle-arbor-work-center system is identified, around which various excitation frequencies are selected for the main tests. Having the excitation added at each of those frequencies, the main test is conducted which consists of tape-recording the horizontal vibration of the work disc with respect to the capacitance-type displacement pickup mounted on the cross slide, at each of the idle running times, the cutting at the critical depth, and the cutting at the depth 0.05 mm less than the critical depth. Every recorded vibration is played back later for the spectral density analysis, and the amplification factor is calculated from the readings at the excitation frequency by $M=$(amplitude at cutting)/(amplitude at idle running). On the other hand, the dynamic response $R(Z)$ is measured using a electrodynamic exciter to facilitate the interpretation of the test results.

3.3. Equipments and materials for the experiment.

Machine tool: Mitsubishi-Oericon DMO/ engine lathe; swing over bed 460 mm, maximum distance between centers 1500 mm; main motor equipped is a pole change induction motor (LOW/HIGH, 1800/3600 rpm, 10/15 HP); only LOW is selected for the experiment.

Tool: Indexable type turning tool originally for left-handed facing is used for right-handed peripheral turning. ISO P10 cemented carbide insert, 12 mm square and 4.5 mm thick, is used in the tool geometries of −5 deg end and side rake angles, 5 deg end and side relief angles, 45 deg end and side cutting edge angles, and 0.8 mm nose radius.

Work piece: 101 to 132 mm diameter and 48 mm thick disc is prepared of 0.45% *C* plain steel having 225 Brinell hardness number.

Cutting condition: work speed 80 rpm (cutting speed 26 to 34 m/min), feed 0.315 mm/rev.

Instruments: Vibratory displacement is detected by a capacitance type pickup, magnified by a dc amplifier, and recorded by a four-channel data recorder (FM modulation, dc to 1 kc). Spectral density of the recorded vibration is analyzed by a variable frequency filter having 20 db to 1/3 octave and 30 db to 1 octave separation. The frequency of the excitation or that of the filtered signal is identifed by a digital counter. To monitor the wave shape of the vibration, a two-channel synchroscope is used.

Exciter: The magnetic and electrodynamic exciters are driven by an oscillator and a power amplifier.

3.4. Test results and the interpretations.

Reducing the depth of cut at 0.05 mm intervals in repeating cutting tests without the excitation, a 0.25 mm depth was found to be the boundary where the self-excited chatter disappears; therefore, 0.25 mm and 0.2 mm depths of cut were selected for the main tests. Also it was found that the principal frequency of the self-excited chatter was 79.0 c/s, around which a range of 75.8 c/s to 80.8 c/s was selected as the excitation frequencies.

The main tests were conducted at intervals of every 0.2 c/s frequencies between the above range at a random order. The amplification factor M was obtained for every test condition, and the whole data of M indicated that its difference due to the change in the depth of cut was not recognized, but that due to the variation in the excitation frequency was apparent, such that peaks in M appeared at five different frequencies as seen in the upper graph of Fig. 8. The intervals between the peaks range from 0.7 c/s to 1.6 c/s and they agree to the predicted 1.3 c/s for the case.

From the measured dynamic response shown in the lower graph of Fig. 8, the resonance frequency as defined by a 90 deg phase lag is found to be 77.5 c/s. Consulting with the upper graph, it is seen that below the resonance frequency, the peak values of M are relatively small, and vice versa at the higher frequencies.

As long as the frequency is less than the resonance frequency so that the point G stays on the right side of the point B in Fig. 6, the point G cannot come so close to

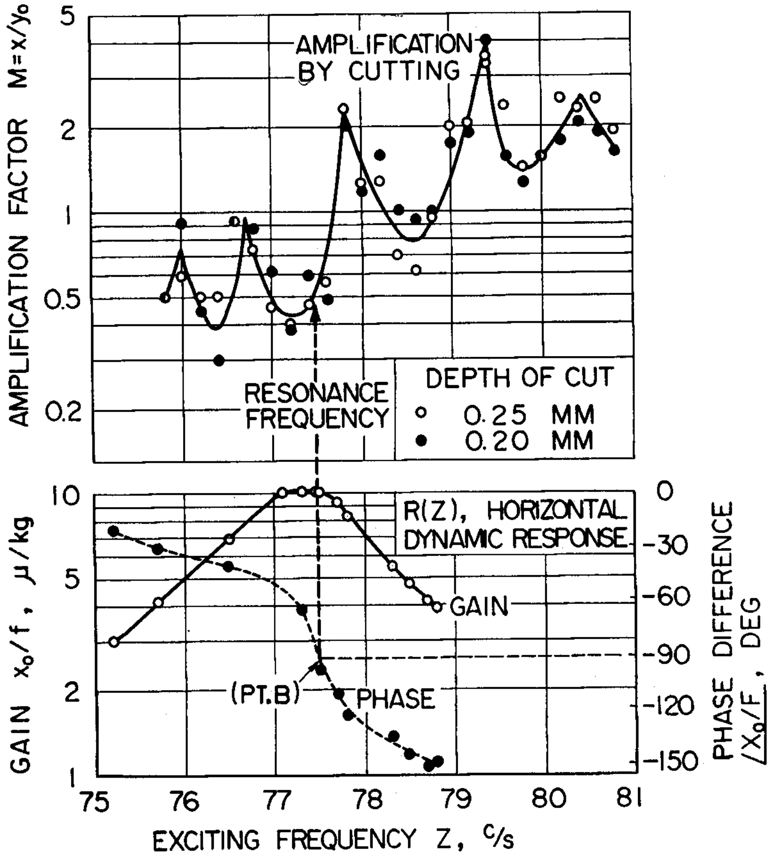

Fig. 8. Results of the cutting test and the dynamic response test. (Upper graph) Amplification factor M of the horizontal vibration of the work with respcct to the cross slide, tested in turning with the magnetic excitation at various forcing frequencies. (Lower graph) Horizontal dynamic response, $R(Z)=X_0/F$ of the machine structure tested by the electrodynamic exciter.

the point H that the peak of M can attain a very high value. On the contrary, when the point G has passed the point B and is on the left of it, the peak of M is considerably high, because the points G and H can get quite close to each other.

In the test result shown in Fig. 8, the amplification factor which is greater than 1 means that the forced vibration is magnified by the cutting. The gratest amplification factor was $M=4$ in these experiments, which occurred when exciting at 79.4 c/s. The spectral density analysis of the vibrations in this case is shown by the three curves in Fig. 9, the lowest curve is that for when running idle without the excitation, that is the true idle time vibration. However, in this study, the above plus the vibration given by the magnetic exciter is assumed as the idle time vibration,

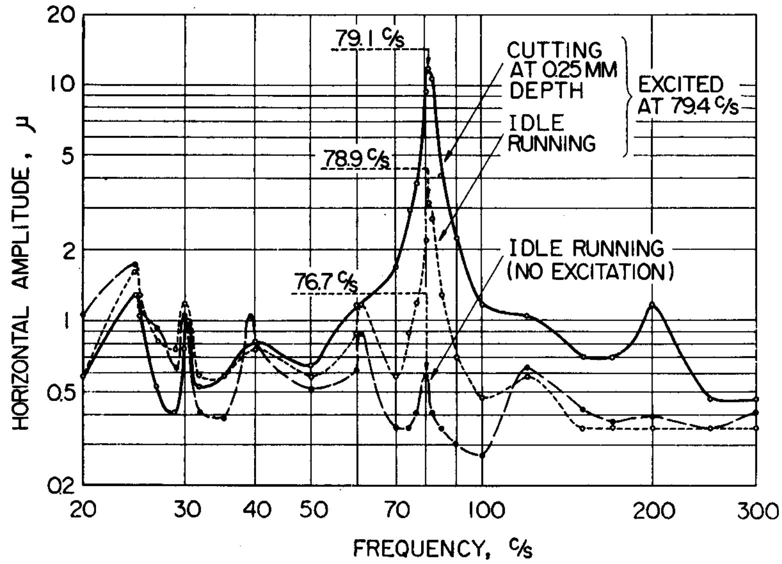

Fig. 9. Spectral density data of the relative vibrations between the work and the cross slide.

which is shown by the middle curve of Fig. 9, and, as is seen, it contains a vibration of 3 μ(1 μ=1 micrometer=10^{-3} mm) in the amplitude at the excited frequency of 79.4 c/s. With the cutting started from this condition, the vibration during the cut is shown by the upper curve and its amplitude at the forced frequency is amplified to 12 μ.

Different from the self-excited chatter, the chatter which is the amplified outcome of the forced vibration is supposed to disappear if the source vibration is eliminated. This was ascertained by a cutting experiment in which the current to the magnetic exciter was switched on and off alternately and the chatter mark appeared or disappeared accordingly as seen in Fig. 10. In this case an excitation at 78.2 c/s brought up the idle time vibration of 9.5 μ in the amplitude, which was magnified into 73 μ by cutting, a gain of 7.7 times.

4. Conclusions.

From the presently conceived theory of the self-excited chatter, the theory of the so-called forced chatter was derived. Since it was expected from this theory that the vibration at the idle running time would be magnified at the cutting time in certain cases, it was examined by a turning test with a forced vibration added by a magnetic exciter. The test proved that the amplification factor of the forced vibration varies with the forcing frequency in the way predicted by the thory.

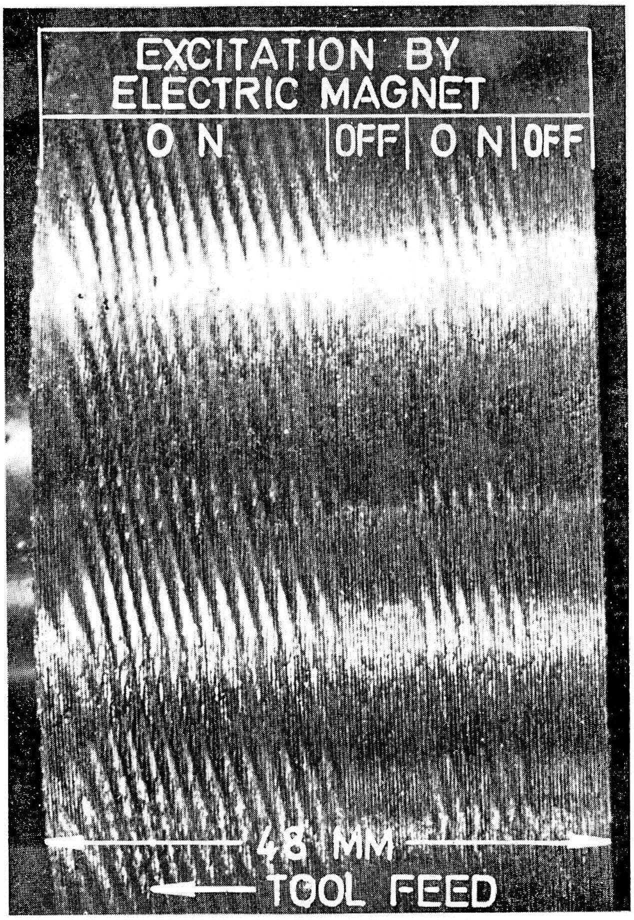

Fig. 10. Chatter mark produced by the forced vibration magnified through the cutting.

Integrating the knowledges obtained from the theory and the experiments, the following conclusions are drawn concerning the chatter in the metal cutting.

1) The forced vibration which exists at the idle running time relative between the tool and the work is able to be amplified into an intense chatter when the frequency of the forced vibration is above the resonance frequency but in the resonance range.

2) Even when the above requirement is not met, the forced vibration appears at the cutting time; however, its amplitude is generally less than that at the idle running time. Such small amplification is yet supposed to cause trouble when a fine finish is requisite.

3) So long as the above condition 1) is satisfied, a highly amplified vibration appears during the cut, only when the cutting conditions are taken quite close to the

stability boundary of the self-excited chatter as determined by the discussion on a harmonic response loci. When this happens, since the resulting chatter is not distinguishable from the self-excited chatter, it seems as if the stability borderline has been lowered.

4) To the two requirements for the chatter free machine structure which have been deduced from the previous theories, namely:

(i) a small maximum real negative part in the harmonic response loci of the structure [5],

(ii) a high resonance frequency which corresponds to the maximum real negative part [2],

a third requirement should be added, that is:

(iii) the absence of a forced vibration of a frequency that lies in the resonance range but higher than the resonance frequency corresponding to the maximum real negative part.

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Nomenclatures

Z	:	Frequency of vibrations, in c/s.
Y_0	:	Vibratory displacement relative between the tool and the work existing at the idle running time, measured in the direction of the depth of cut, namely mormal to the cutting movement, with the positive sign when the distance increases between the tool and the work.
X_0	:	Vibratory displacement relative between the tool and the work which resulted from the deflection of the mechanical structure due to the vibratory cutting force F, measured in the same manner as Y_0.
X	:	Resultant vibratory displacement relative between the tool and the work existing at the cutting time, measured in the same manner as Y_0.
y_0, x_0, x	:	Amplitudes of the vibrations Y_0, X_0, and X respectively, in μ (micrometers).
F	:	Vibratory cutting force.
f	:	Amplitude of F, in kg.
$R(Z)$	:	Dynamic response of the machine structure described as a function of the frequency Z, with the imput F and the output X_0.
μ	:	Overlap factor.
t	:	Time, in sec.
T	:	Time for a rotation of the work, in sec.
r	:	Specific cutting force in dynamic condition, in kg/mm^2.
b	:	Width of cut in orthogonal cutting, or depth of cut in conventional cutting, in mm.
φ	:	Phase difference between the two successive surface waves produced by the chatter, in deg.
ε	:	Fraction of the number of chatter marks per revolution of the work.
N	:	Integer of the number of chatter marks per revolution of the work.
R_{pm}	:	Speed of work revolution, in rpm.
M	:	Amplification factor of the forced vibration by the cutting.