# Mechanism of Friction and Wear of Steel Plate Against Solidified Sandy Soil

By

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In this paper the mechanism of the frictional resistance and the wear of steel plate for solidified sandy soil are cleared and shown. The solidified sandy soil was used to make reappear the heaviest wear in actual earth moving operation. It resulted that the frictional resistance increases linearly with increasing of contact pressure, and the relation between the contact pressure and the amount of wear of metal can be calculated from the depth of penetration of soil particle into metal plate. And the amount of wear of metal increases in proportion to about 0.84~1.00 power of the contact pressure.

## 1. Introduction

In the earth moving work with construction machinery many parts of excavating tools are heavily worn. Especially, the wear of cutting edge of dozer, fluke of excavating bucket or drill of earth auger due to solidified sandy soil or gravel are inconceivably severe in actual operation. And it causes not only the decrease of the output of machine, but the degradation of working rate because of exchanging or repairing of tools. So the problems caused by wear of tools become a difficulty in the rationalization of construction. Therefore, to investigate the mechanism of wear of metal caused by sandy soil and to consider how to deal with it rationally, they have great worth in production and design of excavating equipments and in planning to revise tools effectively.

In this paper, the mechanisms of the frictional resistance and the wear of steel plate for solidified sandy soil are cleared and exemplified.

The wear of metal plate due to solidified sandy soil is principally caused by the abrasive action of soil particle. Hitherto, the mechanical properties of abrasive wear have been studied. Especially, J. Goddard and others[1),2)] established on abrasive test of metal plate against grain particles, that the amount of wear on the metal was proportional to contact pressure and to difference between the coefficient of friction of metal against grain and that of metal against metal fragment removed.

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In case of solidifed sandy soil, the wear phenomena are to a certain degree similar to that of grain. But they are not similar at the view-points of crushing of soil particle under high contact pressure, vanishing of mutual bond force of soil particles and packing due to crushed soil particle. However, both the coefficient of friction and the contact pressure between steel plate and solidified sandy soil are the most important factors which control the amount of wear.

Here, the mechanism of friction has been analysed and the relations between frictional resistance and contact pressure of steel plates against soil particles have been shown from results of the scratch test which was done by a unit particle on the steel plate, assuming that the particle is not crushed when it moves on the plate. And the mechanism of abrasive wear has been studied theoretically at the view point of streak formation due to the abrasive action of soil particle.

The relations between the amount of wear and the contact pressure have been able to be derived from the depth of penetration of soil particle which penetrates into metal plate under load.

So, the amount of wear can be calculated by means of the coefficient of friction and the contact pressure, and those relations have been shown.

Especially, the coefficient of friction and the amount of wear have been considered and discussed from the view point of grain-size distribution of solidified sandy soil and crushing of soil particle under high contact pressure.

## 2. Analysis of Friction and Wear Mechanism

(1) Mechanism of friction

The sliding friction between the surface of metal and sandy soil is mainly due to the abrasive action of soil particles; microscopically, it is a phenomenon of the abrasion on the surface of metal which is caused by sharp edge of soil particles. The roughness of steel surface after wear is influenced by the degree of penetration of soil particles. After metal is worn out enough, streaks on the surface are all parallel to the running-direction of soil particles; therefore, any soil particle which touches the metal does not cross the streaks. Now, here we assume that soil particles cut into the surface of steel is defined to be average plane of its roughness.

The edges of sandy soil are considered to have various shapes, here we assume that the shape is regular pyramid as shown in Fig. 1, $\theta$ shows the average angle of opposite faces on a summit of a soil particle. Now we suppose that a soil particle applied to a normal force $N$ penetrates into the surface of steel by the depth of $\alpha$. The angle between one of the horizontal edges of a soil particle and its progressing direction is supposed to be $\phi$ $(-45°<\phi<45°)$, the cross section of a streak $a_v$

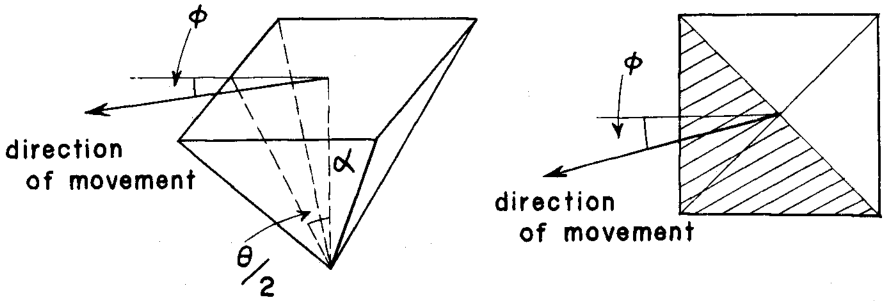


Fig. 1. Movement of Regular Quadrangular Pyramid.

which is the area projected to the progressing direction is given as follows;

$$a_v = \sqrt{2}\,\alpha^2 \cos(45^\circ-\phi)\tan\theta/2 \qquad \cdots\cdots\cdots\cdots\cdots(1)$$

The coefficient of friction $\mu$ between a soil particle and steel plate is considered to be divided into a term $\mu_p$ which is due to abrasive action by the soil particle and another term $\mu_a$ which is due to action of adhesion on contact of the soil particle and steel.

Supposing that $p_m$ is the normal stress which acts on the soil particle and $p'_m$ is the moving resistance of the soil particle on the steel plate and $s$ is the adhesive resistance which acts tangentially on the contact area between the soil particle and the steel plate, the following equations are obtained.

$$\mu_p = p'_m a_v / N$$

$$= (p'_m/p_m)\frac{\sqrt{2}}{2}\cos(45^\circ-\phi)\cot\theta/2 \qquad \cdots\cdots\cdots\cdots\cdots(2)$$

at $-45^\circ<\phi<45^\circ$, the average is

$$\bar{\mu}_p = (p'_m/p_m)\frac{2}{\pi}\cot\theta/2 \qquad \cdots\cdots\cdots\cdots\cdots(3)$$

Samely,

$$\mu_a = \frac{\sqrt{2}}{2}\frac{s}{p_m}\cos(45^\circ-\phi)\operatorname{cosec}\theta/2 \qquad \cdots\cdots\cdots\cdots\cdots(4)$$

$$\bar{\mu}_a = \frac{2}{\pi}\frac{s}{p_m}\operatorname{cosec}\theta/2 \qquad \cdots\cdots\cdots\cdots\cdots(5)$$

Therefore, the coefficient of friction of $\mu_p$ and $\mu_a$ between steel plate and solidified soil particle is influenced by the average angle of opposite faces on summit of the soil particles: the coefficient of friction increases as the edge of soil particle becomes sharper. As the results of abrasive test by unit particle show later, the coefficient of friction becomes larger at the angle $\phi=0^\circ$ (cutting cross section of steel is minimum) rather than $\phi=45°$ (cutting cross section of steel is maximum). In other words, the differences of angles of contact between soil particles and steel vary considerably the moving resistance $p'_m$, when a soil particle cuts into the surface of steel. Therefore, the coefficient of friction must be calculated at $p'_m$ which is observed in various cases.

a) Abrasive test by a unit particle

As mentioned above, the shape of soil particle is assumed to be a regular quadrangular pyramid. We tested the abrasive action of soil particle on steel plate, using a diamond cone as a representative unit particle which cannot be crushed.

Two kinds of angle of opposite faces of a regular quadrangular pyramidal diamond cone were chosen; $\theta=136°$ and $100°$, each was tested on steel plate of different hardness and the coefficient of friction ($\mu=\mu_p+\mu_a$) was directly investigated.

Fig. 2 and Fig. 3 show the relations between abrasive resistance and normal force, when the two kinds of diamond cone ($\theta=136°$ and $100°$) are drawn toward the base and diagonal line of the pyramid. As each of abrasive resistance increases linearly with increase of normal force, the ratio of each is expressed by constant coefficient $\mu'$. Generally, the coefficient $\mu'$ decreases a little with increase of Vicker's hardness of metal: the tendency is exceeding as the angle of opposite faces of a diamond cone is less. These relations are described in Table 1.

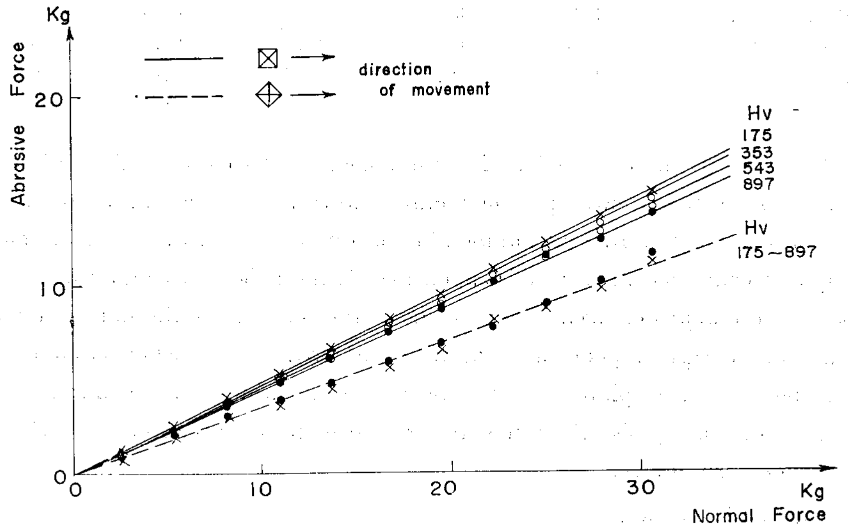


Fig. 2. Abrasive Test by Diamond Cone ($\theta=136$ degrees).

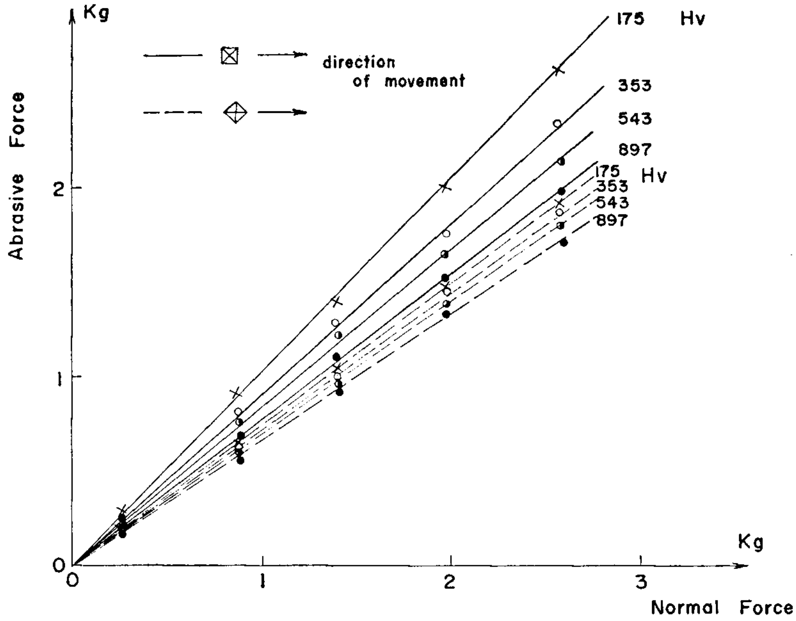


Fig. 3. Abrasive Test by Diamond Cone ($\theta=100$ degrees).

Table 1. Abrasive Coefficient $\mu'$ at Abrasive Test by Diamond Cone.

| Angle of opposite faces | | 136 degrees | | 100 degrees | |
|---|---|---|---|---|---|
| Direction of movement | | ⊠ → | ⊕ → | ⊠ → | ⊕ → |
| Vicker's hardness *Hv* kg/mm² | 175 | 0.49 | 0.35 | 1.05 | 0.75 |
| | 353 | 0.48 | 0.35 | 0.90 | 0.72 |
| | 543 | 0.47 | 0.35 | 0.84 | 0.70 |
| | 897 | 0.45 | 0.35 | 0.77 | 0.67 |

Then, the streaks on which a diamond cone passed are observed by microscope, and it is shown that a large part of metal is flown and heaped up on both sides of the streaks and only a little part of metal is cut off and worn away. Fig. 5 shows an example of examined amount of heaped metal: the height of heaped metal comes to 80% of the depth of streak. Fig. 6 shows the relations of the normal force and the ratio of the depth of penetration at static and movable state when a diamond cone penetrates into steel plate. As the result, at movable state a diamond cone penetrates only 60% of static state without relating to the direction of drawing, kind of metal or normal force.

(2) Mechanism of wear

a) Contact pressure and penetration of soil particle into metal

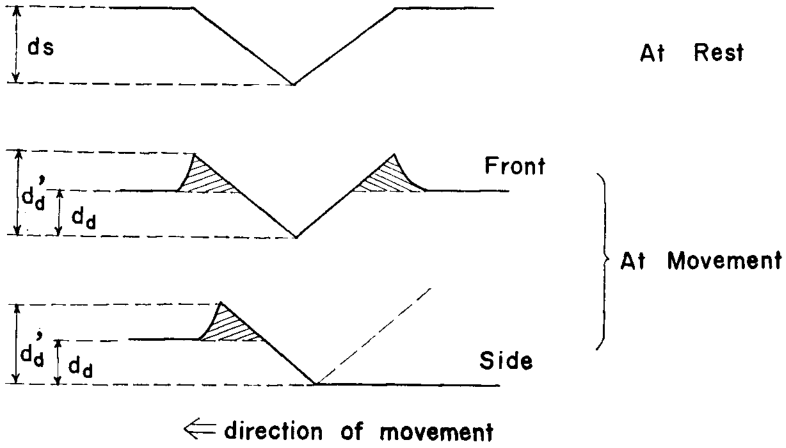


Fig. 4. Abrasion of Metal by Soil Particle.

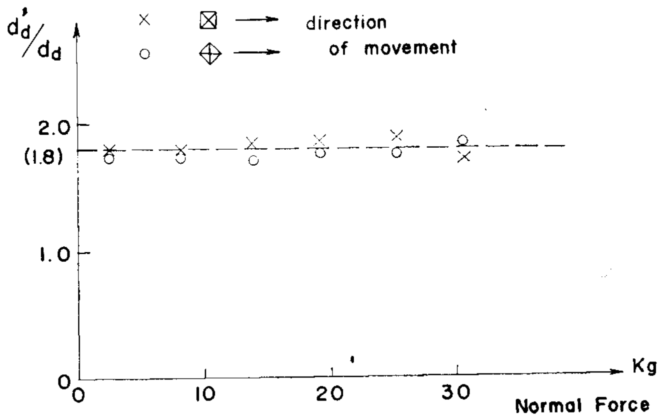


Fig. 5. Relation of Normal Force and $d_d'/d_d$ at Movement of Diamond Cone on Steel Plate.

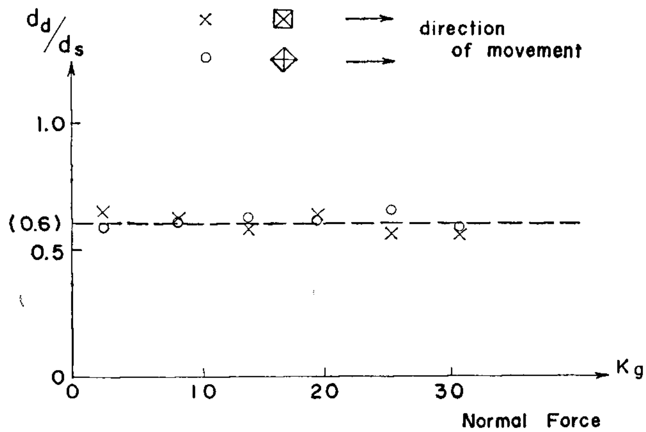


Fig. 6. Relation of Normal Force and Ratio of Depth of Penetration at Movement to at Rest.

When steel plate moves on solidified sandy soil with considerably high contact pressure, the metal is worn away by soil particles; this mechanism of wear is analysed under the following several basic assumptions.

(1) All the shapes of soil particles are supposed to be octahedron, as shown in Fig. 7(a), which are composed of two symmetrical regular quadrangular pyramids. And the length of a side of square which is a basic plane of regular quadrangular pyramid is written as $a_1$, $a_2$, $\cdots$, $a_s$, which is assumed to be equal to grain-size of soil particles. Here, $a_i$ $(i=1, 2, \cdots, s)$ is central value of a grain-size group which is equally divided by weight in $s$ parts on grain-size distribution of sandy soil. And, $\theta$ is the angle of opposite faces of the octahedral soil particle.

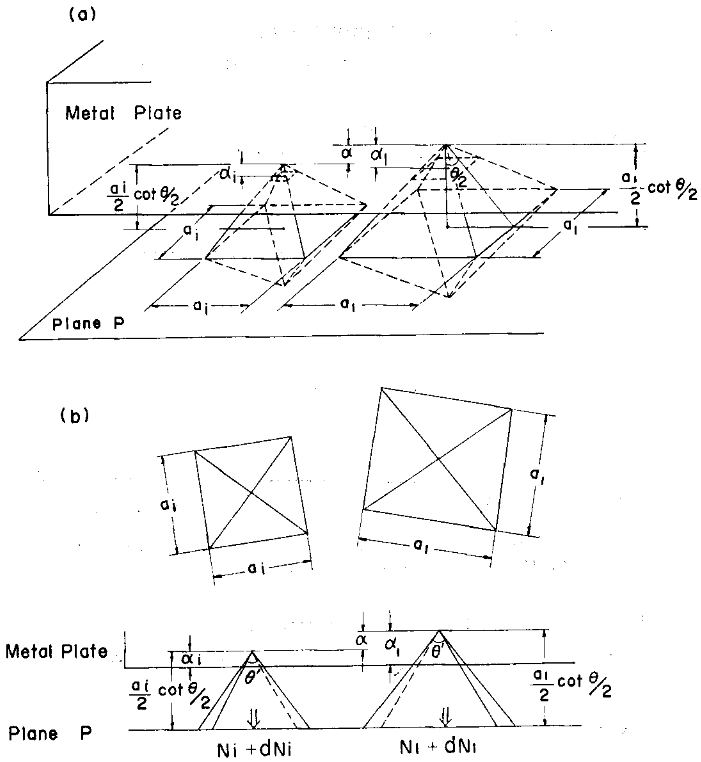


Fig. 7. Penetration of Octahedron Particle into Metal Plate.

(2) As the same Figure shows, the basic values of pyramids that include centres of gravity of octahedral soil particles are supposed to be on a plane $P$ which is parallel to steel plate.

(3) As a soil particle is projected to the plane which stands at right-angl toward the direction of movement, an equilateral triangle is made, as Fig. 7(b) shows. The vertical angle of the triangle is equal to that of streak on steel plate. As the

probability of direction of each soil particle is supposed to be equal, its average vertical angle is assumed to be $\theta'$. (In fact, the angle has been shown to be from 120° to 140°.)

(4) When steel plate moves on solidified sandy soil, soil particle is not crushed. And the solidified sandy soil is not packed by crushed fragment of soil particle. When steel plate gets to a certain load, it touches first on a soil particle of the largest grain-size. As the load increases, the soil particle penetrates gradually into steel plate, and the metal touches on next grain-size of soil particle and number of points of contact increase one after another. Here, the number of each soil particle of grain-size $a_1, a_2, \cdots, a_s$ is assumed $n_1, n_2, \cdots, n_s$, and the normal force which is acted on each soil particle is $N_1, N_2, \cdots, N_s$, and the increased part of the normal force is assumed to be $dN_1, dN_2, \cdots, dN_s$.

The order of grain-size $a_i$ obeys the next rule: $a_1 > a_2 > \cdots > a_s$; weight of the group of soil particles equally divided in $s$ parts is equal to each other, and therefore next equation is obtained. (in the case of the group having same density)

$$a_i^3 n_i = a_j^3 n_j = \text{const.} \qquad \cdots\cdots\cdots\cdots\cdots (6)$$

For $dp$, that is the increment of contact pressure $p$ between solidified sandy soil and steel plate, next equations are obtained.

$$\begin{aligned} & n_1 dN_1 + n_2 dN_2 + \cdots + n_s dN_s \\ & = (n_1 a_1^2 + n_2 a_2^2 + \cdots + n_s a_s^2)(1+e')dp \\ & = A_s(1+e')dp \end{aligned} \qquad \cdots\cdots\cdots\cdots\cdots (7)$$

Here,
$$A_s = n_1 a_1^2 + n_2 a_2^2 + \cdots + n_s a_s^2 \qquad \cdots\cdots\cdots\cdots\cdots (8)$$

And $e'$ is the void ratio of projected parts when pyramids are projected on plane $P$. $n_i$ is calculated from Eq. (8)

$$n_i = A_s / a_i^3 \sum_{j=1}^{s} (1/a_j) \qquad (i=1, 2, \cdots, s) \qquad \cdots\cdots\cdots\cdots\cdots (9)$$

As load acts on steel plate, the soil particles, grain-size $a_1$, penetrate first into the metal. Next, as the load is increased, penetration of $a_1$ becomes large, and for the contact pressure $p_1$, when the metal reaches to the next soil particle, grain-size $a_2$, following equation is obtained.

$$dN_1 > 0, \quad dN_2 = dN_3 = \cdots = dN_s = 0$$

from Eq. (7)

$$n_1 dN_1 = A_s(1+e')dp$$

Therefore,
$$p_1 = n_1 N_1 / A_s(1+e') \qquad \cdots\cdots\cdots\cdots\cdots (10)$$

When the soil particle of largest grain-size $a_1$, acting a certain load, moves along steel plate, the depth of penetration of the soil particle into steel plate is assumed to be $\alpha_1$. And the depth of penetration of the next soil particle, grain-size $a_2$ into steel plate is assumed to be $\alpha_2$; samely, the followings are assumed to be $\alpha_3$, $\alpha_4$, $\cdots$, $\alpha_s$. When hardness of metal and normal force $N$ are constant, contact area $A$ supporting the normal force between a soil particle and metal should be equal either at static state or at moving state. Therefore, as shown in Fig. 4, when a soil particle moves on, the contact area between metal and a soil particle is limited only in the front area of the soil particle. And in moving state the depth of penetration $\alpha$ decreases considerably to compare with that of static state $\alpha_0$, because of scratch fragments of metal which is mounted in front area of the soil particle. That is,

$$\alpha = k\alpha_0 \qquad (0<k<1) \qquad \cdots\cdots\cdots\cdots\cdots(11)$$

Here, $k$ is decided by experimental data.

Assuming the length of diagonal of penetrated square mark on metal which is marked by a soil particle at static state, following equation is obtained:

$$d_0 = 2\sqrt{2}\,\alpha_0 \tan\theta/2$$

and,

$$H_v = N/A = (2N\sin\theta/2)d_0^2$$

so,

$$\alpha_0^2 = \frac{N\sin\theta/2}{4\,H_v\tan^2\theta/2}$$

Therefore, when a soil particle moves along steel plate, depth of penetration of the soil particle into metal $\alpha$ is calculated as follows.

$$\alpha = \left\{\frac{k^2N\sin\theta/2}{4H_v\tan^2\theta/2}\right\}^{1/2} \qquad \cdots\cdots\cdots\cdots\cdots(12)$$

The contact pressure between solidified soil particle and steel plate $p_1$, can be obtained by Eq. (10) calculating $N_1$ in Eq. (12) for the depth of penetration of a soil particle,

$$\alpha_1 = (a_1-a_2)/2\tan\theta/2$$

Then, for the contact pressure $p_2$ at which the steel plate comes into contact with both of soil particles, of grain-size $a_1$, $a_2$, simultaneously, and the plate arrives at the next soil particles of grain-size $a_3$, next conditions are satisfied.

$$dN_1>0\,,\quad dN_2>0\,,\quad dN_3 = dN_4 = \cdots = dN_s = 0$$

And from Eq. (7), next equation is obtained.

$$n_1 dN_1 + n_2 dN_2 = A_s(1+e')dp \qquad \cdots\cdots(13)$$

First, the calculation is started from the next condition.

$$N_1 > 0\,, \quad N_2 = 0$$

Now, $d\alpha_1$ and $d\alpha_2$ are assumed to be the increments of depth of penetration of soil particles of grain-size $a_1$ and $a_2$ into steel plate respectively and $dN_2$ is assumed to be the increment of normal force acting on the soil particle of grain-size $a_2$, when the normal force $N_1$ acting on the soil particle of grain-size $a_1$ increases to $N_1+dN_1$. From Eq. (12), $d\alpha_1$ and $d\alpha_2$ are calculated as follows;

$$d\alpha_1 = \frac{1}{2}\left\{\frac{k^2 N_1 \sin\theta/2}{4H_v \tan^2\theta/2}\right\}^{1/2}\left(\frac{dN_1}{N_1}\right)\left(1-\frac{1}{4}\frac{dN_1}{N_1}\right) \qquad \cdots\cdots(14)$$

And,

$$d\alpha_2 = \left\{\frac{k^2 dN_2 \sin\theta/2}{4H_v \tan^2\theta/2}\right\}^{1/2} \qquad \cdots\cdots(15)$$

Here,

$$d\alpha_1 = d\alpha_2$$

Then, $dN_2$ is obtained from Eq. (14) and (15).

$$dN_2 = \left[\frac{1}{2}\frac{dN_1}{N_1}\left(1-\frac{1}{4}\frac{dN_1}{N_1}\right)\right]^2 N_1 \qquad \cdots\cdots(16)$$

Moreover, $d\alpha_1$ and $d\alpha_2$ vary as follows, when the normal force $N_1$ and $N_2$ increase.

$$d\alpha_1 = \frac{1}{2}\left\{\frac{k^2 N_1 \sin\theta/2}{4H_v \tan^2\theta/2}\right\}^{1/2}\left(\frac{dN_1}{N_1}\right)\left(1-\frac{1}{4}\frac{dN_1}{N_1}\right) \qquad \cdots\cdots(17)$$

$$d\alpha_2 = \frac{1}{2}\left\{\frac{k^2 N_2 \sin\theta/2}{4H_v \tan^2\theta/2}\right\}^{1/2}\left(\frac{dN_2}{N_2}\right)\left(1-\frac{1}{4}\frac{dN_2}{N_2}\right) \qquad \cdots\cdots(18)$$

$dN_1/N_1$ and $dN_2/N_2$ are induced as follows by Eq. (17) and (18), assuming that $d\alpha_1 = d\alpha_2 = d\alpha$

$$\frac{dN_1}{N_1} = 2\left[1-\left\{1-2\,d\alpha\Big/\left(\frac{k^2 N_1 \sin\theta/2}{4H_v \tan^2\theta/2}\right)^{1/2}\right\}^{1/2}\right] \qquad \cdots\cdots(19)$$

$$\frac{dN_2}{N_2} = 2\left[1-\left\{1-2\,d\alpha\Big/\left(\frac{k^2 N_2 \sin\theta/2}{4H_v \tan^2\theta/2}\right)^{1/2}\right\}^{1/2}\right] \qquad \cdots\cdots(20)$$

Therefore, $dN_2$ is calculated by Eq. (16) through $dN_1$ mentioned first, and also $d\alpha_2$ is obtained by Eq. (15). Then $dN_1$, $dN_2$ is obtained by Eq. (19), (20) for a given $d\alpha$ and the calculated $N_1$, $N_2$. By repeating this calculation, the contact pressure $p_2$ is calculated by $N_1$ and $N_2$, when $\alpha_1$ reaches $(a_1-a_3)/2\tan\theta/2$.

$$p_2 = (n_1 N_1 + n_2 N_2)/A_s(1+e') \qquad \cdots\cdots\cdots(21)$$

When contact points increase with the increase of normal force, the relation between contact pressure and depth of penetration of a soil particle into metal is obtained by repeating these calculations.

b) Relation between depth of penetration of a soil particle and amount of wear

The wear of metal occurs after the surface of metal is scratched by keen soil particles and the amount of wear of metal is decided by the number of soil particles and the depth of penetration of the soil particles into metal. Edges of soil particles, however, do not penetrate into an entire plane. As Fig. 8 shows, when soil particles

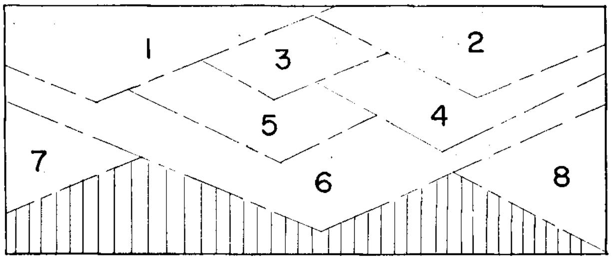

Fig. 8. Process of Forming Streak.

which penetrate into metal, $\alpha_1$, $\alpha_2$, $\cdots$, $\alpha_q$ move simultaneously on the surface of metal streaks of wear of $\alpha_2$, $\alpha_3$, $\cdots$, $\alpha_q$ are almost included by the streak of wear of $\alpha_1$. That is, the streak which occurs by soil particle of a certain grain-size is scratched away by the following larger soil particle rather than that of the grain-size. Therefore, in case of group of soil particles moving on the surface of steel plate infinitely, larger parts of amount of wear occur by the soil particles of the largest grain-size. The other amounts of wear which occur by soil particles of less grain-size are supposed to be calculated as the products of amounts of wear of the soil particle on a streaked plane by the largest soil particle of grain-size $a_1$ by ratio of $\alpha_i$ to $\alpha_1$ the depth of penetration of the soil particle, because they cut only the projected part of streaked surface of metal by the former $\alpha_1$.

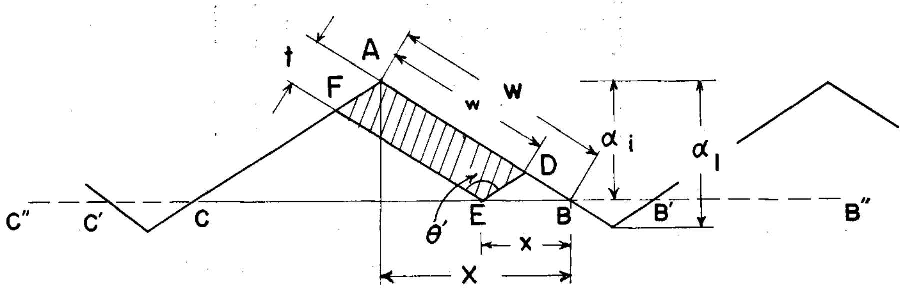

Fig. 9. Depth of Penetration and Cutting Width by Soil Particle.

As in Fig. 9 shows, the streaks on the surface of steel plate by the largest soil particles of grain-size $a_1$ are assumed to be close to each other. And, when the streak ADEF by other soil particle of less grain-size $a_i$ cut into the part of triangle $\triangle ABC$, the depth of cut $t$ and the cutting width $w$ are given as follows.

$$\overline{AB} = W, \quad \overline{MB} = X, \quad \overline{EB} = x$$

$$w = W\left(1 - \frac{x}{2X}\right), \quad t = x \cos \theta'/2$$

Probability which pointed end $E$ of the hatched part comes on the line $MB$ is equal. Hence,

$$\left.\begin{aligned} w_{mi} &= \frac{1}{X}\int_0^x w_i\,dx = \frac{3}{4}W_i = \frac{3}{4}\alpha_i \sec \theta'/2 \\ t_{mi} &= \frac{1}{X}\int_0^x t_i\,dx = \frac{1}{4}W_i \sin \theta' = \frac{1}{2}\alpha_i \sin \theta'/2 \end{aligned}\right\} \quad \cdots\cdots(22)$$

Then, the average area of the streak ADEF is as follows;

$$w_{mi}\,t_{mi} = \frac{3}{8}\alpha_i^2 \tan \theta'/2 \quad \cdots\cdots(23)$$

Therefore, amount of wear $M$ per unit area is calculated as follows from the number of soil particle of each grain-size per unit area $n_i$, and the depth of the penetration, $\alpha_i$, when the soil particles penetrate into steel plate under a certain contact pressure.

$$M = K\rho \tan \theta'/2\left(n_1\alpha_1^2 + \frac{3}{8}\sum_{i=2}^{s} n_i\alpha_i^2 \cdot \frac{\alpha_i}{\alpha_1}\right) \quad \cdots\cdots(24)$$

Here, $\rho$ is the density of metal, and $K$ is a constant, which is decided by the influence of partial charge of load caused by packing with crushed soil particles, the crush of bonding force of soil particles, and the rate of isolation of metal fragment from streak etc.. Furthermore, the depth of penetration $\alpha_i$ varies not only by the contact pressure, but also by the hardness of metal, and the number of soil particles $n_i$ varies with the distribution of grain-size. Therefore, the amount of wear varies according to these factors.

## 3. Results of Calculation of the Amount of Wear

(1) Grain-size distribution of solidified sandy soil

As the most representative grain-size distribution of soil particles, following 4 groups of soil have been chosen. Each of them takes the shape of logarithmic normal distribution and the shape is shown in Fig. 10. The deviations of grain-

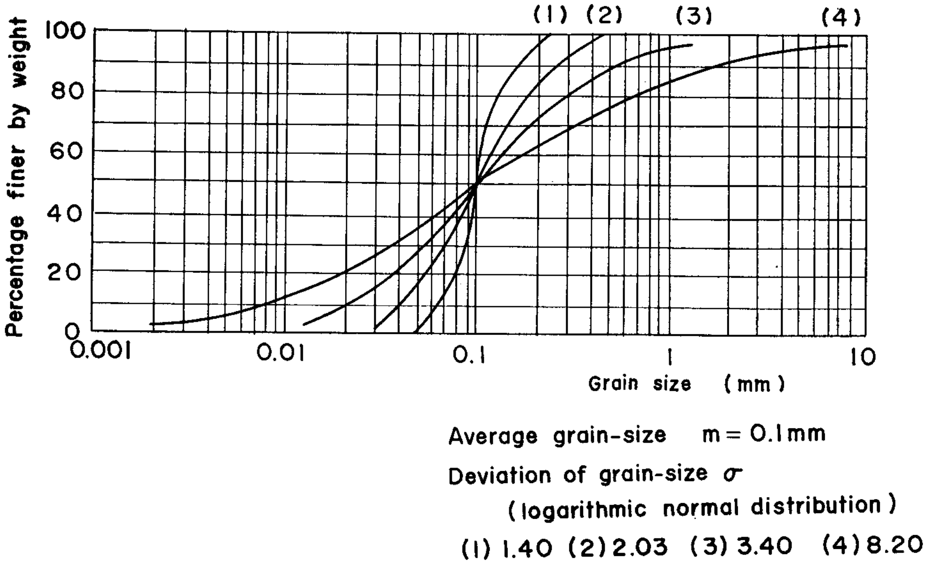


Fig. 10. Grain-size Distributions for 4 cases.

size $\sigma$ are 1.40, 2.03, 3.40 and 8.20 respectively. Table 2 shows the grain-size $a_1, a_2, \cdots, a_{40}$ $(a_1 > a_2 > \cdots > a_{40})$, which is the central value of grain-size of parts equally devided into 40 groups the percentage finer by weight of grain-size distribution and all the average grain-size is 0.1 mm.

(2) Relation of amount of wear and contact pressure for various distribution of grain-size

As mentioned before, the amount of wear of metal against solidified sandy soil can be calculated by Eq. (24), giving the depth of penetration of soil particle against metal plate. The depth of penetration $\alpha_i$ can be calculated as follows;

$$\alpha_i = (a_1 - a_{i+1})/2 \tan \theta/2 \qquad (i=1, 2, \cdots, 40) \quad \cdots\cdots\cdots\cdots\cdots (25)$$

Here, $a_1$ is a largest grain-size given in Table 2. And the contact pressure $p_1$ can be calculated by Eq. (21), summing up the products of the normal force $N_i$ acting on each soil particle by the number of the soil particle $n_i$ for the given depth of penetration $\alpha_i$. In such a way, the relation of amount of wear and contact pressure can be found through the depth of penetration of soil particle against metal plate $\alpha_i$. Now the relations, which are shown in Fig. 11, are calculated for the solidified sandy soil consisting of the 4 cases of grain-size distribution, through the parameter of hardness of metal (Vicker's Hardness), assuming that soil particle is not crushed under any load and the shape of soil particle is usual. $(\theta = 110$ degrees) From Fig. 11, the amount of wear of metal seems to decrease gradually with the increase of

Table 2. Grain-Size Distributions ($m$=0.1 mm)

| | $\sigma$=1.40 | $\sigma$=2.03 | $\sigma$=3.40 | $\sigma$=8.20 |
|---|---|---|---|---|
| $a_1$ | 0.21249 | 0.48842 | 1.55065 | 11.14150 |
| $a_2$ | 0.18201 | 0.35267 | 0.88314 | 4.23242 |
| $a_3$ | 0.16756 | 0.29631 | 0.65370 | 2.52314 |
| $a_4$ | 0.15783 | 0.26125 | 0.52579 | 1.73525 |
| $a_5$ | 0.15041 | 0.23607 | 0.44133 | 1.28411 |
| $a_6$ | 0.14439 | 0.21664 | 0.38043 | 0.99476 |
| $a_7$ | 0.13926 | 0.20073 | 0.33348 | 0.79313 |
| $a_8$ | 0.13478 | 0.18741 | 0.29610 | 0.64653 |
| $a_9$ | 0.13080 | 0.17594 | 0.26551 | 0.53600 |
| $a_{10}$ | 0.12719 | 0.16585 | 0.23975 | 0.44971 |
| $a_{11}$ | 0.12384 | 0.15684 | 0.21768 | 0.38093 |
| $a_{12}$ | 0.12076 | 0.14872 | 0.19858 | 0.32529 |
| $a_{13}$ | 0.11788 | 0.14136 | 0.18190 | 0.27972 |
| $a_{14}$ | 0.11515 | 0.13455 | 0.16703 | 0.24158 |
| $a_{15}$ | 0.11270 | 0.12860 | 0.15445 | 0.21114 |
| $a_{16}$ | 0.11010 | 0.12243 | 0.14187 | 0.18245 |
| $a_{17}$ | 0.10772 | 0.11694 | 0.13106 | 0.15921 |
| $a_{18}$ | 0.10543 | 0.11178 | 0.12122 | 0.13922 |
| $a_{19}$ | 0.10322 | 0.10689 | 0.11221 | 0.12190 |
| $a_{20}$ | 0.10106 | 0.10224 | 0.10390 | 0.10680 |
| $a_{21}$ | 0.09896 | 0.09781 | 0.09625 | 0.09364 |
| $a_{22}$ | 0.09689 | 0.09355 | 0.08912 | 0.08204 |
| $a_{23}$ | 0.09485 | 0.08946 | 0.08250 | 0.07183 |
| $a_{24}$ | 0.09283 | 0.08551 | 0.07630 | 0.06281 |
| $a_{25}$ | 0.09083 | 0.08168 | 0.07049 | 0.05481 |
| $a_{26}$ | 0.08873 | 0.07777 | 0.06475 | 0.04736 |
| $a_{27}$ | 0.08685 | 0.07432 | 0.05987 | 0.04139 |
| $a_{28}$ | 0.08483 | 0.07074 | 0.05498 | 0.03575 |
| $a_{29}$ | 0.08281 | 0.06724 | 0.05036 | 0.03074 |
| $a_{30}$ | 0.08074 | 0.06376 | 0.04594 | 0.02625 |
| $a_{31}$ | 0.07863 | 0.06030 | 0.04171 | 0.02224 |
| $a_{32}$ | 0.07645 | 0.05684 | 0.03766 | 0.01866 |
| $a_{33}$ | 0.07419 | 0.05336 | 0.03377 | 0.01547 |
| $a_{34}$ | 0.07181 | 0.04982 | 0.02999 | 0.01261 |
| $a_{35}$ | 0.06927 | 0.04616 | 0.02629 | 0.01005 |
| $a_{36}$ | 0.06649 | 0.04236 | 0.02266 | 0.00779 |
| $a_{37}$ | 0.06336 | 0.03828 | 0.01902 | 0.00576 |
| $a_{38}$ | 0.05968 | 0.03375 | 0.01530 | 0.00396 |
| $a_{39}$ | 0.05494 | 0.02836 | 0.01132 | 0.00236 |
| $a_{40}$ | 0.04706 | 0.02048 | 0.00645 | 0.00090 |

$\sigma$ : Deviation of grain-size (logarithmic normal distribution)
Unit: mm

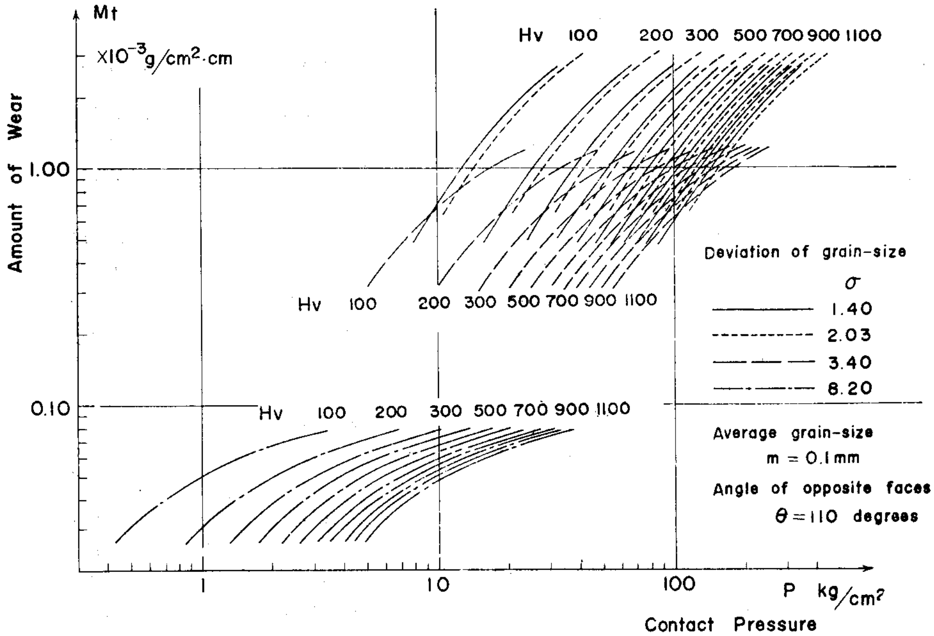


Fig. 11. Relations of Amount of Wear $M_t$ and Contact Pressure for 4 Distributions of Grain-size and Various Hardnesses of Metal, assuming that Soil Particles are not crushed.

deviation of grain-size. However, the fact differs from the actual phenomena.

That is, soil particles are actually crushed for high contact pressure and we cannot neglect this fact. Then the crushing of soil particle is discussed as follows; Rupture test of soil particle is practiced by using compressive test apparatus. Fig. 12 shows the results of rupture test of grain-size 0～10 mm for sand collected from Lake Biwa. Generally it is found that strength of rupture increases in proportion to 1.5 power of grain-size.

If the normal force acted on soil particle is larger than the strength of rupture, the soil particle is crushed and contact pressure is acted on the residual soil particles. In such a way, the first, second and third soil particles vanish one after another, as contact pressure increases and soil particles are crushed. Fig. 13 shows how to vary the relations of amount of wear and contact pressure due to crushing of soil particle. Number of zero shows that soil particles are not crushed and number of $i$ shows that all the soil particles of grain-size from $a_1$ to $a_i$ are crushed. On the process of calculation about the crushing of soil particle, only the soil particle of largest grain-size on the fresh surface of solidified sandy soil should be noticed respectively, because the normal force acted on the others decreases due to increase of the number of contact point.

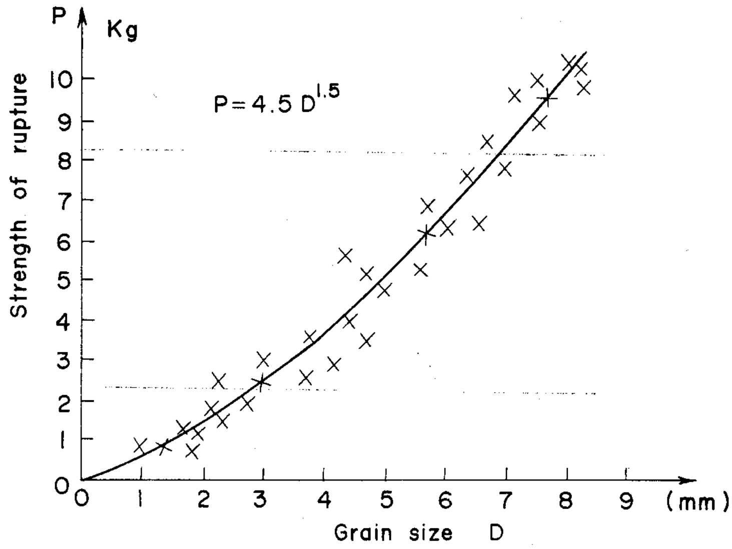


Fig. 12. Results of Rupture Test of Soil Particle.

When the soil particle of largest grain-size is crushed, next grain-size of particle should be checked for the following curves of relations of wear amount and contact pressure.

Therefore, considering the phenomena of crush of soil particle, the relations of amount of wear of metal and contact pressure are found by means of connecting the contact pressure at which each soil particle of largest grain-size is crushed one after another. Fig. 14 shows the results of calculation.

First, we consider the influences of Vicker's hardness of metal for a constant deviation of grain-size $\sigma$. As the Vicker's hardness of metal become high level, numbers of crushed soil particles increase. Because, for constant depth of penetration of soil particle into metal plate, contact pressure should be increased at high level of Vicker's hardness of metal. At the same time normal forces acted on soil particles become large and the soil particles are easily crushed. And it is found that the relations of amount of wear and contact pressure approach gradually the envelope shown in Fig. 13 with the increase of hardness of metal.

Secondly, we consider the influences of deviation of grain-size of solidified sandy soil for a constant Vicker's hardness of metal. As the deviation of grain-size $\sigma$ becomes large, numbers of crushed soil particles increase. Because, for a constant contact pressure, normal force acted on a soil particle increases rapidly due to decrease of the number of soil particles. And it is found that the relations of amount of wear and contact pressure approach gradually the envelope shown in Fig. 13 with the increase of deviation of grain-size.

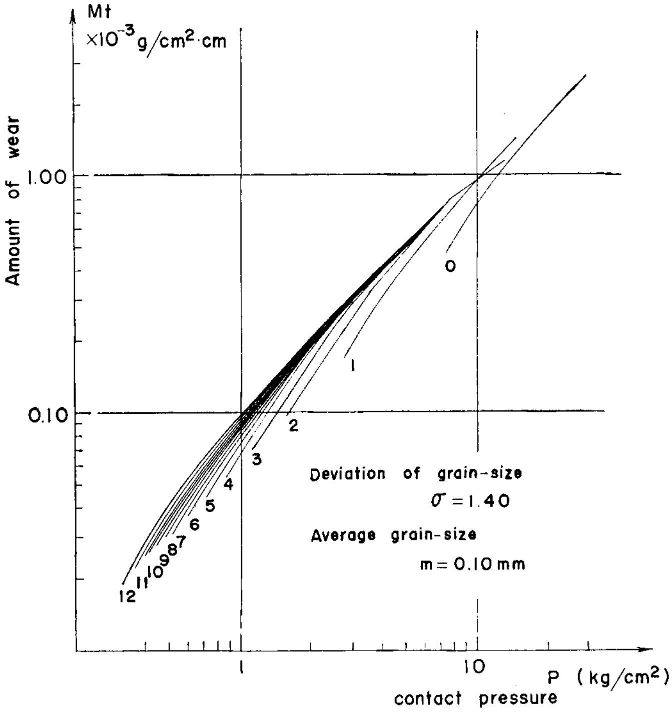


Fig. 13 (a)

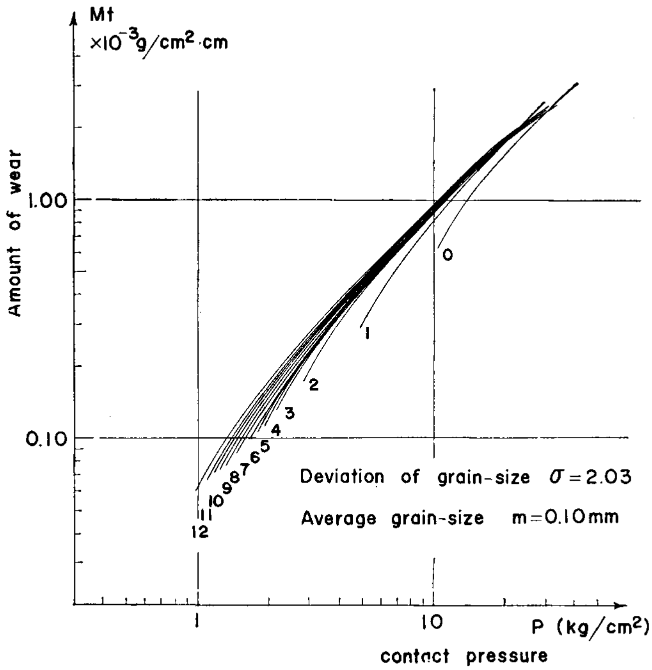


Fig. 13 (b)

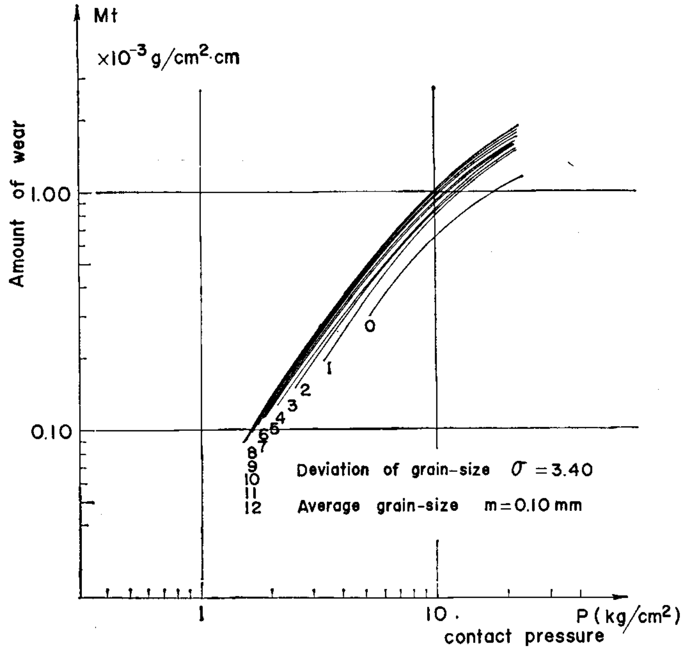


Fig. 13 (c)

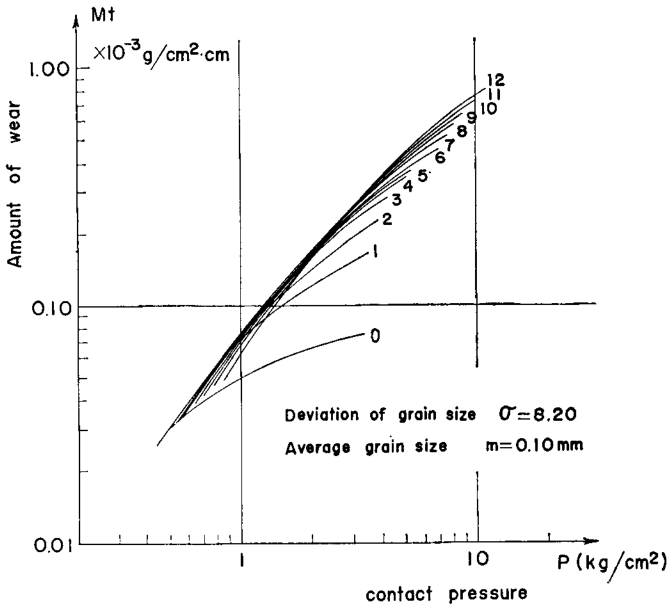


Fig. 13 (d)

Fig. 13. Relations of Amount of Wear and Contact Pressure, which are calculated theoretically by Eq. (24) assuming that Soil Particles are crushed one after another under high Contact Pressure. Number of Zero shows that Soil Particles are not crushed and Number of $i$ shows that all the Soil Particles of Grain-size from $a_1$ to $a_i$ are crushed. (Vicker's Hardness $H_v = 100$)

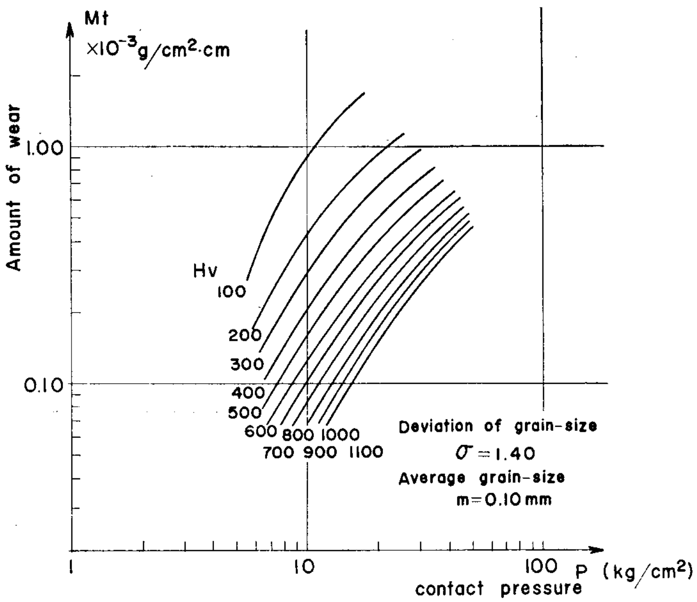


Fig. 14 (a)

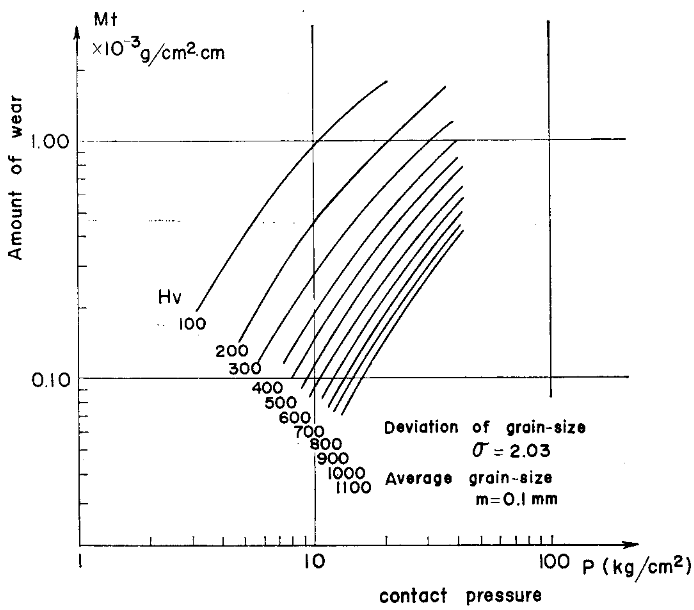


Fig. 14 (b)

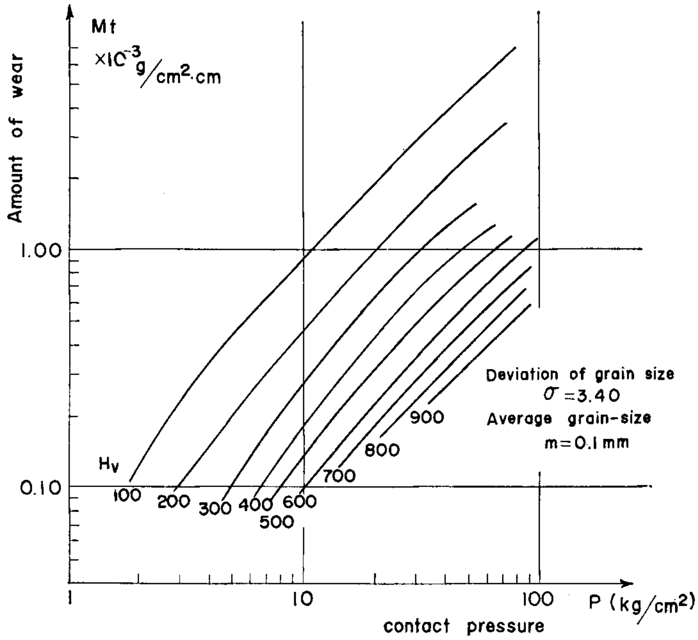


Fig. 14 (c)

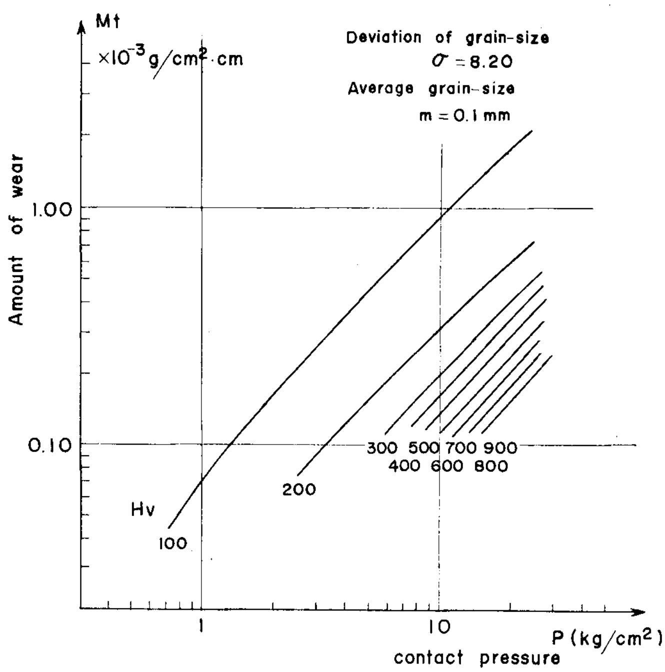


Fig. 14 (d)

Fig. 14. Relations of Amount of Wear and Contact Pressure for 4 Distributions of Grain-size and various Hardness of Steel Plate replotted from Fig. 13, assuming that Soil Particles are crushed

Consequently, the relations of amount of wear and contact pressure approach the envelope shown in Fig. 13 with the increase of deviation of grain-size $\sigma$ and Vicker's hardness of metal.

(3) Relation of amount of wear and hardness of metal

As mentioned above, the relations of amount of wear and contact pressure approach gradually the envelope shown in Fig. 13 with the increase of Vicker's hardness of metal. And the curves of envelope do not increase linearly with the increase of contact pressure as shown in Fig. 13. Therefore, the relation of amount of wear and hardness of metal is not constant for all the contact pressure, but is calculated for the given contact pressure. Fig. 15 shows the relation of amount of

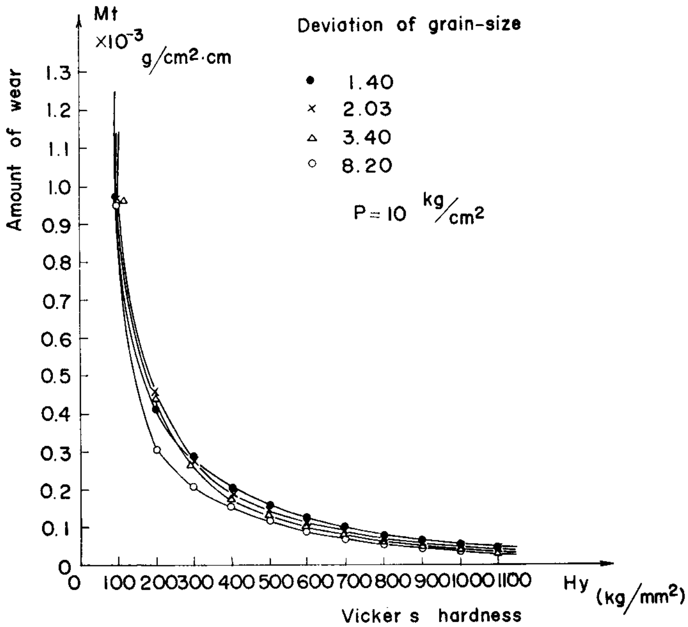


Fig. 15. Relations of Amount of wear and Vicker's hardness of Metal for constant contact pressure 10 kg/cm².

wear and hardness of metal for several deviations of grain-size at contact pressure 10 kg/cm². For all cases, the amount of wear decreases hyperbolically with the increase of the hardness of steel plate, and it is found that the amount of wear decreases considerably for greater hardness of metal over 600 in Vicker's hardness.

In accordance with those considerations, the amount of wear of metal can be calculated theoretically or read by Fig. 14, if the hardness of metal, contact pressure, grain-size distribution of soil particles and the shape of soil particles are given. However, the amount of wear of metal calculated theoretically should be corrected by several factors for actual amount of wear, for example, the influence of partial loading caused by packing with crushed soil particle, the vanishment of bonding-force of soil particles and the rate of isolation of metal fragment from streak, etc..

## 4. Experimental Investigation

(1) Apparatus and method of experiment

In order to exercise the friction and wear test for high contact pressure, the apparatus shown in Fig. 16 was constructed. The rotary steel table which has 1800 mm of diameter and 50 mm of thickness can revolve at the speed of friction from

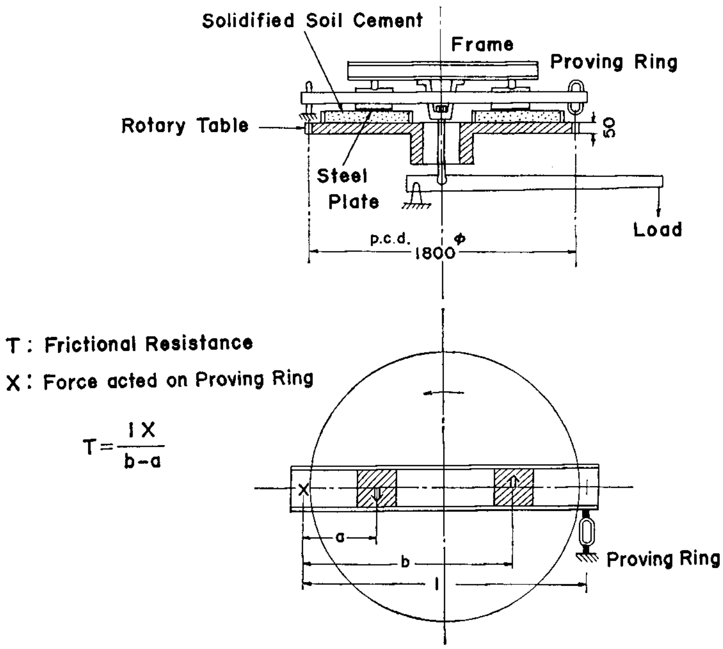


Fig. 16. Friction and Wear Testing Machine.

1.6 to 75 cm/sec by using stepless variable speed motor of 20 HP. On the table, soil samples are offered for testing. The samples were made by molding soil cement in circular mold in 70 mm thick as soil-cement (cement content 6%) in the state of optimum moisture content, and they were cured for seven days, after smoothing the surface. Test pieces of steel plate which have various hardness are put on the soil-cement and worn under various contact pressure. In order to measure the resistance of friction between solidified sandy soil and steel plate, a frame has been constructed on the table. As Fig. 16 shows, the frame is connected with hinge support at one edge and with proving ring at the other end, and the test plates are placed at equal distance from the center of the table, and are set to get the same contact pressure simultaneousely. Therefore, the resistance of friction between solidified sandy soil and steel plate is calculated by the arm length of the frame and the composed resistance measured by proving ring, assuming that the two resistances of friction are equal.

Now, the grain-size whose percentage finer by weight comes to 50% is supposed to be the average grain-size. The soil particles which have larger grain-size than the average is named coarse sandy soil, and in the same way the soil particles which have smaller grain-size than the average are named fine sandy soil. Both of these coarse and fine sandy soil are divided by the aid of sieve. For the convenience of later calculation, the centre value of grain-size divided into 40 parts of soil particles $a_1, a_2, \cdots, a_{40}$ are shown in Table 3. ($a_1 > a_2 > \cdots > a_{40}$). Vicker's hardnesses of steel

Table 3. Grain-Size Distributions of Soil Samples Deviation of grain-size $\sigma=1.55$ (for logarithmic normal distribution) Average grain-size $m=0.70$ mm

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Coarse Sand | | | | | | | |
| $a_1$ | 1.86950 | $a_2$ | 1.52752 | $a_3$ | 1.37118 | $a_4$ | 1.26835 |
| $a_5$ | 1.19125 | $a_6$ | 1.12956 | $a_7$ | 1.07750 | $a_8$ | 1.03105 |
| $a_9$ | 0.99304 | $a_{10}$ | 0.95740 | $a_{11}$ | 0.92486 | $a_{12}$ | 0.89494 |
| $a_{13}$ | 0.86724 | $a_{14}$ | 0.84117 | $a_{15}$ | 0.81790 | $a_{16}$ | 0.79340 |
| $a_{17}$ | 0.77120 | $a_{18}$ | 0.74993 | $a_{19}$ | 0.72947 | $a_{20}$ | 0.70965 |
| Fine Sand | | | | | | | |
| $a_{21}$ | 0.69048 | $a_{22}$ | 0.67171 | $a_{23}$ | 0.65338 | $a_{24}$ | 0.63537 |
| $a_{25}$ | 0.61760 | $a_{26}$ | 0.59910 | $a_{27}$ | 0.58252 | $a_{28}$ | 0.56501 |
| $a_{29}$ | 0.54752 | $a_{30}$ | 0.52981 | $a_{31}$ | 0.51180 | $a_{32}$ | 0.49343 |
| $a_{33}$ | 0.47453 | $a_{34}$ | 0.45475 | $a_{35}$ | 0.43380 | $a_{36}$ | 0.41133 |
| $a_{37}$ | 0.38633 | $a_{38}$ | 0.35735 | $a_{39}$ | 0.32078 | $a_{40}$ | 0.26210 |

(Unit: mm)

plates are divided into 4 groups; 175, 353, 543 and 897: chemical composision of steel is as follows;

$$\text{C: } 0.3\sim0.4\%, \quad \text{Si: } <0.3\%, \quad \text{Mn: } 0.6\sim0.8\%$$
$$\text{S: } <0.06\%, \quad \text{P: } <0.06\%,$$

(2) Relation between coefficient of friction and amount of wear

The amount of wear of steel plate is recognized that it does not vary practically within 10∼30 cm/sec of friction speed. This is also proved from the fact that the resistance of friction in the same range of friction speed does not vary. Therefore, in the latter experiment, friction speed of steel plate is kept constant as 10 cm/sec.

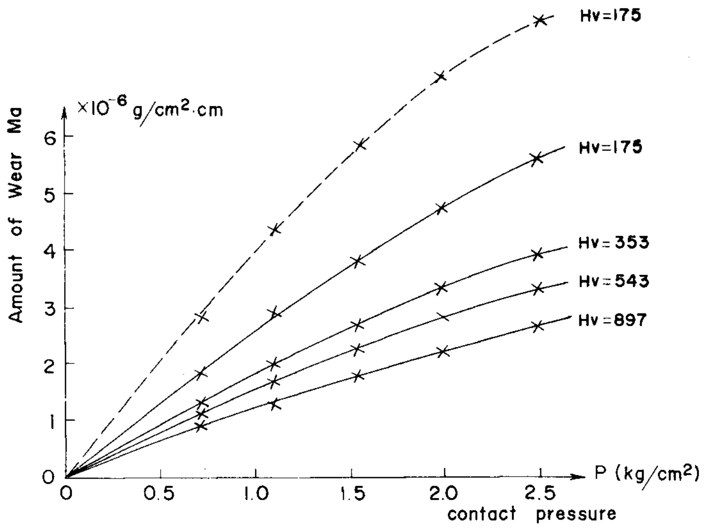


Fig. 17. Relations of amount of wear and contact pressure for various hardness of steel plate.

Relation between amount of wear and contact pressure for each test piece is shown in Fig. 17. The unit of amount of wear is represented as the amount of weight loss of metal by unit area and the movement of steel plate on unit distance, that is, $g/cm^2 \cdot cm$. Each plot is the average value of twelve measurements. In this case, the variations of measurement are $\pm 5\%$ in average. As Fig. 17 indicates, it is clear that in constant contact pressure the amount of wear decreases with the increase of hardness of steel plate, and the rate of increase of amount of wear decreases with the increase of contact pressure. That is, the following experimental equation is formed between the amount of wear and the contact pressure.

$$M_a = K_1 p^x \qquad \cdots\cdots\cdots\cdots\cdots(26)$$

Here, $x$ is about 0.84 and $K_1$ is varied by the coefficient of friction.

On the other hand, frictional resistance which is measured at the same time for the test of wear, as Fig. 18 shows, increases in direct proportion with increase of contact pressure. The relation can be indicated with linear equation. Also, frictional coefficient decreases a little with increase of hardness of steel plate. In any case, the reason why these straight lines swerve a little from the origin in Fig. 18 is considered at the movement of steel plate on the soil cement some degree of adhesive action between the steel plate and the packings by fine crushed soil particles on the solidified sandy soil influences on abrasive action by soil particles. In order to clear the influence of packings, the same test has been done with the condition that

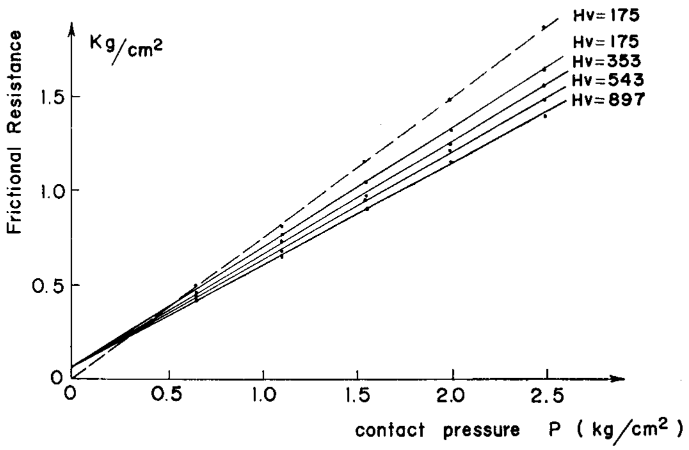


Fig. 18. Relations of frictional resistance and contact pressure for various hardness of steel plate.

crushed soil particles are excluded and steel plate moves on fresh soil samples at any time. The result is that, as shown in Fig. 17 and Fig. 18, with dotted lines frictional resistance relation is straight line through the origin and the coefficient of friction is bigger than that of the former. Amount of wear also increases with increasing of abrasive action, however, in this case the rate of increment of wear decreases also with increase of contact pressure.

Next, from these relations between coefficient of friction and amount of wear and contact pressure, the relations of coefficient of friction and amount of wear for constant contact pressure are shown in Fig. 19. Amount of wear $M_a$ increases parabolically with increasing coefficient of friction $\mu_a$ for every contact pressure. Amount of wear becomes zero at $\mu_a = \mu_0$ (in this case $\mu_0 = 0.25$). That is, as frictional resistance caused by abrasive action of soil particles is extinguished already for less coefficient of friction than $\mu_0$, wear is almost supposed not to occure. The experimental relation between amount of wear $M_a$ and coefficient of friction $\mu_a$ for constant contact pressure is shown as follows;

$$M_a = k_2 \, (\mu_a - \mu_0)^{3.1} \qquad \cdots\cdots\cdots\cdots\cdots (27)$$

Combining Eq. (26) and Eq. (27), next experimental equation is given between amount of wear, contact pressure and coefficient of friction.

$$M_a = k' \, (\mu_a - \mu_0)^{3.1} p^{0.84} \qquad \cdots\cdots\cdots\cdots\cdots (28)$$

Here, $k'$ is a constant, which is decided by experimental conditions.

(3) Relations between distribution of grain-size and friction, and wear

In order to investigate the variations of coefficient of friction and amount of

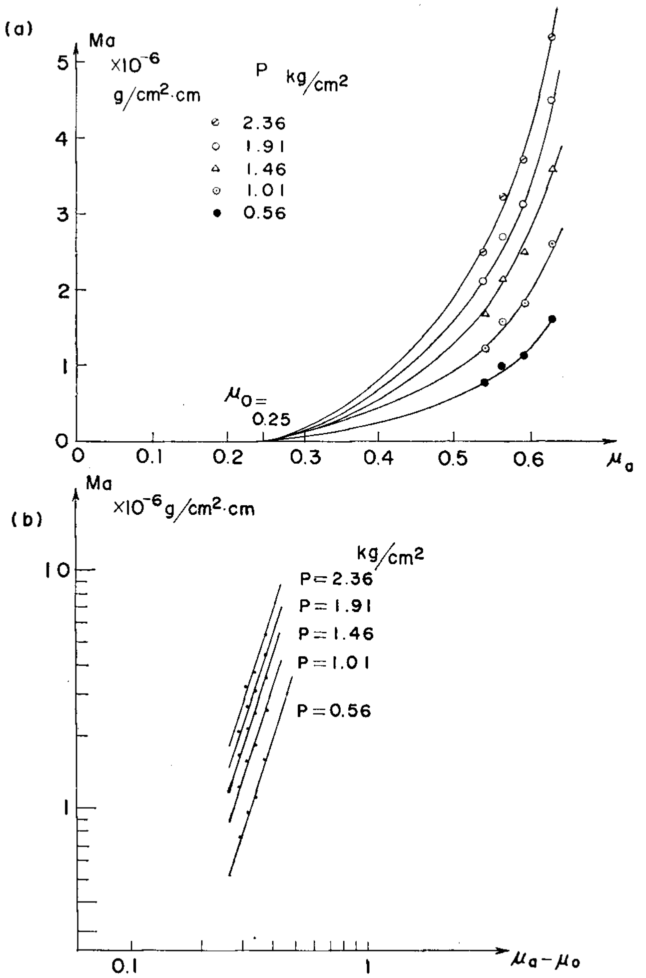


Fig. 19. Relations of amount of wear and coefficient of friction for constant contact pressure.

wear with differences of distribution of grain-size, test has been done on steel plate of Vicker's hardness 175 only. As Fig. 20 shows, the amount of wear increases in proportion to about 0.84 power of contact pressure; absolute amount of wear is larger in the coarse sandy soil. While, Fig. 21 is the relation between frictional resistance and contact pressure; both of them can be approximated by a straight line. Assuming their slope to be the coefficient of friction, that of coarse soil looks larger than fine soil. The reason why such difference occurs in general is considered to be that the rate of partial charge of load on the part of solidified soil pcrticles and on the part of packing ones by crushing should be different in coarse and fine sandy soil.

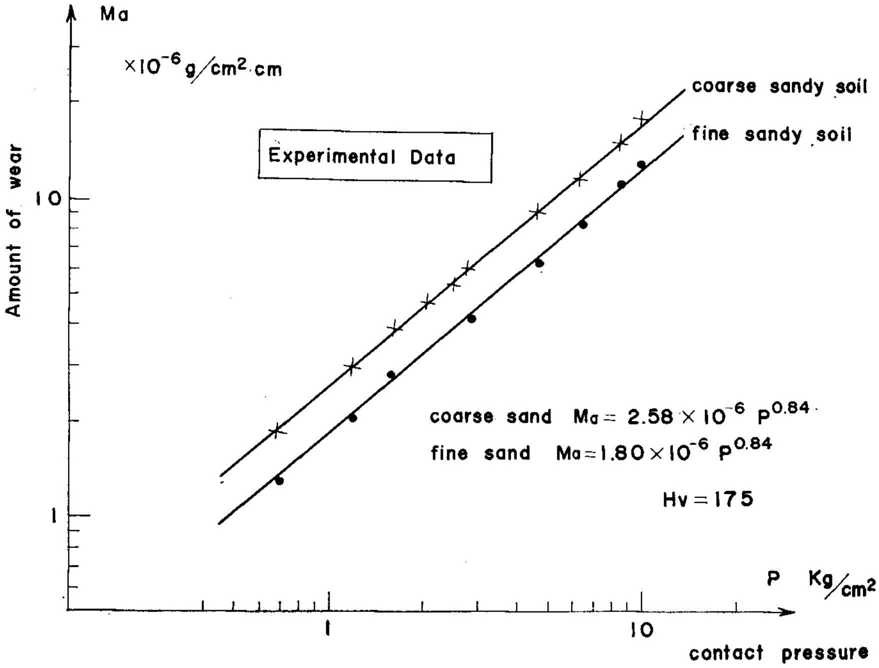


Fig. 20. Relations of amount of wear and contact pressure for coarse and fine sandy soil.

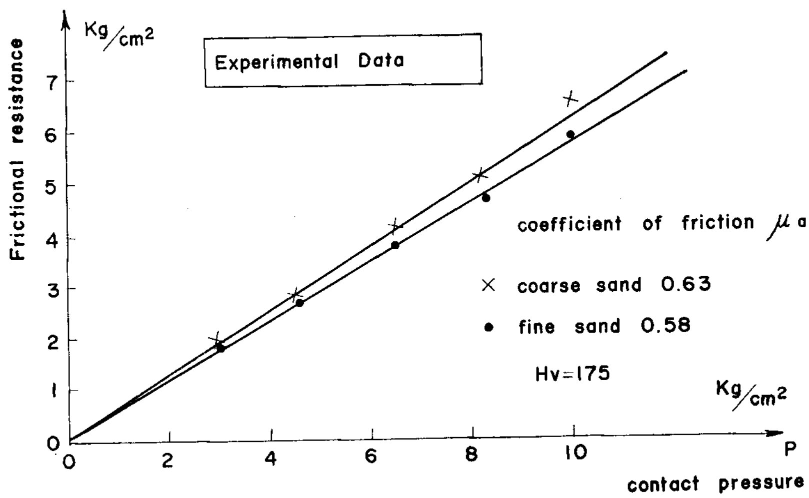


Fig. 21 Coefficient of Friction for coarse and fine sandy soil.

## 5. Results of Experiment and its Consideration

(1) Coefficient of friction between steel plate and solidified soil particles

The shape and opposite angle of soil particles are variously different from each other even if the distribution of grain-size of soil samples were given, and it is difficult to measure these values. Assuming that the shape of soil particle is regular quodrangler pyramid, average angle of opposite face of soil particles is equal to that of relevant soil particle, when $\mu'$ agrees with $\mu_t$: $\mu'$ is obtained from abrasive test of unit particle having various kinds of angle of opposite face and $\mu_t$ is the coefficient of friction between soil particles and steel plate tested without packings by crushed particles. That is, coefficient of friction $\mu_t$, which is tested without packing by crushed particles is 0.74 in Section 4(2). As Fig. 22 shows, compared

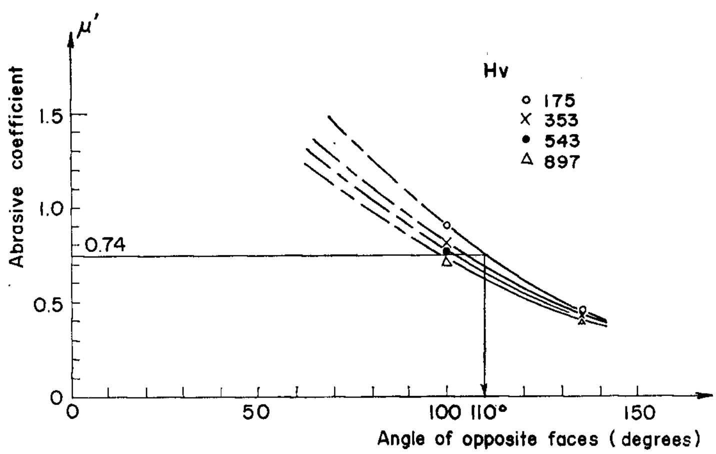


Fig. 22. Relation of Abrasive Coefficient $\mu'$ and Angle of Opposite Faces of Diamond Cone.

$\mu_t$ with coefficient $\mu'$ which is obtained from abrasive test exercised on the same metal by diamond cone with two kinds of angle of opposite face, average angle of opposite face of soil samples may be estimated at 112 degrees.

Therefore, assuming that packing by crushed particles does not occur, coefficient of friction between soil particles and other metals of different Vicker's hardness can also be calculated theoretically for average angle of opposite face formerly described. The results which are represented as $\mu_t$ are compared with actually tested coefficient of friction $\mu_a$ as in Table 4.

The difference between calculated value $\mu_t$ and actually tested value $\mu_a$ is caused by partial charge of load through the packing of crushed soil particles. That is, it is supposed that the contact pressure $p$ between solidified soil particles and

Table 4. Rate of Partial Charge of Load ($f$)

| $H_v$ | $\mu_t$ | $\mu_t-0.25$ | $\mu_a$ | $\mu_a-0.25$ | $f$ |
|---|---|---|---|---|---|
| 175 | 0.740 | 0.490 | 0.630 | 0.380 | 0.776 |
| 353 | 0.678 | 0.428 | 0.586 | 0.336 | 0.785 |
| 543 | 0.650 | 0.400 | 0.566 | 0.316 | 0.790 |
| 897 | 0.613 | 0.363 | 0.541 | 0.291 | 0.802 |

steel plate is divided into $fp$ and $(1-f)p$. $f$ is the rate of partial charge of load, $fp$ is supported by solidified soil particles and $(1-f)p$ by crushed soil particles. It is caused by crushing of soil particles under high contact pressure and piling among the solidified soil particles. Assuming that the coefficient of friction $\mu_a$ obtained from actual test consists of the calculated coefficient of friction $\mu_t$, which is tested without packing, and the coefficient of friction $\mu_0$ between crushed soil particles and steel plate, next equation is established.

$$\mu_a pA = f \cdot \mu_t pA + (1-f)\mu_0 pA$$

Here, A shows the area of steel plate. Therefore,

$$\mu_a = f\mu_t + (1-f)\mu_0 \qquad \cdots\cdots\cdots\cdots\cdots (29)$$

And, the rate of partial charge of load is as follows;

$$f = (\mu_a - \mu_0)/(\mu_t - \mu_0) \qquad \cdots\cdots\cdots\cdots\cdots (30)$$

Apparent coefficient of friction $\mu_a$ is decided by the rate of partial charge of load; but it is difficult to calculate theoretically the rate of partial charge of load. Therefore, $f$ cannot but be investigated indirectly from calculated and actually tested values. Moreover, the coefficient of friction $\mu_t$ which is decided by abrasive test without crushed particles varies with shape of soil particles, direction of their movement and hardness of metal plate etc.. As these factors are already considered in Eq. (24) mentioned before, the amount of wear of metal is not directly influenced by the coefficient of friction $\mu_t$.

(2) Amount of wear and the primary factors

Amount of wear of steel plate for solidified soil can be calculated from depth of penetration of soil particles into steel plate as described already in 2(2). Provided that it is calculated as $k=0.6$, $\theta=110°$ at Eq. (12), the results of calculation of contact pressure $p$ and amount of wear $M_t$ about the depth of penetration of soil particles into steel plate of various hardness $H_v$ on the coarse sandy soil are shown in Table 5. $M_t$ is the value as $K=1$, $\theta'=130°$ at Eq. (24) and is calculated, as a matter of course, by considering the crush of soil particles under high contact

Table 5. Relations of Amount of Wear and Contact Pressure for Coarse Sandy Soil (crush of soil particle is considered)

Contact pressure $p=\sum_i n_i N_i$ kg/cm²
Amount of wear $M$ g/cm²·cm

| $H_v=175$ | | $H_v=353$ | | $H_v=543$ | | $H_v=897$ | |
|---|---|---|---|---|---|---|---|
| $p$ | $M$ | $p$ | $M$ | $p$ | $M$ | $p$ | $M$ |
| 4.0 | $15.3\times10^{-5}$ | 4.9 | $8.4\times10^{-5}$ | 5.5 | $6.3\times10^{-5}$ | 6.6 | $4.5\times10^{-5}$ |
| 5.5 | 30.2 | 7.1 | 18.2 | 8.5 | 13.7 | 10.6 | 10.8 |
| 6.7 | 41.0 | 9.2 | 25.5 | 11.4 | 19.3 | 14.0 | 15.2 |
| 9.2 | 57.0 | 12.1 | 34.2 | 14.7 | 25.0 | 17.5 | 19.2 |
| 13.0 | 80.0 | 16.6 | 46.0 | 19.3 | 33.0 | 22.6 | 24.3 |

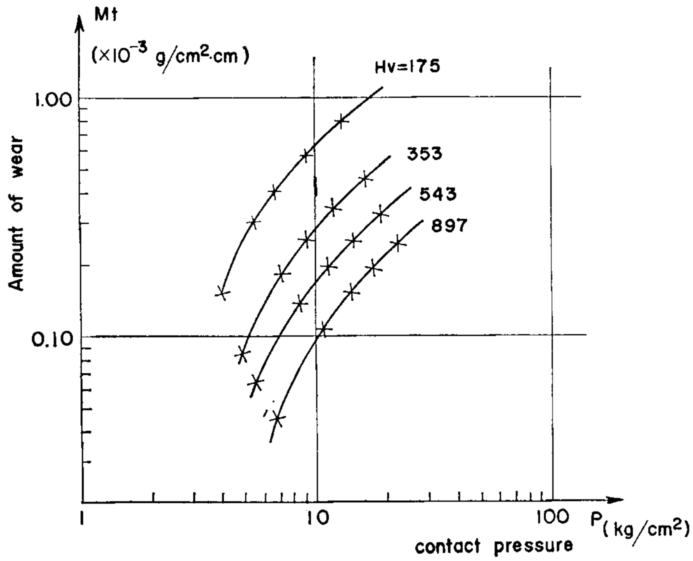


Fig. 23. Relations of Amount of Wear and Contact Pressure for 4 Test Pieces which are calculated theoretically considering the crush of soil particles.

pressure. As Fig. 23 shows, it is proved theoretically by the calculation that amount of wear of metal for solidified soil increases in proportion to about from 0.84 to 1.00 power of contact pressure. Absolute values of wear is different with calculated value from actually tested, and this difference is considered to be due to packing of crushed particles.

In other words, wear of metal for soil particles occurs from the part of contact pressure $fp$ with solidified soil particles, and not by the part of packing of crushed particles. As mentioned before, when amount of wear $M_t$ is assumed to be calculated without packing of crushed soil particles, next equation is established.

$$M_t = K_1 p^x \qquad \cdots\cdots\cdots\cdots\cdots(31)$$

$x$ is exponent of the value from 0.8 to 1.0. To calculate the amount of wear $M_t'$ considering the phenomena of packing, it is needed to substitute $fp$ for $p$ in above equation.

$$M_t' = K_1(fp)^x = f^x M_t \qquad \cdots\cdots\cdots\cdots\cdots(32)$$

Furthermore, actually tested amount of wear $M_a$ is not only influenced by the partial charge of load by packing, but by the distribution of streaks of wear given at the surface of steel plate which is different because of crush of soil particles or extinguishing of bond force of each soil particle during movement on the steel plate. Therefore, it is proper to consider that $M_a$ is equal to $M_t'$ multiplied by constant value $K_2$.

$$M_a = K_2 M_t' = K_2 f^x M_t \qquad \cdots\cdots\cdots\cdots\cdots(33)$$

For example, the relation between distribution of grain-size and amount of wear will be discussed. For constant hardness of metal and solidified sandy soil without crush of soil particles, it is considered that the coefficient of friction between the solidified sandy soil and steel plate does not vary for coarse and fine sandy soil. However, the fact that the coefficient of friction for coarse sand is actually larger than that of fine sand is caused by the difference of the rate of partial charge of load between them by the crushed soil particles.

The coefficients of friction in the calculation and the test and the rate of partial charge of load are shown in Table 6 for coarse and fine sand. About the

Table 6. Calculation of Rate of Partial Charge of Load for Coarse and Fine Sandy Soil

| Sample | Coefficient of friction | | Rate of partial charge of load $f$ (%) |
|---|---|---|---|
| | Calculated value | Tested value | |
| Coarse sand | 0.74 | 0.63 | 0.776 |
| Fine sand | 0.74 | 0.58 | 0.673 |

amount of wear, the results of calculation from Eq. 24 are shown in Fig. 24 for each distribution of grain-size.

for coarse sand
$$M_t = 9.20\times10^{-5}p^x$$
$$M_a = 2.58\times10^{-6}p^{0.84}$$

for fine sand
$$M_t = 8.80\times10^{-5}p^x$$
$$M_a = 1.80\times10^{-6}p^{0.84}$$

For Eq. (33), the coefficient of distribution of streaks is as follows;

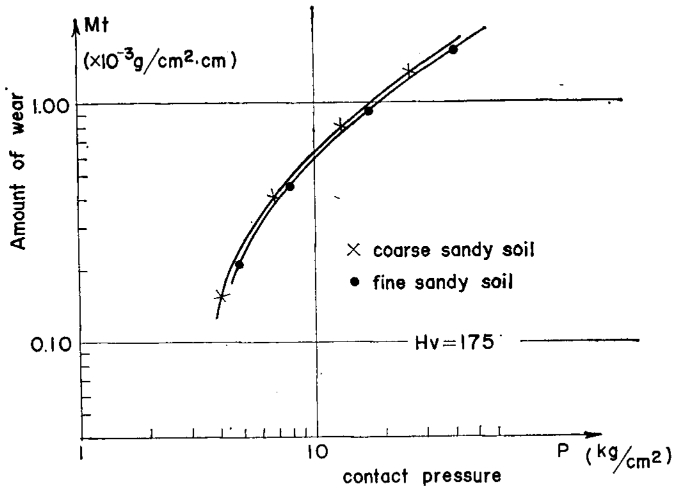


Fig. 24. Relations of Amount of Wear and Contact Pressure for Coarse and Fine Sandy Soil which are calculated theoretically considering the crush of soil particles.

for coarse sand $K_2 = 0.0347$ , for fine sand $K_2 = 0.0285$

The fact that the amount of wear varies with the distributions of grain-size is caused mainly by the differences of the rate of partial charge of load between them by the crush of soil particles.

## 6. Conclusion

The mechanism of the frictional resistance and the wear between steel plate and solidified sandy soil have been cleared and shown. The results are summarized as follows.

1) The relations between frictional resistance and contact pressure of steel plates and solidified soil have been exemplified from results of the scratch test which was done by a unit particle on the steel plate, assuming that the particle is not crushed when it moves on the plate.

It resulted that the frictional resistance increases linearly with increasing of the contact pressure even to high pressure, and this linear equation has been proved from the experiment in which the contact pressure was increased to 10 kg/cm$^2$. Actual frictional resistance is, however, less than that of the scratch test because of crushing of soil particles. That is, the frictional resistance consisted of the cutting resistance on metal by soil particles and the friction between crushed soil particle and metal. And the resistance varies with the rate of supported load on soil particles to total load.

2) The shape of soil particle is assumed to be an octahedron that is constructed with two symmetrical regular quodrangular pyramids, and all bases are assumed to be fixed on the same plane pararelled to the steel plate.

Furthermore, it is assumed that the grain-size of soil particle is presented with the size of the base, and when steel plate moves on the solidified soil particles, they are not crushed.

The depth of penetration of some soil particle to the steel plate $\alpha_i$ when it moves on the plate, is given as follows;

$$\alpha_i = \{k^2 N_i \sin(\theta/2)/4H_v \tan^2(\theta/2)\}^{1/2} \qquad \text{(i)}$$

Here, $H_v$ is the Vicker's hardness of the given steel plate, the suffix $i$ is named to each particle and $N_i$ is normal force acted on a soil particle of number $i$. $k$ shows the ratio of the penetration depth when soil particles scratch on the steel plate and when they penetrate at rest. And $\theta$ shows the average angle of opposite faces on a summit of a soil particle.

The amount of wear of metal $M$ is formulated in the equation by use of $\alpha_i$, considered that the scratch groove is scratched by larger soil particle.

$$M = K\rho \tan(\theta'/2)\left(n_1\alpha_1^2 + \frac{3}{8}\sum_{i=2} n_i\alpha_i^2 \cdot \frac{\alpha_i}{\alpha_1}\right) \qquad \text{(ii)}$$

Here, $\alpha_1$ is the depth of penetration of a soil particle of the largest grain-size, $\rho$ is the density of metal, $\theta'$ is average vertical angle of streaks. $K$ is a constant which is decided by the rate of partial charge of load, by the interruption of streaks with crushing of soil particles at moving state, by the extinction of bonding force of each soil particle and by the separation ratio of metal fragment from the streak etc.. Thus the relation between the contact pressure and the amount of wear of metal can be calculated from the depth of penetration of $\alpha_i$.

As a result, it is seen that the amount of wear of metal $M_t$ increases in proportion to about $0.84 \sim 1.00$ power of the contact pressure $p$, considering the phenomenon of crush of soil particles under high contact pressure. This relations are shown in Fig. 14 for various deviations of grain-size distribution of soil particles and various hardnesses of steel plates, and at the same time they have been proved theoretically for both cases.

3) In the meantime, it has been shown by several experiments that the actual amount of wear of metal $M_a$ increases in proportion to about 0.84 power of the contact pressure $p$. It is proved that this coefficient varies proportionally to 3.1 power of $(\mu_a - \mu_0)$, in which $\mu_a$ is actual coefficient of friction and $\mu_0$ is that between crushing soil particle and metal. The experimental equation is shown

as follows;

$$M_a = k'(\mu_a - \mu_0)^{3.1} p^{0.84} \quad \text{(iii)}$$

4) Considering that the amount of wear $M_a$ varies with the rate of partial charge of load $f$, the next equation is calculated from the theoretical amount of wear $M_t$ which is given from Eq. (ii), when $K$ is 1, $\theta' = 130$ degrees and is calculated as a matter of course, by considering the crush of soil particles under high contact pressure.

$$M_a = K_2 f^x M_t \quad \text{(iv)}$$

5) As the Vicker's hardness of metal decreases, the amount of wear of metal against solidified sandy soil increases hyperbolically.

As mentioned above, the coefficient of friction between solidified sandy soil and steel plate, and the amount of wear of metal are influenced considerably by crushing soil particles which fill spaces of solidified particles. Actual amount of wear of metal $M_a$ can be calculated from Eq. (i), (ii) and (iv), by means of the ratio $f$ which is given by the actual coefficient of friction. Or, it can be calculated directly from the experimental equation (iii) by using of the actual coefficient of friction $\mu_a$.

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