Interfacial Mixing in Two-Layered System of Fresh and Salt Waters

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This paper deals with the physical behaviours of the two-layered system of fresh and salt waters in the light of the mixing at the interface through experimentations. The result obtained in the study reveals that the fetch will be a significant characteristic in the physical process of the interfacial mixing.

1. Introductory Statement

In a two-layered system of fresh and salt waters, where the lower salt water is stationary and very deep in its depth, the upper flow is characterized in kinematics and dynamics by its densimetric Froude number. For smaller values than unity in this similitude parameter, a distinct interface will be observed and the interfacial mixing is a mechanical process resulting from the breaking of internal waves. The volumetric exchange from the lower layer to the upper layer is little. When the discontinuity of density becomes obscure, no evidence of the distinct interface will result. The entrainment theory[1] in jet flows will be a physical simulation in describing the dynamic process.

The present paper deals with the interfacial mixing of the two-layered system characterized by small densimetric Froude numbers in terms of the fetch imposed in the physical system. Because the underlying hydrodynamic behaviours will be analogous to those of wind-driven waves.

2. Salt Concentrations due to Mixing in Upper Layer

During experimentations for the two-layered system of fresh and salt waters, the observation may be made that the internal wave will break at and near the wave crest, the salt water is mostly flowing downstream without any mixing, whereas some will be intruded into the upper fresh water. The physical charac-

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teristics will be made use of three simplified models:

a. the upper fresh water layer, in which the flow velocity may be assumed to be independent of the depth,

b. the transitional layer where the salt concentration and thus flow velocity will change rapidly in their magnitudes, and

c. the lower stationary layer where the concentration will be constant. In the lower layer, very weak reverse current may be partly observed.

The use of such physical models gives the mathematical analysis a severe complexity, and furthermore the depth of transitional layer will be very thin. The two-layered system of the upper fresh water and the lower salt water may be approximated as an idealized model. Next problem arises from the actual determination of the interface between both layers. In the present study, the interface will be defined as the depth of 5% concentration of the lower salt water. The velocity in the upper layer will be then constant in the vertical direction.

The two-dimensional mixing process in the upper layer may be formulated by the following equation:

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=\frac{\partial}{\partial x}\left(K_x\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial c}{\partial y}\right) \tag{1}$$

in which u and v: velocity components in the directions of x- and y-axes, c: salt concentration, and K_x and K_y: longitudinal and lateral eddy diffusivities. Under the assumptions that the longitudinal mixing will be small and thus ignored and K_y may be in average signified by K as a reference, Eq. (1) will be simplified:

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial v}=K\frac{\partial^2 c}{\partial y^2} \tag{2}$$

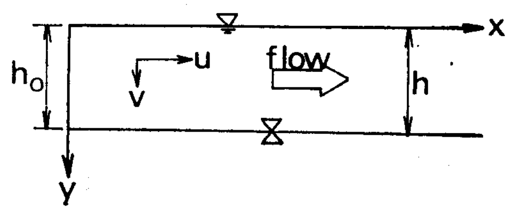

Fig. 1. Definition Sketch.

The convective motion of the salt water in the upper fresh water will be solved by introducing the velocity profiles of the upper flow. The motion of a mixture of fresh and salt waters classified as a two-component flow in fluid mechanics may be approximated by the flow of single component of fresh water, because of small concentrations in the upper layer. The gradient of stream lines is then given by

$$\frac{v}{u}=\frac{y}{h}\frac{dh}{dx} \tag{3}$$

The use of the following dimensionless parameters of

$$\xi=\frac{x}{h_0}, \quad \eta=\frac{y}{h}$$

makes (2) Eq. the following equation of one-dimensional heat conduction:

$$\frac{\partial c}{\partial \xi}=\frac{Kh_0}{uh^2}\frac{\partial^2 c}{\partial \eta^2} \tag{4}$$

where h and h_0 are references in length, the former being the upper layer depth at any section and the latter that at $x=0$. The solution of Eq. (4) will be treated under conditions that

1) at the upstream inflow section, there is no salt concentration in the upper layer, which is

$$c=0, \quad \text{at} \quad \xi=0 \tag{5}$$

2) no mass flux of salt will result at the free surface, that is

$$\frac{\partial c}{\partial \eta}=0, \quad \text{at} \quad \eta=0 \tag{6}$$

3) and the salt concentration at the interface is constant

$$c=c_0, \quad \text{at} \quad \eta=1 \tag{7}$$

where c_0 is 5% concentration of the lower salt layer.

The result obtained under three conditions of Eqs. (5), (6), and (7) is

$$\frac{c}{c_0}=1+\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{2n+1}\cos\left(\frac{2n+1}{2}\pi\eta\right)\exp\left[-\frac{(2n+1)^2}{4}\pi^2\int_0^{\xi}\frac{K}{q}\frac{h_0}{h}d\xi'\right] \tag{8}$$

where $q=uh$. The longitudinal mass flux of salt is defined by

$$Q_s=\int_0^h cu\,dy \tag{9}$$

and the dimensionless mass flux of salt becomes

$$\frac{Q_s}{c_0 q}=1-\frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\exp\left[-\frac{(2n+1)^2}{4}\pi^2\int_0^{\xi}\frac{K}{q}\frac{h_0}{h}d\xi'\right] \tag{10}$$

The rate of salt volume intruded through a unit length of interface, F, is

$$F=\frac{dQ_s}{dx}=c_0u\left[\frac{2K}{q}\sum_{n=0}^{\infty}\exp\left(-\frac{(2n+1)^2}{4}\pi^2\int_0^{\xi}\frac{K}{q}\frac{h_0}{h}d\xi'\right)\right] \tag{11}$$

Fig. 2 is a diagram showing Q_s/c_0q and F/c_0u for $\xi=x/h_0$ in terms of various values of K/q.

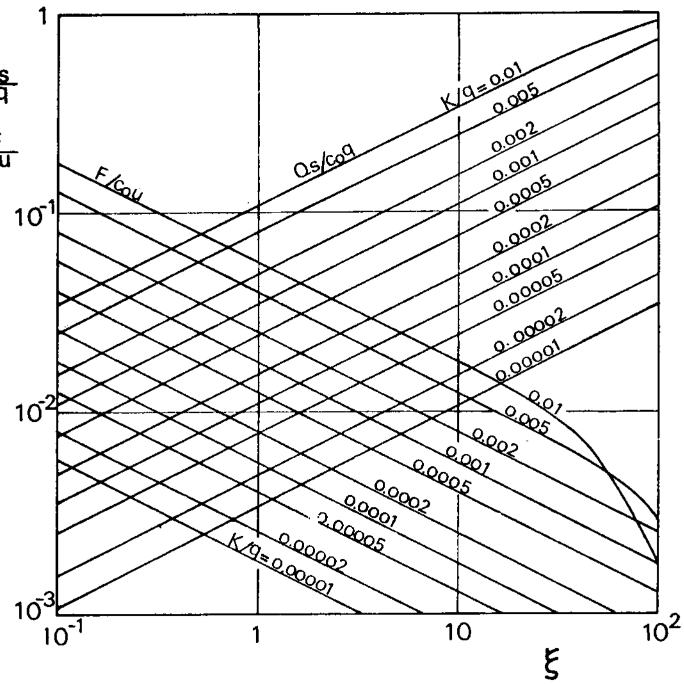

Fig. 2. Relation of Q_s/c_0q and F/c_0u to ξ in Terms of K/q.

3. Experimental Verifications

The experimental work has been done at the Hydraulics Laboratory, Department of Civil Enginnering. The test flume used is of rectangular shape and 0.25 m in width, 0.35 m for the central portion and 0.18 m for both upstream and downstream portions in depth, and 11.00 m in length. The total fetch of the interface is 5.50 m, as seen in Fig. 3. The density of the salt water used in the test runs is 1.02 gr/cm^3. The salt water was steadily supplied from a storage tank to hold the constant interface. The salt concentration was estimated through the measurement of electric conductivities. The samples were simultaneously extracted at six sections of $x=0.05$, 0.10, 0.20, 0.50, 1.00 and 2.00 m along the center line of the flume.

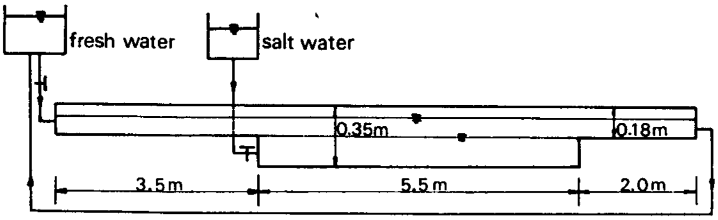

Fig. 3. Experimental Set-up.

(1) Stability of two-layered system

Keulegan[2] obtained the stability criterion for the internal wave in a form of

$$\begin{aligned} \theta &= 0.127 \quad \text{for} \quad R_{e_1} < 450 \\ \theta &= 0.178 \quad \text{for} \quad R_{e_1} > 450 \end{aligned} \tag{12}$$

in which

$$\theta = \frac{(\varepsilon g \nu_2)^{1/3}}{u} \tag{13}$$

$$R_{e_1} = \frac{u_1 h_1}{\nu_1} \tag{14}$$

$$\varepsilon = \frac{\rho_2 - \rho_1}{\rho_2} \tag{15}$$

and the subscripts of 1 and 2 indicate values of the upper and lower layers, respectively. Obviously, θ is known as the Keulegan number indicating the stability of a two-layered system.

Observations of the characteristics of internal waves by eyes are indicated in terms of the Keulegan number and the Reynolds number in Fig. 4. The ex-

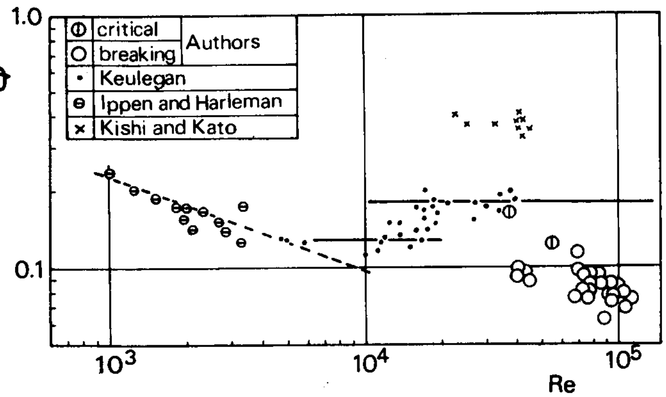

Fig. 4. Relation between Keulegan Numbers and Reynolds Numbers.

perimental results of Ippen-Harleman[3] and Kishi-Kato[4] are also plotted in the same figure. The solid line is the Keulegan criterion for the stability of internal waves and the broken line that of Ippen-Harleman. The results obtained in the present study will support the Keulegan criterion.

(2) Intrusion velocity into fresh water layer

The vertical intrusion velocity of the salt water into the fresh water across a unit area of the interface is

$$V = \frac{1}{c_2}\int_0^h cu\,dy \tag{16}$$

An empirical formula of Keulegan is

$$V = C(u - 1.15\,u_c) \tag{17}$$

where u_c is the critical velocity of u for the stable interface. C is an empirical constant and estimated as 3.5×10^{-4}. Fig. 5 described the vertical intrusion velocity V obtained by the increase of the mass flux of salt water between $x=1.00$ m and 1.00 m as a function of the horizontal velocity in the upper layer. Numbers in the figure express the Richardson number of

$$R_i{}^* = \frac{\varepsilon gh}{u^2} \tag{18}$$

which is inversely proportional to the densimetric Froude number. The critical

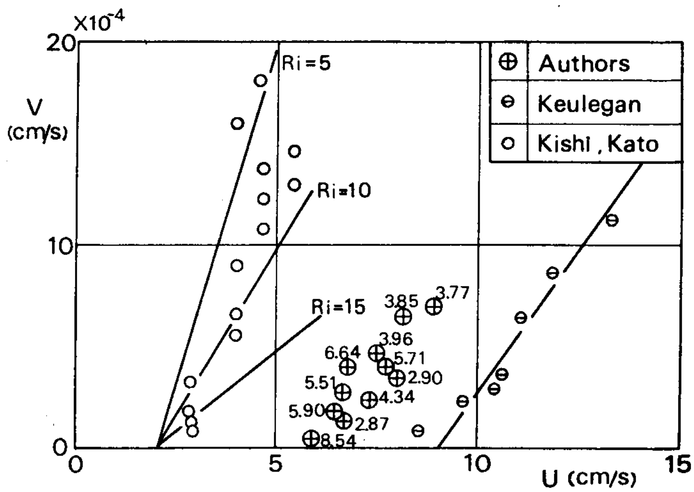

Fig. 5. Vertical Intrusion Velocities and Upper Undisturbed Velocities in Fresh Water Layer.

velocity u_c in the present experiment evaluated by the Keulegan method is 4.57 cm/sec and will be a little small value compared with that estimated by $V=0$ in Fig. 5. Nevertheless, the discrepancy between these two values will be admissible. C estimated by this xperiment is of same order of that of Keulegan and will be decreased with the increase of $R_i{}^*$. It is then concluded that the expression of Eq. (17) will be plausible and practical at the present state where the detailed mechanism of mixing has not been disclosed. However, the interfacial mixing will result from the breaking of internal waves and must be closely related to the growth of internal waves. There will still remain unsolved problems in the simulated expression of Eq. (17).

(3) Vertical distribution of concentration

Fig. 6 is plots of the salt concentration measured at various locations along the channel course. Numerical calculations have also made and plotted in the same figure. In this figure,

$$\alpha = \int_0^\xi \frac{K}{q} \frac{h_0}{h} d\xi \tag{19}$$

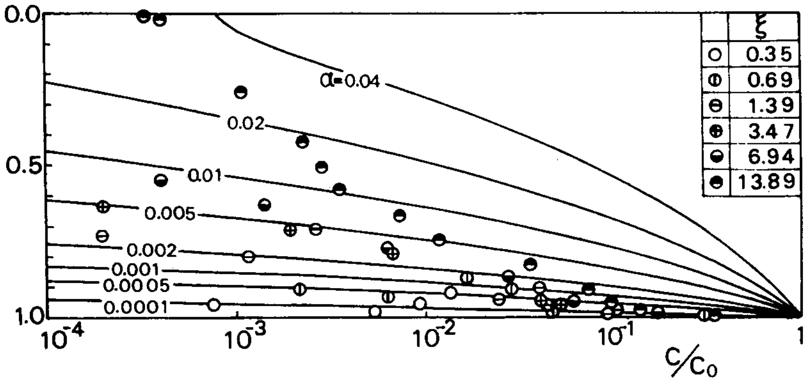

Fig. 6. Salt Concentration in Upper Fresh Water Layer.

In the neighbourhood of the interface, use may be made of Eq. (11) as a characteristic for the vertical distribution of concentration, whereas Eq. (11) will fail in other zones, because of the underlying assumption of the constant eddy diffusion coefficient. The estimation of the coefficients in this model by the use of Fig. 6 will be difficult. Nevertheless, c/c_0 is very small near the free surface and thus the estimation of the eddy diffusion coefficient as a measure of mean mass flux of salt integrated will be effective.

(4) Mass flux of salt

The plots of the dimensionless mass flux of salt, Q_s/c_0q, and the dimensionless fetch, $\xi=x/h_0$, experimentally obtained are shown in Fig. 7. If Q_s/c_0q is assumed to be proportional to ξ^{σ}, the vertical intrusion velocity of salt is also proportional to $\xi^{\sigma-1}$. A glance at Fig. 7 will show that σ is less than unity and therefore the vertical intrusion velocity will decrease with the increase of the fetch.

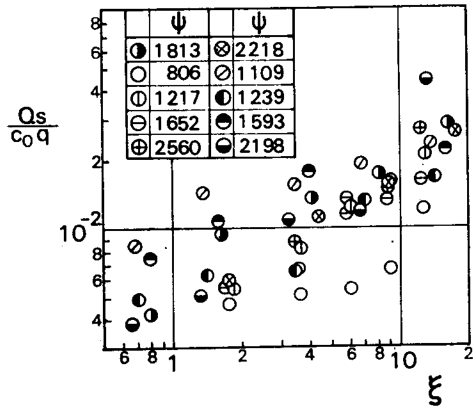

Fig. 7. Relation between Q_s/c_0q and α.

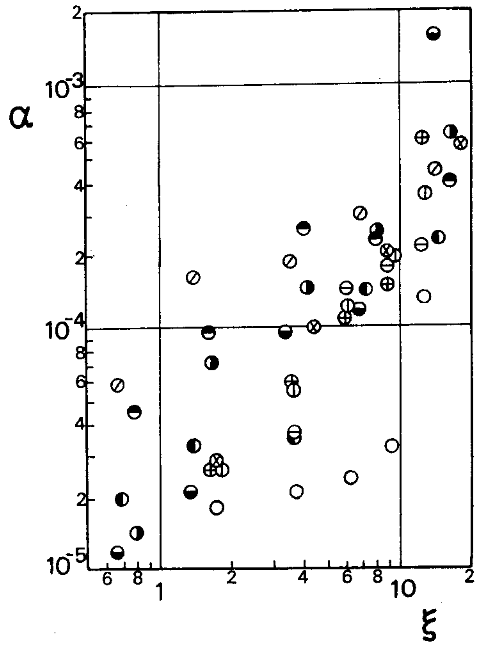

Fig. 8. Change of α with Increase of ξ.

The estimation of a through the measurement of Q_s/c_0q is shown in Fig. 8. In this experiment, dh/dx is of order of 10^{-3}, h_0/h is nearly equal to unity for $\xi<10^2$, and then α will become proportional to ξ under the assumption of constant K in Eq. (19). In this case, $\alpha\propto\xi^m$ and $K\sim\xi^{m-1}$. Fig. 9 describes a general trend between m

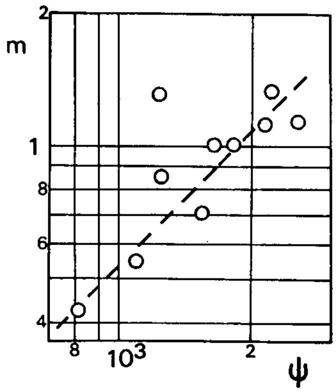

Fig. 9. m and ψ.

and $\psi(=\theta^{-3})$. In response to the increase of ψ and that the interface will become gradually unstable, $m-1$ will change its sign from negative to positive. This figure will predict that the change of the eddy diffusion coefficient with the increase of the fetch will be closely associated with the stability of internal waves and ψ will be also a characteristic parameter in the interfacial mixing between two layers.

4. Closing Remarks

The paper described herein is a progress report in the research program for the interfacial mixing between the two-layered system of fresh and salt waters. Careful observation reveals that the mixing will be largely influenced by the fetch of the two-layered interface and the Keulegan number will be a main parameter in this process. Further research must be directed to the physical growth of internal waves which will play a vital role in the interfacial mixing.

References

1) Ellison, T.H. and J.S. Turner: Jour. Fluid Mech., Vol. 6, (1959)
2) Keulegan, G.H.: Research Paper RP 2040, National Bureau of Standards, U.S. Government, Vol. 43, (1949)
3) Ippen, A.T. and D.R.F. Harleman: Gravity Wave Symposium, Circulation 521, National Bureau of Standards, U.S. Government, (1951)
4) Kishi, T. and M. Kato: Proc. 14th Coastal Engineering Conference in Japan, (1967)