Traffic Assignments in Road Network with Flow Dependent Travel Times

By

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In this paper, first, the author will comment on the principles of traffic assignment used generally so far, that is, the principle of minimizing total travel times, of equal travel times, and of travel times ratio. Secondarily, new assignment methods will be proposed respectively according to those principles, on the assumption that the travel times along any road section depends linearly upon the traffic volume at that section. But in this second point, the discussion is confined to computational procedure of those system solutions.

1. Introduction

Traffic assignment is the process by which journey, the origins and destinations of which are known, are distributed to the given whole network. It has been found that, of many factors affecting driver's choice of route, journey time is the most important and a large number of assignments have been carried out on this basis. Recently, attention has been given to methods of assigning traffic to a network in which the journey time along each road section of network increases as the flow on the road section increases. So a systems solution is needed.

The principles of traffic assignment used generally are classified into the following three types.

1. The principle of minimizing total travel time[1][2]
2. The principle of equal travel times[3][4][5][6]
3. The principle of travel times ratio[7][8]

In this paper the author will set forth systems solutions respectibly as regards the three principles above, on the assumption that the travel time along any road section depends linearly upon the traffic volume at that section.

Of these, the first principle is formulated in the way of Quadratic Programming (Q. P.) from the aspect of the transportation problem of multi-commodity in network.

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Since the object of this principle is maximization of the road network efficiency, so travellers are obliged to behave in such a way that they consider the overall situation and not just what suits themselves. Therefore, this solution has the disadvantage that it may result in a few drivers being very seriously delayed. And so the situation caused by this principle seems unlikely to be achieved in practice, except by enforcement and automatic diversion signs. But this principle would supply useful information for operating traffic signals and other traffic control devices, for setting tolls, for recommending routes and other measures of influence traffic distribution.

The second, the principle of equal travel times states that times are identical on all routes used between two zones and less than (equal to) the travel times on all unused routes. Therefore, the situation achieved by this principle seemes to be desirable for travellers. However, from the fact of imperfect information, drivers can have only little knowledge about the traffic condition over the routs to be chosen, or on account of being unable to compare the routes simultaneously, they cannot judge which one is the most favourable i.e. the quickest. And for mathematical reason, the restriction with respect to the choice of routes should be severely imposed, so that the principle of equal travel times may be satisfied. From this, it is probably impossible to accept that such behaviour corresponds to what actually happens. But the flow pattern may become close to such a situation, provided that the technique of the information deliverly makes still more progress, or traffic demands remain constant even in future.

Consequently, we cannot say that these two principles reflect practical traffic behavior, from the fact that drivers choose their route arbitrarily and variously.

Meanwhile, comparing the routes of a OD pair according to traffic studies observed so for, it has been made clear that traffic is distributed to each routes in proportion to an inverse n-th power ratio of its travel times. For there exist a great number of travellers who absolutely wish to arrive as fast as possible. Hence, we can say that the principle of travel times ratio is the most practical one.

As mentioned above, the characteristics of each principle has been discussed.

Now, in the process of assignment calculation, we can have three ways as to how the variables are dealt with.

The first, it is the way using path flow, of which assignment is performed by taking traffic demands over the routes between each OD pair as variables[9]. The second is the way using arc (link) flow, which is the traffic demands of each OD pair along each road section[10]. Therefore in this method, we need continuity condition that the inflow traffic vloume is equal to the outflow traffic volume at each turning point. The third is the way treating the traffic demands on each road section as variables, which is the very result of this problem[11].

In using path flow or arc flow, the traffic demands on each road section are obtained by summing up these flows of all OD. From the view of the number of variables treated in computational procedure, the third method is the most desirable. And, follows the arc flow method and path flow method.

In this paper, the principle of minimizing total travel time is discussed by way of arc flow, and the other two principles are debated by way of path flow. But this time, the author does not intend to discuss the way of taking traffic demand itself on each road section as variable.

2. Assignment on the basis of the principle of minimizing total travel time[12]

Let us consider the network with n nodes, m directed arcs, and q pairs of origin and destination. Let y_{ij}^k denote the flow value of k th OD along arc ij from node i to node j. The conditions that every y_{ij}^k is required to satisfy are as follows.

The flow value of each OD along each arc must be non-negative,

$$y_{ij}^k \geqq 0 \qquad \begin{pmatrix} k=1, 2, \dots\dots, q \\ ij=1, 2, \dots\dots, m \end{pmatrix} \tag{1}$$

The node conservation requirements are

$$\sum_j (y_{ij}^k - y_{ji}^k) = \begin{cases} S^k & (i=\text{origin node}) \\ -S^k & (i=\text{destination node}) \\ 0 & (i=\text{otherwise}) \end{cases} \tag{2}$$

$$(i=1, 2, \dots\dots, n-1 : \quad k=1, 2, \dots\dots, q)$$

This means that at each node, input flow is equal to output flow for each OD.

And the capacity constraints that must be also satisfied along each arc, are written as

$$\sum_{k=1}^{q} y_{ij}^k \leqq C_{ij} \qquad (ij=1, 2, \dots\dots, m) \tag{3}$$

where, C_{ij} is the capacity of arc ij.

Here let us consider such column vectors and the matrix shown below.

$$\boldsymbol{y}_k = [y_1^k, y_2^k, \dots\dots, y_{ij}^k, \dots\dots, y_m^k]' \tag{4}$$

$(k=1, 2, \dots\dots, q)$ m dimensional column vector

$$\boldsymbol{d}_k = [0, \dots\dots, S^k, 0, \dots\dots, -S^k, 0, \dots\dots, 0]' \tag{5}$$

Origin node (at S^k) Destination node (at $-S^k$)

$(k=1, 2, \dots\dots, q)$ $(n-1)$ dimensional column vector

$$\boldsymbol{C} = [C_1, C_2, \dots\dots, C_{ij}, \dots\dots, C_m]' \tag{6}$$

m dimensional column vector

where, ′ mark expresses transposed form.

$$\boldsymbol{B}=[\boldsymbol{B}_1, \boldsymbol{B}_2, \ldots\ldots, \boldsymbol{B}_m] \tag{7}$$

is the incidence matrix of the network with $(n-1)$ rows and m columns, and $\boldsymbol{B}_1$, $\boldsymbol{B}_2$, ... , $\boldsymbol{B}_m$ are $(n-1)$ dimensional column vectors which compose the incidence matrix $\boldsymbol{B}$. Furthermore, let us introduce the following column vector. Where, $\boldsymbol{s}$ is slack.

$$\boldsymbol{y}^*=[\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots\ldots, \boldsymbol{y}_q, \boldsymbol{s}]' \tag{8}$$

$(q+1)m$ dimensional column vector

$$\boldsymbol{d}=[\boldsymbol{C}, \boldsymbol{d}_1, \boldsymbol{d}_2, \ldots\ldots, \boldsymbol{d}_q]' \tag{9}$$

$\{m(n-1)\}$ dimensional column vector

Then, eq. (1) and eq. (2) (3) are reduced as

$$\boldsymbol{y}^* \geqq 0 \tag{10}$$

$$\begin{pmatrix} \boldsymbol{I} & \boldsymbol{I} & \cdots & \boldsymbol{I} & \boldsymbol{I} \\ \boldsymbol{B} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ & \boldsymbol{B} & \ddots & & \vdots \\ \boldsymbol{0} & & & \boldsymbol{B} & \boldsymbol{0} \end{pmatrix} \boldsymbol{y}^*=\boldsymbol{d} \tag{11}$$

Now we have assumed the relation between the travel time T_{ij} and the flow X_{ij} along arc ij shown as

$$T_{ij}=a_{ij}X_{ij}+b_{ij} \tag{12}$$

where a_{ij} and b_{ij} are positive constants peculiar to arc ij. And, as X_{ij} is expressed by

$$X_{ij}=\sum_{k=1}^{q} y_{ij}^k \tag{13}$$

So, the total travel time T over the whole network is written as

$$\begin{aligned} T&=\sum_{ij=1}^{m} T_{ij}X_{ij} \\ &=\sum_{ij=1}^{m}\left\{\left(a_{ij}\sum_{k=1}^{q} y_{ij}^k+b_{ij}\right)\sum_{k=1}^{q} y_{ij}^k\right\} \end{aligned} \tag{14}$$

In order to represent this objective function by matrices, let us introduce such following vectors and matrix as

$$\boldsymbol{y}=[y_1^1, y_1^2, \ldots\ldots, y_1^q, \ldots\ldots, y_{ij}^k, \ldots\ldots, y_1^m, y_2^m, \ldots\ldots, y_m^q, s_1, s_2, \ldots\ldots, s_m]' \tag{15}$$

$(q+1)m$ dimensional column matrix

$$\boldsymbol{b}=[-b_1, \ldots\ldots, -b_1, \ldots\ldots, -b_{ij}, \ldots\ldots, -b_m, \ldots\ldots, -b_m, 0, \ldots\ldots, 0] \tag{16}$$

$(q+1)m$ dimensional row vector

$$
\mathbf{A} = \begin{bmatrix}
\begin{array}{|c|}\hline 2a_1 \cdots\cdots 2a_1 \\ \cdots\cdots\cdots \\ 2a_1 \cdots\cdots 2a_1 \\ \hline \end{array} & & & & \\
& \ddots & & \mathbf{O} & \\
& & \begin{array}{|c|}\hline 2a_{ij} \cdots\cdots 2a_{ij} \\ \cdots\cdots\cdots \\ 2a_{ij} \cdots\cdots 2a_{ij} \\ \hline \end{array} & & \\
& & & \ddots & \\
& \mathbf{O} & & \begin{array}{|c|}\hline 2a_m \cdots\cdots 2a_m \\ \cdots\cdots\cdots \\ 2a_m \cdots\cdots 2a_m \\ \hline \end{array} & \\
& & & & \begin{array}{|c|}\hline 0 \cdots\cdots 0 \\ \cdots\cdots\cdots \\ 0 \cdots\cdots 0 \\ \hline \end{array}
\end{bmatrix} \tag{17}
$$

$(q+1)m$ dimensional square matrix

By using the above vectors and matrix, the objective function can be written as follows.

$$F(\boldsymbol{y}) = -T = \boldsymbol{by} - \frac{1}{2}\boldsymbol{y}'\boldsymbol{Ay} \tag{18}$$

Here, the order of element of $\boldsymbol{y}^*$ in eq. (10) and (11) are different from that of $\boldsymbol{y}$ in eq. (18), so let them be rearranged like $\boldsymbol{y}$. Then eq. (10) (11) are shown as belows.

$$\left.\begin{aligned} \boldsymbol{y} &\geqq \mathbf{0} \\ \boldsymbol{Qy} &= \boldsymbol{d} \end{aligned}\right\} \tag{19}$$

where

$$
\mathbf{Q} = \left[\begin{array}{ccc|ccc|c|ccc|c|ccc|ccc}
 & \mathbf{1}_1 & & & \mathbf{1}_2 & & \cdots & & \mathbf{1}_{ij} & & \cdots & & \mathbf{1}_m & & & \mathbf{I} & \\ \hline
\mathbf{B}_1 & & & \mathbf{B}_2 & & & & \mathbf{B}_{ij} & & & & \mathbf{B}_m & & & \mathbf{O} & & \\
 & \mathbf{B}_2 & & & \mathbf{B}_2 & & \cdots & & \mathbf{B}_{ij} & & \cdots & & \mathbf{B}_m & & & \mathbf{O} & \\
 & & \ddots & & & \ddots & & & & \ddots & & & & \ddots & & & \ddots \\
 & & \mathbf{B}_1 & & & \mathbf{B}_2 & & & & \mathbf{B}_{ij} & & & & \mathbf{B}_m & & & \mathbf{O}
\end{array}\right] \tag{20}
$$

and $\mathbf{1}_l$ $(l=1, 2, \ldots\ldots, m)$ is the martix with m rows and q columns of which the element in l th row are all unity and other elements are all zero.

Consequently, it is required to obtain $\boldsymbol{y}$ which maximizes eq. (18) subject to eq. (19). Since the objective function shown in eq. (18) is a concave function, it is possible in this probelm to obtain the unique set of optimum solutions by applying any technique which has been developed so far to solve the problem of Quadratic Programming.

3. Assignment on the basis of the principle of equal travel times[13)]

The assignment procedure by this method begins from the selection of paths of each OD over which travel times are equal on the basis of results of the preliminary assignment, in which every OD pair has only one route between them. But, the way in which the preliminary assignment should be performed in order to gain solutions in accordance with the principle, has not yet been found out.

Anyway, if paths of equal travel times are searched out, the formulation of the assignment is represented as follows.

The first condition that every path flow is required to satisfy, is that the sum of path flows of each OD must be equal to the traffic demands of that OD. It is called a OD condition.

$$S^k = \sum_p x_p{}^k \qquad (k=1, 2, \ldots\ldots, q) \tag{21}$$

where $x_p{}^k$ is the path flow of k th OD over p th.

The traffic demand X_{ij} on the road section ij, is denoted as follows using path flows.

$$X_{ij} = \sum_{k, p\in ij} x_p{}^k \tag{22}$$

Then, eq.(12) which represents the relation between travel time and traffic volume is written as

$$T_{ij} = a_{ij} \sum_{k, p\in ij} x_p{}^k + b_{ij} \tag{12'}$$

Since we can get the travel time over p th path of k th OD by summing up that of all road sections along the path, it follows that

$$T_p{}^k = \sum_{ij\in k, p} T_{ij} = \sum_{ij\in k, p} (a_{ij} \sum_{k, p\in ij} x_p{}^k + b_{ij}) \tag{23}$$

Assuming that there exist n_k paths between k th OD, the second condition, so-called equal travel times condition, that the principle is realized, is expressed by

$$T_1{}^k = T_2{}^k = \ldots\ldots = T_{n_k}{}^k \tag{24}$$

These are equivalent to

$$\sum_{ij\in k,1}(a_{ij}\sum_{k,p\in ij}x_p{}^k+b_{ij})=\sum_{ij\in k,2}(a_{ij}\sum_{k,p\in ij}x_p{}^k+b_{ij})$$
$$=\cdots\cdots\cdots\cdots \tag{25}$$
$$=\sum_{ij\in k,n_k}(a_{ij}\sum_{k,p\in ij}x_p{}^k+b_{ij})$$
$$(k=1,\ 2,\ \ldots\ldots,\ q)$$

In eq. (25), if attention is paid to a certain kth OD, the number of linear independent equations is apparently (n_k-1). From this, with refference to only kth OD, there exist one OD condition and (n_k-1) equal travel times conditions. That is to say, the number of equations is equal to that of variables, or that of paths. Therefore it is possible that the path flows as solutions are obtained by solving simultaneous linear equations composed of eq. (21) and (25).

But in general, if all pairs of Origin and Destination are superimposed, the linear independence of the equations above is seldom established.

So let us define the long distance OD if both ends of the other OD are included in the path, and the short distance OD if not.

Then, because the equations of equal travel times condition of the long distance OD can be formed by that of the short distance OD, the number of the linear independent equations solving the problem is less than $\sum_{k=1}^{r} n_k$ and the rank of the simultaneous linear equations degenerates. It means that it is impossible, even if required, to obtain uniquely the set of path flows according to the principle. However, it is made clear that the set of flows on road section is unique, if flows on road sections satisfying the principle of equal travel times exist. That is, even if the set of flows on road sections is obtained and determined, the OD composition of flows on road sections is arbitrary. Therefore in this paper, two standpoints, that of uniform information and of non-uniform information are taken into account in the assignment calculation in order to determine the flows on road sections and the OD composition of them uniquely.

(1) the standpoint of non-uniform information

As the equations of equal travel times conditions of the long distance OD are formed by that of the short distance OD, so the path flows of the long distance OD may be decided primarily, if satisfying the OD condition and being non-negative. Accordingly, prior decision of the path flows of the long distance OD is carried out on the standpoint, such that, the shorter the distance between a OD is, the more the demand of information about the path, for example, traffic congestion, increases.

On this standpoint, as the distance between the OD becomes longer, the assignment is given on the basis of a distance. On the contrary, as the distance becomes shorter, it is given on the basis of travel times. Like this, obtaining the set of solutions of the long distance OD in advance, that of the short distance OD unknown are gained by the simultanoues linear equation described above.

(2) the standpoint of uniform information

The standpoint of uniform information is on assumption that the demand of information about paths between an OD is equal regardless of the distance of the OD. Therefore, from this standpoint it may seem to satisfy the condition of the assignment ratio that, if the route diverges at a point, trips of any OD pair passing through there, are distributed similarly over the paths. Since the conditions of assignment ratio can be supplemented by the number of conditions of equal travel times degenerated, on this standpoint, the set of solutions of path flows are also determined uniquely. Nevertheless, in case of uniform information, the simultaneous equations are in higher order, and it becomes very difficult to obtain solutions. So the calcutaneous procedure starts from the standpoint of non-uniform information, and each path flows is gained by the iterative procedure in the way satisfying the condition of assignment ratio.

4. Assignment on the basis of the principle of travel times ratio

Then, the assignment over the whole network can be obtained by considering each single OD pair simultaneously through the use of an iteration method.

Let us consider a single OD pair with r paths and, denote traffic demand and travel time over the path as x_i and T_i respectively. Of course, the sum of the path flows should coincide with the number of distributed trips of the OD (OD condition).

$$\sum_{i=1}^{r} x_i = Q \tag{26}$$

Assuming that the percentage of ith route chosen, m_i, is determined in accordance with a converse ratio of its travel time (nth power), m_i is formulated as follows

$$m_i = \frac{x_i}{Q} = \frac{(T_i)^{-n}}{\Sigma (T_i)^{-n}}, \qquad (i=1, 2, \ldots\ldots, r) \tag{27}$$

if $n=6$ and $r=2$, eq (27) is identical with the equation of conversion ratio curve shown by AASHO. From eq. (12), the relation between T_i and X_i is written as

$$T_i = a_i X_i + b_i \tag{12''}$$

Substituting eq. (12″) into eq. (27), it follows that

$$\frac{x_i}{Q} = \frac{(a_i X_i + b_i)^{-n}}{\Sigma (a_i X_i + b_i)^{-n}} \qquad (i=1, 2, \ldots\ldots, r) \tag{28}$$

Since the simultaneous equations (28) are homogeneous, however, one of these equations should be replaced by eq. (26), Then, the assignment is achieved of solving the simultaneous higher order equations. But in case of such higher order equations, we may have a number of sets of solutions. But, it is the solution such that path flows are all positive and the sum of them satisfies the OD conditions, that we need now. Therefore it must be made clear whether the set of solutions exists uniquely, because planners can not have a criterion finding out which set of solutions to be adopted if there exist various sets of solutions.

So, let us prove the existence of a solution, and its uniquencess, which is available for the calculation.

Primarily, the case of only two paths is considered. Eliminating x_2 from eq. (26) and (28), it follows that

$$x_1(a_1x_1+b_1)^n=(Q-x_1)[a_2(Q-x_1)+b_2]^n \tag{29}$$

Here, let us take into account such $f(x_i)$ as follows.

$$f(x_1)=x_1(a_1x_1+b_1)^n-(Q-x_1)[a_2(Q-x_1)+b_2]^n \tag{30}$$

Examining the characteristics of the function $f(x_1)$, it is found that

$$\left.\begin{aligned} &f(0)=-Q(a_2Q+b_2)<0 \qquad \text{at} \quad x_1=0 \\ &f(Q)=Q(a_1Q+b_1)>0 \qquad \text{at} \quad x_1=Q \\ &\frac{df(x_1)}{dx_1}=(a_1x_1+b_1)^n+x_1\cdot n\cdot(a_1x_1+b_1)^{n-1}\cdot a_1+[a_2(Q-x_1)+b_2]^n \\ &\qquad\qquad +(Q-x_1)\cdot n\cdot[a_2(Q-x_1)+b_2]^{n-1}\cdot a_2>0 \quad \text{at} \quad 0\leqq x_1\leqq Q \end{aligned}\right\} \tag{31}$$

Consequently, we can say that x_1 satisfying $f(x_1)=0$ exists uniquely. That is, the set of solutions of which the elements being all positive and the sum being equal to the traffic demands of the OD, can be determined uniquely. However, in the case of r paths, it is impossible to eliminate variables one by one like this. Then the proof is achieved as follows.

Eq. (28) is equivalent to

$$x_1(a_1x_1+b_1)^n=x_2(a_2x_2+b_2)^n=\ldots\ldots=x_r(a_rx_r+b_r)^n \tag{32}$$

Since each term of eq. (32) is monotone increasing function at the extent of positive x_i, if a certain variable x_j is given a positive value conveniently, the unique set of the other variables x_i corresponding to this x_j, of which all elements are positive, can be obtained. Therefore, let us denote this relation between x_i and x_j here as

$$x_i=h_i(x_j) \qquad (i\neq j,\ i=1, 2, \ldots\ldots, r) \tag{33}$$

Besides,

$$\frac{dh_i(x_j)}{dx_j} \geqq 0 \qquad \text{at} \quad x_j \geqq 0 \tag{34}$$

Substituting eq. (33) into eq. (26), let us introduce $G(x_j)$ as

$$G(x_j) = x_j + \sum_{i \neq j}^{r} h_i(x_j) - Q \tag{35}$$

As in the case of two paths, the characteristics of the function $G(x_j)$ is examined.

$$\left.\begin{aligned} &G(0) = -Q < 0 \qquad \text{at} \quad x_j = 0 \\ &G(Q) = \sum_{i \neq j}^{r} h_i(Q) > 0 \qquad \text{at} \quad x_j = Q \\ &\frac{dG(x_j)}{dx_j} = 1 + \sum_{i \neq j}^{r} \left[\frac{dh_i(x_j)}{dx_j} \right] > 0 \qquad \text{at} \quad 0 \leqq x_j \leqq Q \end{aligned}\right\} \tag{36}$$

From eq. (36) as it is shown above, we can make clear the existence and uniquencess of the solution. Next, we shall show a computational procedure to obtain such a set of solutions.

First, let us introduce such function $F(x_i)$ as below with respect to eq. (33). And it should be carried on to search out x_i until satisfying $F(x_i) = 0$.

$$\begin{aligned} &F(x_i) = x_i(a_i x_i + b_i)^n - x_j(a_j x_j + b_j)^n \\ &\qquad (i \neq j, \ i = 1, 2, \ldots\ldots, r) \qquad x_j \text{ is given} \end{aligned} \tag{37}$$

Since $F(x_i)$ is a monotone increasing and a convex function at $x_i > 0$, x_i can be found easily as follows. (see Fig. 1)

Q' such that $F(Q')$ becomes positive, is searched out expediently, then, by means

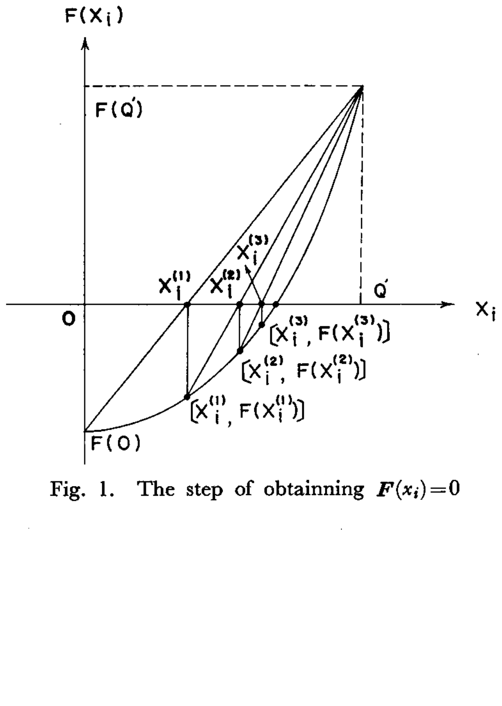

Fig. 1. The step of obtainning $F(x_i) = 0$

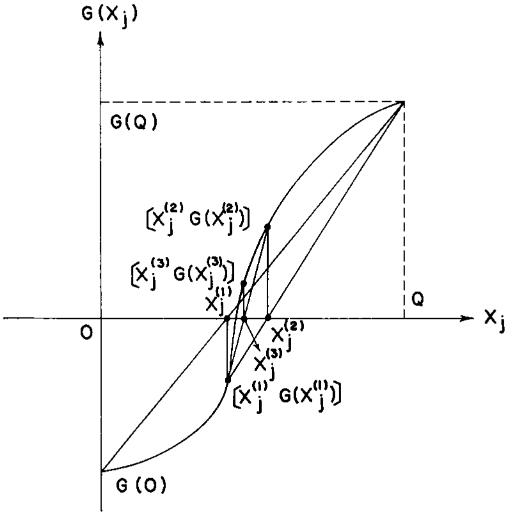

Fig. 2. The step of obtainning $G(x_j)=0$

of connecting between the points $[Q', F(Q')]$ and $[0, F(0)]$ by a straight line, a first point of intersection $x_i^{(1)}$ on x_i axis is gained. Henceforth, the point $[Q', F(Q')]$ remaining fixed, a point of intersection $x_i^{(n+1)}$ on x_i axis is obtained by a straight line connecting two points, $[Q', F(Q')]$ and $[x_i^{(n)}, F(x_i^{(n)})]$. And it is arrived at, if $F(x_i^{(n)})$ becomes nearly zero. In this case, instead of this method, Newton's may be used.

In succession, we shall show the procedure with regard to x_j. At the beginning of this calculation, the point of intersection $x_j^{(1)}$ of x_j axis is gained from a straight line between two points $[x_j=0, G(0)]$, $[x_j=Q, G(Q)]$. And, returnning to the above procedure after this, x_i according to $x_j^{(1)}$ is calculated by using eq. (37), and the value of $G(x_j^{(1)})$ is examined by substituting the set of x_i into eq. (35). If it is negative, the straight line between $[x_j^{(1)}, G(x_j^{(1)})]$ and $[Q, G(Q)]$ gives the next point of intersection $x_j^{(2)}$. And next, examining the value of $G(x_j^{(2)})$ as shown just above if it is positive, this time the line between two points $[x_j^{(1)}, G(x_j^{(1)})]$, $[x_j^{(2)}, G(x_j^{(2)})]$ presents $x_j^{(3)}$. (see Fig. 2)

Because it is known that $G(x_j)$ is not such a function as convex or concave but a monotone increasing function, the procedure in the calculation is performed in such a way that two points are connected by a straight line, of which one is positive and the other is negative, having the greatest number of iterations respectively.

It has been discussed about a single OD, but this method is also available in the case of multiple OD pairs. Namely, since x_i is composed of the path flow of various OD pairs in this case, we may have path flows of all OD pairs fixed except for a peculiar OD pair, and the iteration procedure is carried on until travel times ratio is realized, transferring a peculiar fixed OD pair to be calculated.

Thus, the traffic assignment is performed on the basis of the principle of travel times ratio.

5. Conclusion

Since the first principle of minimization of the total travel times places great emphasis on maximization of the road network efficiency, so, when asked if the assignment on this basis is most useful in design of a road netwrork, we can not always say, yes. Because it is impossible to accept the fact that the principle accords with what actually happens. Consequently, considering randomness and variety of driver's choice of routes, the third principle of travel times ratio seems to be the most practical.

Besides, the assignment on the basis of the principle of equal travel times has been confined to triangulate road networks in order to avoid the theoretical complexity. But, henceforth the author intends to develope the theory on the general type of network.

Traffic assignment calculation in network with flow dependent travel times is complicated and heavy.

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