Hydraulic Efficiency of Flood Detention Pools Evaluated by Means of Mathematical Simulation

By

Yoshiaki IWASA, Kazuya INOUE and Yoshiaki TSUNEMATSU*

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The hydraulic efficiency to secure optimization of the flood detention pool system will be treated through mathematical simulation. The mathematical model is approximated by the point-sink virtually located at the center of the deversoir section.

Numerical computations show that the optimization of the system in hydraulic efficiency will be given through a combination of the deversoir length and the initial diversion discharge. Furthermore, it is seen the whole pool volume will be nearly constant for any possible combination.

Recognition of Underlying Problem

A system of flood detention pool is a physical device, which bridges, in technical aspect, the surface water during flood as water resources, and the flood control as a need of socio-economic life. Excess water in capacity of the downstream channel is charged temporarily in the pool and it will be discharged later when the flood has passed and regional security will be obtained.

The physical structure of the system consists of pool, inlet, outlet and other appurtenant structures along the main channel course and frequently near the confluence of two rivers. The main characteristic of the system in physical behaviour will be exhibited in the inlet structure, through which the main channel and the pool are connected. The lateral deversoir (weir) will be mostly used as the inlet.

The inflow-outflow relationship between the main flow and the pool water through the inlet will be then a function of the weir length (L), the weir height (D) and the capacity of pool (V) as structural elements. However, the weir height will be related to the main flow discharge at the initial diversion. This type of inflow-outflow relationship will be established at the uncontrolled lateral deversoir, which can show no man-made regulation against the diverted flow. The flood

* Department of Civil Engineering

control project by means of the detention pool system will be designed under extremely rare hydrologic conditions, in order to warrant the socio-economic life in human-ecological, regional and national levels. This is one of the reasons why the inlet structure is uncontrolled. More effective use of the flood flow as water resources by changing the physics from natural to man-made with the system matrix of the detention pool will be expected.

Mathematical Simulation of Physical Models

A simplified model of the original system will be seen in Fig. 1-a, where a design flood hydrograph as input will be divided into the main downstream channel

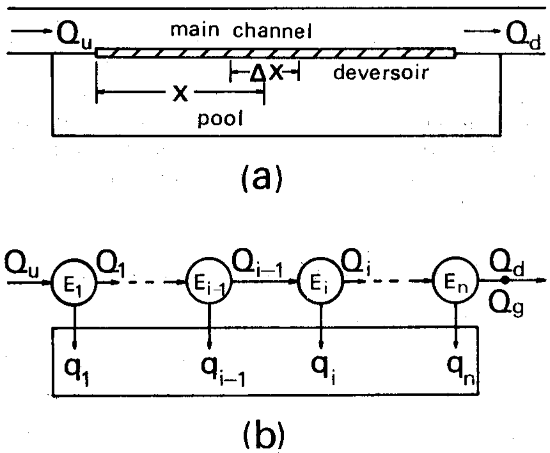

Fig. 1. Flood Detention Pool System

flow and the charge in the detention pool through the lateral deversoir as the inlet. A mathematical abstraction in the one-dimensional theoretical hydraulics from the physical model gives a set of partial differential equations with two independent variables of the time (t) and the distance travelled (x) and two dependent variables of the flow depth (H) and the mean velocity (v) or the discharge (Q), which is the so-called system of unsteady flow equations with variable discharge. The solution will be obtained through the method of characteristics or by the use of the Lax-Wendroff scheme.[1] However, the numerical procedure is so complicated that an approximation is the most that can be expected. The dynamic programming for the non-serial system will not be applied because of the physical structure in diversion.

The quasi-steady flow assumption will be then introduced without making any verification. The problem becomes simplified and the following set of ordinary differential equations gives the mathematical simulation[2] at every moment:

$$\frac{dH}{d\xi}=i-\frac{n^2Q^2}{R^{3/4}A^2}+\frac{\alpha pqQ}{2gA^2} \qquad (1)$$

$$\frac{dx}{d\xi}=1-\frac{\alpha Q^2}{gA^3}\frac{\partial A}{\partial H} \qquad (2)$$

$$\frac{dQ}{d\xi}=-q\left(1-\frac{\alpha Q^2}{gA^3}\frac{\partial A}{\partial H}\right) \qquad (3)$$

where, adding to the previous notations, R: the hydraulic radius, A: the flow area in the main channel, q: the diverted discharge per unit length of the lateral deversoir, n: the Manning roughness, i: the channel slope, α: the Coriolis coefficient, p: the coefficient due to the energy diversion, g: the acceleration of gravity, and ξ: the parameter with length dimension. The constraints for the system are

$$Q_g \geqq Q_d = Q_u-\int q\,dx \qquad (4)$$

$$V \geqq \iint q\,dxdt \qquad (5)$$

where Q_g: the design discharge in the downstream main channel, Q_d: the actual downstream discharge, Q_u: the upstream discharge, and V: the volumetric capacity of pool. The integral with respect to the time must be made throughout the flood duration.

As described in the above, the main characteristic of the physical system will be reproduced in the expression of q. The deversoir will be 10^2m or more in length and thus q will be variable from location to location along the deversoir. This description enables us to approximate the original distributed system in a lumped system schematically shown in Fig. 1-b. The set of Eqs. (1)—(3) is used for each component of the lumped system and usually is replaced by the set of finite difference equations.

Most hydraulic practices require no such detailed behaviour in each system component. In reality, the Japanese standard for river improvement works gives technical information concerning channel and hydraulic geometries at every 200 m. The lumped system may be then replaced by a single component, through which the relationship among inflow, outflow and diversion will be established. The hydraulic performance of the lateral deversoir will be simply simulated by a point sink. The present procedure of analysis will be based on this simulation.

General geometric patterns of the flood detention pool system are 1) a single pool system, 2) a serial system of pools, 3) a non-serial system of pools and so on.

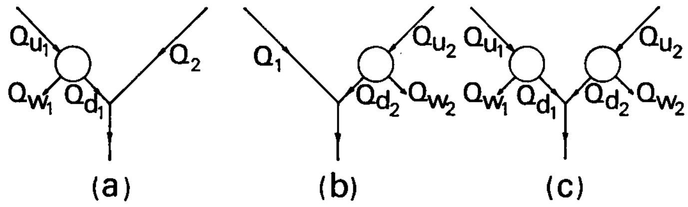

Fig. 2. Typical Geometric Patterns of System

The system is a physical device for flood control, so that it will locate near the confluence of two rivers. Typical geometric patterns of the system will be shown in schematic figure 2. The mathematical simulation will be then made, in order to warrant two constratints of (4) and (5). Because of underlying characteristics of the inflow-outflow relationship, the downstream discharge Q_d will be

$$Q_d = Q_d(L, V, Q_D) \tag{6}$$

where Q_D: the main flow discharge at initial diversion stage. In Eq. (6), L, V and Q_D will be considered to be system parameters. However, as described in the foregoing, the flood control project aims the security of downstream basin, and therefore the capacity of the pool must be so lrge that the maximum value of the downstream discharge after control $(Q_d)_{max}$ is less than the design flood discharge Q_g. In technical aspect, it means that V will be assumed to be constant. Then, the hydraulic efficiency of the detention pool system may be evaluated by

$$\frac{\partial Q_d}{\partial L} = 0 \quad \text{and} \quad \frac{\partial Q_d}{\partial Q_D} = 0 \tag{7}$$

Actual mathematical procedures, for example, in the case of a single pool system near the confluence will be in the following:

1) Determine design hydrographs of both rivers after hydrologic analysis, and the stage-discharge relationship at the confluence.
2) Assume Q_{d_1} and add it to Q_2 at a given instant, to obtain the discharge at the confluence. Estimate the stage at the confluence for $Q_{d_1}+Q_2$ through the given relationship.
3) Trace the surface profiles of water with the use of Eqs. (1)—(3) under the initial value estimated at the confluence.
4) Obtain the stage at the center of lateral deversoir where a point sink will be virtually assumed to exist.
5) Estimate the diverted discharge (Q_w) at the stage obtained through the experience or the use of a proper hydraulic formula.

6) Obtain the upstream discharge Q_{u_1} by summing Q_{d_1} and Q_{w_1}, and compare with the given design hydrograph.
7) If the initial assumption for Q_{d_1} is not correct, repeat the same procedure until the correct answer is obtained.
8) At the next instant after passing Δt, the same computation will be made, until the complete calculation is finished.

The flow chart in mathematical computation will be seen in Fig. 3. In the case of a non-serial system shown in Fig. 2-c, a little modification must be made. Q_{d_1} and Q_{d_2} will be initially assumed and the same procedure will be used. Check must be made through $Q_{d_1}+Q_{w_1}=Q_{u_1}$ and $Q_{d_2}+Q_{w_2}=Q_{u_2}$. The hydraulic performance of the pool system to the main flow pattern will then be evaluated after establishing the modification in flow patterns.

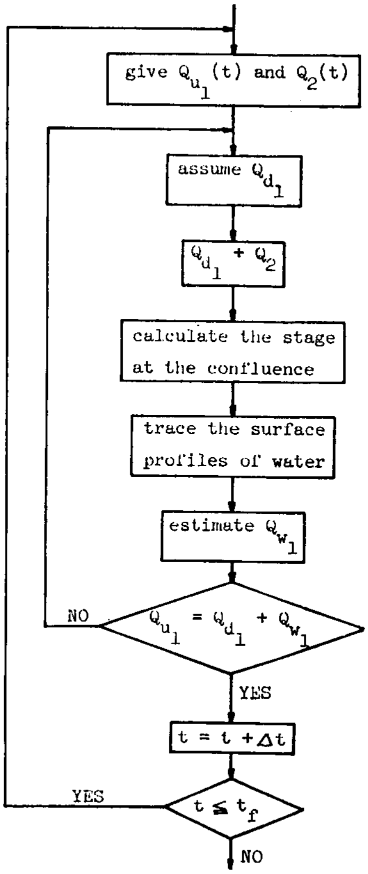

Fig. 3. Flow Chart of Mathematical Computations

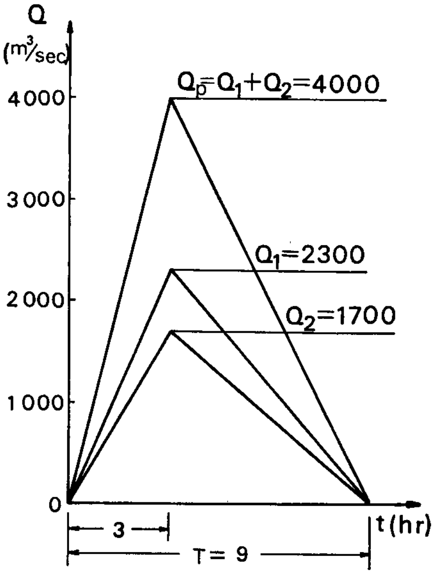

Fig. 4. Design Hydrographs

Numerical Evaluation of Hydraulic Efficiency Through Mathematical Simulation

Some numerical calculations for a simplified channel system, of which the hydraulic and geometric characteristics have been extracted from a real complicated system, will next be shown. The channel geometry of all three rivers is of rectangular shape with 100 m in width, 0.035 in Manning roughness and 1/1000 in slope. The design hydrographs used in the numerical analysis will be shown in Fig. 4. The bottom elevation of the pool is assumed to be the same as the channel. The capacity of pool per unit depth C is 4.00×10^5 m^3. Furthermore, the point sink will be assumed to be 0.6 km upstream from the confluence.

Numerical computations have been proceeded with the time increment of 15 min. (=900 sec). Fig. 5–7 are graphical representations for the maximum controlled downstream discharge Q_d' against Q_D after dividing by Q_p as parametric values in deversoir length. Fig. 5 corresponds to the pattern (*a*) in Fig. 2 and Figs. 6 and 7 to those of (*b*) and (*c*), respectively. When the length of deversoir L will be given, the optimized discharge Q_d' will be determined for a particular discharge at an initial diversion Q_D. This is the graphical indication of Eq. (7). The three-dimensional representation among L, Q_D and Q_d' shown in Fig. 8 will give a better

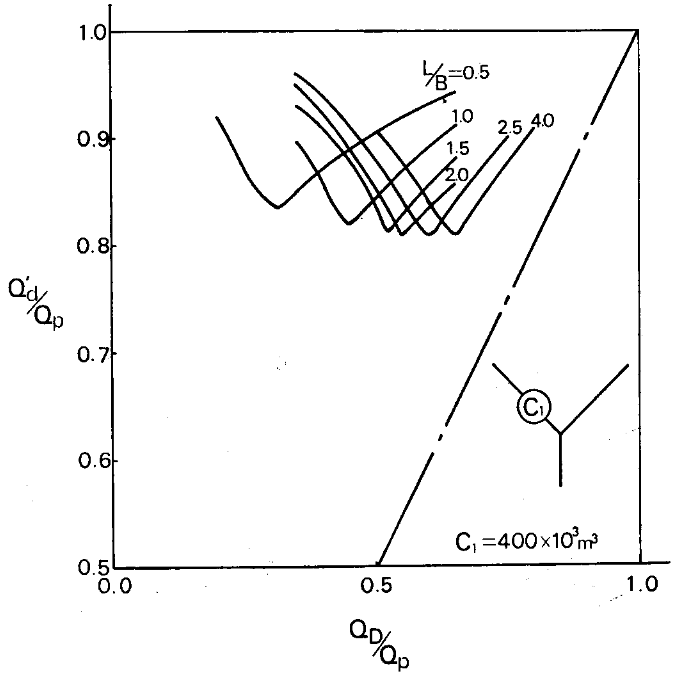

Fig. 5. Q_d' versus Q_D for Pattern (a)

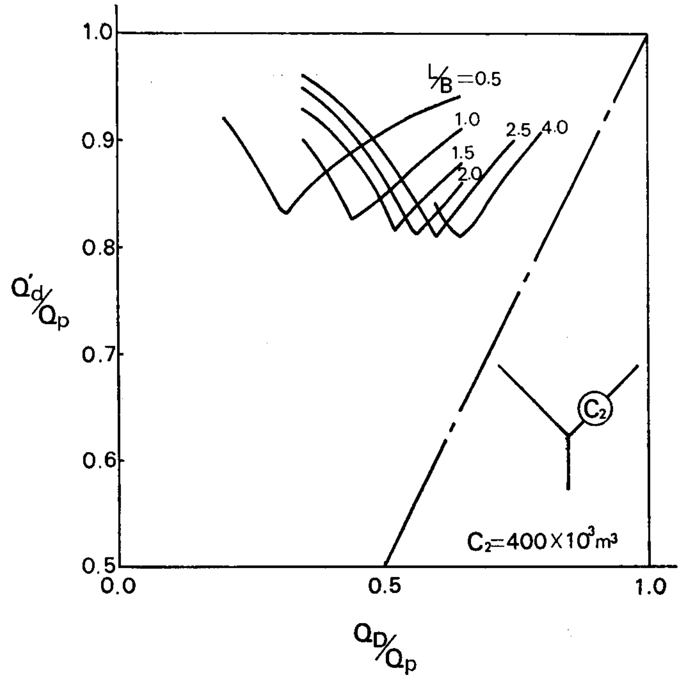

Fig. 6. Q_d' versus Q_D for Pattern (b)

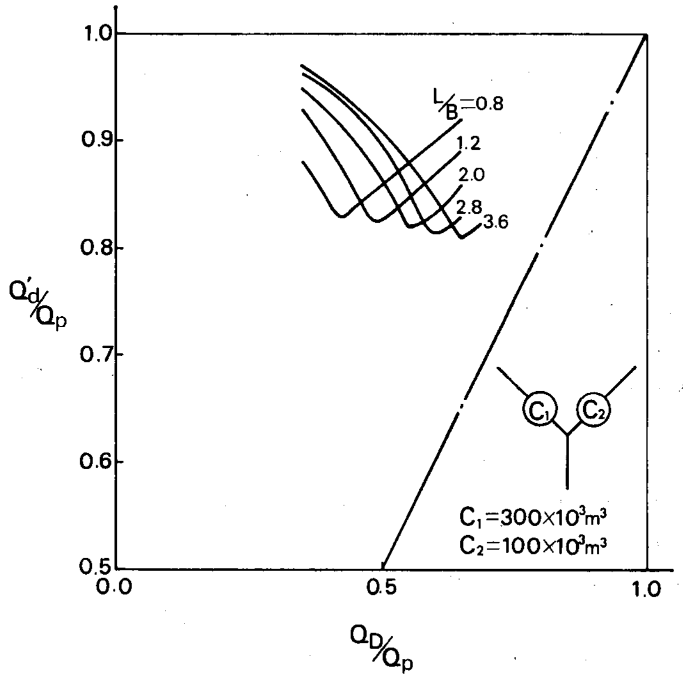

Fig. 7. Q_d' versus Q_D for Pattern (c)

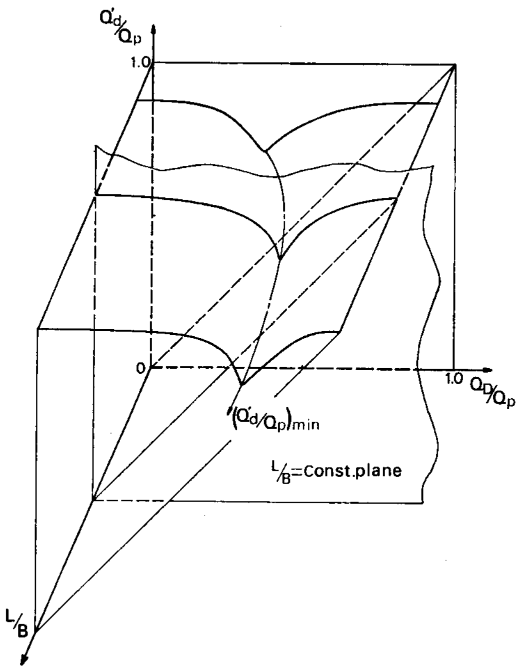

Fig. 8. Three-dimensional Representation for Hydraulic Efficiency

understanding for the optimization in the hydraulic efficiency of the pool system to the design hydrograph.

A suitable combination of the deversoir length and the initial diversion discharge to warrant the optimization of the system or the minimum downstream discharge $(Q_d')_{min}$ will be thus obtained, if $Q_g \geqq Q_d'$. On the contrary, if $Q_d' > Q_g$, the pool capacity must be increased.

Fig. 9 is a graphical indication for the necessary pool volume V_{max} numerically calculated under the condition of $C = 4.00 \times 10^5$ m^3. In the same figure, a relationship among $(Q_d')_{min}$, L, and Q_D is also shown. It will be understood that various combinations among $(Q_d')_{min}$, L, and Q_D give a nearly constant V_{max}. This figure will be a practical contribution to the design problem. Much calculation also give the information that the confluence effect to the hydraulic efficiency will be substantial.

Fig. 10 shows the relationship between Q_d' and Q_D as parametric expressions of L and C. For a particular value of L, all the increasing limbs correspond to each other. This is an indication that the maximum efficiency of the system will be warranted by making a suitable combination of C and Q_D.

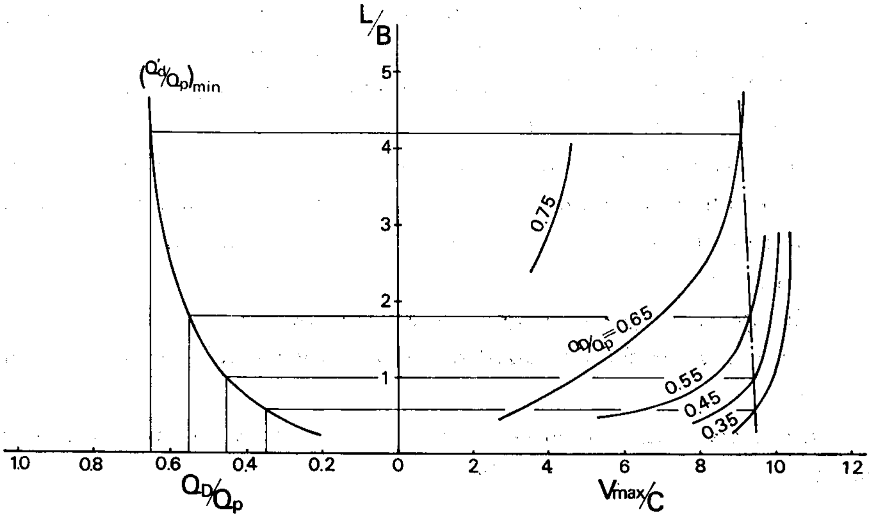

Fig. 9. Necessary Pool Volume $V_{\max}$

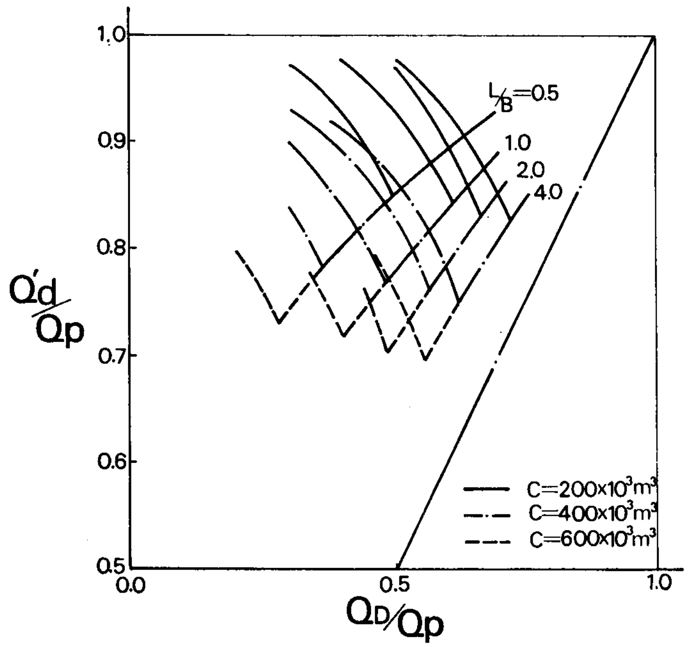

Fig. 10. Q_d' versus Q_D in Terms of L and C

Conclusive Description

The present study has described the hydraulic efficiency of the detention pool system through mathematical simulation. A number of computations with the use of FACOM 230-60 at the Data Processing Center, Kyoto University give the following conclusive information:

1. For a given whole volume of the pool, the maximum hydraulic efficiency of the system, which warrants the minimum controlled downstream discharge $(Q_d')_{min}$, will be obtained as a combination of the deversoir length and the initial diversion discharge.

2. The optimum system designed for a particular hydrograph gives the same controlled discharge as the optimized discharge, when small hydrograph will be experienced.

3. If C is given, the optimized efficiency will be easily obtained, after several combinations of L and Q_D are examined.

References

1) Lax, P. and B. Wendroff, Comm. Pure Appl. Math., Vol. 13, pp. 217-237, 1960.
2) Iwasa, Y., Hydraulics, Asakura Civil Eng. Series No. 3, p. 124, 1967.