Optimization Theory of Hall MHD Generator Duct

By

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With the intention of designing an optimum Hall generator duct with constant velocity, constant or distributed mach number, the authors derive a new digital calculation which makes a integral given by Carter minimum that expresses the duct size under the condition of extracting a needed output power, when applied magnetic flux density, mass flow, inlet stagnation pressure and temperature in the duct are held constant. Moreover the authors confirm that their optimization theory can be applied to not only the diverging rectangular duct but also the annular one.

1. Introduction

In designing an MHD generator, it is very important that the generator duct is shaped to give optimum performance, for example, the duct size, that is length, surface area or volume, is minimized under the condition of extracting a needed output power, when applied magenetic flux density, mass flow, stagnation pressure and temperature in the duct inlet or outlet are kept constant, by means of optimizing the distributions of gas pressure, temperature, velocity, loading parameter and etc. along the flow. According to such an idea and applying calculus of variations to the quasi one-dimensional MHD equations and Carter integral,[1] which expresses the duct size to be minimized, Carter[1),2)] and others[3),4),5)] have proposed a new optimization theory for an ideally segmented electrode Faraday generator duct with constant velocity, mach number or distributed mach number.

On the other hand, recently Hall generator has become more attractive than Faraday generator, because the former has more simple construction and can generate much higher output voltage than the latter and it needs only a pair of electrodes to be connected with the external circuit. However the gas flow performances in Hall duct have been investigated much less than the ones in Faraday duct and the optimization of Hall duct has been not yet discussed except in the case of the authors' reports.[6),7)] Therefore, in this paper, with respect to

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the ideally segmented electrode Hall generator duct using combustion gas with constant velocity, constant or distributed mach number, the authors intend to propose the digital computation method, which makes the above mentioned Carter integral minimum under a needed output power, when magnetic flux density, mass flow, inlet stagnation pressure and inlet in the generator duct are assumed constant. Moreover the authors confirm that their optimization theory can be applied to not only the diverging rectangular duct but also the annular one.

Finally in this paper, the authors treat the one-dimensional gas flow, assume that the working fluid obeys the perfect gas law, and neglect the effect of ion ship, the thermal and frictional losses.

2. Basic Equations

The magnetohydrodynamic flow in the ideally segmented electrode Hall generator duct (Fig. 1) is described by the following set of the basic equations

$$\rho u A = \rho_0 u_0 A_0 = m_0 \quad : \text{continuity equation,} \tag{1}$$

$$\rho u \frac{du}{dx} + \frac{dp}{dx} = J_y B \quad : \text{momentum equation,} \tag{2}$$

$$\rho u \frac{d}{dx}\left(c_p T + \frac{1}{2} u^2\right) = J_x E_x \quad : \text{energy equation,} \tag{3}$$

$$p = \rho R T \quad : \text{state equation,} \tag{4}$$

$$J_x A = J_{x0} A_0 \quad : \text{current continuity equation,} \tag{5}$$

$$J_x = \frac{\sigma (E_x + \beta u \beta)}{1+\beta^2} = \frac{\sigma u B (1-\kappa_h)}{\beta (1+\beta^{-2})}, \tag{6}$$

$$J_y = \frac{\sigma (\beta E_x - u B)}{1+\beta^2} = -\frac{\sigma u B (\kappa_h + \beta^{-2})}{1+\beta^{-2}}. \tag{7}$$

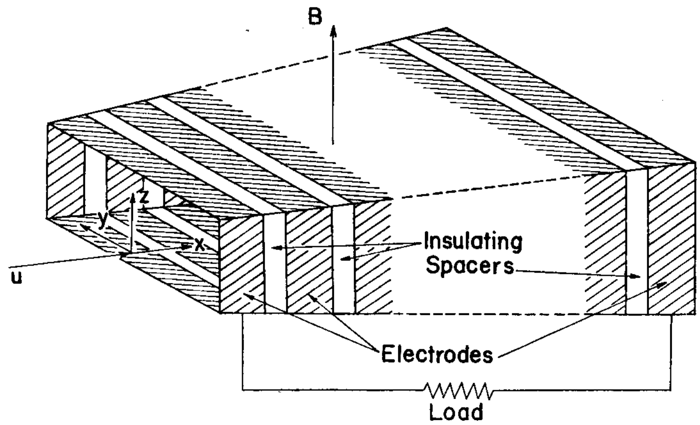

Fig. 1. Sketch of diverging rectangular Hall generator duct.

In these equations and Fig. 1

A: duct cross-sectional area,
B: magnetic flux density*,
$c_p=\alpha R$:specific heat* at constant pressure,
c_v: specific heat* at constant volume,
E_x: x-component of electric field intensity, which is equal to total one,
J_x and J_y: x- and y-components of current density respectively,
m_0: mass flow rate*,
p: gas pressure,
R: gas constant*,
T: gas temperature,
u: gas velocity,
$\alpha=\gamma/(\gamma-1)$*,
β=Hall parameter with respect to electron,
$\gamma=c_p/c_v$: specific heat ratio*,
$\kappa_h=-E_x/\beta uB$: loading parameter,
σ=electrical conductivity,
suffix 0 and 1 : show the quantities in duct inlet and outlet respectively,
*: shows the quantities which are assumed constant in analysis. (8)

Here we assume that σ is expressed by the following expression

$$\sigma=cp^m T^n \exp(-T_i/T), \tag{9}$$

where

C, m and n: constants,
$T_i=\varepsilon_i/2k$,
ε_i: equivalent ionization energy,
k: Boltzmann constant, (9)′

for combustion gases with no elevation of electron temperature. Next we can assume that β is expressed by the following equations

$$\beta=c'p^{m'}T^{n'}, \tag{10}$$

where

c', m' and n': constants. (10)′

3. Optimization Theory of Diverging Rectangular Duct

3.1 Constant Velocity Duct[8)]

Let us discuss the optimization in the case of constant velocity, viz.

$$u=u_0. \tag{11}$$

In this case, using Eq.s (2) to (7) except (5), we get

$$\frac{1}{p}\frac{dp}{dx}=-\frac{\sigma u_0 B^2(\kappa_h+\beta^{-2})}{p(1+\beta^{-2})}\quad, \tag{12}$$

$$\frac{\alpha}{T}\frac{dT}{dx}=-\frac{\sigma u_0 B^2(1-\kappa_h)\kappa_h}{p(1+\beta^{-2})}\quad. \tag{13}$$

Now putting

$$\xi=\log\ (p_0/p), \tag{14}$$

$$\zeta=\log\ (T_0/T), \tag{15}$$

Eq.s (12) and (13) are transformed as follows:

$$\frac{d\xi}{dx}=\frac{\sigma u_0 B^2(\kappa_h+\beta^{-2})}{p(1+\beta^{-2})}\quad, \tag{16}$$

$$\frac{d\zeta}{dx}=\frac{\sigma u_0 B^2(1-\kappa_h)\kappa_h}{\alpha p(1+\beta^{-2})}\quad. \tag{17}$$

From these two equations, we obtain

$$\frac{d\xi}{d\zeta}=G\ (\xi,\zeta)\,, \tag{18}$$

$$\left.\begin{aligned}
&G(\xi,\zeta)=\alpha\{K_1^{-1}\ \exp\ (q'\xi+r'\zeta-T_i^*\ \exp\ \zeta)-1\}/\kappa_h,\\
&K_1=\beta_0 J_{x0}/\sigma_0' u_0 B=(\beta_{s0} J_{x0}/\sigma_{s0}' u_0 B)\ (1-\delta_{s0}u_0^2)^{(-m+m')\alpha-n+n'},\\
&q'=-m+m'+1,\\
&r'=-n+n'-1,\\
&T_i^*=T_i/T_0=T_{is}^*/(1-\delta_{s0}u_0^2),\\
&T_{is}^*=T_i/T_{s0},\\
&\beta_0=c' p_0^{m'} T_0^{n'},\ \beta_{s0}=c' p_{s0}^{m'} T_{s0}^{n'},\\
&\delta_{s0}=1/(2c_p T_{s0}),\ \delta_{s1}=1/(2c_p T_{s1})\\
&\zeta_1=\log\ T_0/T_1=\log\{T_{s0}(1-\delta_{s0}u_0^2)/T_{s1}(1-\delta_{s1}u_0^2)\},\\
&\sigma_0'=cp_0^m T_0^n,\ \sigma_{s0}'=cp_{s0}^m T_{s0}^n,\\
&\text{Suffix } s\text{: shows the quantities at the stagnation point.}
\end{aligned}\right\} \tag{18$'$}$$

and also Eq.s (1), (4), (5), (11), (14) and (15) give

$$\kappa_h=1-J_{x0}T_0 p\beta(1+\beta^{-2})/u_0 B p_0 T\sigma$$
$$=1-K_1(1+\beta^{-2})\ \exp\{-(q'\xi+r'\zeta-T_i^*\ \exp\ \zeta)\}, \tag{18$''$}$$

in which

$$\beta=\beta_0\ \exp\{-(m'\xi+n'\zeta)\},$$

Next let us consider Carter integral

$$I_N=\int_0^l A^N dx\ , \tag{19}$$

which expresses the duct size, namely

$$N=\begin{cases}0\ :\ \text{duct length } l\\ 1/2:\ (\text{duct surface area } S)/4,\\ 1:\ \text{duct volume } V\end{cases} \tag{19$'$}$$

to be minimized.

From Eq.s (17) and (19), we obtain

$$I_N=\int_0^{\zeta_1} A^N \frac{dx}{d\zeta} d\zeta=\int_0^{\zeta_1} F_N(\xi,\zeta)\,d\zeta, \tag{20}$$

where

$$\left.\begin{aligned}&F_N(\xi,\zeta)=(A_0{}^N/K_0\kappa_h)\ \exp\{-(q\xi+r\zeta)\},\\&A_0=m_0RT_0/p_0u_0=(m_0RT_{s0}/u_0p_{s0})\,(1-\delta_{s0}u_0{}^2)^{1-\alpha},\\&K_0=\beta_0J_{x0}B/\alpha p_0=(\beta_{s0}J_{x0}B/\alpha p_{s0})\,(1-\delta_{s0}u_0{}^2)^{(m'-1)\alpha+n'},\\&q=-N-m',\\&r=N-n'+1.\end{aligned}\right\} \tag{20$'$}$$

when in a constant velocity duct the outlet temperature T_1 and so ζ_1 is kept constant, the output power becomes constant, too. Therefore I_N is the integral which is intended to make the duct size minimum under a determinded ouptut power.

When we can solve Eq. (18) by an appropriate digital calculation as Runge-Kutta-Gill method, we can determine the quantitative relation between ξ and ζ or p and T. Then by using the relation in Eq. (20), we can compute numerically the value of I_N. When we give u_0 and κ_{h0} the various values as a parameters, we can find numerically the optimum value of u_0 and κ_{h0} which make I_N minimum.

Next I_N gives the duct length when $N=0$ as shown in Eq. (19)$'$ and consequently putting ζ instead of ζ_1 in Eq.s (20), we can obtain the relations between ζ or T and x, i.e.

$$x=\int_0^{\zeta} F_0(\xi,\zeta)\,d\zeta. \tag{21}$$

Therefore we see that the numerical solution of x has been already acquired on the way of the numerical computation of I_0 in Eqs. (20). Moreover the duct area is evaluated by the equation

$$A=m_0/\rho u_0, \tag{22}$$

which is obtained by Eq.s (1) and (11).

The other quantities, for example, σ, β and κ_h can be digitally calculated by substituting the numerical values of p and T, which are obtained by the above-mentioned method, into Eq.s (9), (10) and (18)$''$ respectively.

3.2. Constant Mach Number Duct

In this article, we shall derive an optimization theory for a constant mach number generator duct.

We can rewrite the basic flow equations (2) and (3) in terms of the stagnation values by use of the adiabatic law

$$\log\,(p_s/p)=\alpha\,\log\,(T_s/T)=\alpha\,\log\,(1+X), \tag{23}$$

where

$$\left.\begin{aligned}&X=u^2/(2c_pT)=(\gamma-1)M^2/2,\\&M=u/\sqrt{\gamma RT}:\ \text{mach number},\end{aligned}\right\} \tag{23$'$}$$

as follows:

$$\frac{1}{p_s}\frac{dp_s}{dx}+\frac{\alpha X}{T_s}\frac{dT_s}{dx}=-\frac{\sigma u B^2(\kappa_h+\beta^{-2})}{p(1+\beta^{-2})}\ , \tag{24}$$

$$\frac{\alpha(1+X)}{T_s}\frac{dT_s}{dx}=-\frac{\sigma u B^2(1-\kappa_h)\kappa_h}{p\,(1+\beta^{-2})}\ . \tag{25}$$

Here using

$$\xi_s=\log(p_{s0}/p_s), \tag{26}$$

$$\zeta_s=\log(T_{s0}/T_s), \tag{27}$$

from Eq.s (24) and (25) we get

$$\frac{d\xi_s}{dx}=\frac{\sigma u B^2}{p(1+\beta^{-2})}\left\{(\kappa_h+\beta^{-2})-\frac{(1-\kappa_h)\kappa_h X}{1+X}\right\}, \tag{28}$$

$$\frac{d\zeta_s}{dx}=\frac{\sigma u B^2(1-\kappa_h)\kappa_h}{\alpha p\,(1+X)\,(1+\beta^{-2})}\ . \tag{29}$$

Eq.s (28) and (29) yeild

$$\frac{d\xi_s}{d\zeta_s}=G(\xi_s,\zeta_s,X), \tag{30}$$

where

$$\left.\begin{aligned}
&G(\xi_s,\zeta_s,X)=\alpha[\{K_{s1}^{-1}(1+X)^{-t'}\ \exp\ (q'\xi_s+r'\zeta_s-T_{s}{}^*\overline{1+X}\ \exp\zeta_s)\\
&-1\}\,(1+X)/\kappa_h-X],\\
&\kappa_h=1-K_{s1}(1+X)^{t'}(1+\beta^{-2})\ \exp\{-(q'\xi_s+r'\zeta_s-T_s{}^*\overline{1+X}\ \exp\ \zeta_s)\},\\
&K_{s1}=\beta_{s0}J_{x0}X_0^{-1/2}(1+X_0)^{\alpha-1/2}/(\sigma_{s0}'B\sqrt{2\alpha RT_{s0}}\ ),\\
&q'=-m+m'-1,\\
&r'=-n+n'-1,\\
&t'=-q'\alpha+r',\\
&\beta=\beta_{s0}(1+X)^{-(m/\alpha+n')}\ \exp\{-(m'\xi_s+n'\zeta_s)\},\\
&\beta_{s0}=c'P_{s0}^{m'}T_{s0}^{n'},\\
&\sigma_{s0}'=cp_{s0}^{m}T_{s0}^{n}.
\end{aligned}\right\}\ (30)'$$

Also Carter integral, which expresses the duct size, is given by Eq. (19). With Eq.s (26), (27) and (29), Eq. (19) is transformed as follows:

$$I_N=\int_0^{\zeta_{s1}}A^N\frac{dx}{d\zeta_s}d\zeta_s=\int_0^{\zeta_{s1}}F_N(\xi_s,\zeta_s,X)\,d\zeta_s, \tag{31}$$

where

$$\left.\begin{aligned}
&F_N(\xi_s,\zeta_s,X)=A_{s0}^{N}X^{s}(1+X)^{t}\ \exp\{-(q\xi_s+r\zeta_s)\}/K_{s0}\kappa_h\\
&A_{s0}=m_0RT_{s0}/p_{s0}\sqrt{2\alpha RT_{s0}}\ ),\\
&K_{s0}=\beta_{s0}J_{x0}BX_0^{-1/2}(1+X_0)^{\alpha-1/2}/\alpha p_{s0},\\
&q=-N-m',\\
&r=(N+1)/2-n',\\
&s=-(N+1)/2,\\
&t=N(\alpha-1/2)+1/2+(m'\alpha+n').
\end{aligned}\right\}\ (31)'$$

Now when

$$X = X_0 = \text{constant}, \tag{32}$$

namely in constant mach number duct, we can determine the quantitative relation between ξ_s and ζ_s if we can solve Eq. (30) with a numerical calculation. Using the relation, we can carry out the numerical integration of Eq.s (31) when we put $X = X_0$. When we give X_0 and κ_{h0} the various values as a parameter, we are able to find the value of X_0 and κ_{h0} which make I_N minimum.

Moreover, as described in the preceding article, substituting $N=0$, $X=X_0$ and ζ_s in place of ζ_{s1} in Eq.s (31), we have

$$x = \int_0^{\zeta_s} F_0(\xi_s, \zeta_s, X_0)\, d\zeta_s. \tag{33}$$

The values of A, σ, β and κ_h can be obtained in the same way as the preceding case.

3.3 Distributed Mach Number Duct

Here let us discuss the optimization of a duct, along which the mach number M and therfore X varies. For this case, Carter integral I_N is already given by Eq.s (31).

When Eq. (30) i.e.

$$\xi_s' - G = 0, \tag{34}$$

where

$$\left.\begin{aligned} &\xi_s' = d\xi_s/d\zeta_s, \\ &G = G(\xi_s, \zeta_s, X), \end{aligned}\right\} \tag{34$'$}$$

is adopted as one subsidiary condition, our problem that let us minimize I_N becomes one that we solve the following simultaneous Euler equations

$$F_{N\xi_s} - \lambda(\zeta_s) G_{\xi_s} - \frac{d\lambda(\zeta_s)}{d\zeta_s} = 0, \tag{35}$$

$$F_{NX} - \lambda(\zeta_s) G_X = 0, \tag{36}$$

where

$$\lambda(\zeta_s): \text{Lagrange multiplier.} \tag{37}$$

By eliminating $\lambda(\zeta_s)$ from Ep.s (35) and (36) and combining the result with Eq. (34), we can get the simultaneous differential equations to determine the optimum distribution of X in the duct for a special values of X_0 and κ_{h0} as follows:

$$\left.\begin{aligned} &F_{N\xi_s} - \frac{F_{NX}}{G_X} G_{\xi_s} - \left(\frac{\partial}{\partial\zeta_s} + \xi_s' \frac{\partial}{\partial\xi_s} + X' \frac{\partial}{\partial X}\right)\left(\frac{F_{NX}}{G_X}\right) = 0, \\ &\xi_s' - G = 0. \end{aligned}\right\} \tag{38}$$

These equations can be solved numerically be an appropriate digital calculation. Moreover by applying the numerical calculation results for the various values of X_0 and κ_{h0} to Carter integral, we are able to find the optimum value of X_0 and κ_{h0}, which minimize the integral.

Next substituting $N=0$ and ζ_s for ζ_{s1} in Eq.s (31), we obtain

$$x=\int_0^{\zeta_s} F_0(\xi_s, \zeta_s, X)\,d\zeta_s\,. \tag{39}$$

4. Optimization Theory of Diverging Rectangular Duct When the Cross-sectional Area is Assumed to Increase Linearly: Prearranged Cross-sectional Area Duct

In this section, we shall derive an optimization theory for a duct, whose cross-sectional area is assumed to vary according to a prearranged form, here as one example, to increase linearly from inlet to outlet.

Now we assume that the cross-sectional area is expressed by the following equation

$$A=A_0(1+gx), \tag{40}$$

where

g: duct gradient. (40)′

From Eq.s (1), (4) and (40), we get

$$\frac{A}{A_0}=\frac{\rho_0 u_0}{\rho u}=\frac{p_0 u_0 T}{p u T_0}=\frac{p_0 u_0 (T_s-u^2/2c_p)}{pu\,(T_{s0}-u_0^2/2c_p)}=1+gx. \tag{41}$$

So putting

$$\left.\begin{aligned} &p^*=p/p_0\,,\\ &T_s^*=T_s/T_{s0}\,,\\ &u^*=u/u_0\,, \end{aligned}\right\} \tag{42}$$

Eq.s (41) and (42) yields

$$u^*=\sqrt{\{(\delta_{s0}^{-1}u_0^{-2}-1)/2\}^2 p^{*2}(1+gx)^2+\delta_{s0}^{-1}u_0^{-2}T_s^*}-\{(\delta_{s0}^{-1}u_0^{-2}-1)/2\}\,p^*(1+gx), \tag{43}$$

where

$$\delta_{s0}=1/(2c_pT_{s0})\,. \tag{43$'$}$$

Next using Eq.s (42), Eq.s (2) and (3) are transformed to the following simultaneous differential equations

$$\frac{dp^*}{dx}=F_h(x,p^*,T_s^*)\,, \tag{44}$$

$$\frac{dT_s^*}{dx}=G_h(x,p^*,T_s^*)\,, \tag{45}$$

where

$$\left.\begin{aligned} F_h(x,p^*,T_s^*)=&-\{RT_{s0}u_0^{-2}u^{*-2}(T_s^*-\delta_{s0}u_0^2u^{*2})\,(1-M^2)\}^{-1}\\ &\times[p^*(T_s^*-\delta_{s0}u_0^2u^{*2})^{-1}G_h(x,p^*,T_s^*)-gp^*(1+gx)^{-1}\\ &-K'_{s2}u^{*-1}p^{*m}(T_s^*+\delta_{s0}u_0^2u^{*2})\,(T_s^*-\delta_{s0}u^{*2}u_0^{*2})^n(\kappa_h+\beta^{-2})\\ &\times(1+\beta^{-2})^{-1}\exp\{-T_u^*(T_s^*-\delta_{s0}u_0^2u^{*2})^{-1}\}],\\ G_h(x,p^*,T_s^*)=&K'_{s1}\kappa_h p^{*m'-1}(T_s-\delta_{s0}u_0^2u^{*2})^{n'+1}(1+gx)^{-1},\\ \kappa_h=&1-K'_{s3}p^{*-m+m'}(T_s^*-\delta_{s0}u_0^2u^{*2})^{-n+n'}u^{*-1}(1+gx)^{-1}(1+\beta^{-2}) \end{aligned}\right\} \tag{46}$$

$$\times \exp\{T_u^*(T_s^*-\delta_{s0}u_0^2u^{*2})^{-1}\},$$
$$K'_{s1}=-\beta'_{s0}BJ_{x0}/\alpha p_0,$$
$$K'_{s2}=-\sigma''_{s0}B^2RT_{s0}/p_0u_0,$$
$$K_{s3}=\beta'_{s0}J_{x0}/\sigma''_{s0}u_0B,$$
u^*: given by Eq. (43),
$$\beta=\beta'_{s0}p^{m'}(T_s^*-\delta_{s0}u_0^2u^{*2})^{n'},$$
$$\beta'_{s0}=c'p_0^{m'}T_{s0}^{n'},$$
$$\sigma''_{s0}=cp_0^{m}T_{s0}^{n},$$

When we can solve the simultaneous differential equations (44) and (45) by an appropriate digital calculation, we can get the numerical values of p and T_s^* with respect to x. While the digital calculation is carried on, we can acquire the value of x for T_{s1}, which can be determined by a needed output power, T_{s0} and m_0 assumed constants, i.e. the duct length l. When l is obtained in such a way, the surface area S and volume V can be also numerically calculated by the following equations

$$S=4I_{1/2}=\frac{2K_s^{1/2}}{3g}\{(1+gl)^{3/2}-1\} \tag{47}$$

$$V=I_1=\frac{K_s}{2}l(2+gl), \tag{48}$$

where

$$K_s=(Rm_0T_{s0}/p_0u_0)\ (1-\delta_{s0}u_0^2) \tag{49}$$

which are obtained by substituting Eq. (40) in (19).

When we give u_0 and κ_{h0} the various values as parameter in solving Eq.s (44) and (45), we can find the optimum value of u_0 and κ_{h0} which make I_N minimum.

5. Optimization Theory of Diverging Annular Duct

Let us investigate the annular Hall generator duct (Fig. 2). As is well known, Ohm law considering Hall effect is given as follows:

$$\boldsymbol{J}=\frac{\sigma}{1+\beta^2}\left\{\boldsymbol{E}'-\frac{\beta}{B}(\boldsymbol{E}'\times\boldsymbol{B})+\frac{\beta^2}{B^2}(\boldsymbol{E}'\boldsymbol{B})\boldsymbol{B}\right\}, \tag{50}$$

where

$$\left.\begin{array}{l}\boldsymbol{E}'=\boldsymbol{E}+\boldsymbol{u}\times\boldsymbol{B},\\ \boldsymbol{B}\text{: magnetic flux density vector,}\\ \boldsymbol{E}\text{: electric field intensity vector,}\\ \boldsymbol{J}\text{: current density vector,}\\ \boldsymbol{u}\text{: velocity vector.}\end{array}\right\} \tag{50$'$}$$

When we adopt the cylindrical coordinate system (r, θ, x) as shown in Fig. 2 and assume the quasi one-dimentional flow as in the rectangular duct described in Sections 3 and 4, we have

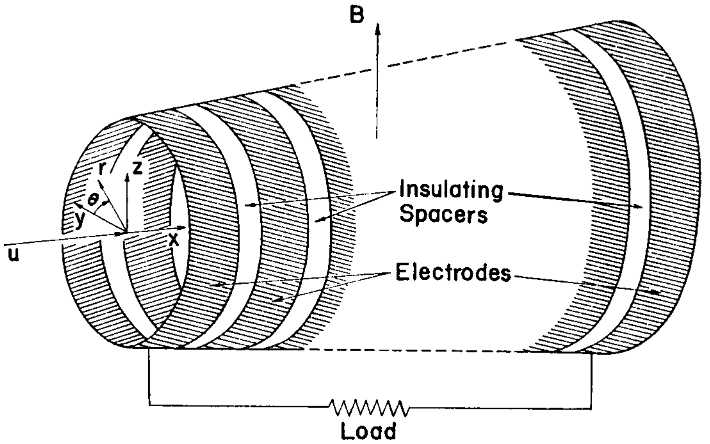

Fig. 2. Sketch of diverging annular Hall generator duct.

$$\left.\begin{aligned} B &= (B_r,\ B_\theta,\ 0) = (B\sin\theta,\ B\cos\theta,\ 0), \\ \boldsymbol{E} &= (0,\ 0,\ E_x), \\ \boldsymbol{J} &= (J_r,\ J_\theta,\ J_x), \\ \boldsymbol{u} &= (0,\ 0,\ u), \end{aligned}\right\} \tag{51}$$

where

suffixes r, θ and x: show the quantities in r-, θ- and x-direction respectively.

Susbtitution of Eq.s (51) in (50) gives

$$\left.\begin{aligned} J_r &= \frac{\sigma(\beta E_x - uB)\cos\theta}{1+\beta^3}, \\ J_\theta &= -\frac{\sigma(\beta E_x - uB)\sin\theta}{1+\beta^2}, \end{aligned}\right\} \tag{52}$$

$$J_x = \frac{\sigma(E_x+\beta uB)}{1+\beta^2} = \frac{\sigma uB(1-\kappa_h)}{\beta(1+\beta^{-2})}, \tag{53}$$

where

$\kappa_h = -E_x/\beta uB$: loading parameter. (53)′

Using Eqs. (52), y-component of current density becomes

$$J_y = J_r\cos\theta - J_\theta\sin\theta = -\frac{\sigma uB(\kappa_h+\beta^{-2})}{1+\beta^{-2}}. \tag{54}$$

As Eq.s (53) and (54) are identical with Eq.s (6) and (7) respectively, the basic equations for the annular duct with respect to the rectangular coordinate system are entirely the same as the ones for the rectangular duct. Therefore we can apply the optimization theory introduced in Sections 3 and 4 to the annular generator duct.

6. Conclusion

In Section 3, with respect to the constant velocity and constant mach number ducts the numerically solvable differential equations between pressure and temperature have been derived from the basic MHD equations. When the differential equations can be solved in regard to the various values of u_0 or X_0 and κ_{h0} and the results are applied to Carter integral, we can find the optimum values of u_0 or X_0 and κ_{h0} which make the integral minimum.

Next about the distributed mach number duct the authors could obtain the numerically solvable differential equations for ξ_s, ζ_s and X, which make Carter integral minimum for special values of X_0 and κ_{h0}. Moreover, when the equations can be digitally solved with the various values of X_0 and κ_{h0} and the results are put in Carter integral, we can find the optimum values of X_0 and κ_{h0} which make the integral minimum.

Next in Section 4, the authors have introduced the digitally solvable simultaneous differential equations of p^* and $T_s{}^*$ for the rectangular duct, whose cross-sectional area is assumed to increase linearly from inlet to outlet. When the equations are solved for the various values of u_0 and κ_{h0} and the results are substituted in Carter integral, we can find the optimum value of u_0 and κ_{h0} which make the integral minimum.

Also in Section 5, the authors have confirmed that their optimization theory can be applied to not only the rectangular duct but also the annular one.

Lastly the above derived numerical calculation can be easily transformed into the one for the case when the outlet conditions are assumed.

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