An Equilibrium Model of Commuter Flow in Urbanized Area

By

Kozo AMANO* and Yoshitaka AOYAMA*

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Commuter traffic volumes generated and absorbed in urbanized area can be estimated by existing models using some economic indices. But in those methods, equilibrium between areas and correlation between generated and absorbed traffic volumes are not taken into consideration. Furthermore, the effects caused by improvement of traffic facilities cannot be included into estimation.

In order to solve these problems, we propose an estimation model composed of additional factors such as generative and attractive potentials in each urbanized area.

1. Introduction

Transportation volume is usually estimated by the following four steps:

1. The estimation of generated and absorbed transportation volume
2. Transportation distribution
3. Transportation assignment
4. Cross section traffic volume

There are some methods of estimating these traffic volumes, but it is more important to estimate the volumes of generated and absorbed transportation in each zone. However, because these volumes are affected by the improvement of transit facilities, they are difficult to estimate. In addition, the first step is not well assured, so the distributed traffic volume and assigned traffic volume will be estimated with low accuracy. This generated and absorbed traffic volume is usually estimated by the functional model of linear equation, but in this case, the greatest problem is to know whether or not the parameter of each equation will be constant. Even if we assume that the parameters will not be constant, and that we can estimate it by extrapolation of time-series tendency, it is difficult to obtain sufficient data.

The usual way to estimate generated and absorbed traffic volume independently is by some economic indices, but in those methods, the equilibrium with other areas and the correlation between generated and absorbed traffic volume are not taken into consideration. Furthermore, the influence of traffic facility impro-

* Department of Transportation Engineering

vements cannot be included directly. Thus, we propose another estimation model in which these defects have been improved.

2. Indices which are used in the model

When we estimate generated and absorbed traffic volume, usually we use economic indices such as residential population, number of workers, industrial production, number of stores, area of land by use, area of building floor by use, number of offices, number of houses, etc. From these indices, area of land by use is considered to be the most basic factor, especially as it can be considered that the traffic volume which is composed of human beings, depends upon the relationship with the land use. Thus, we adopt each land area by its land use as indices for the estimation. In this case, it may be thought that it would be preferable to use area of floor space by use rather than area of land by use, because we could get a more accurate estimate of the future city which will develop with more and more buildings. Then, we could set the generated traffic volume in zone i as A_i, each floor space in the commercial area, industrial area, quasi-industrial area, residential-commercial area, etc., as X_i^1, X_i^2, $X_i^3, \cdots$. In the past, this linear equation was used:

$$A_i = H_0 + H_1 X_i^1 + H_2 X_i^2 + H_3 X_i^3 \cdots \tag{1}$$

When we set absorbed traffic volume in zone j as B_j, we can obtain the formula in the same way as formula (1)

$$B_j = K_0 + K_1 X_j^1 + K_2 X_j^2 + K_3 X_j^3 + \cdots \tag{2}$$

Where H_k, K_k ($k=0, 1 \cdots n$) are constants which represent generated and absorbed traffic volume per unit area. As a result of practical adaptation, it was found that this equation has enough accuracy. For example, we compared real values and calculated values of A_i, B_j of eight zones in Kobe regarding the commuters in 1965 by multi-regression analysis. We found that the multi-correlation coefficients of A_i, B_j are 0.9211 and 0.9954 respectively, i.e., very high. In this case we have made the simple assumption of three main land uses - commercial, industrial and residential. But when we use H_k, K_k directly in the future estimation of A_i and B_j, there is a tendency to estimate over the value. Therefore, it is not suitable to adopt the estimated value of H_k and K_k directly to the future estimation. This is because the area of floor per one person will be enlarged as the income increases. Therefore, parameters H_k, K_k will become smaller in the future. T stands for the total generated transportation volume which is equivalent to the total absorbed transportation volume A_i or the total of B_j. Thus, we can obtain the following formula: (see Figure 1)

O \ D	1	· · · ·	j	· · · ·	m	Total
1	T_{11}	· · · ·	T_{1j}	· · · ·	T_{1m}	A_1
⋮	⋮				⋮	⋮
i	⋮	· · · ·	T_{ij}	· · · ·	⋮	A_i
⋮	⋮				⋮	⋮
m	T_{m1}	· · · ·	T_{mj}	· · · ·	T_{mm}	A_m
Total	B_1	· · · ·	B_j	· · · ·	B_m	T

Fig. 1. Example of OD Commuter Traffic Table.

$$T = \sum_i A_i = \sum_j B_j \tag{3}$$

If we set the rate of generated traffic volume which zone i bears as a_i, and the rate of absorbed traffic volume which zone j bears as b_j,

$$\left.\begin{aligned} a_i &= A_i/T \\ b_j &= B_j/T \end{aligned}\right\} \tag{4}$$

Then a_i and b_j have the same meaning as generated traffic volume and absorbed traffic volume. Hence, we deal with a_i and b_j in further estimation.

Indices of land use related with a_i and b_j are the rates of area of each floor by use in zone i and j to their total floor space, i.e.,

$$\left.\begin{aligned} p_i(x_1) &= X_i^1/X_0^1 \ , \ p_i(x_2) = X_i^2/X_0^2 \ , \ p_i(x_3) = X_i^3/X_0^3 \ , \ \cdots\cdots \\ p_j(x_1) &= X_j^1/X_0^1 \ , \ p_j(x_2) = X_j^2/X_0^2 \ , \ p_j(x_3) = X_j^3/X_0^3 \ , \cdots\cdots \end{aligned}\right\} \tag{5}$$

where,

$$X_0^1 = \sum_i X_i^1 = \sum_j X_j^1 \ , \ X_0^2 = \sum_i X_i^2 = \sum_j X_j^2 \ , \ X_0^3 = \sum_i X_i^3 = \sum_j X_j^3 \ , \ \cdots\cdots \tag{6}$$

Then, from the same way as formula (1) and (2).

$$a_i = h_0 + h_1 p_i(x_1) + h_2 p_i(x_2) + h_3 p_i(x_3) + \cdots\cdots \tag{7}$$

$$b_j = k_0 + k_1 p_j(x_1) + k_2 p_j(x_2) + k_3 p_j(x_3) + \cdots\cdots \tag{8}$$

According to this formula, even if the floor space will greatly increase in the future, the increase of floor space per one person will be denied relatively, because we deal with the rate to total area of floor, as shown in formula (6).

The problem of future change of value of H_k and K_k in formula (1) and (2) can be solved in formula (7) and (8). That is, we evaluate the rate of generated and absorbed traffic volume to the total volume as a linear equation of the rate of each floor space by use to each total floor space. But when those rates are adopted with future estimation, it is a_i and b_j that can be obtained. So it is necessary to know the future value of T in order to estimate A_i and B_j. As T is the total number of commuters in the city, we would be better to estimate it exogenously with the future economic indices. We adapted these formulas (7) and (8) to Kobe city in 1965 and found that the multi-correlation coefficients were the same as before.

According to formulas (7) and (8), A_i and B_j are estimated independently of each other. The influence of future traffic facility improvements and the relation to other areas are not taken into consideration. In order to include them, we consider the following "accessibility", which stands for the relation to other areas. There are various definitions about "accessibility", but in short it means "to be able to approach" or "being easy to approach".

A typical accessibility model of the gravity type is as follows:

$$R_i = \sum_{\substack{j=1 \\ (i \neq j)}}^{m} \frac{S_j}{t_{ij}^{\ r}} = \sum_{\substack{j=1 \\ (i \neq j)}}^{m} S_j t_{ij}^{\ -r} \tag{9}$$

where

R_i = relative accessibility of zone i

S_j = scale of urban activity in zone j

t_{ij} = time-distance between zone i and j

r = constant which shows the influence of distance

We can consider that it is an index of movability which expands the conception of population potential. According to this concept, we define accessibility in this paper as follows: (see Figure 2)

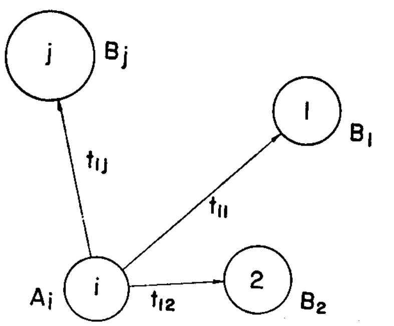

Fig. 2. The Generative Potential in Zone i to Some Attractive Zones.

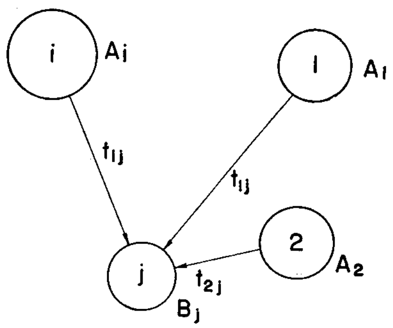

Fig. 3. The Attractive Potential in Zone j from Some Generative Zones.

First, as an accessibility concerned with the generated traffic volume, we take B_j as index S_j and $r=1$, and then the generated potential α_i will be expressed as the following formula:

$$\alpha_i = \frac{\sum_j B_j t_{ij}^{-1}}{\sum_i \sum_j B_j t_{ij}^{-1}} \tag{10}$$

In order to apply formulas (4) and (6), $\sum_i \alpha_i$ is arranged to 1 in this formula. This generated potential will be explained as follows:

In order to generate traffic volume from zone i, the attracting district must be located around it. The nearer the attracting area is, and the more attractive it is, (in other words, the smaller t_{ij} is, and the bigger B_j is,) the easier the generation of traffic volume from zone i is.

Regarding the absorbed traffic volume, we take A_i as S_i and $r=1$ in the same way. Then we express it as the following absorbed potential:

$$\beta_j = \frac{\sum_i A_i t_{ij}^{-1}}{\sum_i \sum_j A_i t_{ij}^{-1}} \tag{11}$$

The standardization of $\sum_j \beta_j = 1$ is the same as α_i. This absorbed potential can be explained as follows: (see Figure 3)
That is, the residential area must be located closely so that zone i may absorb traffic volume. The nearer the residential area is, and the more generative the area is, (in other words, the smaller t_{ij} is, and the bigger A_i is), the easier the absorption of traffic volume into zone j is.

As mentioned above, for the estimation of both future generated and absorbed traffic volume, the following three kinds of indices are used: floor space by use, time-distance between zone i and j, and the total number of commuters. These values give us the indices as in formulas (6), (10) and (11).

3. Presumption model for generated and absorbed traffic volume and its calculation process

Using various indices mentioned above, we propose a presumption model for generated and absorbed traffic volume as in the following formula:

$$a_i = h_0 + h_1 p_i(x_1) + h_2 p_i(x_2) + h_3 p_i(x_3) + \cdots + h_\alpha \alpha_i \tag{12}$$

$$b_j = k_0 + k_1 p_j(x_1) + k_2 p_j(x_2) + k_3 p_j(x_3) + \cdots + k_\beta \beta_j \tag{13}$$

When we adopt this model to the analysis of trip distribution in Kobe city in 1965, each multi-correlation coefficient is 0.9528 and 0.9996. In this formula, h_0, h_1, $\cdots h_\alpha$, k_0, $k_1,\cdots k_\beta$ are parameters. As each index has been standardized, it can be thought that the required accuracy will be kept, even if we assume these parameters are constant. Both total number of commuters T, and the area of floor by use in the future $X_i^l(l=1, 2,\cdots\cdots n)$ can be given exogenously by urban planning. Also, time-distance t_{ij} between zone i and j will be given by future traffic networks exogenously. Therefore, the values in equations (14) and (15) will be constant.

$$h_0+h_1p_i(x_1)+h_2p_i(x_2)+h_3p_i(x_3)+\cdots\cdots+h_np_i(x_n) \tag{14}$$

$$k_0+k_1p_j(x_1)+k_2p_j(x_2)+k_3p_j(x_3)+\cdots\cdots+k_np_j(x_n) \tag{15}$$

The rate of generated traffic volume a_i, generated potential α_i, rate of absorbed traffic volume b_j and absorbed potential β_j have been connected with each other as shown in formulas (16), (17), (18) and (19). So we can estimate these future values by trial and error calculation. The calculation procedure is shown in a flow-chart in Figure 4. In this figure, formulas from (16) to (19) are expressed briefly as follows:

$$\alpha_i = \alpha(B_j, t_{ij}) \tag{16}$$

$$\beta_j = \beta(A_i, t_{ij}) \tag{17}$$

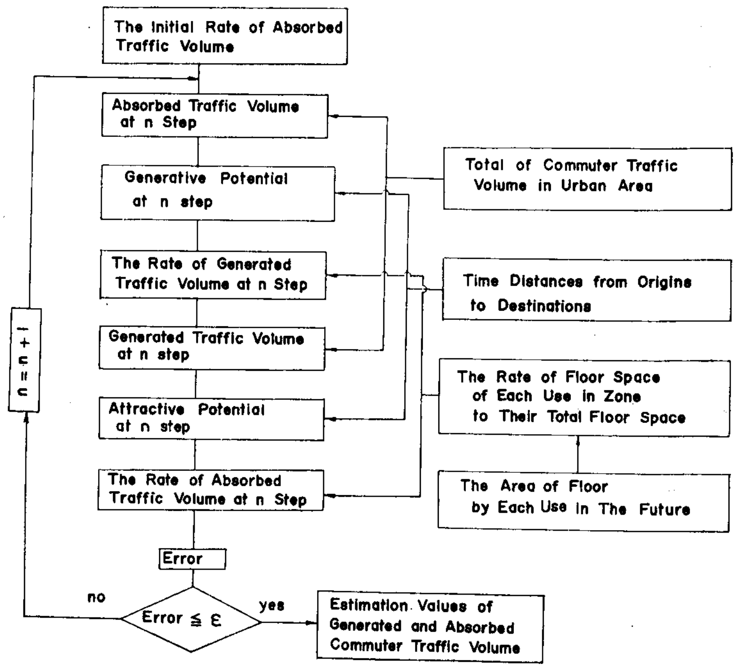

Fig. 4. Estimation Procedure in The Equilibrium Model.

$$a_i = a(p_i, \alpha_i) \tag{18}$$

$$b_j = b(p_j, \beta_j) \tag{19}$$

1. First of all we give an initial value to b_j. Though b_j can be taken at any value, we find it better to give the present value of b_j in order to promote the convergence.
2. We multiply b_j by T that is given exogenously and we are able to get B_j.
3. Then we obtain α_i with the value of B_j and t_{ij}, which is given exogenously.
4. The value of a_i will be obtained from this α_i and p_i of future land use.
5. We multiply a_i by T to make it A_i.
6. We calculate β_j from A_i and t_{ij}.
7. b_j can be obtained from β_j and p_j.
8. By using this b_j, we do the second calculation in the same order.
9. Repeat this calculation until the difference D will become a certain permitted extension, that is to say, $D \leqq \varepsilon$.

Here, D is

$$D = \sum_{i,j} \{|a_i(n) - a_i(n-1)| + |b_j(n) - b_j(n-1)|\} \tag{20}$$

4. Conclusion

This model has the advantage over the existing models, such as the gravity, present pattern model, etc., because by applying this model to the estimation of trip distribution, we can forecast the generative and attractive potential distribution in each area caused by future improvements of rapid transit networks.

This advantage is important for the fundamental solution of the rapid transit problem, especially in expanding large cities.

References

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