Study on Process of Construction Planning and its Application

By

Kazuhiro YOSHIKAWA* and Mamoru HARUNA*

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In this paper the planning process based on the network techniques is dealt with for a construction project where the project is defined as a function of sequence among jobs included in the project.

The planning process consists of three processes: In the first process the set of alternatives of job sequences which satisfy the restriction for the completion time is obtained by the use of devised branch and bound algorithm.

In the second process every kind of performance which is useful for the evaluation of construction project is computed for each alternative and further actual restrictions are established for all performances.

In the last process alternatives which satisfy all restrictions are obtained by the filtering procedure and an optimal project is selected from them by a definite criterion.

An application of this planning process is shown for the elevated railway construction of the New Sanyo Trunk Line.

1. Introduction

During recent years network techniques for planning and scheduling of construction projects such as PERT and CPM have become a useful tool for managers who associate with construction projects. Many governments and construction industries require the preparations and use of these techniques when they plan a construction project. While it is well known that these techniques are more effective for managing a project than the traditional technique called bar charts, it appears that little effort has been made to establish a practical integrating process of the construction planning.

Though PERT/COST, PERT/MANPOWER and CPM are developed for the integration of the project based on PERT/TIME, they are not satisfactory to be used for the construction project because in these techniques the process to make a project network is little discussed or it is assumed that a project

* Department of Civil Engineering

network is already obtained. On the other hand, it is very important to make a project network for managing a large project, since it is often found that various project networks can be proposed for the same construction project.

All network systems for planning and scheduling involve the use of the project network model which is essentially a graphical representation of the job flow, and the job flow is based on the structure of the job sequence in the project. It is clear that the reason why various project networks can be constructed for the same construction project is based on the existence of the various job sequences.

In this paper, we will analyze these job sequences and establish a mathematical representation of the job sequence. Since the schedule of construction project is only a function of job sequence if the duration time of job in the project is given, all the resource requirements, project cost and any other characteristics of the project are computed as a function of job sequence.

In order to obtain a feasible project, a mathematical model will be establised, in which project network is defined as a function of job sequence and constraints are put for all restricted performances of characteristic in the project and to solve it the devised branch and bound method which has originally been developed as an optimal searching method and at the same time the sequencial filtering procedure will be introduced.

Feasibility can be easily examined by comparing the computational result of the characteristic of the project with actual restrictions. But the optimal project cannot uniquely be selected among those feasible alternatives until establishing optimality criterion.

When optimality criterion is established by the project manager, the optimal project can be uniquely selected. In this paper, methods for selecting an optimal project are discussed.

Applying this process to the planning of elevated railway construction on the New Sanyo Trunk Line in Hyogo district it was made clear that this integrating process is a general procedure which works well for construction planning.

2. The Job Sequence in the Project

If and only if the structure of job sequence is unique, the project network is also unique on the assumption that all job characteristics such as the duration time, requirements of every kind of resource and the cost of implementation and so on are predetermined by a certain procedure.

In the general construction project this structure is not unique. Constructing operations have to be performed according to a certain job sequence which is predetermined by the constructing technique strongly associated with a structural design and by the physical condition of a construction site.

We will show as an example of this, a job sequence in the foundation work with piling foundation. The job of driving piles has to precede the job of placing the base concrete as that of driving pile has preceded the job of erecting the form. Those job sequences always exist in all constructing works in spite of the difference of the scale of the construction and the amount of the operation. The assumption that this sequence is uniquely determined may be established without any trouble, if and only if only one construction technique is adopted.

We will call this job sequence the technical job sequence and represent it by the following sequence matrix X_0.

$$X_0=(x_{ij}^0), \tag{1}$$

where x_{ij}^0 implies the technical sequence between job i and job j such that

$$x_{ij}^0=\begin{cases}1, \text{ if } i<j,\\ 0, \text{ otherwise},\end{cases} \tag{2}$$
$$i,\ j\in J_P.$$

In the above equation the relation $i<j$ implies that job i has to precede job j with respect to constructing technique and that J_P is the set of jobs in a project P.

Each job in the project always needs several kinds of resource and the allocation of resources to those jobs have to be well managed for the implementation of efficient construction. The allocation of resources to jobs in the project is represented as the sequence of jobs which need these resources.

If and only if such an assumption is acceptable that the technical sequence X_0 is uniquely determined, the problem of the project planning is to find an optimal sequence by which resources are actually allocated to jobs in project.

We will call this job sequence the resource sequence and represent it by the sequence matrix X_k for the resource k such that

$$X_k=(x_{ij}^k),\quad k=1, 2, \cdots\cdots, r, \tag{3}$$

where x_{ij}^k implies the resource sequence for the resource k between the job i and j such that

$$x_{ij}^k=\begin{cases}1, \text{ if } i<j,\ i\in J_k \text{ and } j\in J_k,\\ 0, \text{ otherwise},\end{cases} \tag{4}$$
$$i,\ j\in J_P.$$

In this case the relation $i<j$ implies that the resource k allocated to job i

before job j and J_k is the set of jobs which need only the resource k satisfying the following relations.

$$J_k \cap J_h = \Phi \text{ (empty)}, \quad k \neq h, \quad k, h = 1, 2, \cdots\cdots, r. \tag{5}$$

Using these definitions, the structure of job sequence X_P is determined by adding the whole sequence matrices.

$$X_P = X_0 + X_R = X_0 + \sum_{k=1}^{r} X_k. \tag{6}$$

In the above equation the following arithmetic rule is applied.

$$\left.\begin{array}{l} 1+1=1, \\ 1+0=0+1=1, \\ 0+0=0. \end{array}\right\} \tag{7}$$

It is necessary that the resource sequence in the project network satisfies the following feasibility.

(1) Condition for Continuous Operation

It is necessary that the allocation of resource k to jobs included in the competitive job set J_k is feasible. When one competitive job which was allocated the resource k is finished, if and only if this job is not finally allocated one in the project, resource k has to be allocated to the next job in J_k.

This is the condition for the continuous operation and is represented as follows:

$$\left.\begin{array}{l} \sum_{i=1}^{N} x_{ij}^{k} \leqq 1, \\ \sum_{j=1}^{N} x_{ij}^{k} \leqq 1, \end{array}\right\} \tag{8}$$

where N is the total number of jobs in the project.

(2) Acyclic Condition

The acyclic condition is important for the project network. If the project network has a cycle, not only the project becomes infeasible, but also it will cause trouble when computing the schedule, namely these projects lead to the infinite or meaningless completion time.

As will be stated in detail in section **4**, successive approximating method for the computation of schedules is applied, where the acyclic condition is examined by a simple criterion. Therefore only the symbolic description that X_P and X_k are acyclic is shown as

$$L(X_P) = L(X_k) = 0. \tag{9}$$

(3) Condition for the Number of the Sequence

Connected with the condition for the continuous operation, the total number of resource sequences is restricted. Since the resource sequence divides competitive jobs which need this resource into several subsets same in number as the amount of each resource, the number of total resource sequences for resource k is to be determined as equal to the difference between the number of competitive job n_k and the amount of resource q_k.

$$\sum_{j=1}^{N}\sum_{i=1}^{N} x_{ij}^{k} = n_k - q_k. \tag{10}$$

3. Outline of the Sequencial Filtering Procedure

If it is assumed that jobs and their characteristics such as the amount of operational cost, the duration time and every kind of requirement for their operations are predetermined and further the technical sequence among them is established, the problem in the project planning is to find the resource sequence which satisfies all restrictions and the optimal criterion.

Before implementing a filtering procedure, we should obtain characteristics of the project which are associated with characteristics of jobs included in the project. These characteristics are classified into two groups; one group associated with the project duration and the project cost, which has to be evaluated to make the project better, and the other associated with requirements of the resources and the storage space, which has to be examined to see if they satisfy actual restrictions. The former is called the object of the project and the latter the restriction. Through many experiences and various analyses based on many experiments, we know there are an allowable range for the former, and quite the same with the latter. Actually there are many cases where we cannot give a longer project duration and the cost is limited because of financial reasons. Therefore in order to make systematic the planning process of the project by the filtering procedure, we put an allowable range or allowable state whose attainment is the aim of the project for all characteristics, and we represent them as restrictions, and thus we are going to evaluate effectiveness and feasibility of the project.

Let us assume m kinds of restriction as

$$RC = (RC_1, RC_2, \cdots\cdots, RC_m). \tag{11}$$

We can find the set of resource sequences A_i such that the resource sequence $X_R \in A_i$ is assured to satisfy the effectiveness or feasibility for only one restriction RC_i.

The set of resource sequences A^* in which the sequence $X_R \in A^*$ may satisfy all restrictions is obtained as an intersection of the set A_i.

$$A^* = \bigcap_i A_i. \tag{12}$$

If the set A^* is empty, there exists no resource sequence which is effective and feasible, then it is necessary to examine all restrictions and to improve them until the set A^* which is not empty is found.

From the equation (12) it is clear that set A^* is the subset of the set A_i for all i.

$$A^* \subseteqq A_i. \tag{13}$$

Let us define the set A as the set of resource sequences which satisfy only restrictions on the sequence represented as equations (8), (9) and (10).

From these relations we will construct the sequencial filtering procedure for finding the set A^* from the set A.

In the beginning of the procedure we obtain the set RA_1 of resource sequences which satisfy only the restriction RC_1. In this case the set RA_1 equal to the set $A_1 \subseteqq A$. If the set RA_1 is obtained, we can obtain the set RA_2 of resource sequences which are included in RA_1 and at the same time satisfy the restriction RC_2.

Since the set RA_2 is represented as

$$RA_2 = A_1 \cap A_2, \tag{14}$$

it is clear that the set A^* is included in the set RA_2.

$$A^* \subseteqq RA_2 \subseteqq RA_1. \tag{15}$$

If the set RA_2 is obtained, we can obtain the set RA_3 for the restriction RC_3 by the implementation of the same procedure. Thus we continue to implement this procedure for all restrictions so that the sequence of set RA_i, $(RA_1, RA_2 \cdots\cdots, RA_m)$, is obtained.

The set RA_i always satisfies the following relations;

$$\left.\begin{aligned} &RA_i = \bigcap_{k=1}^{i} A_k, \\ &A^* \subseteqq RA_i \subseteqq RA_{i-1}, \end{aligned}\right\} \tag{16}$$

it is clear that the set A^* is obtained as the set RA_m.

This is the outline of the sequencial filtering procedure. In each step of the recurrence of this procedure, resource sequences which are evaluated are extremely reduced from A_i to RA_i as the index i increases. Further it is possible to deal with the evaluation problem which is difficult to solve (because the technique for solving it is undeveloped) and it is also possible to select the alternative very efficiently.

4. Sequencing Problem for Finding Minimum Completion Time

As a general job shop scheduling problem, it is possible to obtain an optimal feasible job sequence with minimum completion time if the amount of useful resources in the project is given. In this section we will deal with the mathematical model in order to obtain an optimal resource sequence and develop the algorithmic method for solving this problem. Further, this method will be adopted in order to establish the first process of the sequencial filtering procedure.

4.1 Formulation of the Problem

Let λ be the completion time of the project. The completion time λ is a function of only the job sequence X_P when all of the duration time of job are given. Furthermore, since X_0 which composes X_P is predetermined and constant, λ becomes a function of only the set of resource sequences $X_1, X_2, \cdots\cdots, X_r$. Then we may set

$$\lambda=\lambda(X_1, X_2, \cdots\cdots, X_r).$$

Since the objective function of this problem is defined as the above equation, the problem to find the optimal resource sequence is formulated as follows: Minimize

$$\lambda=\lambda(X_1, X_2, \cdots\cdots, X_r) \tag{17}$$

subject to

$$\left.\begin{aligned}
&\sum_{i=1}^{N} x_{ij}^k \leqq 1,\\
&\sum_{j=1}^{N} x_{ij}^k \leqq 1,\\
&L(X_P)=L(X_k)=0,\\
&\sum_{j=1}^{N}\sum_{i=1}^{N} x_{ij}^k = n_k - q_k,\\
&k=1, 2, \cdots\cdots, r.
\end{aligned}\right\} \tag{18}$$

4.2 Branch and Bound Algorithm

We can view the problem of minimizing equation (17) subject to equation (18) as a combinatorial problem of each resource sequence. Since the objective function of this model is non-linear, it will be efficient to adopt the branch and bound algorithm.

Before establishing the branch and bound algorithm, we will rearrange competitive jobs and non-competitive jobs such that

$$\left.\begin{aligned} &J_1=\{j_l\,;\ l=1,2,\cdots\cdots,l_1\},\\ &J_2=\{j_l\,;\ l=l_1+1,l_1+2,\cdots\cdots,\ l_2\},\\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ &J_r=\{j_l\,;\ l=l_{r-1}+1,l_{r-1}+2,\cdots\cdots,l_r\},\\ &J_P-\bigcup_{k=1}^{r}J_k=\{j_l\,;\ l=l_r+1,l_r+2,\cdots\cdots,N\}, \end{aligned}\right\} \tag{19}$$

where l_k $(k=1,2,\cdots\cdots,r)$ is defined as

$$l_k=\sum_{i=1}^{k} n_i. \tag{20}$$

Let us suppose that l_r be a decision process for this combinatorial problem and introduce in each decision process the variables $I(l)$, $Y(l)$ and $Z(l)$ as follows:

$$\left.\begin{aligned} &\mathrm{I}(l)=\begin{cases} i, \text{ if } x_{ij_l}^k=1 \text{ and } x_{hj_l}^k=0 \text{ for all } h\neq i,\\ 0, \text{ if } x_{ij_l}^k=0 \text{ for all } i, \end{cases}\\ &Y(l)=\begin{cases} 1, \text{ if } I(l)\neq 0,\\ 0, \text{ othewise}, \end{cases}\\ &Z(l)=(I(1),I(2),\cdots\cdots,I(l)). \end{aligned}\right\} \tag{21}$$

Using these variables, we may set constraints on each process as shown in the following equations.

$$\left.\begin{aligned} &I(l)\neq I(l'),\ \text{if } Y(l')=1,\\ &\sum_{l=l_{k-1}}^{l_k} Y(l)=n_k-q_k,\ \text{for all } k,\\ &L(Z(l))=0. \end{aligned}\right\} \tag{22}$$

The objective function will be put

$$\lambda=\lambda(Z(l)). \tag{23}$$

If the number of variables in this problem is not many, it is efficient to obtain the solution by the enumeration method having a tree like diagram with branch and node.

In this diagram we define l as the level of node, where l is the number equal to that of branches included in the path from the starting node 0 to this node and assign each node a different combination of variables $Z(l)$.

In branch and bound algorithm we will use the tree like diagram implicitly. The level of node corresponds to each decision process l and the branch implies the new addition $I(l+1)$ to the combination $Z(l)$ obtained at source node. Suppose the combination $Z(l)$ corresponds to the node s where the level of node s is l, and E_s is the set of nodes which have node s as the source node of the branch, the combination $Z(l+1)$ at the sink node $t\in E_s$ is set

$$Z(l+1)=(Z(l), I(l+1)). \tag{24}$$

Setting the combination $Z(0)$ the empty set, we will proceed with the recurrence of the branching procedure which was stated above under the restriction shown in equation (22). In each process we compute the schedule S_l by using $Z(l)$, X_0, and duration time such as

$$S_l=S(Z(l), X_0, D), \tag{25}$$

where D denotes the set of duration time of jobs.

When the branching procedure is finished, all the feasible combinations that are feasible job sequences in the project are obtained at the final level l_r and it is easy to find the optimal combination with the minimum completion time from them.

Since the number of the feasible combinations extremely increases with the number of the competitive jobs, it is impossible to obtain the optimal solution by the enumeration method. In construction project there exist so many competitive jobs that it is almost impossible to adopt this enumeration method directly.

In the branch and bound algorithm the lower bound of the objective function will be adopted so as to reduce the number of enumeration efficiently. The lower bound of this model will be the completion time which as is referred to above and is obtained at each node that is computed by using the combination $Z(l)$.

Suppose $\lambda(Z(l))$ is the completion time at node s of level l, the completion time $\lambda(Z(l+1))$ at the node $t\in E_s$ is obtained as

$$\lambda(Z(l+1))=\lambda(Z(l))+\Delta\lambda(I(l+1)). \tag{26}$$

The increment $\Delta\lambda(I(l+1))$ is computed by the following equation:

$$\left.\begin{aligned}&\Delta\lambda(I(l+1))=\max\{EF_i^{(l)}-ES_{j_{l+1}}^{(l)}-TF_{j_{l+1}}^{(l)}, 0\},\\&I(l+1)=i,\end{aligned}\right\} \tag{27}$$

where $ES_{j_{l+1}}^{(l)}$, $EF_i^{(l)}$ and $TF_{j_{l+1}}^{(l)}$ are the earliest start time of job j_{l+1}, the earliest finish time of job i and the total float of job j_{l+1} respectively and they are obtained from the result of schedule S_l.

Since $\Delta\lambda(I(l+1))$ is non-negative, the completion time never decreases as the increase of level, it is possible to adopt this completion time as the lower bound.

Let $LB^{(s)}$ and $LB^{(t)}$ be the lower bound at node s and t respectively, the following relation is obtained for all $t\in E_s$.

$$LB^{(s)}\leqq LB^{(t)}, \tag{28}$$

where $LB^{(s)}$ is equal to $\lambda(Z(l^{(s)}))$ and $LB^{(t)}$ to $\lambda(Z(l^{(t)}))$. Furthermore the relation between these combinations is obtained as

$$Z(l^{(s)}) \subset Z(l^{(t)}), \tag{29}$$

where $l^{(s)}$ and $l^{(t)}$ are the level of node s and t respectively and $l^{(t)}=l^{(s)}+1$ if $t \in E_s$.

The relation in equation (28) can be extended to the general case in which $l^{(s)}<l^{(t)}$ if and only if equation (29) is satisfied.

Suppose that $LB^{(*)}$ is the current minimum of the completion time which is obtained at the node of the final level l_r and the relation

$$LB^{(s)} \geqq LB^{(*)} \tag{30}$$

is obtained at node s, the following relation is generally satisfied.

$$LB^{(t)} \geqq LB^{(*)}, \quad \text{if} \quad Z(l^{(s)}) \subset Z(l^{(t)}). \tag{31}$$

This relation is also useful in the case in which node t is at the final level l_r.

Since the object of this branch and bound algorithm is to find the combination with the lower bound which is less than the current minimum $LB^{(*)}$ and is to proceed the branching procedure until finding at least one optimal combination, we can introduce these relations as criteria in branching procedure, where all combinations are evaluated by those criteria and decided whether necessary to proceed by the branching procedure from the node s or not.

In the following, we will show the branch and bound algorithm. Let us define the following sets.

$$V_1=\{s\,;\ Z(l^{(s)}) \text{ is feasible and } LB^{(s)}<LB^{(*)}\},$$

$$V_2=\{s\,;\ Z(l^{(s)}) \text{ is feasible and } LB^{(s)} \geqq LB^{(*)}\},$$

$$V_3=\{s\,;\ Z(l^{(s)}) \text{ is infeasible}\}.$$

The branching procedure is used only for the node s having the lower bound $LB^{(s)}$ which satisfies the relation such that

$$LB^{(s)}<LB^{(*)}. \tag{32}$$

Further when we proceed with the branching, it is not necessary to compute the schedule for all nodes which only need the value of the lower bound, except the node which will be the source node for the next branching. The lower bound can be easily computed by equations (26) and (27), where only the schedule of the source node is useful.

The lower bound $LB^{(*)}$ is replaced by the lower bound $LB^{(s)}$ if and only if node s is at the final level l_r and satisfies the equation (30). Proceeding in this way we can find the optimal combination which gives the current minimum lower bound at the node of final level, if any node which satisfies equation (32) does not exist.

The set V_1 includes nodes from which it is necessary to proceed with the branching procedure and both the set V_2 and V_3 include nodes from which it is not necessary to proceed with the branching procedure. Of course in the initial state all of them are empty and in the final state only the set V_1 is empty.

step 1

Set the initial condition as follows:

1. $LB^{(*)}=\infty$ (infinite),
2. $s=0$,
3. $l^{(s)}=0$,
4. $Z(l^{(s)})=\Phi$ (empty),
5. $V_1=V_2=V_3=\Phi$ (empty).

step 2

Compute the schedule at node s by using $Z(l^{(s)})$, X_0 and D.

step 3

If $L(Z(l^{(s)}))\neq 0$, replace V_3 by $s\cup V_3$ and V_1 by V_1-s and go on to step 8. If $L(Z(l^{(s)}))=0$ ($Z(l^{(s)})$ gives a cyclic job sequence), go to the next step.

step 4

Branch from node s and construct the set E_s. For all $t\in E_s$ compute the level $l^{(t)}$ and the combination $Z(l^{(t)})$ by using equation (24).

step 5

Decide each combination whether feasible or infeasible by the constraint (22) corresponding to resource k where $l_{k-1}\leqq l^{(t)}<l_k$, and rearrange V_3 by adding to V_3 the node which has an infeasible combination.

step 6

Compute the lower bound $LB^{(t)}$ for all $t\in E_s-(E_s\cap V_3)$ by equations (26) and (27).

step 7

Rearrange V_1 and V_3 corresponding to the relation between $LB^{(t)}$ and $LB^{(*)}$. If V_1 is empty, go to step 11. If V_1 is not empty, go to the next step.

step 8

Determine the set of nodes W from which the source node for the next branching will be selected and go to the next step. The set W is obtained such that

$$W=\begin{cases} E_s\cap V_1, & \text{if } E_s\cap V_1\neq\Phi \text{ (empty)},\\ V_1\ , & \text{if } E_s\cap V_1=\Phi. \end{cases}$$

step 9

Select the source node t^* for the next branching by the following equation.

$$LB^{(t^*)}=\min\{LB^{(t)}\ ;\ t\in W\}.$$

step 10

If $l^{(t^*)}$ is equal to final level l_r, compute the schedule at node t^*. If $L(Z(l^{(t^*)})) \neq 0$, replace V_1 by $V_1 - t^*$, V_3 by $t^* \cup V_3$ and go back to step 8 and if not, replace $LB^{(*)}$ by $LB^{(t^*)}$, W by V_1 and go back to step 9. If $l^{(t^*)}$ is less than l_r, replace s by t^* and go back to step 2.

step 11

Compute the schedule S^* by using Z^* which gives the current minimum lower bound $LB^{(*)}$.

As was stated previously, we can find the optimal combination and schedule with less effort by searching the optimal feasible combinations by the branch and bound algorithm. The algorithm, however, can be made more efficient by the device concerned with the initial value of the lower bound $LB^{(*)}$; it is not necessary to put $LB^{(*)}$ infinite value at the initial state if we can find at least one feasible combination by trial and error method or by various kinds of approximation method which has been developed already. Sometimes we can obtain the feasible combination with ease, which suggests us that it is worth trying to find a feasible combination before starting the procedure.

4.3 Computation of the Schedule by the Successive Approximation Method

Though the mathematical basis and techniques of PERT have been well established, its program is of little efficiency for the network which has node numbers without topological ordering as well as for the network of project graph type as used in this study, because of such tremendous implementations to draw the network into the arrow diagram and to make the topological ordering. In the following, therefore, we will devise the computation technique.

The successive approximating method based on Bellman's maximum principle will be used to make the program more efficient and to scan the acyclic condition by the use of a certain criterion.

Let us consider a schedule that may be found by a certain method. If and only if the project network has no cyclic sequence, the earliest finish time satisfies the following functional equation.

$$\left.\begin{aligned} &EF_j = D_j + \max_i \{x_{ij} EF_i\}, \\ &i, j = 1, 2, \cdots\cdots, N, \end{aligned}\right\} \quad (33)$$

where D_j is the duration time of job j.

Since we contemplate solving the equation (33) by means of various successive approximation techniques, it is important to show the uniqueness of the solution. In this case, however, it is easy to prove the uniqueness of the solution. The earliest finish time EF_j is a maximum length of path selected among paths from such jobs to job j that these jobs have no preceding job. The length of the minimum path is finite and unique if the network has no cyclic sequence, since the number of jobs which are included in this path are finite and the length is only the summation of the duration time of those jobs.

We can also use the successive approximation method to establish the existence of solution in the system of equations (33) and to provide a practical computational scheme. As our initial approximation $EF_j^{(0)}$ of job j, we make them equal to its duration time such that

$$EF_j^{(0)}=D_j, \quad j=1, 2, \cdots\cdots, N. \tag{34}$$

The higher-order approximations are obtained in the usual way,

$$\left.\begin{aligned} &EF_j^{(k+1)}=D_j+\max_i\{x_{ij}EF_i^{(k)}\}, \\ &i, j=1, 2, \cdots\cdots, N, \end{aligned}\right\} \tag{35}$$

for $k=1, 2, \cdots\cdots$. From this equation it follows that the approximations are monotone increasing,

$$EF_j^{(k)} \leqq EF_j^{(k+1)}, \quad j=1, 2, \cdots\cdots, N, \tag{36}$$

which is easy to establish by induction. Since, if the network has no cyclic sequence, the schedule is feasible and each $EF_j^{(k)}$ has a finite value, the convergence of the approximating sequence is established. As a matter of fact, however, a path which includes the maximum number of jobs exists and its number is at most $N-1$.

Since at stage k the length of certain paths which include jobs in themselves will converge, the convergence of the process is assumed after at most $N-1$ stages. After convergence at stage $\bar{k}$, we can find the following equation;

$$EF_j^{(\bar{k})}=EF_j^{(\bar{k}+1)}, \quad j=1, 2, \cdots\cdots, N. \tag{37}$$

If the project network has cyclic sequences, the convergence of the process cannot be assumed over $N-1$ stages since the length of path will be infinite. Thus we can scan the acyclic condition of the project network to know whether the number of recursive stages is less than $N-1$ or not.

Concerning the latest start time, the same procedure can be applied. In this case, however, the initial approximation $LS_i^{(0)}$ of job i is set so that

$$LS_i^{(0)}=\lambda, \quad i=1, 2, \cdots\cdots, N, \tag{38}$$

where λ is the completion time of the project and inducted as a maximum value of earliest finish time.

$$\lambda=\max_j\{EF_j\}, \quad j=1, 2, \cdots\cdots, N. \tag{39}$$

The higher-order approximations are obtained as follows:

$$\left.\begin{array}{l} LS_i^{(k+1)} = -D_i + \min_j\{x_{ij} LS_j^{(k)}\}, \\ i, j = 1, 2, \cdots\cdots, N. \end{array}\right\} \quad (40)$$

The convergence in this case is assumed after at most $N-1$ stages as well as in the case of EF_j.

Other schedules such as the earliest start time ES_i, the latest finish time LF_i, the total float TF_i and the free float of job FF_i will be computed by the following equations since both the earliest finish time and the latest start time are already obtained.

$$\left.\begin{array}{l} ES_i = EF_i - D_i, \\ LF_i = LS_i + D_i, \\ TF_i = LF_i - ES_i - D_i, \\ FF_i = -EF_i + \min_j\{x_{ij} ES_j\}, \\ i, j = 1, 2, \cdots\cdots, N. \end{array}\right\} \quad (41)$$

5. Planning Process

5.1 Constructing Alternatives

Since we have discussed the sequencial filtering procedure as a general procedure in section 3, it is not satisfactory to apply this procedure to deal with the general evaluation in the construction planning. There are some characteristics of the project which have strong interrelation and characteristics have different properties such that some of them can be evaluated at the same time and others cannot. Further we can divide them by knowing whether they are important or not in view of integrating the plan.

There are many characteristics which are to be evaluated in the construction project. However, if the project is defined as a function of resource sequence on the assumption that all job characteristics have already been obtained, all of the project characteristics are represented as functions of resource sequence.

As referred above, it is efficient to divide them into a set of characteristics which must be evaluated when we construct the alternatives of the job sequence as well as the set of characteristics which are easier to evaluate after obtaining alternatives of the job sequence.

In the practical filtering procedure as is dealt with in this study, we construct a set of feasible and effective alternatives, RA_1, of the job sequence under the restriction RC_1 for specified characteristics which act on the project as a strong restriction, and compute values of conditions of characteristics for the performance of them for every alternative $X_R \in RA_1$ of the job sequence and evaluate the alternative $X_R \in RA_1$ by examining whether computational results satisfy the restriction RC_2 in order to obtain the set of alternatives $A^* = RA_2$.

In this study we adopt project characteristics which have the restriction RC_1, the completion time of the project and the combination of both efficiency and amount of machines, and that of both the class and amount of facilities and equipments.

These characteristics act strongly on the effectiveness of the project. Concerning the completion time, the longest allowable completion time is often given from planners and further the project manager may want a short completion time for the purpose of reducing the project cost, since the completion time is associated strongly with the project cost. Thus there exists an allowable range for the completion time.

Recently, the machine, facility and equipment became so expensive that we must avoid wasting their operations. Further since their allocations are represented directly as resource sequences, the evaluation of this characteristic must precede others. Since not so much freedom is admissible in selecting their combination, it is strongly restricted from the technical point of view and the restriction of their working areas, and it is necessary to treat the combination B as a parameter in the project as $(B_1, B_2, \cdots\cdots)$.

In the following we will show the procedure for constructing alternatives RA_1. At the beginning the combination B_1 is given for the model which was dealt with in section **4** as the input. We can soon find that the optimal resource sequence with the minimum completion time LB_{opt} by branch and bound algorithm.

If we obtain the minimum completion time LB_{opt}, we replace the current lower bound $LB^{(*)}$ operationally by T, where $T > LB_{opt}$. The brach and bound algorithm is continued with the new lower bound $LB^{(*)} = T$ and hence forth this lower bound $LB^{(*)}$ may be kept until the end of the algorithm.

When the algorithm finishes, only the set of resource sequences, $RA_1^{(1)}$, with the completion time λ which satisfies the relation,

$$LB_{opt} \leqq \lambda < T, \tag{42}$$

are obtained at final level l_r.

The upper bound of the completion time, T, is sometimes the longest completion time T_g which is externally given from planners, but it can be given in trial when the manager can decide this based on his experience that it has an allowable range δ such that the range satisfies the relation

$$T_g \geqq T = LB_{opt} + \delta. \tag{43}$$

After the set of resource sequences $RA_1^{(1)}$ for combination B_1 was obtained, we continue the same procedure for the next combination B_2 in order to obtain the set $RA_1^{(2)}$.

When the procedure finishes for all combinations, the set of the resource sequences RA_1 is obtained by the following equation.

$$RA_1 = \bigcup_k RA_1^{(k)}. \tag{44}$$

5.2 Computation of Characteristics of the Project

We have obtained the set of alternatives of the resource sequence which satisfies restrictions concerned with the completion time and the combination of the machine, facility and equipment.

In the next step we will compute all characteristics in order to know their performances through the project duration. Since restricted conditions are established beforehand for each performance of the characteristics, their effectiveness and feasibility can be easily evaluated by comparing the computational result with these restricted conditions.

If the performance of jobs in the project during their implementations is obtained, each performance through the project duration is easily computed. The performance of jobs in the project is shown in the schedule, which is computed by the use of the resource sequence X_R included in the set of alternatives RA_1.

Let us define $\alpha_i(t)$ for job i as the index which has the value 1 if job i is implemented at time t and has value 0 if not. The performance of the characteristic k at time t is given as

$$R_k(t) = \sum_{i=1}^{N} \alpha_i(t) r_{ik}, \tag{45}$$

where r_{ik} denotes the value of the characteristic k which is given to job i.

Using this information $R_k(t)$, we are going to evaluate the performance of all characteristics from the viewpoint of integration.

The performance which is useful for the evaluation of the construction project is given for each characteristic.

(1) Project Cost

The project manager should try to implement the construction project with the minimum project cost and combine the time and cost dimensions to forecast the rate of spending on a project. A construction schedule may be used to estimate the amount of funds that a manager must provide in financing a project during construction.

The information about them can be easily obtained by computing the direct cost and indirect cost which contribute to the total project cost.

The direct cost is spent for implementations of all jobs included in the project. Since all kinds of cost spent for implementations of each job are already estimated as the job characteristics, the direct cost spent for one day is obtained as

$$C(t)=\sum_{i=1}^{N}\alpha_i(t)C_i, \tag{46}$$

where C_i denotes the summation of all kinds of cost which is spent on the implementation of job i for one day.

For the evaluation of the general performance of the direct cost it is more convenient to get its cumulative $C_D(t)$ at the project time t. The cumulative of the direct cost is obtained as

$$C_D(t)=\int_0^t C(\tau)d\tau. \tag{47}$$

The indirect cost also contributes to the total project cost such as overhead and distributives and perhaps a penalty for not completing the project or a portion of it by a certain time. These external costs must also be taken into account when the manager plans how the project should be implemented relative to over-all objectives. Since the major portion of the external cost usually varies only with duration of the project, we define this cost by $C_I(\lambda)$.

The total project cost $C(\lambda)$ is obtained by using these costs as follows:

$$C(\lambda)=C_D(\lambda)+C_I(\lambda). \tag{48}$$

If the manager cannot provide the fund $F(t_2)$ at project time t_2 for the expenditure $C(t_2)-F(t_1)$, such that

$$F(t_2)\geqq C(t_2)-F(t_1), \tag{49}$$

where $F(t_1)$ is the cost already provided by the manager at time $t_1<t_2$, the project cannot be implemented without providing the fund equal to the shortage $C(t_2)-F(t_1)-F(t_2)$ additionally.

(2) Manpower

In recent years the amount of skilled labour comes to be short compared to the increment of the amount of construction and it becomes a serious problem to secure sufficient skilled labours which are needed by the construction project. When a construction project is planned, it is important to get the information beforehand concerning when and how many skilled labours we can secure. Therefore at the planning stage of a project the maximum available amount of skilled labour is to be regarded as a restriction.

Further the pattern of day by day requirements of each skilled labour is very important for securing it. If the pattern has a smooth wave with a large period, the manager can secure skilled labours and easily prepare the facilities for them.

To evaluate these restrictions it is necessary to get the information on day by day requirements of various kinds of skilled labour and the maximum requirement during the project duration.

If we designate the required amount of skilled labour k as l_{ik} by which job i is implemented, the total requirement of skilled labour at project time t and the maximum requirement L_k^* are represented as follows:

$$\left.\begin{aligned} L_k(t) &= \sum_{i=1}^{N} \alpha_i(t) l_{ik}, \\ L_k^* &= \max_t \{L_k(t)\}. \end{aligned}\right\} \tag{50}$$

(3) Machine, Facility and Equipment

The machine, facility and equipment, especially big machines and special equipment, become so expensive that the manager will try to reduce the cost by reducing their idle time. Further the manager finds it so difficult to deliver them to the construction site as soon as they are needed. Therefore the schedule of their operation must be done so that they may begin to operate when needed and their idle time may shift to the minimum.

In order to evaluate the schedule of the machine, facility and equipment performances of them during the project duration have to be prepared as the relation between the day by day requirement and the available amount prepared for the project.

Since the day by day requirement is obtained by the equation (45), the total waste is computed as W_k:

$$W_k = Q_k(t_2 - t_1) - \int_{t_2}^{t_1} R_k(t)dt, \tag{51}$$

where Q_k is the maximum available and the time t_1 and t_2 are the start time and the finish time of its operation in the construction site respectively. Further the maximum requirement can never exceed Q_k.

(4) Material

Materials should be delivered before they are needed. Excessive early delivery is, however, not desirable because of the possibility that the materials might deteriorate or might congest working areas in which storage space is limited.

Since the material m_{ik} which is needed for one day when job i is implemented, the total requirement of material k at the project time t is computed as a cumulative $M_k(t)$ in the following fashion.

$$M_k(t)=\int_0^t\left\{\sum_{i=1}^{N}\alpha_i(\tau)m_{ik}\right\}d\tau. \tag{52}$$

If the cumulative of the amount of the delivered material at the project time t is $O_k(t)$, $M_k(t)$ should never exceed $O_k(t)$ and storage is obtained as $O_k(t)-M_k(t)$. The manager always has to order the material k so that the storage space $O_k(t)-M_k(t)$ can be secured.

5.3 Finding the Optimal Project

In 5.2 it was shown what performance of each characteristic should be evaluated and what conditions should be established as restrictions on the cost, manpower, machine, facility, equipment and material.

As is shown in the filtering procedure we can find a set of feasible alternatives A^* on examining whether restrictions are satisfied or not.

If we know that A^* is not empty, we can proceed to the optimality test, but if A^* is empty, we have to extend the allowable range of the restrictions in order to obtain at least one feasible project.

There may be priority among restrictions when the manager extends the allowable range where the extention is prior to others as the priority level increases. In general priority can be established as follows:

priority level 1 cost,
priority level 2 manpower,
priority level 3 machine,
priority level 4 otherwise.

Since every alternative included in A^* satisfies all restrictions, no trouble occurs in implementation of the project. But the manager wants to select the optimal project prior to other alternatives.

There may be two different methods for selecting the optimal project. One is the general method, in which only one object such as the total project cost is established and at least one alternative is selected so that its value is minimum.

In the case of the other method every performance is classified into h states and each state of performance is assigned a certain value which is selected from discrete numbers 1 to h so that its value increases with the increase of its desirableness. The weighted value U_{ik} is assigned to performance k of alternative i according to its class.

The weighted function U_i is defined for every alternative i as

$$U_i=\sum_k U_{ik}. \tag{53}$$

Then the optimal project can be selected so that the value of its weighted function may have a maximum value.

It is possible that both the priority and weighted value are uniquely established as the policy, and also possible that the manager adopts them according to his advantage.

6. Application to the Planning of Elevated Railway Construction of the New Sanyo Trunk Line

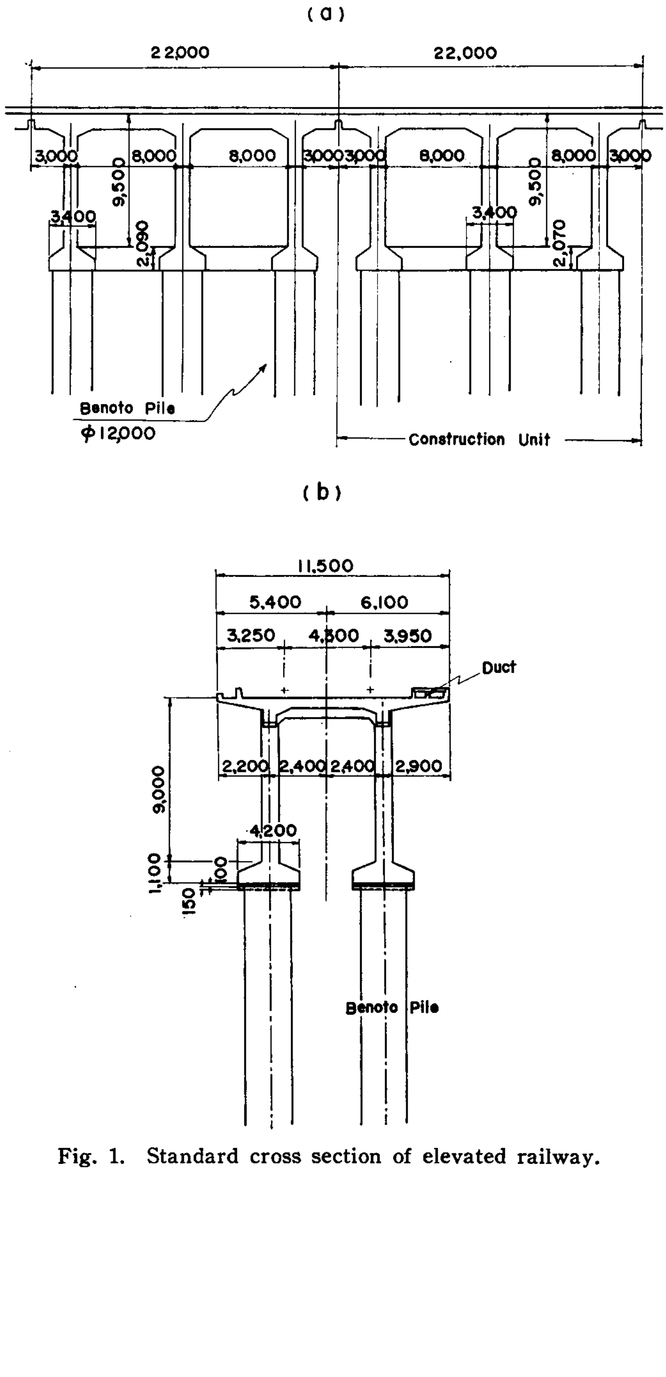

Fig. 1. Standard cross section of elevated railway.

In the post war days Japan has made great economical progress, so the amount of transportation of both passengers and freight across its country has seen tremendous increases. Then it is forecasted that the capacity of the transportation throughout the country will be insufficient in the future and to solve this problem as well as to promote social and economical advancement, a project has recently been planned, which aims at arranging and promoting systematically the transportation network throughout the country. As is well known, the New Tokaido Trunk Line was constructed between Tokyo and Osaka which is now contributing much to the society by transporting a great many passengers and much freight. For the same purpose as the New Tokaido Trunk Line, the New Sanyo Trunk Line was planned aiming at the increase of transportation in Western Japan and is now under construction between Osaka city and Okayama city.

Using the planning process proposed in this paper, we will plan the elevated railway construction of the New Sanyo Trunk Line in Hyogo district.

The structure adopted for the elevated railway is shown in Fig. 1 using a reinforced concrete structure with a benoto piling foundation and there are 36 spans with the same structure.

On implementation of the construction continuous three spans of the structure are defined as a construction unit, therefore the project includes twelve construction units. Each unit has the same kind of job and is constructed in the same technical job sequence as is shown in Fig. 2.

The project includes all jobs by which each construction unit is constructed and also includes the preparatory work which precedes all the other jobs and the finishing work succeeding all other jobs. For the preparation to implement

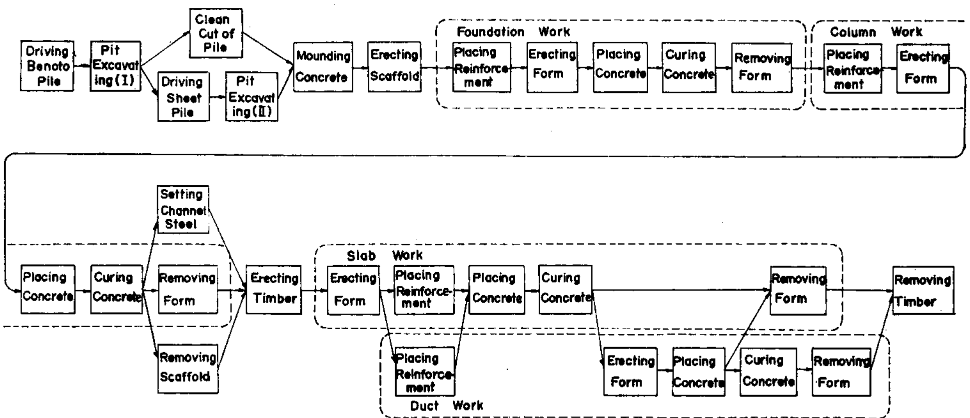

Fig. 2. Job sequence in construction unit.

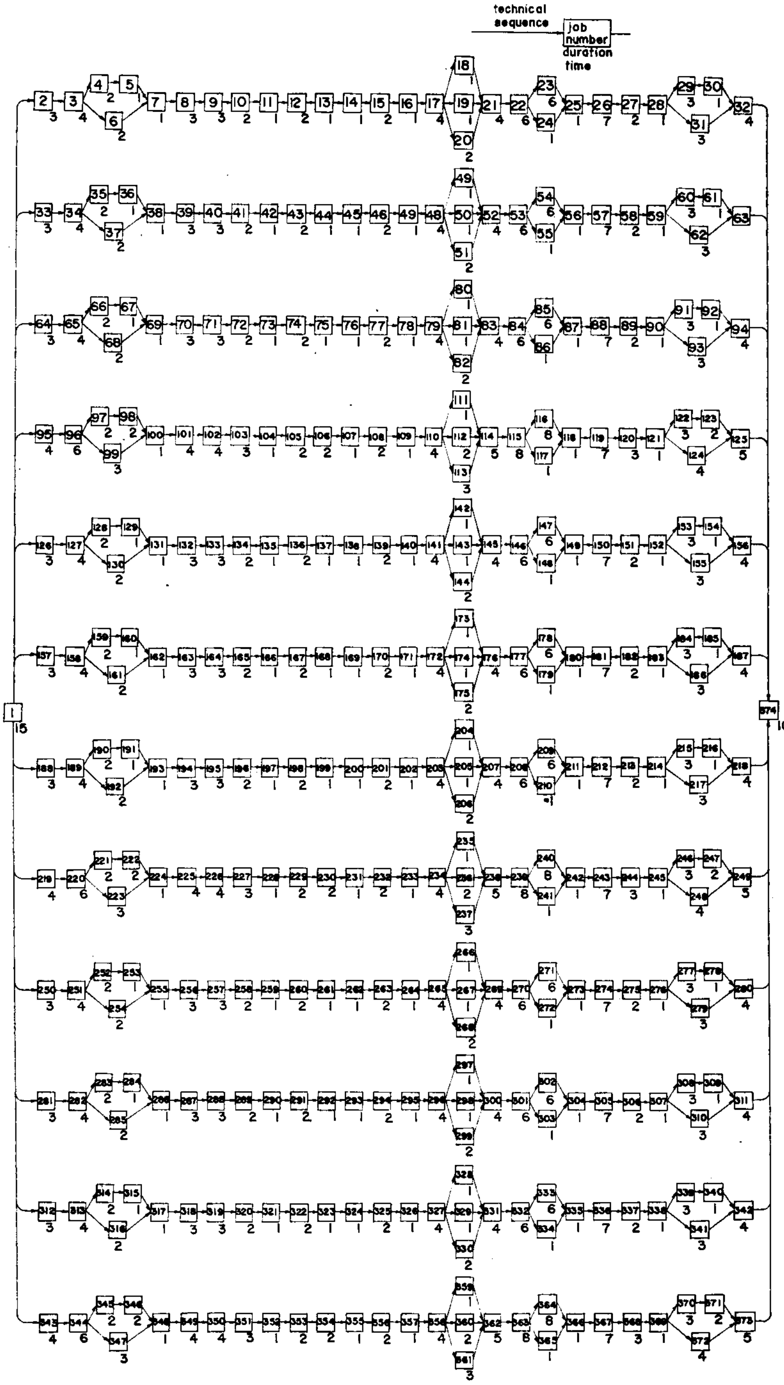

Fig. 3. Network based on technical job sequence.

Table 1 Twelve alternatives of resource sequence.

Alteranative																							
1		2		3		4		5		6		7		8		9		10		11		12	
I	J	I	J	I	J	I	J	I	J	I	J	I	J	I	J	I	J	I	J	I	J	I	J
2	95	2	64	2	95	2	64	2	33	2	33	95	2	95	2	64	2	64	2	95	64	2	33
95	33	64	95	95	64	64	33	33	95	33	64	2	64	2	33	2	95	2	33	64	2	33	64
33	64	95	33	64	33	33	95	95	64	64	95	64	33	33	64	95	33	33	95	2	33	64	95
64	126	33	126	33	126	95	126	64	126	95	126	33	219	64	219	33	188	95	188	33	219	95	126
126	219	126	188	126	219	126	188	126	157	126	157	219	126	219	126	188	126	188	126	219	188	126	157
219	157	188	219	219	188	188	157	157	219	157	188	126	188	126	157	126	219	126	157	188	126	157	188
157	188	219	157	188	157	157	219	219	188	188	219	188	157	157	188	219	157	157	219	126	157	188	219
188	250	157	250	157	250	219	250	188	250	219	250	157	343	188	343	157	312	219	312	157	343	219	250
250	343	250	312	250	343	250	312	250	281	250	281	343	250	343	250	312	250	312	250	343	312	250	281
343	281	312	343	343	312	312	281	281	343	281	312	250	312	250	281	250	343	250	281	312	250	281	312
281	312	343	281	312	281	281	343	343	312	312	343	312	281	281	312	343	281	281	343	250	281	312	343
3	65	3	65	3	65	3	65	3	65	3	65	3	65	3	65	3	65	3	65	65	3	3	65
65	96	65	96	65	96	65	96	65	96	65	96	65	96	65	96	65	96	65	96	3	96	65	96
96	34	96	34	96	34	96	34	96	34	96	34	96	34	96	34	96	34	96	34	96	34	96	34
34	127	34	127	34	127	34	127	34	127	34	127	34	127	34	127	34	127	34	127	34	189	34	127
127	189	127	189	127	189	127	189	127	189	127	189	127	189	127	189	127	189	127	189	189	127	127	189
189	220	189	220	189	220	189	220	189	220	189	220	189	220	189	220	189	220	189	220	127	220	189	220
220	158	220	158	220	158	220	158	220	158	220	158	220	158	220	158	220	158	220	158	220	158	220	158
158	251	158	251	158	251	158	251	158	251	158	251	158	251	158	251	158	251	158	251	158	313	158	251
251	313	251	313	251	313	251	313	251	313	251	313	251	313	251	313	251	313	251	313	313	251	251	313
313	344	313	344	313	344	313	344	313	344	313	344	313	344	313	344	313	344	313	344	251	344	313	344
344	282	344	282	344	282	344	282	344	282	344	282	344	282	344	282	344	282	344	282	344	282	344	282
13	72	13	72	13	72	13	72	13	72	13	72	13	72	13	72	13	72	13	72	13	72	13	72
75	103	75	103	75	103	75	103	75	103	75	103	75	103	75	103	75	103	75	103	75	103	75	103
106	41	106	41	106	41	106	41	106	41	106	41	106	41	106	41	106	41	106	41	106	41	106	41
44	134	44	134	44	134	44	134	44	134	44	134	44	134	44	134	44	134	44	134	44	134	44	134

137	196	137	196	137	196	137	196	137	196	137	196	137	196	137	196	137	196	137	196	137	196	137	196
199	227	199	227	199	227	199	227	199	227	199	227	199	227	199	227	199	227	199	227	199	227	199	227
230	165	230	165	230	165	230	165	230	165	230	165	230	165	230	165	230	165	230	165	230	165	230	165
168	258	168	258	168	258	168	258	168	258	168	258	168	258	168	258	168	258	168	258	168	258	168	258
261	320	261	320	261	320	261	320	261	320	261	320	261	320	261	320	261	320	261	320	261	320	261	320
323	351	323	351	323	351	323	351	323	351	323	351	323	351	323	351	323	351	323	351	323	351	323	351
354	289	354	289	354	289	354	289	354	289	354	289	354	289	354	289	354	289	354	289	354	289	354	289
18	77	18	77	18	77	18	77	18	77	18	77	18	77	18	77	18	77	18	77	18	77	18	77
80	108	80	108	80	108	80	108	80	108	80	108	80	108	80	108	80	108	80	108	80	108	80	108
111	46	111	46	111	46	111	46	111	46	111	46	111	46	111	46	111	46	111	46	111	46	111	46
49	139	49	139	49	139	49	139	49	139	49	139	49	139	49	139	49	139	49	139	49	139	49	139
142	201	142	201	142	201	142	201	142	201	142	201	142	201	142	201	142	201	142	201	142	201	142	201
204	232	204	232	204	232	204	232	204	232	204	232	204	232	204	232	204	232	204	232	204	232	204	232
235	170	235	170	235	170	235	170	235	170	235	170	235	170	235	170	235	170	235	170	235	170	235	170
173	263	173	263	173	263	173	263	173	263	173	263	173	263	173	263	173	263	173	263	173	263	173	263
266	325	266	325	266	325	266	325	266	325	266	325	266	325	266	325	266	325	266	325	266	325	266	325
328	356	328	356	328	356	328	356	328	356	328	356	328	356	328	356	328	356	328	356	328	356	328	356
359	294	359	294	359	294	359	294	359	294	359	294	359	294	359	294	359	294	359	294	359	294	359	294
30	89	30	89	30	89	30	89	30	89	30	89	30	89	30	89	30	89	30	89	30	89	30	89
92	58	92	58	92	58	92	58	92	58	92	58	92	58	92	58	92	58	92	58	92	58	92	120
61	120	61	120	61	120	61	120	61	120	61	120	61	120	61	120	61	120	61	120	60	120	123	58
123	151	123	151	123	151	123	151	123	151	123	151	123	151	123	151	123	151	123	151	123	151	61	151
154	213	154	213	154	213	154	213	154	213	154	213	154	213	154	213	154	213	154	213	154	213	154	213
216	182	216	182	216	182	216	182	216	182	216	182	216	182	216	182	216	182	216	182	216	182	216	244
185	244	185	244	185	244	185	244	185	244	185	244	185	244	185	244	185	244	185	244	185	244	247	182
247	275	247	275	247	275	247	275	247	275	247	275	247	275	247	275	247	275	247	275	247	275	185	275
278	337	278	337	278	337	278	338	278	338	278	338	278	338	278	338	278	338	278	338	278	338	278	337
340	306	340	306	340	306	340	306	340	306	340	306	340	306	340	306	340	306	340	306	340	306	340	368
309	368	309	368	309	368	309	368	309	368	309	368	309	368	309	668	309	368	309	368	309	368	371	306

(In this table I and J are denoted as predecessor and successor respectively) the process we obtain the network constructed only according to the technical sequence among all jobs (Fig. 3) and all characteristics of jobs.

The following five kinds of characteristic are adopted as resources in the mathematical model dealt with in section **4**.

k	resource	amount q_k
1	the benoto piling machine	1
2	the pit excavator	1
3	the form for placing the foundation concrete	1
4	the form for placing the column concrete	1
5	the form for placing the duct concrete	1

In order to obtain the set of resource sequences RA_1 by applying the filtering procedure we set the restriction on the completion time as the longest duration of 200 days and adopt only one combination of resources in this case. As the computational result we can obtain the twelve alternatives shown in Table 1.

For every alternative we compute all performances which are shown in section **5** by using the information of job characteristics.

After excluding the infeasible project by the filtering procedure two alternatives included in $A^*=RA_2$ are obtained, namely the alternative 1 and 11.

In order to select the optimal project we define the total project cost as the object which is desirable to minimize. Since the total project cost of the alternative 11 (146 million yen) is less than that of the alternative 1, the optimal project is the alternative 11.

The network of optimal project is shown in Fig. 4. Performance of project cost (cumulative), day by day requirement of carpenters, the form for placing slab concrete and reinforcement and plans for them are illustrated in Fig. 5.

7. Concluding Remarks

In this paper we have dealt with the planning process of the construction project. The planning process consists of three processes. In the first process the set of alternatives which satisfy the restriction on the completion time is obtained by the use of the devised branch and bound method. In the second process every kind of performance which is useful for the evaluation of construction project is computed for each alternative and further restrictions are established for all performances. In the last process alternatives which satisfy all restrictions are obtained by the filtering procedure and the optimal project is selected by a certain criterion.

In the application of this planning process, computation has been executed

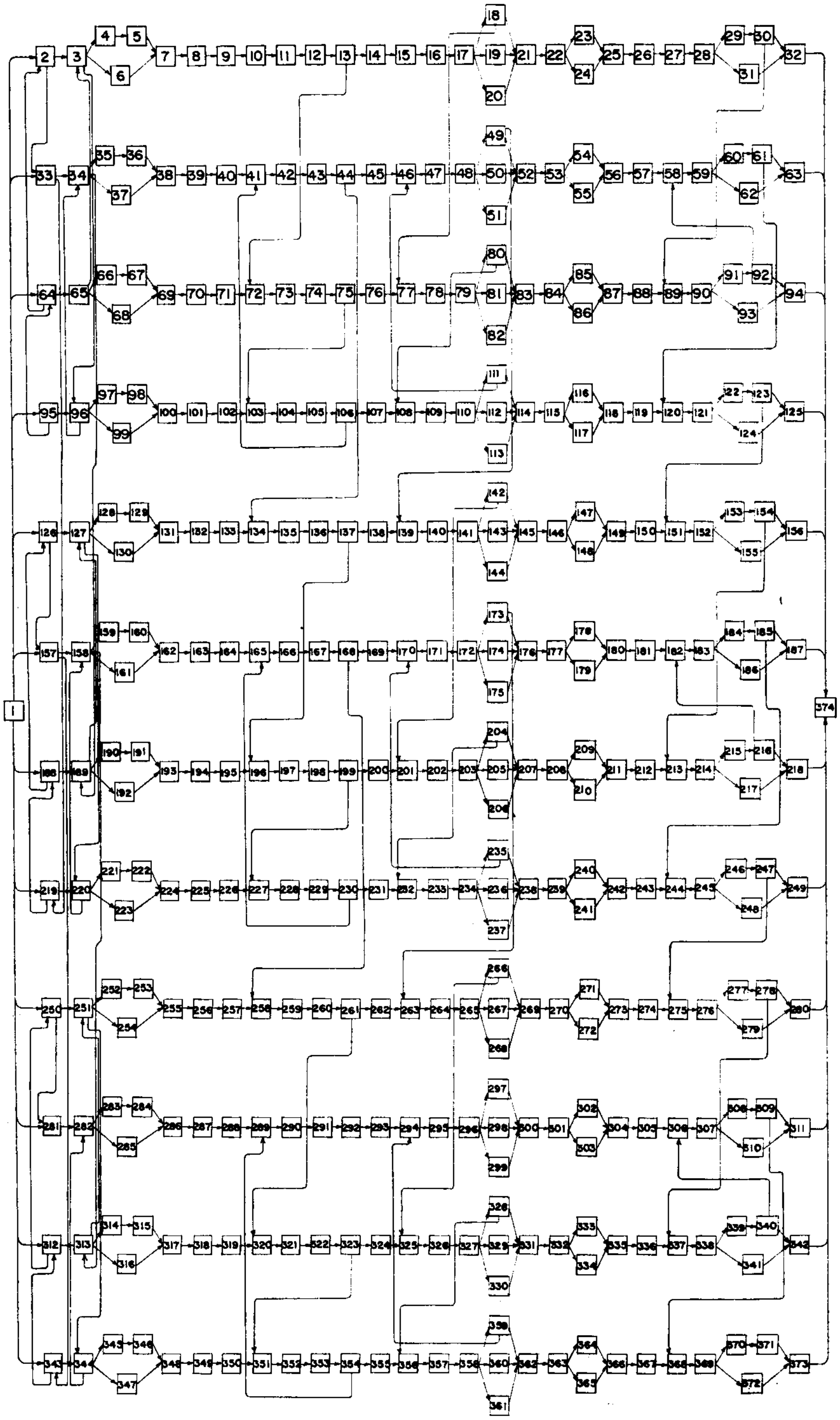

Fig. 4. Optimal project network.

on an automatic degital computer FACOM 230-60 at Kyoto University.

In this study we adopted the man-machine system through computation because of the shortage of computing time and a little difficulty of evaluation, but it is desirable to develop the automatic planning system.

In order to develop the automatic planning system by the automatic degital

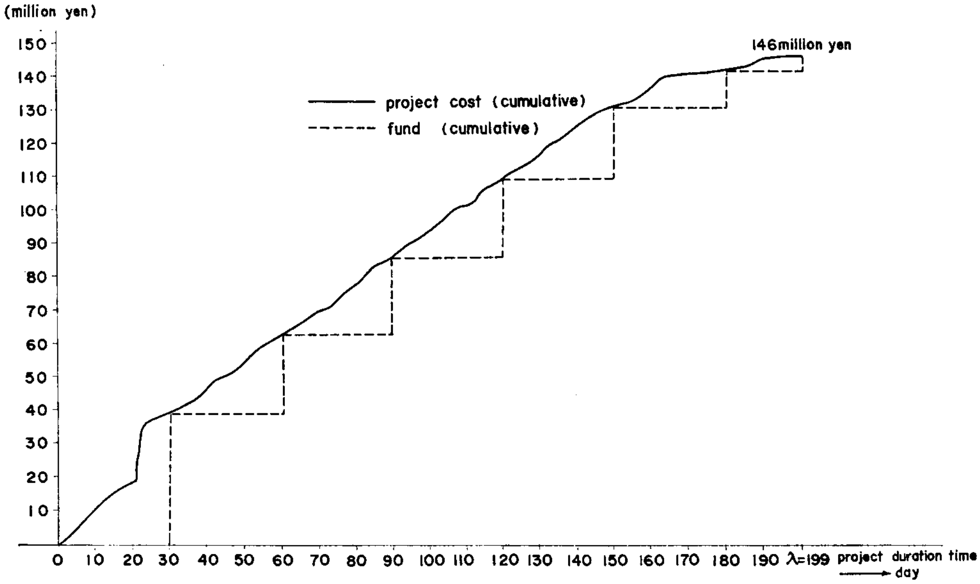

Fig. 5 (a). Project cost curve

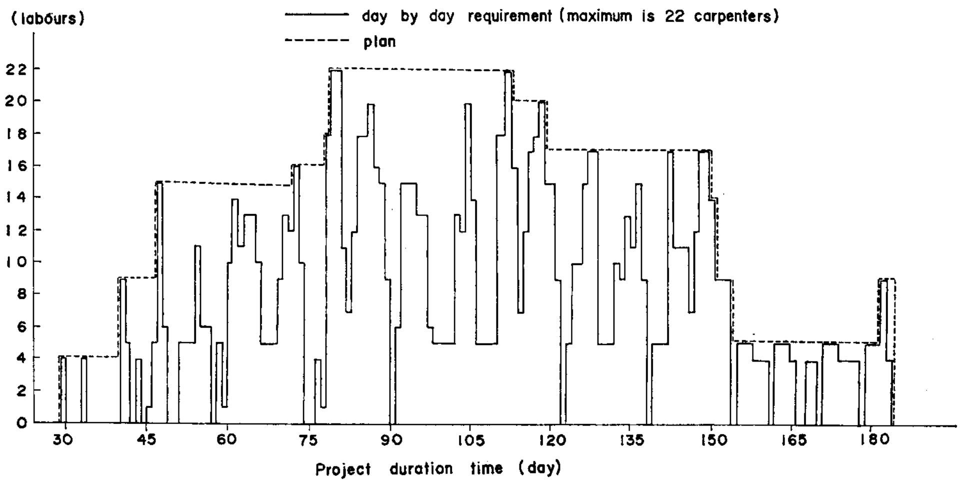

Fig. 5 (b). Carpenter

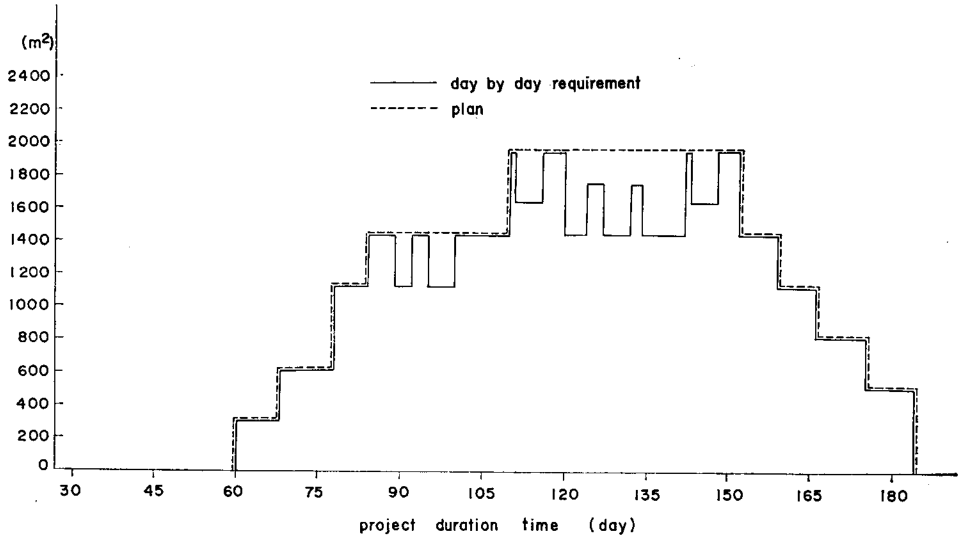

Fig. 5 (c). Form for placing slab concrete

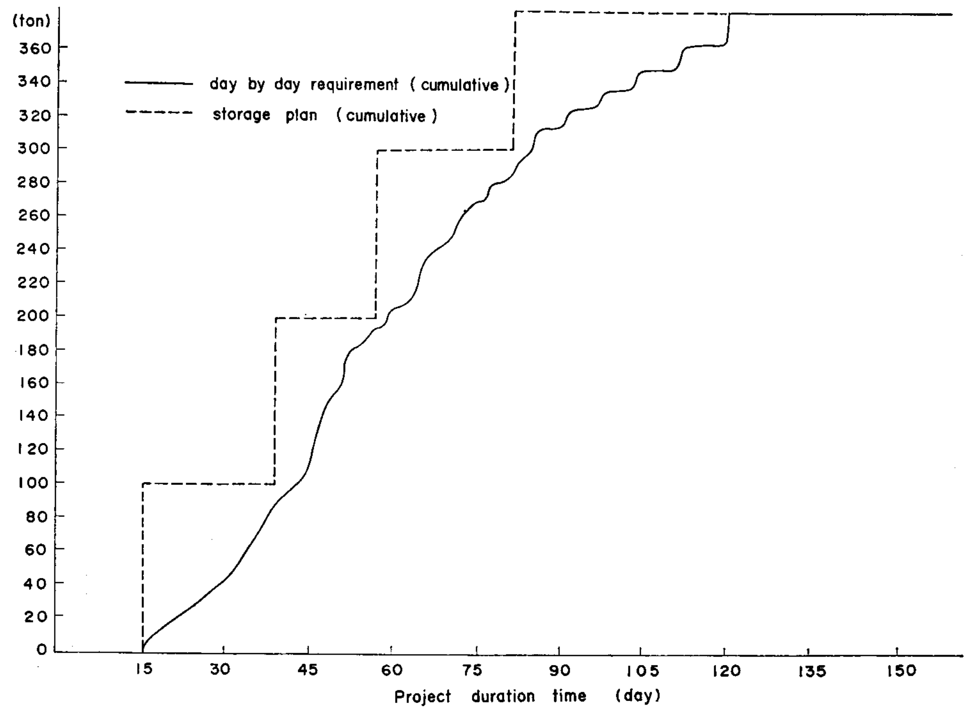

Fig. 5 (d). Reinforcement

Fig. 5. Performances of project characteristic and their plans.

computer, a new research project has already been started by our research group. The next paper by the authors in near future will deal with a new program with the automatic evaluation of the performance of every characteristic in the construction project.

Acknowledgment

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