A Procedure of Graph-partitioning for the Mixed Analysis of Electrical Networks

By

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A procedure to get a graph-partitioning for the mixed anaysis of Electrical Networks with the minimum number of equilibrium equations is described. The graph representing the given electrical network is partitioned into subgraphs each of which have a certain specified property, and a partial ordering is given to these subgraphs. The graph-partitioning is then acquired according to the partial ordering. A method to manipulate the diagram showing the partial ordering is given simplifying the partitioning.

1. Introduction

It has been shown that the required number of equilibrium equations for the nodal or loop analysis where the variables in the equations are either all voltages or all currents respectively, can, in certain cases, be reduced by choosing a suitable set of variables containing both voltages and currents. The analysis using such variables is called mixed analysis. The minimum number of equations required is equal to the topological degree of freedom of the network.[1] For the mixed analysis the graph representing the given electrical network is partitioned to two edge-disjoint subgraphs associated with voltage variables and current variables respectively. The optimal partitionings (denoted P_d for brevity) which lead to the minimum number of equilibrium equations are closely related to maximally distant trees and the principal partition of the graph.[2]

There are three subgraphs uniquely determined in the principal partition, though some of which may be an empty graph. In order to get a P_d the principal subgraph with respect to common chords of a pair of maximally distant trees should be included in the subgraph associated with voltage variables, and the principal subgraph with respect to common (tree) branches should be in the subgraph associated with current variables. The third subgraph of the principal partition is called the principal subgraph of disjoint branches. In general it is subdivided into smaller subgraphs corresponding to the minimal graphs consisting of a pair of

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edge-disjoint trees (PET), and a partial ordering can be given to these smaller subgraphs together with the other two principal subgraphs.[3] As for achieving a P_d, these smaller subgraphs may belong to the subgraph associated with voltage variables or to that associated with current variables, provided a certain restriction arising from the partial ordering is observed. Thus there may be more than one P_d of the given graph and to choose one of them some criterion other than the number of equations may be introduced.

We describe a method which enables us easily to choose the optimal P_d for some criterions, as well as a method to obtain the partially ordered set of subgraphs.

2. The Partially Ordered Set of Subgraphs

We denote the graph representing the given electrical network G, and the principal subgraph with respect to common chords, with respect to common branches, and of disjoint branches G_1, G_2 and G_0 respectively. We also denote the subgraphs associated with voltage and current variables G_n and G_m respectively. We assume without loss of generality that G is connected.

The principal subgraph G_1 may consist of several smaller subgraphs corresponding to the non-separable graphs obtained when the edges of G_2 and G_0 are opened. Similarly G_2 may consist of several smaller subgraphs corresponding to the non-separable graphs obtained when the edges of G_1 and G_0 are shortened.

If the edges of G_1 are shortened and those of G_2 opened, G_0 becomes a graph consisting of a PET. If it contains no proper subgraph also consisting of a PET, it is a minimal graph of graphs consisting of a PET.[3] If it is not minimal, it may contain one or more minimal subgraphs. These minimal subgraphs have neither an edge nor more than one vertex in common. There is no closed chain of such minimal subgraphs contained in the graph. When we shorten the edges of these minimal subgraphs, we get a single vertex if the graph consists entirely of minimal subgraphs, or otherwise again get a graph consisting of a PET. In the latter case the graph obtained may itself be a minimal graph or may contain one or more minimal subgraphs. We repeat the shorting operation on the edges of the minimal subgraphs again and so on until we get a single vertex. The original graph is called k-compound if k shorting operations are needed to bring it to a vertex. The edges of G_0 corresponding to these minimal subgraphs appeared in the above process form the subgraphs of G_0. The order of appearance of the minimal subgraphs also corresponds to the partial ordering of the subgraphs of G_0. It may be helpful in finding a minimal graph to note that a dual of a minimal graph, if exits, is also minimal.

In order to obtain those subgraphs and the relation between them we adopt the following algorithms based on the algorithms given in 1) and 2). We assume a pair of maximally distant trees (T_1, T_2) has already been obtained. By A1 and A2 given below we are going to determine sets of edges denoted L_x and C_y respectively.

A1

step 1. Choose a chord, say x, of T_1. Set $L_x=\{x\}$. Find the fundamental loop defined by x and T_1. Add to L_x the other egdes of the loop besides x.

step 2. Find the fundamental loops defined by the newly added edges and T_2. Add to L_x the edges in the loops which are not in L_x. If no new edge is added, stop. Otherwise go to step 3.

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A 2

step 1. Choose a tree branch, say y, of T_2. Set $C_y=\{y\}$. Find the fundamental cutset defined by y and the cotree of T_2. Add to C_y the other edges of the cutset besides y.

step 2. Find the fundamental cutsets defined by the newly added edges and the cotree of T_1. Add to C_y the edges in the cutsets which are not in C_y. If no new edge is added, stop. Otherwise go to step 3.

step 3. Find the fundamental cutsets defined by the newly added edges and the cotree of T_2. Add to C_y the edges in the cutsets which are not in C_y. If no new edge is added, stop. Otherwise go to step 2.

We can always find the fundamental loops or cutsets if we start the algorithms from proper edges.

We observe the following properties of the sets of edges obtained by the application of A1 and A2 to edges x,y and z.

(1) $L_x \supseteq L_x$.

(2) If $L_x \supseteq L_y$ and $L_y \supseteq L_x$, then $L_x=L_y$.

From the way A1 goes we see that

(3) if $L_x \supseteq L_y$ and $L_y \supseteq L_z$, then $L_x \supseteq L_z$.

Similar relations hold for C_x, C_y and C_z. Besides

(4) $L_x \supseteq L_y$ if and only if $C_y \supseteq C_x$.

Property (4) can be proved by observing that if $L_x \supseteq L_y$ then there must be a string of edges $x=x_1, x_2, \ldots, x_i, x_{i+1}, \ldots, x_n=y$ such that x_{i+1} is included in the fundamental loop defined by x_i and thus x_i is included in the fundamental cutset defined by x_{i+1}, which means $C_y \supseteq C_x$, etc. We also see that

(4)' $L_x \supset L_y$ if and only if $C_y \supset C_x$.

We denote $L_x \cap C_x = S_x$. Then

(5) $L_x = L_y$ or $C_x = C_y$ if and only if $y \in S_x$ or $x \in S_y$.

For if $y \in S_x$, $L_x \supseteq L_y$ and $C_x \supseteq C_y$. From (4), (2) and the definition of S_x, we get $L_x = L_y$ and $C_x = C_y$ and so on.

We first apply A1 to each of the common chords of (T_1, T_2) and determine sets of edges. If any two of these sets have at least one element in common, they are replaced by their union. Repeat the process until all sets are mutually disjoint. These sets thus obtained correspond to the subgraphs of G_1. Similarly we apply A2 to the common branches and then get the subgraphs of G_2.

Now $G_0 = G - G_1 - G_2$. We are going to find the subgraphs of G_0 and their partial ordering. Suppose we get L_x and C_x starting A1 and A2 respectively from an edge x of G_0. Then the edges in S_x form a subgraph of G_0. It is denoted G_{0x}. We repeat the same operation on an edge y in $G_0 - G_{0x}$ to get G_{0y}, and then on an edge z in $G_0 - G_{0x} - G_{0y}$ to get G_{0z} and so on. This process can be shortened by use of the above mentioned properties. Set of edges L_x includes only the disjoint edges of (T_1, T_2), and the subgraph formed by the edges in L_x is the smallest subgraph which contains x and consists of a PET. If L_y is the largest proper subset contained in L_x, then $L_x - L_y = S_x$ from (4)' and (5), and thus we see S_x corresponds to the minimal subgraph described before. Hence if, at some step to get L_x or C_x, an edge is found to be in S_y which has already been determined, the algorithm may be stopped immediately, since the edges in S_y never belong to S_x. Now either $L_x \supset L_y$ or $L_y \supset L_x$ and from (1), (2) and (3) we can define the partial ordering of subgraphs of G_0, that is, $G_{0x} > G_{0y}$ if and only if $L_x \supset L_y$ or $C_y \supset C_x$. Similar conditions are used to define the ordering of the subgraphs of G_0 and those of G_1 and G_2; that is if L_x contains some edges of a subgraph of G_1, say G_{11}, then $G_{0x} > G_{11}$, etc. The ordering of the subgraphs of G_1 and those of G_2 is determined likewise.

Now we draw a diagram showing the partial ordering of the subgraphs with a greater one at a higher level. The subgraphs of G_1 are always placed at the lowest level and those of G_2 at the highest. There are k levels between them if G_0 is k-compound. Such a diagram shows the configuration of G in the sense of the topological degree of freedom. An example of a graph and its configuration is shown in Fig. 1.

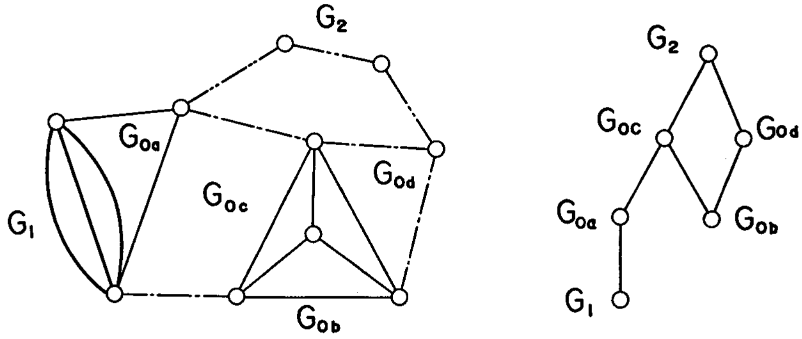
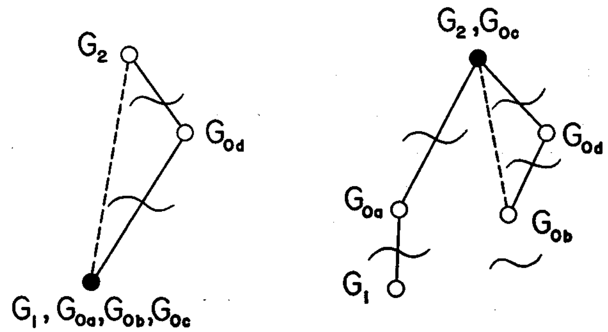


Fig. 1. (a) G (b) Configuration of G

3. Partitioning for the Mixed Analysis

It was shown[4] that the graph-partitioning for the mixed analysis with the minimum number of equilibrium equations can be reduced to the partitioning of the diagram of the partial ordering of the subgraphs. The subgraphs of G_1 and those of G_2 should be included in G_n and G_m respectively. As for the subgraphs of G_0, they may be in either G_n or G_m, but if $G_{0x} > G_{0y}$ and if G_{0x} is included in G_n, then G_{0y} should also be included in G_n, or if G_{0y} is included in G_m, then G_{0x} should also be included in G_m. There are, in general, many possibilities for partitioning. Now if the diagram is such that all the subgraphs are on a straight vertical line, all the possibilities for partitioning can be easily seen. In order to bring a diagram to such a form, we consider the two cases separately in which a subgraph is included in G_n and G_m respectively. No definite algorithm for this process has been found, but in general we pick a subgraph with many lines indident to it in the diagram. Then we draw two new diagrams for each of the two cases. For example, we pick G_{0c} in Fig. 1 (b). If we include G_{0c} in G_n, we get Fig. 2 (a), and if in G_m, we get Fig. 2 (b). If a line appears which connects two subgraphs with some other route



(a) (b)

Fig. 2. Possibilities for partitioning of G

between them (like that shown by a dotted line in Fig. 2), it can be deleted. We repeat this operation until the diagrams become simple enough for us to make a decision. As shown in Fig. 2 (a), there are two possibilities for partitioning in this case. In Fig. 2 (b) we see $2\times3=6$ possibilities, and thus there are $2+6=8$ possibilities for partitioning in all. To choose one of them we need some other criterion as stated before. The last process is related to Theorem 12 of 1), which is based on the operations on edges. It may be much easier to handle with the subgraphs and their partial ordering diagram.

4. Conclusion

A procedure of graph-partitioning for the mixed analysis is given. It may not be the simplest, but it is very easy to follow, since we can work entirely on the original graph G. The process to find the subgraphs may be simplified if the foldant algorithm is introduced. For instance when G_1 and G_2 are found, their edges are shortened and opened respectively to get the graph consisting of a PET. The edges of its subgraphs are also shortened or opened if the subgraphs are found to be locally greatest or locally least of the partial ordering. The minimal subgraphs of the graphs consisting of a PET can sometimes be recognized without applying A1 or A2, and can be utilized.

References

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