Asynchronous Operation and Resynchronization of a Synchronous Machine in a Power System

By

Takashi HIYAMA*, Chikasa UENOSONO** and Muneaki HAYASHI**

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The large signal performance of a multimachine power system is described by a set of differential equations of the form $\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x})$.

The system disturbance caused by asynchronous operation of a synchronous machine in a power system was investigated by solving the system equations with the method of Runge-Kutta-Gill and the possibility of the resynchronization of an asynchronously operating machine was studied, using the method of phase plane analysis. The effects of control systems on the resynchronization of a machine were estimated.

1. Introduction

The accepted practice in power-station operation is that, in the event of an alternator losing synchronism, either owing to loss of excitation or following a system disturbance, the machine is immediately disconnected from the system, and after the system disturbance has been cleared, it is set back on to the busbar. But the process of shutting down and bringing the set back on to the busbar requires a considerable period of time; this is undesirable from the point of view of the maintenance of the power supply. Furthermore, owing to the continued increase in the rating of generator units, the power which a single machine supplies to the system has become larger; therefore, it is also undesirable from the point of view of system reliability to disconnect a large capacity machine from the system for a long period of time.

In many such cases, it would be very desirable if the machine were able to operate asynchronously for a short period until synchronism could be restored. Full scale tests carried out about 40 years ago, and others conducted more recently[1~3] have shown that, under certain conditions and for limited periods, it is permissible to leave the machine connected to the system until

* formerly, a graduate student of the Institute of Electrical Engineering; at present, an assistant of Kumamoto University.

** Institute of Electrical Engineering.

some action can be taken to bring the machine back into synchronism. During the last ten years, in the power systems of the Soviet Union and, later in a number of systems in other countries, the asynchronous running of machines which fall out of step and their subsequent resynchronization, without interference from the service personnel or by the governor control of the dropped-out unit has begun to be widely used.

In this paper, the asynchronous performance and resynchronization of a synchronous machine which is connected to a multimachine power system were studied by simulating the system in the form of a mathematical model on a general purpose digital computer.

The model system contains an arbitrary number of synchronous machines and a transmission network of an arbitrary topological form, including impedance loads. The transmission system[4] is expressed in the rotating common reference frame; synchronous machines are expressed using Park's quantities in a rotating reference frame fixed to their rotors. After both sets of equations; *i. e.*, the transmission system and machine equations, were obtained, the quantities for the transmission system were projected into the frames fixed to the rotors of each synchronous machine. The axis-transformation[4] described above, enables the whole system to be expressed using Park's quantities; the large signal performance of a multimachine power system is described by non-linear first order simultaneous differential equations of the form $\dot{x}=f(x)$. The large signal performance, for instance, the transient stability, asynchronous operation, and resynchronization of synchronous machines, was studied by solving these equations by the Runge-Kutta-Gill method.

A simple model of a 3-machine system was used in the actual calculations; for the most part, the effects of control systems on the operation described above were investigated in this paper. The conditions required for resynchronization were investigated using the method of phase plane analysis of a one machine power system.

2. Representation of Systems

The equations describing the large signal performance of the entire system were derived on the basis of a hybrid reference frame. Each machine was described by Park's quantities in the frame fixed to its rotor. The complete description included the governor and excitation systems. The interconnected network and impedance loads were expressed[4] in the relation between the busbar voltages and the currents in the common reference frame fixed to the rotor of an imaginary machine rotating with a synchronous angular velocity

$\omega_0(=2\pi f_0)$. At the busbar, voltages and currents in the two reference frames were related to one another by axis transformation and the whole system was described using Park's quantities. During any disturbance, the speeds of the machines change and hence their individual reference frames oscillate with respect to the synchronously rotating common reference frame.

Axis Transformation

The axis transformation used in this paper was based mainly on Park's transformation[5]. Let W_{Dj} be the column vector of quantities at the j-th bus expressed in the common reference frame rotating with synchronous angular velocity ω_0, and W_{dj} be the column vector of their Park's quantities with respect to the rotating reference frame of the j-th machine. Symmetrical conditions exist in the system, allowing the zero-sequence quantities to be neglected, so that the vectors W_{Dj} and W_{dj} become second order column vectors. The phaser relations between the two reference frames are shown in Fig. 1.

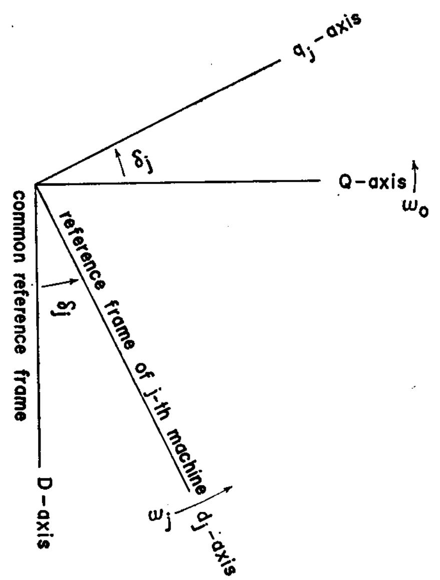

Fig. 1. Phasor relation between two reference frames.

Then the relationship between W_{Dj} and W_{dj} becomes:

$$W_{dj}=P(\delta_j)\cdot W_{Dj} \tag{1}$$

$$W_{Dj}=P(\delta_j)^{-1}\cdot W_{dj} \tag{2}$$

where δ_j is the displacement angle of the j-th machine's rotor from the rotating common reference frame, and $P(\delta_j)$ is the transformation matrix whose elements are shown in Appendix 1. Angle δ_j becomes:

$$\delta_j=(\omega_j-\omega_0)\cdot t+\delta_{j0}=\Delta\omega_j\cdot t+\delta_{j0} \tag{3}$$

where δ_{j0} is the initial value of δ_j.

In a steady state the angular velocity ω_j is equal to the synchronous speed ω_0, and the angle δ_j is not changed, so the displacement angle δ_{ij}, equal to $\delta_i-\delta_j$, is also unchanged. In a transient state ω_j, $\Delta\omega_j$, and δ_j are all changed owing to the unbalanced powers, and:

$$p\delta_j=\omega_j-\omega_0=\Delta\omega_j \tag{4}$$

Description of Transmission Network

The interconnected network and impedance loads were expressed in the relation between the busbar voltages and the currents in the common reference frame. The relationship between the voltages and the currents becomes:

$$\boldsymbol{I}_{Dj}=\sum_{k=1}^{n}\boldsymbol{Y}_{jk}\cdot\boldsymbol{V}_{Dk} \quad (j=1\sim n) \tag{5}$$

where the vectors $\boldsymbol{I}_{Dj}$ and $\boldsymbol{V}_{Dj}$, respectively, express the current and the voltage of the j-th bus, and $\boldsymbol{Y}_{jk}$ is a second order matrix consisting of a short circuit transfer conductance G_{jk} and susceptance B_{jk} between the j-th bus and the k-th bus. $\boldsymbol{I}_{Dj}$, $\boldsymbol{V}_{Dj}$, and $\boldsymbol{Y}_{jk}$ are described as follows:

$$\boldsymbol{I}_{Dj}=\begin{bmatrix} I_{Dj} \\ I_{Qj} \end{bmatrix} \quad \boldsymbol{V}_{Dj}=\begin{bmatrix} V_{Dj} \\ V_{Qj} \end{bmatrix} \quad \boldsymbol{Y}_{jk}=\begin{bmatrix} G_{jk}, & -B_{jk} \\ B_{jk}, & G_{jk} \end{bmatrix} \tag{6}$$

The zero-sequence quantities may be neglected here, because of the symmetrical condition of the system.

The equivalent circuit of the transmission network need not be altered if the frequency of the system is only slightly affected by the system disturbance. But, if the change of frequency is appreciable, the calculation should be corrected[6] by multiplying the inductive reactance by ω/ω_0 and correspondingly reducing the capacitive reactance. In this paper, it was assumed that the equivalent circuit is not affected by the change of system frefuency.

Description of a Synchronous Machine

The complete description of the dynamic behaviour of a synchronous machine requires a consideration of its electrical and mechanical characteristics as well as those of associated control systems. In this section the electrical and mechanical characteristics are described using Park's quantities in per-unit form[4,7]. Owing to the symmetrical condition of the machine, it is not necessary to consider the zero-sequence variables. Furthermore, the terms $p\psi_d$, $p\psi_q$, and ψ_{kd} are considered[8,9] only in the asynchronous power, *i.e.*, damping power.

Induced Voltage in Armature Circuit:

$$V_d = -r \cdot I_d + x_q' \cdot I_q + E_d' \tag{7}$$

$$V_q = -r \cdot I_q - x_d' \cdot I_d + E_q' \tag{8}$$

Change of Flux-linkage in Field Circuit:

$$pE_q' = \{E_{fd} - E_q' - (x_d - x_d') \cdot I_d\}/T_d' \tag{9}$$

$$pE_d' = \{\quad -E_d' + (x_q - x_q') \cdot I_q\}/T_q' \tag{10}$$

Mechanical Equations:

$$M \cdot p^2\delta = P_t - \Delta P_t - (P_s + P_{as})$$

$$p\delta = \Delta\omega \tag{11}$$

where $P_s = V_d \cdot I_d + V_q \cdot I_q,\ V_t = (V_d^2 + V_q^2)^{1/2}$ (12)

$$P_{as} = \frac{1}{2} V_t^2 \left\{ \frac{x_d' - x_d''}{x_d' \cdot x_d''} \cdot \frac{\Delta\omega \cdot T_d''}{1 + (\Delta\omega \cdot T_d'')^2} + \frac{x_q' - x_q''}{x_q' \cdot x_q''} \cdot \frac{\Delta\omega \cdot T_q''}{1 + (\Delta\omega \cdot T_q'')^2} \right\}$$ [6,8,9]

The terms P_s and P_{as}, respectively, express the synchronous and the mean asynchronous power; the first term of P_{as} is a *d*-axis component due to the damper winding and the second term is a *q*-axis component.

In calculating the synchronous e. m. f. in accordance with the eqn. (9), the expression P_s includes the synchronous components, as well as the asynchronous components of the power produced in the excitation winding. But, if the change of synchronous e. m. f. with time is not considered, it is necessary that the third term[6] due to the excitation winding be added to the description of P_{as}; this term is described as follows:

$$\frac{1}{2} V_t^2 \cdot \frac{x_d - x_d'}{x_d \cdot x_d'} \cdot \frac{\Delta\omega \cdot T_d'}{1 + (\Delta\omega \cdot T_d')^2}$$

The term ΔP_t is the change in the power input to the rotor owing to the governor action.

Voltage Regulator:

The automatic voltage regulator is simulated by a widely used model for a continuously acting voltage regulator with a derivative stabilizing trans-

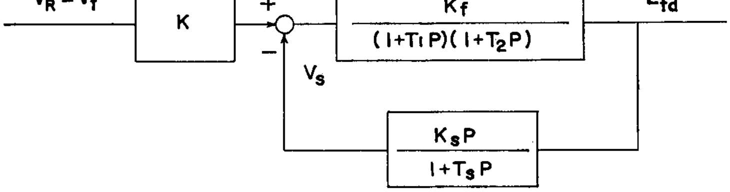

Fig. 2. Block diagram of widely used continuously acting voltage regulator.

former. A block diagram of this model is shown in Fig. 2, and is described as follows:

$$E_{fd}=\frac{K_f}{(1+T_1\cdot p)(1+T_2\cdot p)}\{K(V_R-V_t)-V_s\} \tag{13}$$

$$V_s=\frac{K_s}{1+T_s\cdot p}\cdot E_{fd},\quad V_R=E_{fd0}/K\cdot K_f+V_{t0} \tag{14}$$

The subscript 0 denotes the values in a steady state. The saturation of the amplifier is represented by the limits on its output, so that:

$$E_{fdmin}\leqslant E_{fd}\leqslant E_{fdmax} \tag{15}$$

Speed Governor:

The simplified model shown in Fig. 3 was used; this model is described by:

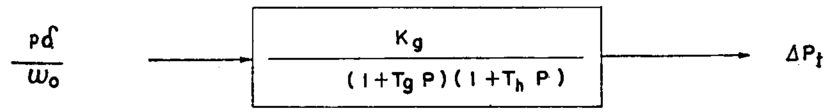

Fig. 3. Block diagram of simplified governor.

$$\Delta P_t=\frac{K_g}{(1+T_g.p)(1+T_h.p)}\cdot\frac{p\delta}{\omega_0} \tag{16}$$

Non-linear First Order Differential System Equations

All the essential first order differential equations describing the dynamic behaviour of synchronous machine are collected in this section.

First the machine and the control system equations which express the large signal performance are given as follows for the case of the j-th machine.

Mechanical Equations:

$$p\delta_j=\Delta\omega_j \tag{17}$$

$$p\Delta\omega_j=\{P_{tj}-\Delta P_{tj}-(P_{sj}+P_{asj})\}/M_j \tag{18}$$

Time Rate of Variation of Direct and Quadrature Axis Flux Linkage:

$$pE_{qj}'=\{E_{fdj}-E_{qj}'-(x_{dj}-x_{dj}')\cdot I_{dj}\}/T_{dj}' \tag{19}$$

$$pE_{dj}'=\{\quad -E_{dj}'+(x_{qj}-x_{qj}')\cdot I_{qj}\}/T_{qj}' \tag{20}$$

Governor Action:

$$p\Delta P_{tj}=(p\Delta P_{tj}) \tag{21}$$

$$p(p\Delta P_{tj})=\{K_{gj}.\Delta\omega_j/\omega_0-(T_{gj}+T_{hj})(p\Delta P_{tj})-\Delta P_{tj}\}/T_{gj}\cdot T_{hj} \tag{22}$$

Voltage Regulator Action:

$$pE_{fdj}=(pE_{fdj}) \tag{23}$$

$$p(pE_{fdj})=[K_{fj}\{K_j(V_{Rj}-V_{tj})-V_{sj}\}-(T_{1j}+T_{2j})(pE_{fdj})-E_{fdj}]/T_{1j}\cdot T_{2j} \quad (24)$$

$$pV_{sj}=\{K_{sj}(_pE_{fdj})-V_{sj}\}/T_{sj} \quad (25)$$

If the j-th machine is asynchronously running owing to the loss of excitation, the voltage regulator action of the j-th machine need not be considered, so that eqns. (23)~(25) may be neglected.

The terminal voltages and currents required for the connection with the network are:

$$E_{qj}'=V_{qj}+x_{dj}'\cdot I_{dj} \quad (26)$$

$$E_{dj}'=V_{dj}-x_{qj}'\cdot I_{qj} \quad (27)$$

The behaviour of the entire power system is expressed by one such set of equations, as described above, for each machine, together with the terminal constraints imposed by the interconnected network.

The interconnected network is expressed as follows from eqn. (5).

$$I_{Dj}=\sum_{k=1}^{n}(G_{jk}\cdot V_{Dk}-B_{jk}\cdot V_{Qk}) \quad (28)$$

$$I_{Qj}=\sum_{k=1}^{n}(G_{jk}\cdot V_{Qk}+B_{jk}\cdot V_{Dk}) \quad (29)$$

Then the terminal constraints become as follows:

$$\begin{bmatrix} V_{dj} \\ V_{qj} \end{bmatrix}=P(\delta_j)\cdot\begin{bmatrix} V_{Dj} \\ V_{Qj} \end{bmatrix} \quad (30)$$

$$\begin{bmatrix} I_{dj} \\ I_{qj} \end{bmatrix}=P(\delta_j)\cdot\begin{bmatrix} I_{Dj} \\ I_{Qj} \end{bmatrix} \quad (31)$$

In the computation processes, the voltages and the angles E_{qj}', E_{dj}', and δ_j which result from the machine equations and appear as the integrable variables in the machine equations, were considered as the input quantities for the solutions of the transmission network equations; whereas the voltages V_{dj}, V_{qj} and the currents I_{dj}, I_{qj} were looked upon as their output quantities. The large signal performance of the power system was studied by solving eqns. (17)~(25) together with eqns. (26)~(31).

In this section the armature resistance was neglected.

A Model Multimachine Power System and Initial Conditions

A multimachine power system contains an extraordinarily large number of system parameters. It would be confusing and also outside the scope of this paper to study all their effects. Hence in the actual calculation, a simple model of a 3-machine system, as shown in Fig. 4, was studied in order to

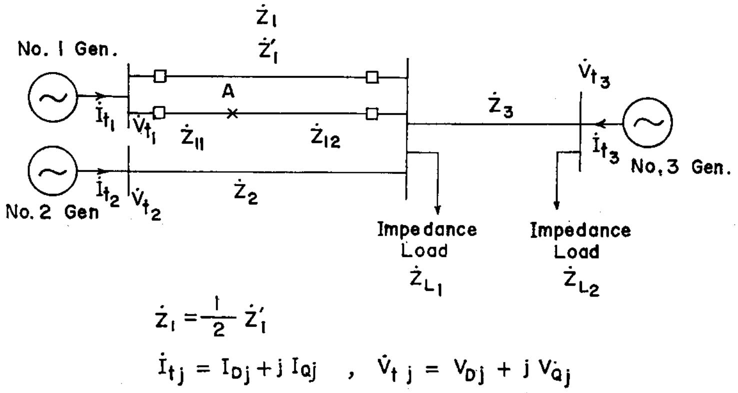

Fig. 4. Model 3-machine system.

investigate the asynchronous operation and the process of resynchronization of a synchronous machine.

Table 1. System parameters.

	machine #1	machine #2	machine #3
x_d	1.15	1.15	0.115
x_d'	0.37	0.37	0.037
x_d''	0.24	0.24	0.024
x_q	0.75	0.75	0.075
x_q'	0.75	0.75	0.075
x_q''	0.34	0.34	0.034
T_d'	1.30	1.30	1.30
T_d''	0.035	0.035	0.035
T_q''	0.035	0.035	0.035
T_g	0.10	0.10	0.10
T_h	0.30	0.30	0.30
K	1.0	1.0	1.0
K_s	0.007	0.007	0.007
T_1	0.20	0.20	0.20
T_2	0.20	0.20	0.20
T_s	0.30	0.30	0.30
J	5.6	5.6	56.0
Z_1	0.0+j 0.2		(K_{f3}= 5.0)
Z_2	0.0+j 0.1		(K_{g3}=95.0)
Z_3	0.0+j 0.1		
Z_{11}	0.0+j 0.1		
Z_{12}	0.0+j 0.3		
Z_{L1}	0.3+j 0.1		
Z_{L2}	0.1+j 0.02		

Table 2. Admittance matrices under several system conditions.

Admittance matrix in steady state			
	1	2	3
1	0.1095−j 4.0511	0.2190+j 1.8978	1.2190+j 1.8978
2	0.2190+j 1.8978	0.4380−j 6.2043	0.4380+j 3.7956
3	0.2190+j 1.8978	0.4380+j 3.7956	10.0533−j 8.1275
Admittance matrix during fault			
	1	2	3
1	0.0257−j 12.2700	0.1029+j 0.9202	0.1029+j 0.9202
2	0.1029+j 0.9202	0.4115−j 6.3193	0.4115+j 3.6807
3	0.1029+j 0.9202	0.4115+j 3.6807	10.0269−j 8.2424
Admittance matrix when faulted line is isolated			
	1	2	3
1	0.0334−j 2.2383	0.1336+j 1.0468	0.1336+j 4.0468
2	0.1336+j 1.0468	0.5345−j 5.8129	0.5345+j 4.1871
3	0.1336+j 1.0468	0.5345+j 4.1871	10.1499−j 7.7360

Table 3. Initial conditions of model system.

	machine #1	machine #2	machine #3
P	1.000	1.000	9.767
Q	0.400	0.579	2.154
V_t	1.000	0.994	0.973
V_D	0.9899	0.9937	0.9732
V_Q	0.1416	0.0348	0.0000
I_t	1.0770	1.1619	10.2766
I_D	1.0465	1.0254	10.0354
I_Q	−0.2544	−0.5463	−2.2132
δ	−0.9055	−1.0506	−0.9870
E_{fd}	1.8392	2.0108	1.6605
E_q'	1.1793	1.2435	1.0850

In this model, machine #3 was conventionally used to represent a large scale power system; it represents a machine equivalent to about ten machines connected to busbar #3. Also, it was assumed that the internal power consumption in this large scale power system was equivalent to the power which is consumed at the shunt impedance load of busbar #3. When machine #1 is asynchronously running, or pulled into step on the resynchronization scheme, the behaviour of the other machine can be determined by an investigation of machine #2.

The parameters of the machines, transmission network, and impedance loads are shown in Table 1. The data for the machines were taken from Kimbark[10], and typical values were used for the control loops. The equivalent circuit yielded admittance matrices of the third order under several conditions of the system as shown in Table 2.

Before the differential equations representing the system behaviour could be solved, it was necessary to determine the initial values of pertinent variables. The initial conditions of this model system, obtained by the method of steepest descent, are shown in Table 3.

3. Asynchronous Operation of Synchronous Machine

Asynchronous operation may be initiated[6] by several causes, such as loss of excitation, loss of steady state stability when underexcited in a heavy load system, or loss of transient stability after a sudden disturbance. Furthermore, short periods of asynchronous operation may also exist while connecting a generator in parallel to a system by the self-synchronization[11] method.

Also, with the continued increase in the rating of generator units, it becomes increasingly desirable, in the event of a generator losing synchronism for some reason, to be able to operate asynchronously at a reduced output until synchronism can be restored. Asynchronous operation as a mean of improving the resultant stability is of considerable importance in giant power stations.

Steady State Asynchronous Operation

A generator may run into stable asynchronous operation under different field circuit conditions, namely;

(1) Field short-circuited on itself

(2) Field short-circuited through the discharge resistor

(3) Field open-circuited

(4) Field excited

If a synchronous machine is running asynchronously, at a certain value of slip, the turbine torque becomes equal to the asynchronous torque, bringing the generator into stable asynchronous operation. The value of slip $s_\infty(-\Delta\omega_\infty/\omega_0)$ at which the steady state asynchronous operation is commenced can be determined graphically as shown in Fig. 5.

Assuming first that the machine is perfectly symmetrical and unexcited (*i.e.*, the cases (1)$\sim$(3)) and hence that the active power supplied by it to the system does not pulsate, the electrical torque corresponding to the asynchronous power balances the mechanical torque supplied by the turbine. Then the slip of the machine has the constant value s_∞ during the steady state asynchronous

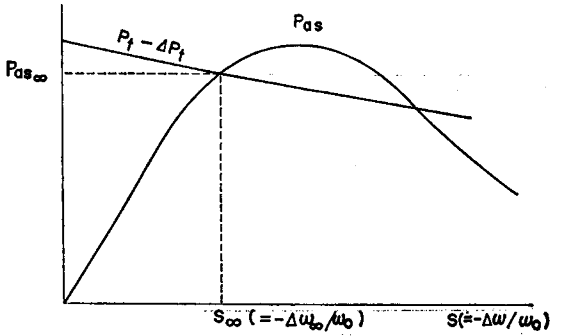

Fig. 5. Asynchronous power and turbine power vs. slip.

operation. But, generally speaking, the generator is not perfectly symmetrical, so there may be a synchronous power component designated the 'reluctance power' due to the saliency of the machine. This power causes the slip to pulsate and the period of this pulsation is equal to π. There are pulsations in various system quantities as will be seen later on. If the machine is excited, in addition to the balanced asynchronous torque, its shaft is also subjected to a synchronous torque. This torque causes the slip pulsation. The amplitude of this pulsation depends on the magnitude of the synchronous component of the torque. An appreciation of this pulsation is important in determining the conditions for resynchronization.

System Disturbance during Asynchronous Operation

The terminal voltage of each machine changes, as shown in Fig. 6, during

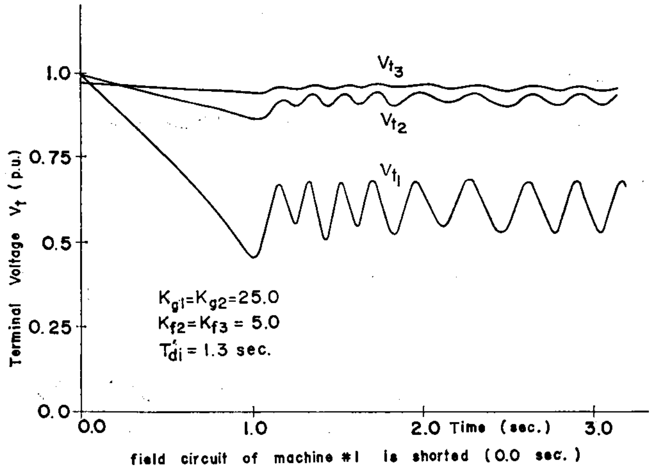

Fig. 6. Changes of terminal voltages during asynchronous operation of machine #1.

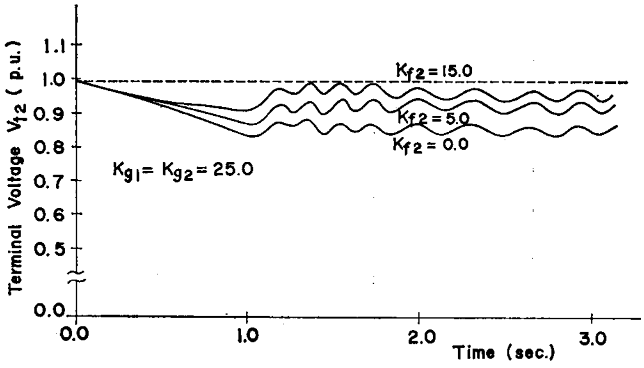

Fig. 7. Effect of voltage regulator on a change in the terminal voltage of machine #2.

the asynchronous operation of machine #1 with the field short-circuited. From this figure it can be deduced that the terminal voltage of machine #1 is reduced to 50~60 % of the rated value, and that of the others is reduced to about 90 % of the rated value, but if the voltage regulator is used, the terminal voltage drop of these other machines may be even further reduced, as shown in Fig. 7.

The pulsation of reactive powers and generator currents are shown in Fig. 8 and Fig. 9. From Fig. 8, it can be seen that machine #1 consumes reactive power during the asynchronous operation of machine #1 with the field short-circuited and unexcited, and this causes a reduction of voltage to the entire power system as described above.

Machines #2 and #3 are running almost synchronously during the asynchro-

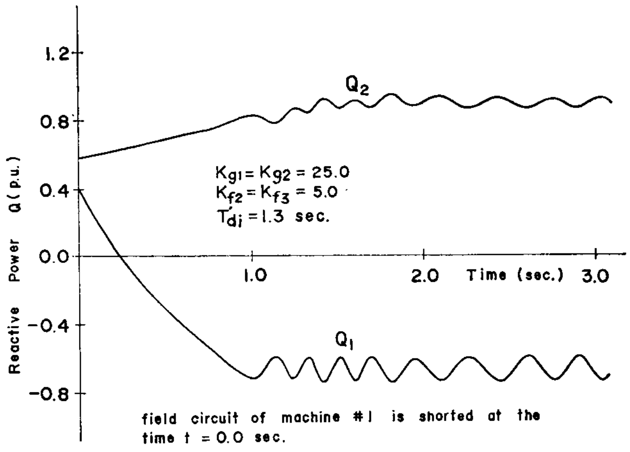

Fig. 8. Changes in reactive power during the asynchronous operation of machine #1.

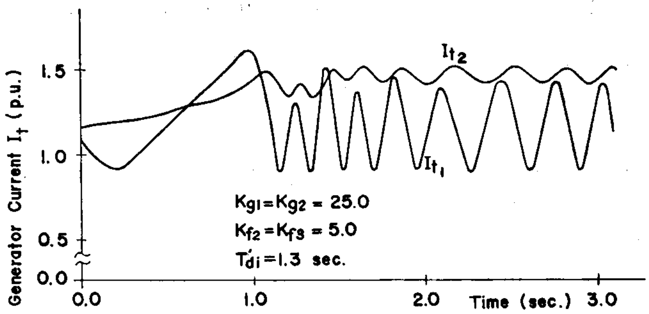

Fig. 9. Changes in generator currents during the asynchronous operation of machine #1.

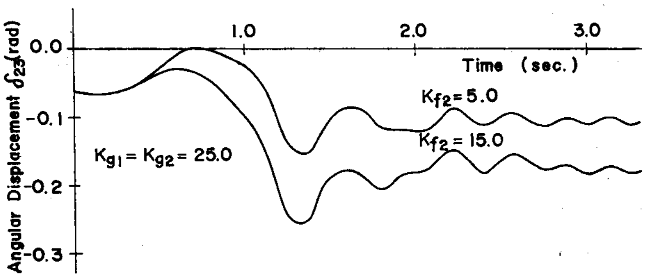

Fig. 10. (a) Relative motion between machines #2 and #3 during the asynchronous operation of machine #1.

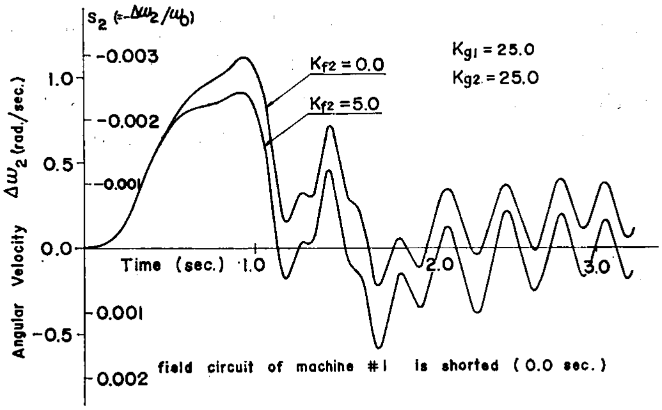

Fig. 10. (b) Motion of machine #2 during the asynchronous operation of machine #1.

nous operation of machine #1 as shown in Fig. 10.

From these curves, it may be deduced that all the system quantities pulsate during the asynchronous operation with the field excited or unexcited as described above.

4. Resynchronization of a Synchronous Machine

If, when a generator falls out of step, the following processes were undertaken, the period of time required for bringing the machine back on to the system would be shortened.

(1) Operating the machine asynchronously by means of the field short-circuited or open-circuited

(2) Adjusting the machine shaft torque by the governing control

(3) Applying the field excitation

In this paper, these processes are designated 'resynchronization of synchronous machine'.

Resynchronization Process

A typical resynchronization process is shown in Fig. 11 (a)~(d). In these figures, the symbols A_1, A_2, A_3, A_4 and A_5 represent the following five conditions:

A_1: Three phase short-circuit fault has occurred at the point A in the model system at the time $t=0.0$ sec.

A_2: Faulted line was isolated by circuit breakers at the time $t=0.30$ sec.

A_3: Faulted line was reclosed at the time $t=0.40$ sec.

A_4: Field circuit is shorted at the time $t=0.42$ sec.

A_5: Field excitation is applied at the time $t=1.42$ sec.

As shown in Fig. 11 (c) and Fig. 11 (d), the active power output of machine #1 pulsates considerably during this process; this is also the case for machine #2. Since the period of this pulsation is about 2 or 3 seconds, it causes little trouble. The relative motion of machines #2 and #3 is shown in Fig. 11 (b).

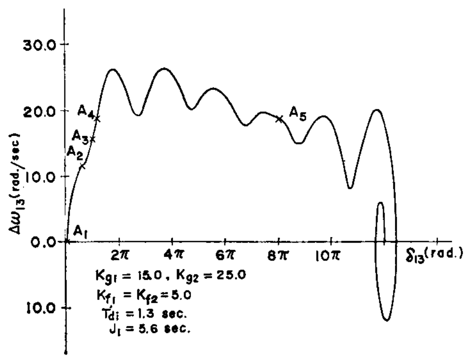

Fig. 11. (a) Typical resynchronization process (Relative motion between machines #1 and #3).

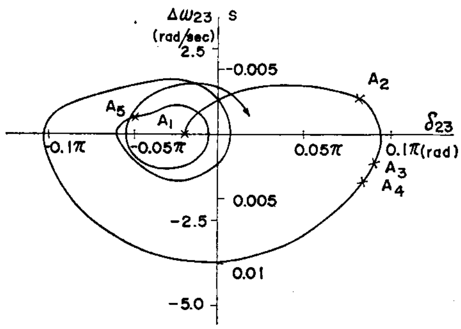

Fig. 11. (b) Typical resynchronization process (Relative motion between machines #2 and #3).

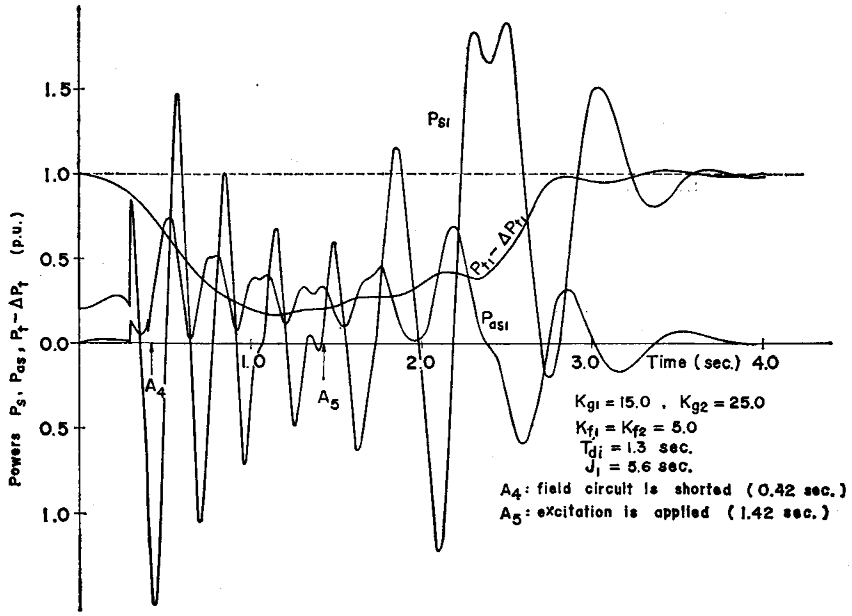

Fig. 11. (c) Typical resynchronization process.
(P_s, P_{as}, P_t-ΔP_t/time characteristics of machine #1)

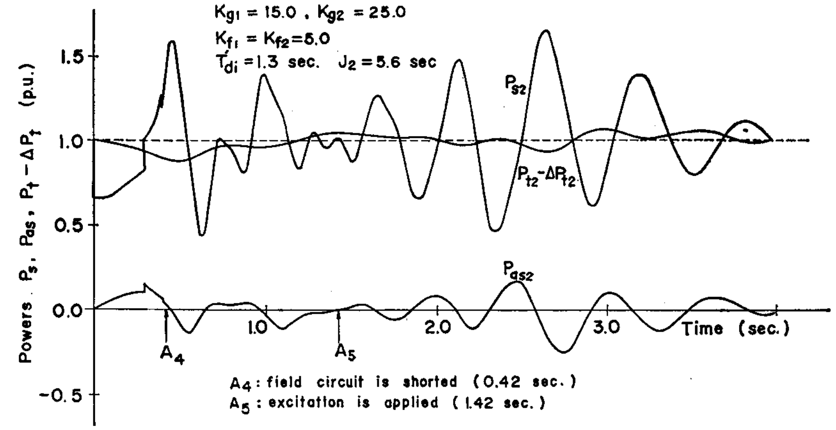

Fig. 11. (d) Typical resynchronization process.
(P_s, P_{as}, P_t-ΔP_t/time characteristics of machine #2)

From the figure it is obvious that machines #2 and #3 are running almost synchronously during the asynchronous operation and the resynchronization of machine #1.

Effect of Various System Parameters

In Fig. 12 (a)~(e), the effects of various system parameters, *i.e.*, governor gain, voltage regulator gain, field circuit time constant, external reactance, and inertia constant, on the resynchronization are shown.

Effect of Governor: During the asynchronous operation the speed governor affects the mean slip and the power output of the machine by controlling the input to the turbine. The greater the governor gain becomes the smaller the mean slip, so the resynchronization of an asynchronously running machine with the field short-circuited is much affected by its governor action as shown in Fig. 12 (a). Furthermore, the governor has no direct effect on the electrical side of the system, so that it is very desirable to increase the governor gain.

Effect of Voltage Regulator: The automatic voltage regulator action also has an effect on the resynchronization as shown in Fig. 12 (b). During the asynchronous operation the terminal voltage of machine #1 is reduced to 50 or 60% of the rated value, so if the excitation is applied to resynchronize the machine, the magnitude of the excitation voltage is increased by the voltage regulator action, so that the magnitude of the synchronous power increases faster than in the case without the voltage regulator. This voltage regulator action shortens the period from the application of the excitation voltage to the resynchronization of the machine as shown in Fig. 12 (b). But the pulsation of slip becomes greater according to the increase of voltage regulator gain.

Effect of Field Circuit Time Constant: As shown in Fig. 12 (c) the smaller the time constant of the field circuit becomes, the shorter the period required to resynchronize a machine. This is explained by the fact that the internal induced voltage increases almost exponentially in accordance with the field circuit time constant when the excitation is applied.

Effect of Inertia Constant: The effect of the inertia constant is shown in

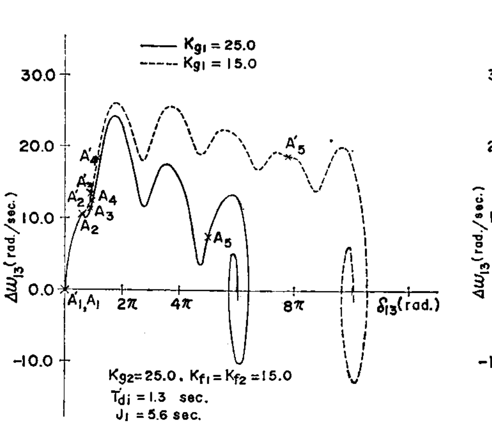

Fig. 12. (a) Effect of the governor gain K_{g1} on resynchronization.

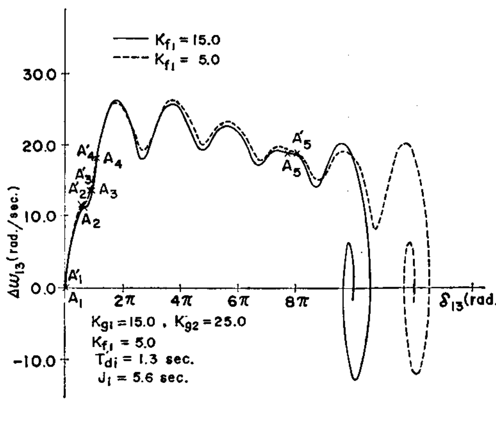

Fig. 12. (b) Effect of the voltage regulator gain K_{f1} on resynchronization.

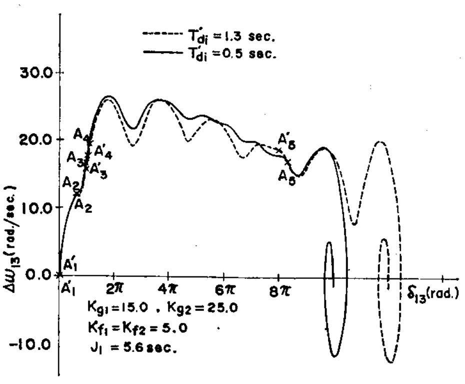

Fig. 12. (c) Effect of the field circuit time constant T_{d1}' on resynchronization.

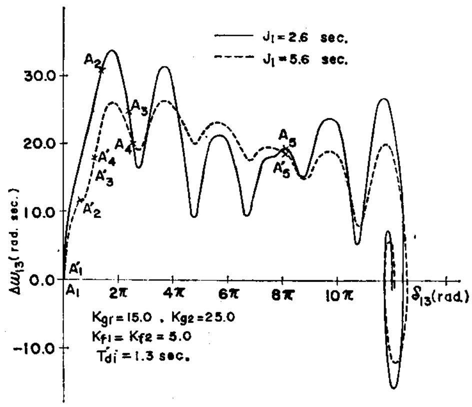

Fig. 12. (d) Effect of the inertia constant J_1 on resynchronization.

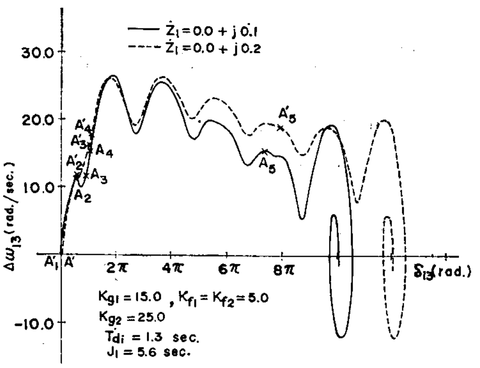

Fig. 12. (e) Effect of the external reactance $\dot{Z}_1$ on resynchronization.

Fig. 12 (d). The total moment of inertia of the machine affects the magnitude of the slip pulsation and the large pulsation of slip helps towards resynchronization; thus a lower moment of inertia generally aids resynchronization.

Effect of External Reactance: The mean asynchronous power decreases in accordance with the increase of external reactance at the same value of slip. This may be deduced from Fig. 12 (e); the machine is resynchronized more easily in the case where the external reactance is small.

5. Conditions for Resynchronization

The conditions required for resynchronization were investigated with the method of phase plane[12] analysis in this section. A simple one machine system was used for the model system and the following assumptions were made.

(1) The time lag of the speed governing control system may be neglected.

(2) The internal induced voltage E_q' increases exponentially in accordance

with the field circuit time constant T_d', when the excitation is applied, and so the magnitude of synchronous power increases likewise.

(3) The voltage regulator action need not be considered.

(4) The armature resistance and the transmission line resistance may be neglected.

Fundamental Equations

The process of resynchronization of a synchronous machine may be represented in the following equations.[13,14]

$$M\cdot p^2\delta+P_d\cdot(1-b\cdot\cos 2\delta)\cdot p\delta+P_m(1-\varepsilon^{-t/T_d'})\sin\delta+P_r\sin 2\delta=P_t \qquad (32)$$

where

$$P_d=\frac{1}{2}V^2\cdot\left\{\frac{(x_d'-x_d'')\cdot T_d''}{(x_d'+x_e)^2}+\frac{(x_q'-x_q'')\cdot T_q''}{(x_q'+x_e)^2}\right\} : \text{Damping Coefficient}$$

$$b=\left\{\frac{(x_d'-x_d'')\cdot T_d''}{(x_d'+x_e)^2}-\frac{(x_q'-x_q'')\cdot T_q''}{(x_q'+x_e)^2}\right\}\Big/\left\{\frac{(x_d'-x_d'')\cdot T_d''}{(x_d'+x_e)^2}+\frac{(x_q'-x_q'')\cdot T_q''}{(x_q'+x_e)^2}\right\} : \text{Pulsating Coefficient}$$

$P_m=E_q'\cdot V/(x_d'+x_e)$: Magnitude of Synchronous Power

$P_r=\frac{1}{2}V^2(x_d'-x_q')/\{(x_d'+x_e)\cdot(x_q'+x_e)\}$: Magnitude of Reluctance Power

If the governor action is considered, eqn. (32) becomes:

$$M\cdot p^2\delta+P_d(1-b\cdot\cos 2\delta)\cdot p\delta+P_m(1-\varepsilon^{-t/T_d'})\sin\delta+P_r\sin 2\delta=P_t+\Delta P_t \qquad (33)$$

$$\Delta P_t=-K_g\cdot\Delta\omega/\omega_0,\quad \Delta\omega=p\delta \qquad (34)$$

The time lag of the speed governing control system may be neglected, so the value of ΔP_t is proportional to the value of slip $-\Delta\omega/\omega_0$. From eqns. (33) and (34), the equation on the phase plane may be expressed as follows:

$$\frac{d\Delta\omega}{d\delta}=\frac{P_t-\{P_d(1-b\cos 2\delta)+K_g/\omega_0\}\Delta\omega-P_m(1-\varepsilon^{-t/T_d'})\sin\delta-P_r\sin 2\delta}{M\cdot\Delta\omega} \qquad (35)$$

After a sufficiently long passage of time, the term $\varepsilon^{-t/T_d'}$ is reduced to zero, then eqn. (35) becomes:

$$\frac{d\Delta\omega}{d\delta}=\frac{P_t-\{P_d(1-b\cos 2\delta)+K_g/\omega_0\}\Delta\omega-P_m\cdot\sin\delta-P_r\cdot\sin 2\delta}{M\cdot\Delta\omega} \qquad (36)$$

The equilibrium points were obtained as follows:

$$P_m\cdot\sin\delta+P_r\cdot\sin 2\delta=P_t \qquad (37)$$

Phase Plane Trajectory and Condition for Resynchronization

The phase plane trajectories, expressed by eqn. (36), were classified into the following four types[14] as shown in Figs. 13 (a)~(d).

(a) The type in which there exist no equilibrium points; the trajectory, which starts from any initial point, converges with the curve which is designated 'limit cycle of synchronous machine'. This curve δ vs. $\Delta\omega$ has the period 2π; this condition is represented by the following relation:

$$P_m \cdot \sin\delta_m + P_r \cdot \sin 2\delta_m < P_t \quad \text{for} \quad \delta_m = \cos^{-1}\left\{-\frac{1}{8}\left(\frac{P_m}{P_r}\right) + \sqrt{\left(\frac{1}{8}\right)^2 \cdot \left(\frac{P_m}{P_r}\right)^2 + \frac{1}{2}}\right\}$$

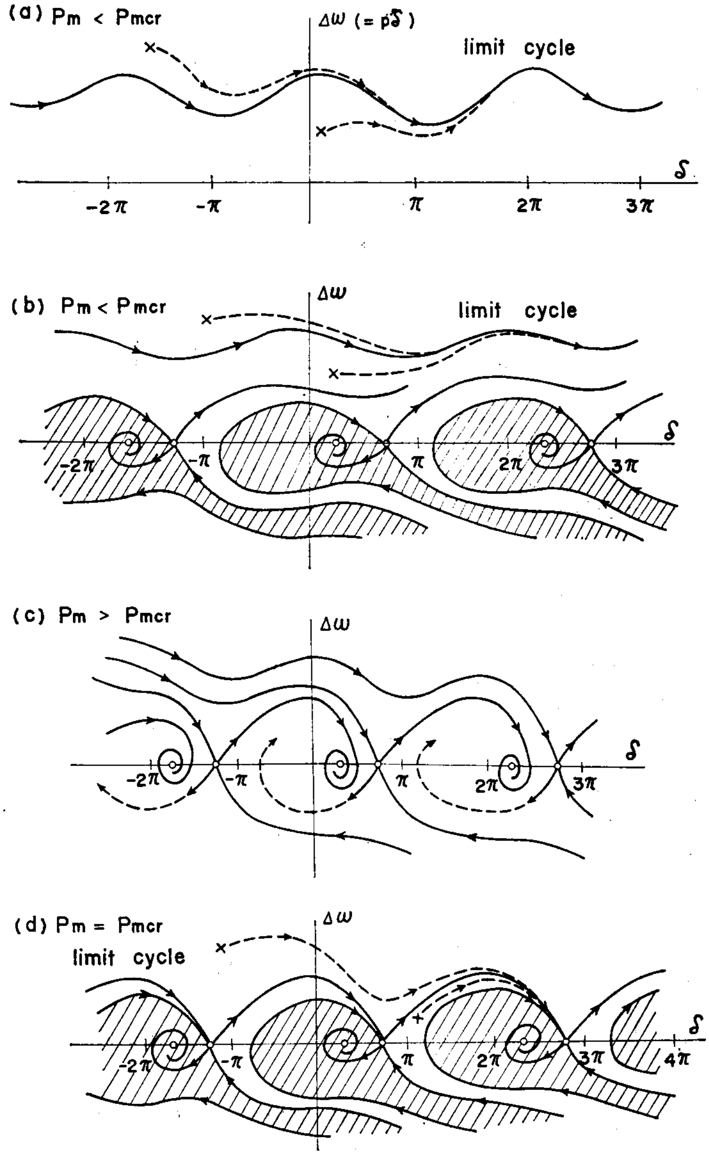

Fig. 13. Classification of phase plane trajectories on a large oscillation of the power system.

(b) The type in which there exist stable and unstable equilibrium points and the limit cycle; the trajectory, which starts from the specified initial points contained in the shaded part of Fig. 13 (b), converges to a stable equilibrium point and in the other case, it converges with the limit cycle.

(c) The type in which there exist equilibrium points but not the limit cycle; in this case the trajectory, which starts from any initial point, converges to a stable equilibrium point.

(d) The case in which there exist equilibrium points and the limit cycle; in this case the limit cycle becomes a curved line drawn from one unstable equilibrium point to the next unstable equilibrium point as shown in Fig. 13 (d) and the trajectory, which starts from the specified initial points contained in the shaded part of Fig. 13 (d), converges to a stable equilibrium point. In the other case, it converges with the limit cycle but after a sufficiently long passage of time the trajectory may converge to a stable equilibrium point.

The condition for resynchronization is represented by type (d), if the field circuit time constant T_d' is not considered. In other words, the extinction of the limit cycle expresses the condition for resynchronization. Furthermore, provided the field circuit time constant is sufficiently small and the initial point is contained in the shaded part of Fig. 13, the trajectory expressed by eqn. (35) converges to a stable equilibrium point. This stable domain, *i.e.*, the shaded part of Fig. 13, was obtained by the method of Liapunov[15]. In this paper, however, the condition for the extinction of the limlt cycle was investigated by solving eqn. (36). Since it is impossible to solve eqn. (36) if the initial point is fixed to the unstable equilibrium point, the initial point was fixed to a point close to the unstable equilibrium point.

The initial point is expressed as follows:

$$\Delta\omega=\left(\frac{d\Delta\omega}{d\delta}\right)_{\delta=\delta_2}\cdot\Delta\delta \tag{38}$$

$$\delta=\delta_2+\Delta\delta \tag{39}$$

where

$$\left(\frac{d\Delta\omega}{d\delta}\right)_{\delta=\delta_2}=\frac{1}{2}\Bigg[-\frac{1}{M}\left\{P_a(1-b\cdot\cos 2\delta_2)+K_g/\omega_0\right\} + \sqrt{\left[\frac{1}{M}\cdot\left\{P_a(1-b\cos 2\delta_2)+K_g/\omega_0\right\}\right]^2-4(P_m\cos\delta_2+2P_r\cos 2\delta_2)}\Bigg]$$

The term $\left(\frac{d\Delta\omega}{d\delta}\right)_{\delta=\delta_2}$ is the gradient according to which the trajectory starts from an unstable equilibrium point. In the actual calculation, the value 0.01 was used as the value of $\Delta\delta$.

Numerical Solution

By solving eqn. (36) using the method of Runge-Kutta-Gill, the effects of the speed governing control, inertia constant, external reactance, saliency, and damping coefficient on the resynchronization of synchronous machine were investigated. The initial conditions are given by eqns. (38) and (39). The calculated results are shown in Fig. 14 (a)~(e). From these figures, it can be seen that there exists a maximum mean slip[16] beyond which it is practically impossible to resynchronize a machine. The maximum of the slip is determined by the magnitude of synchronous power which is determined by the ceiling voltage of the excitation system. Therefore, it is desirable that at the instance of initiating the resynchronization, the mean slip should fall within the predescribed limit.

Effect of Governor and Damping Coefficient: The effect of the governor and damping coefficient is shown in Fig. 14 (a) and Fig. 14 (b). From Fig. 14 (a) it can be seen that the governor action is very effective on the resynchronization of a machine. From eqn. (36), it is evident that the increase of governor gain is equivalent to an increase in the damping coefficient, if the time lag of the speed governing control system is neglegible. It may also be deduced from Fig. 14 (b) that a machine which has a greater value of damping coefficient, is more easily resynchronized.

The damping coefficient depends on the design parameter of the machine, so that this is not controlled, hence the speed governing control system is very useful for the resynchronization.

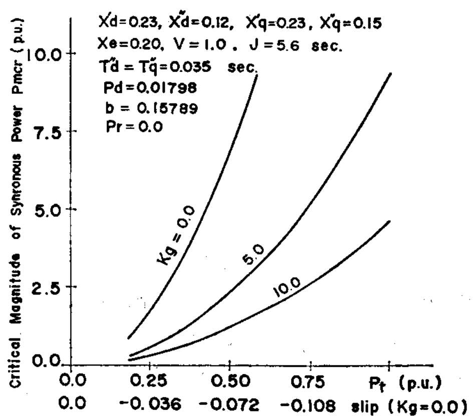

Fig. 14. (a) Effect of the governor gain on the magnitude of synchronous power required to resynchronize a machine.

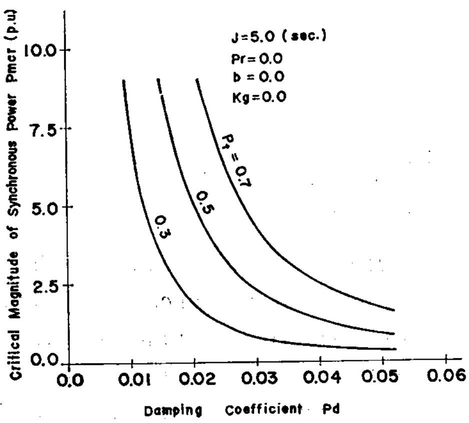

Fig. 14. (b) Effect of the damping coefficient on the magnitude of synchronous power required to resynchronize a machine.

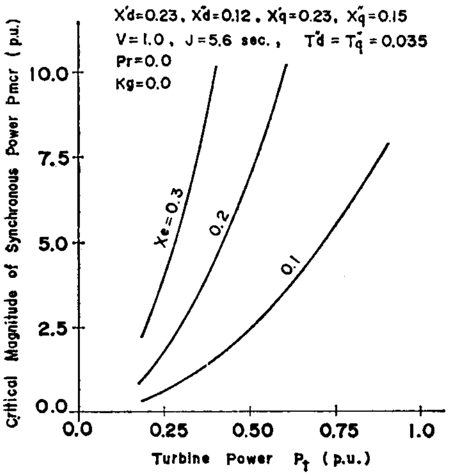

Fig. 14. (c) Effect of the external reactance on the magnitude of synchronous power required to resynchronize a machine.

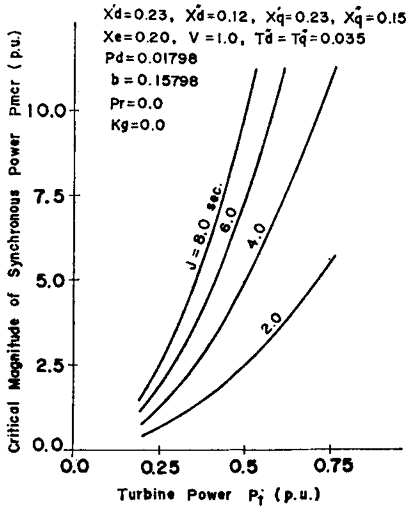

Fig. 14. (d) Effect of the inertia constant on the magnitude of synchronous power required to resynchronize a machine.

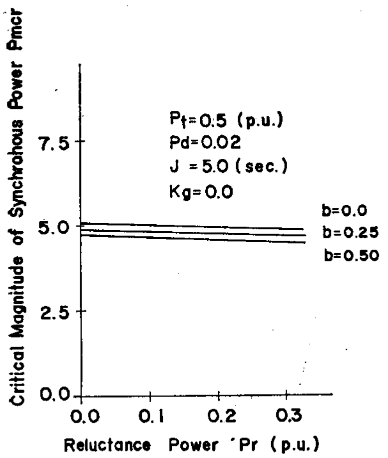

Fig. 14. (e) Effect of saliency on the magnitude of synchronous power required to resynchronize a machine.

Effect of External Reactance: The effect is shown in Fig. 14 (c). A smaller value of external reactance is very effective for resynchronization, since the increase of external reactance decreases the value of the damping coefficient.

Effect of Inertia Constant: As shown in Fig. 14 (d), the smaller the inertia constant becomes, the larger the area in which the machine is successful in resynchronization. This is explained by the fact that the large pulsation of slip helps towards resynchronization and a small value of inertia constant causes a large pulsation.

Effect of Saliency: The effect of reluctance power and pulsating coefficient is shown in Fig. 14 (e). From this figure, it would appear that these terms have no effect on resynchronization.

6. Conclusion

In this paper, the asynchronous operation and the resynchronization of a synchronous machine in a multimachine power system have been studied by solving the system equations using the method of Runge-Kutta-Gill. The conditions required for resynchronization have been also obtained using the method of phase plane analysis of a one machine system.

The results are as follows:

(1) During the asynchronous operation, various system quantities, *i.e.*, active power, reactive power, terminal voltage, and generator current, pulsate and the asynchronous operation causes a drop in terminal voltages; this can be reduced by the voltage regulator action.

(2) In a multimachine power system, the asynchronous operation of a single machine has almost no effect on the behaviour of the other machines; these machines continue running almost synchronously during this asynchronous operation.

(3) The speed governing control system has a great effect on the resynchronization of a synchronous machine and the voltage regulator is also effective during the resynchronization.

(4) The magnitude of mean slip during the asynchronous operation has a great effect on the resynchronization; namely, there exists a maximum mean slip beyond which it is practically impossible to resynchronize a machine. This maximum is derermined by the magnitude of synchronous power, which is determined by the ceiling voltage of the excitation system. The maximum of slip is also determined by the design parameters of the machine, *i.e.*, the inertia constant and damping coefficient, and by the design parameter of the transmission network, *i.e.*, external reactance. But the machine constants are not controlled, hence the speed governing control is very useful for the resynchronization.

Acknowledgement

The authors wish to thank Mr. S. Ihara for his encouragement during this study.

Appendix 1. Transformation Matrix

$$P(\delta_j)=\begin{bmatrix}\cos\delta_j & \sin\delta_j\\ -\sin\delta_j & \cos\delta_j\end{bmatrix},\qquad P(\delta_j)^{-1}=\begin{bmatrix}\cos\delta_j & -\sin\delta_j\\ \sin\delta_j & \cos\delta_j\end{bmatrix}$$

Appendix 2. Martix equation which determines the conditions of the transmission network in a transient state

$$\begin{bmatrix}V_{D1}\\ \vdots\\ V_{Dn}\\ I_{D1}\\ \vdots\\ I_{Dn}\end{bmatrix}=\begin{bmatrix}P(\delta_1) & 0 & 0 & XD_1 & 0 & 0\\ 0 & \ddots & 0 & 0 & \ddots & 0\\ 0 & 0 & P(\delta_n) & 0 & 0 & XD_n\\ Y_{11} & \cdots & Y_{1n} & -U & 0 & 0\\ \vdots & \ddots & \vdots & 0 & \ddots & 0\\ Y_{n1} & \cdots & Y_{nn} & 0 & 0 & -U\end{bmatrix}^{-1}\times\begin{bmatrix}E_{d1}'\\ \vdots\\ E_{dn}'\\ 0\\ \vdots\\ 0\end{bmatrix}$$

where

$$V_{Dj}=\begin{bmatrix}V_{Dj}\\ V_{Qj}\end{bmatrix},\quad I_{Dj}=\begin{bmatrix}I_{Dj}\\ I_{Qj}\end{bmatrix},\quad E_{dj}'=\begin{bmatrix}E_{dj}'\\ E_{qj}'\end{bmatrix},\quad Y_{jk}=\begin{bmatrix}G_{jk}, & -B_{jk}\\ B_{jk}, & G_{jk}\end{bmatrix}$$

$$XD_j=\begin{bmatrix}x_{qj}'\cdot\sin\delta_j & -x_{qj}\cdot\cos\delta_j\\ x_{dj}'\cos\delta_j & x_{dj}\sin\delta_j\end{bmatrix}$$

Symbols

d, q: direct and quadrature axes of reference frame of each machine

D, Q: direct and quadrature axes of common reference frame

ω_0: synchronous angular velocity ($=2\pi f_0$) (rad./sec.)

ω: instantaneous angular velocity (rad./sec.)

$\Delta\omega$: change in angular velocity owing to system disturbance (rad./sec.)

δ: angular displacement between d, q axes of machine and D, Q axes of common reference frame (rad.)

I_D: generator current vector in common reference frame

V_D: terminal voltage vector in common reference frame

G_{jk}: short circuit transfer conductance between the j-th bus and the k-th bus

B_{jk}: short circuit transfer susceptance between the j-th bus and the k-th bus

x_d, x_q: d- and q-axis synchronous reactance

x_d', x_q': d- and q-axis transient reactance

x_d'', x_q'': d- and q-axis subtransient reactance

r : armature resistance
V_d, V_q : d- and q-axis terminal voltage
V_D, V_Q : D- and Q-axis terminal voltage
I_d, I_q : d- and q-axis generator current
I_D, I_Q : D- and Q-axis generator current
V_t : terminal voltage
I_t : generator current
E_d', E_q' : d- and q-axis internal induced voltage in transient state
E_{fd} : excitation voltage referred to armature circuit
T_d', T_q' : d- and q-axis transient time constant (sec.)
T_d'', T_q'' : d- and q-axis subtransient time constant (sec.)
M : angular momentum
P_t : initial power input to rotor
ΔP_t : change in power input to rotor owing to governor action
P_s : synchronous power output
P_{as} : asynchronous power output
K : voltage transformer gain
K_f : voltage regulator open loop gain
K_s : derivative stabilizing gain of voltage regulator
T_1, T_2 : voltage regulator open loop time constant (sec.)
T_s : derivative stabilizing time constant (sec.)
V_R : voltage regulator reference voltage
K_g : governor gain
T_g : governor time constant (sec.)
T_h : time constant representing turbine delay (sec.)
s : instaneous slip $(=-\Delta\omega/\omega_0)$
s_∞ : mean slip in steady state asynchronous operation $(=-\Delta\omega_\infty/\omega_0)$
$P_{as\infty}$: mean asynchronous power in steady state asynchronous operation
x_e : external reactance
P_d : damping coefficient
b : pulsating coefficient
P_r : magnisude of reluctance power
P_m : magnitude of synchronous power
J : inertia constant $(=\omega_0 \cdot M)$ (sec.)
V : infinite bus voltage
t : time (sec.)
p : differential operator $\left(=\frac{d}{dt}\right)$

In this paper the subscript j denotes the j-th machine quantities and the subscript 0 denotes the value in the steady state.

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