Collisional-Radiative Coefficients and Population Coefficients of Hydrogen Plasma

By

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Abstract

Numerical calculation based on the collisional-radiative model is made for rate equations of optically thin hydrogen plasma to give the collisional-radiative and population coefficients. Results are tabulated for the range of plasma parameters of $1\times10^3 \leqslant T_e \leqslant 2.56\times10^5$ °K and $10^6 \leqslant n_e \leqslant 10^{20}$ cm^{-3}. Behavior of all the coefficients is analyzed as a function of electron density and as a result the asymptotic expressions of the coefficients at low and high electron-density limits as well as the approximate expressions of the critical electron densities for these asymptotic behaviors are obtained in terms of the rate coefficients for atomic transitions. Effect of trapping of resonance radiations on the coefficients is discussed.

1. Introduction

Since the collisional-radiative model[1)-3)] was proposed it has been applied to several atomic systems in plasma, and the collisional-radiative and population coefficients of the quasi-steady state solution have been calculated for the rate equation under various conditions of plasma; that is, hydrogen and hydrogenic ion have been treated in the papers by Bates *et al.*[1),2)], by McWhirter *et al.*[3)] and recently by Drawin *et al.*[4),5)], helium by Drawin *et al.*[5)], lithium by Gordiets *et al.*[6)] and cesium by Norcross *et al.*[7)] These coefficients play an important role in the study of plasma spectroscopy because the former coefficients give the effective rate of ionization and recombination of a plasma and the latter ones determine the population density distribution among energy levels of atomic and ionic species.

The collisional-radiative and population coefficients are dependent on atomic coefficients for various transitions such as excitation and ionization cross sections by

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electronic collision, oscillator strengths for optical transitions and so forth. One purpose of the present paper is to calculate these coefficients for optically thin hydrogen plasma using recent data on atomic coefficients and another one is to analyze the behavior of the coefficients as a function of plasma parameters.

In the next section the general method used to calculate the coefficients is briefly described with references to recent data on atomic coefficients and then the results of the numerical calculation are tabulated. In section 3 the behaviors of the coefficients of interest are examined at the low and high electron-density limits. Then the asymptotic forms of the coefficients and the corresponding critical electron densities for these limits are expressed in terms of the rate coefficients for atomic transitions. The last section discusses effect of absorption of radiation in resonance series.

2. Method of Calculation and Results

2.1. Collisional-radiative model[1)-3)]

For optically thin hydrogen plasma, the rate of variation of the population density $n(p)$ of level p is described by

$$\dot{n}(p) \equiv \frac{\mathrm{d}}{\mathrm{d}t} n(p) = \sum_{q=1}^{p-1} C(q,p) n(q) n_{\mathrm{e}} - \left[\left\{ \sum_{q=1}^{p-1} F(p,q) + \sum_{q=p+1}^{\infty} C(p,q) + S(p) \right\} n_{\mathrm{e}} + \sum_{q=1}^{p-1} A(p,q) \right] n(p) + \sum_{q=p+1}^{\infty} \left\{ F(q,p) n_{\mathrm{e}} + A(q,p) \right\} n(q) + \left\{ \alpha(p) n_{\mathrm{e}} + \beta(p) \right\} n_{\mathrm{i}} n_{\mathrm{e}}, \quad (1)$$

where p and q are the principal quantum numbers. Further, the notations in Eq. (1) are as follows:

- n_{e}: electron density,
- n_{i}: ion density,
- $C(q, p)$: rate coefficient for excitation from level q to p by electronic collision,
- $F(p, q)$: that for deexcitation from level p to q,
- $S(p)$: that for ionization from level p,
- $A(p, q)$: Einstein coefficient for radiative transition from level p to q,
- $\alpha(p)$: rate coefficient for three-body recombination to level p,
- $\beta(p)$: that for radiative recombination.

The coefficients $F(p, q)$ and $\alpha(p)$ are related to $C(q, p)$ and $S(p)$, respectively, with the principle of detailed balance.

A set of infinite number of the coupled equations (1) with $p=1, 2, \ldots, \infty$ describes the population densities of all the discrete levels. In general, it can not be solved, and hence the following two assumptions are introduced in order to solve it practically. 1). For all levels p above some high-lying level r, the population density $n(p)$ obeys the Saha-Boltzmann distribution law, *i.e.*, these levels are in LTE, and Eqs. (1) with $p>r$ are not considered below.

$$n(p) = n_E(p) \equiv Z(p) n_i n_e \qquad \text{for } p > r \tag{2}$$

with

$$Z(p) = \{g(p)/2\omega_i\} (h^2/2\pi m k T_e)^{3/2} \exp \{\chi(p)/kT_e\}, \tag{3}$$

where $g(p)$ and ω_i are the statistical weight of level p and the partition function of ion, respectively, $\chi(p)$ is the ionization potential of level p, and m is the electron mass. Other symbols are used in the usual meanings. The population density $n_E(p)$ represents that of level p in LTE. 2). The rate equation can be set equal to zero for all levels other than the ground level ($p=1$);

$$\begin{cases} \dot{n}(1) \neq 0 \\ \dot{n}(p) = 0 \qquad \text{with} \quad p = 2, 3, \ldots\ldots\ldots, r. \end{cases} \tag{4}$$

The normalized population density $\rho(p)=n(p)/n_E(p)$ is defined and Eq. (1) is rewritten in the form;

$$\begin{aligned} \frac{\dot{n}(p)}{n_E(p)} = & \sum_{q=1}^{p-1} F(p, q) \rho(q) n_e \\ & -\Big[\Big\{\sum_{q=1}^{p-1} F(p, q) + \sum_{q=p+1}^{\infty} C(p, q) + S(p)\Big\} n_e + \sum_{q=1}^{p-1} A(p, q)\Big] \rho(p) \\ & + \sum_{q=p+1}^{\infty} \Big\{ C(p, q) n_e + \frac{Z(q)}{Z(p)} A(q, p) \Big\} \rho(q) \\ & + \frac{1}{Z(p)} \{\alpha(p) n_e + \beta(p)\} = 0 \end{aligned} \tag{5}$$

with $p=2, 3, \ldots, r$. The summations to infinity appearing in Eq. (5) are cut off at a sufficiently high-lying level s ($s>r$).

Assuming the solution of the form

$$\rho(p) = r_0(p) + r_1(p) \rho(1) \qquad \text{with} \quad 2 \leqslant p \leqslant r \tag{6}$$

and substituting it into Eq. (5), one has the two sets of $r-1$ equations for $r_0(p)$ and

$r_1(p)$ with $2 \leqslant p \leqslant r$, because Eq. (5) must hold for any value of $\rho(1)$. The solutions $r_0(p)$ and $r_1(p)$ obtained from these two sets of equations give the population density;

$$n(p) = Z(p)r_0(p)n_i n_e + \{Z(p)/Z(1)\} r_1(p)n(1). \tag{6'}$$

The coefficients $r_0(p)$ and $r_1(p)$ are called the population coefficients.

The set of solutions $n(p)$ with $2 \leqslant p \leqslant r$ is substituted into Eq. (1) with $p=1$ and then the equation for the ground level is obtained;

$$\dot{n}(1) = -S_{\mathrm{CR}} n(1)n_e + \alpha_{\mathrm{CR}} n_i n_e, \tag{7}$$

where

$$S_{\mathrm{CR}} = S(1) + \sum_{q=2}^{s} C(1, q) - \frac{1}{Z(1)n_e} \sum_{q=2}^{s} Z(q)r_1(q) \{F(q, 1)n_e + A(q, 1)\} \tag{8}$$

and

$$\alpha_{\mathrm{CR}} = \alpha(1)n_e + \beta(1) + \sum_{q=2}^{s} Z(q)r_0(q) \{F(q, 1)n_e + A(q, 1)\}. \tag{9}$$

Here S_{CR} and α_{CR} are called the collisional-radiative ionization and recombination coefficients, respectively, and they represent the effective rate coefficients for ionization and recombination of the plasma.

2.2. Atomic coefficients

The data on cross sections and optical transition probabilities employed in the present calculation are as follows.

$C(p, q)$: For the excitation of 1s→2p the cross section measured by Fite *et al.*[8] and the one calculated by Burke *et al.*[9] are used. The cross section for the transition of $1 \to p (p \geqslant 3)$ is obtained from that for 1s→2p by making use of the dependence on the threshold energy and the oscillator strength of the transition of interest. For other transitions the semiempirical formula by Drawin[10] is adjusted to fit the calculation by Omidvar.[11] The resulting rate coefficients agree with those calculated by Podlubnyi *et al.*[12] within a factor of 2.

$S(p)$: The cross section for the ionization from the ground level is given by Fite *et al.*[13], and those from excited levels are given by Drawin.[10]

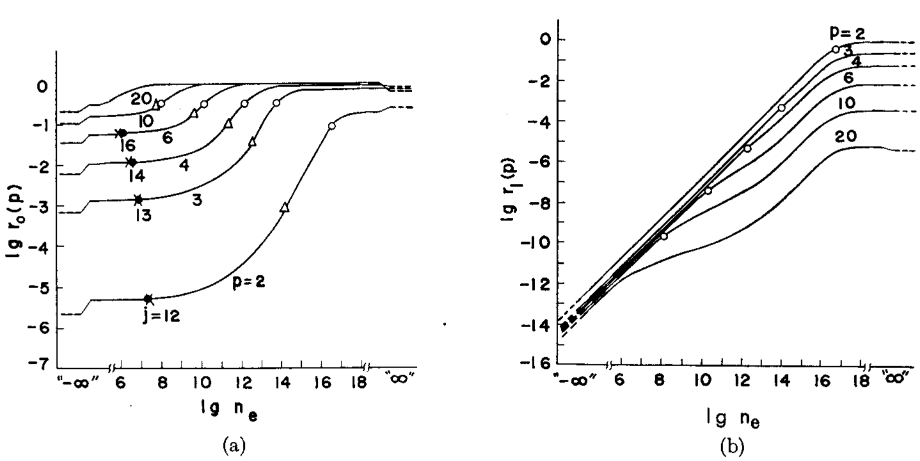
$A(p, q)$: The oscillator strengths for the transitions from a level p lower than 20 are given by Green *et al.*[14] and those concerned with higher levels are given by the well-known approximate formula.[15]

$\beta(p)$: The coefficients for the recombinations into several lower levels are given by Bates *et al.*[16] The approximate formula with the assumption that g-factor is equal to unity is employed for the recombinations into higher levels.

2.3. Results

The calculation is made for the range of plasma parameters of $1\times10^3 \leqslant T_e \leqslant 2.56\times 10^5$ °K and $10^6 \leqslant n_e \leqslant 10^{20}$ cm^{-3}. The limits r and s are 35 and 40, respectively. The results for the population and collisional-radiative coefficients are shown in Tables 1 and 2, respectively. In Table 1, as p increases for a given electron density, the value of $r_0(p)$ tends to unity at some level $p\leqslant 35$. This is consistent with the assumption of $r_0(p)=1$, *i. e.*, LTE, for the levels with $p>35$. The top rows, "$\to -\infty$", in Table 1 indicate the low electron-density limits calculated by the method by Seaton,[17] and the bottom rows, "$\to\infty$", in Table 1 and 2 correspond to the high-density limits, which are calculated with the neglect of the terms of radiative transitions in Eqs. (5)~(9).

Examples of the electron-density dependence of $r_0(p)$ and $r_1(p)$ are shown in Figs. 1(a) and (b), respectively, for $T_e=4\times10^3$ °K. Some of the coefficients S_{CR} and α_{CR} are shown in Figs. 2(a) and (b), respectively.



(a) (b)

Fig. 1. (a) Population coefficient $r_0(p)$ vs. n_e(cm^{-3}) for $T_e=4\times10^3$ °K. Critical electron densities: ●; Eqs. (12), (13) and (17), where j is indicated, ×; Eq. (18). △; Eq. (14), ○; Eq. (17). Limitting values: for lg $n_e\to$ "$-\infty$", ——; the top row in Table 1, - - -; Eq. (22). For lg $n_e\to$ "∞", ——; the bottom row in Table 1, - - -; Eq. (25).
(b) Population coefficient $r_1(p)$ vs. n_e(cm^{-3}) for $T_e=4\times10^3$ °K. Critical densities; ○; Eqs. (20) and (17). Limitting values: for lg $n_e\to$ "$-\infty$", ——; the top row in Table 1, - - -; Eq. (23). For lg $n_e\to$ "∞", ——; the bottom row in Table 1, - - -; Eq. (28) with (29).

Table 1. Population coefficients $r_0(p)$ and (in parenthesis) $r_1(p)$

$T_e = 1\times 10^3$ °K

p / lg n_e	2	3	4	5	7	10	15
$\rightarrow -\infty$	2.24^{-19} ($4.00^{-17}n_e$)	8.79^{-10} ($1.37^{-17}n_e$)	2.35^{-6} ($8.96^{-18}n_e$)	9.90^{-5} ($7.14^{-18}n_e$)	2.98^{-3} ($5.81^{-18}n_e$)	2.14^{-2} ($5.12^{-18}n_e$)	7.23^{-2} ($4.62^{-18}n_e$)
6	2.94^{-19} (4.00^{-11})	1.15^{-9} (1.37^{-11})	3.10^{-6} (8.96^{-12})	1.32^{-4} (7.14^{-12})	4.05^{-3} (5.80^{-12})	3.06^{-2} (5.07^{-12})	1.29^{-1} (4.28^{-12})
8	5.81^{-19} (4.00^{-9})	2.31^{-9} (1.37^{-9})	6.32^{-6} (8.96^{-10})	2.76^{-4} (7.14^{-10})	9.42^{-3} (5.77^{-10})	1.31^{-1} (4.56^{-10})	7.15^{-1} (1.43^{-10})
10	3.26^{-18} (4.00^{-7})	1.35^{-8} (1.37^{-7})	4.01^{-5} (8.96^{-8})	2.25^{-3} (7.14^{-8})	1.91^{-1} (4.86^{-8})	7.49^{-1} (1.44^{-8})	9.65^{-1} (1.97^{-9})
11	1.38^{-17} (4.00^{-6})	6.00^{-8} (1.37^{-6})	2.28^{-4} (8.99^{-7})	2.25^{-2} (7.14^{-7})	4.78^{-1} (3.45^{-7})	8.76^{-1} (8.03^{-8})	9.84^{-1} (1.02^{-8})
12	8.60^{-17} (4.00^{-5})	4.42^{-7} (1.37^{-5})	3.07^{-3} (9.22^{-6})	1.38^{-1} (7.10^{-6})	6.45^{-1} (2.83^{-6})	9.21^{-1} (6.24^{-7})	9.90^{-1} (7.81^{-8})
13	7.33^{-16} (4.00^{-4})	6.37^{-6} (1.41^{-4})	2.59^{-2} (1.07^{-4})	2.46^{-1} (8.08^{-5})	7.07^{-1} (3.12^{-5})	9.36^{-1} (6.81^{-6})	9.92^{-1} (8.51^{-7})
14	8.07^{-15} (3.99^{-3})	8.62^{-5} (1.69^{-3})	5.83^{-2} (1.51^{-3})	2.87^{-1} (1.14^{-3})	7.26^{-1} (4.37^{-4})	9.40^{-1} (9.56^{-5})	9.93^{-1} (1.19^{-5})
15	1.17^{-13} (3.85^{-2})	4.39^{-4} (2.81^{-2})	6.65^{-2} (2.60^{-2})	2.95^{-1} (1.97^{-2})	7.29^{-1} (7.55^{-3})	9.41^{-1} (1.65^{-3})	9.93^{-1} (2.06^{-4})
16	1.08^{-12} (2.86^{-1})	7.03^{-4} (2.73^{-1})	6.77^{-2} (2.55^{-1})	2.96^{-1} (1.93^{-1})	7.29^{-1} (7.40^{-2})	9.41^{-1} (1.62^{-2})	9.93^{-1} (2.02^{-3})
17	3.13^{-12} (8.00^{-1})	7.47^{-4} (7.96^{-1})	6.78^{-2} (7.42^{-1})	2.96^{-1} (5.61^{-1})	7.29^{-1} (2.16^{-1})	9.41^{-1} (4.71^{-2})	9.93^{-1} (5.89^{-3})
18	3.83^{-12} (9.76^{-1})	7.52^{-4} (9.74^{-1})	6.78^{-2} (9.09^{-1})	2.96^{-1} (6.87^{-1})	7.29^{-1} (2.64^{-1})	9.41^{-1} (5.77^{-2})	9.93^{-1} (7.21^{-3})
$\rightarrow \infty$	3.93^{-12} (1.00)	7.52^{-4} (9.99^{-1})	6.78^{-2} (9.32^{-1})	2.96^{-1} (7.04^{-1})	7.29^{-1} (2.71^{-1})	9.41^{-1} (5.91^{-2})	9.93^{-1} (7.39^{-3})

The indices give the power of ten by which the coefficient values must be multiplied.

Table 1 (contd)

$T_e = 2\times10^3$ °K

p / lg n_e	2	3	4	5	7	10	15
$\to -\infty$	1.47^{-10} $(2.59^{-17}n_e)$	9.96^{-6} $(8.87^{-18}n_e)$	5.65^{-4} $(5.90^{-18}n_e)$	3.97^{-3} $(4.82^{-18}n_e)$	2.46^{-2} $(4.00^{-18}n_e)$	7.35^{-2} $(3.50^{-18}n_e)$	1.50^{-1} $(3.08^{-18}n_e)$
6	1.66^{-10} (2.59^{-11})	1.12^{-5} (8.87^{-12})	6.41^{-4} (5.90^{-12})	4.53^{-3} (4.81^{-12})	2.85^{-2} (3.98^{-12})	8.84^{-2} (3.44^{-12})	2.07^{-1} (2.82^{-12})
8	2.34^{-10} (2.59^{-9})	1.60^{-5} (8.87^{-10})	9.27^{-4} (5.90^{-10})	6.71^{-3} (4.81^{-10})	4.56^{-2} (3.92^{-10})	2.17^{-1} (2.95^{-10})	7.57^{-1} (8.00^{-11})
10	6.12^{-10} (2.59^{-7})	4.35^{-5} (8.87^{-8})	2.73^{-3} (5.90^{-8})	2.47^{-2} (4.74^{-8})	3.58^{-1} (2.71^{-8})	8.42^{-1} (6.37^{-9})	9.81^{-1} (7.67^{-10})
11	1.51^{-9} (2.59^{-6})	1.13^{-4} (8.87^{-7})	9.16^{-3} (5.88^{-7})	1.34^{-1} (4.31^{-7})	7.02^{-1} (1.36^{-7})	9.47^{-1} (2.37^{-8})	9.94^{-1} (2.58^{-9})
12	5.36^{-9} (2.59^{-5})	4.88^{-4} (8.90^{-6})	6.88^{-2} (5.73^{-6})	4.52^{-1} (3.07^{-6})	8.70^{-1} (7.05^{-7})	9.79^{-1} (1.12^{-7})	9.98^{-1} (1.18^{-8})
13	2.82^{-8} (2.59^{-4})	4.49^{-3} (9.16^{-5})	3.11^{-1} (5.16^{-5})	6.79^{-1} (2.34^{-5})	9.31^{-1} (4.96^{-6})	9.89^{-1} (7.70^{-7})	9.99^{-1} (8.04^{-8})
14	2.36^{-7} (2.58^{-3})	4.05^{-2} (1.12^{-3})	4.86^{-1} (5.74^{-4})	7.72^{-1} (2.53^{-4})	9.52^{-1} (5.30^{-5})	9.93^{-1} (8.20^{-6})	9.99^{-1} (8.55^{-7})
15	2.96^{-6} (2.52^{-2})	1.46^{-1} (1.68^{-2})	5.64^{-1} (8.53^{-3})	8.09^{-1} (3.74^{-3})	9.60^{-1} (7.84^{-4})	9.94^{-1} (1.21^{-4})	9.99^{-1} (1.26^{-5})
16	2.70^{-5} (2.06^{-1})	1.97^{-1} (1.60^{-1})	5.92^{-1} (8.11^{-2})	8.21^{-1} (3.56^{-2})	9.63^{-1} (7.45^{-3})	9.94^{-1} (1.15^{-3})	9.99^{-1} (1.20^{-4})
17	9.63^{-5} (7.22^{-1})	2.04^{-1} (5.72^{-1})	5.96^{-1} (2.90^{-1})	8.23^{-1} (1.27^{-1})	9.63^{-1} (2.67^{-2})	9.94^{-1} (4.12^{-3})	9.99^{-1} (4.30^{-4})
18	1.29^{-4} (9.63^{-1})	2.05^{-1} (7.65^{-1})	5.97^{-1} (3.88^{-1})	8.23^{-1} (1.70^{-1})	9.63^{-1} (3.57^{-2})	9.94^{-1} (5.51^{-3})	9.99^{-1} (5.75^{-4})
$\to \infty$	1.34^{-4} (1.00)	2.05^{-1} (7.95^{-1})	5.97^{-1} (4.03^{-1})	8.23^{-1} (1.77^{-1})	9.63^{-1} (3.71^{-2})	9.94^{-1} (5.73^{-3})	9.99^{-1} (5.97^{-4})

The indices give the power of ten by which the coefficient values must be multiplied.

Table 1 (contd)

$T_e = 4 \times 10^3$ °K

p / lg n_e	2	3	4	5	7	10	15
$\to -\infty$	4.94^{-6} ($1.77^{-17} n_e$)	1.37^{-1} ($6.04^{-18} n_e$)	1.11^{-2} ($4.12^{-18} n_e$)	3.15^{-2} ($3.40^{-18} n_e$)	8.58^{-2} ($2.80^{-18} n_e$)	1.61^{-1} ($2.40^{-18} n_e$)	2.47^{-1} ($2.08^{-18} n_e$)
6	5.21^{-6} (1.77^{-11})	1.45^{-3} (6.05^{-12})	1.18^{-2} (4.12^{-12})	3.35^{-2} (3.40^{-12})	9.23^{-2} (2.78^{-12})	1.77^{-1} (2.34^{-12})	2.96^{-1} (1.90^{-12})
8	6.16^{-6} (1.77^{-9})	1.72^{-3} (6.05^{-10})	1.43^{-2} (4.11^{-10})	4.12^{-2} (3.37^{-10})	1.20^{-1} (2.70^{-10})	3.06^{-1} (1.98^{-10})	7.82^{-1} (5.99^{-11})
10	1.02^{-5} (1.77^{-7})	2.96^{-3} (6.04^{-8})	2.61^{-2} (4.08^{-8})	8.98^{-2} (3.22^{-8})	4.80^{-1} (1.64^{-8})	8.82^{-1} (3.56^{-9})	9.86^{-1} (4.17^{-10})
11	1.71^{-5} (1.77^{-6})	5.23^{-3} (6.04^{-7})	5.80^{-2} (3.97^{-7})	2.92^{-1} (2.57^{-7})	8.04^{-1} (6.60^{-8})	9.68^{-1} (1.04^{-8})	9.97^{-1} (1.08^{-9})
12	3.76^{-5} (1.77^{-5})	1.41^{-2} (6.02^{-6})	2.49^{-1} (3.31^{-6})	6.68^{-1} (1.35^{-6})	9.38^{-1} (2.42^{-7})	9.91^{-1} (3.39^{-8})	9.99^{-1} (3.35^{-9})
13	1.16^{-4} (1.77^{-4})	7.50^{-2} (5.91^{-5})	6.27^{-1} (2.02^{-5})	8.72^{-1} (6.67^{-6})	9.79^{-1} (1.07^{-6})	9.97^{-1} (1.45^{-7})	1.00 (1.41^{-8})
14	6.00^{-4} (1.77^{-3})	3.70^{-1} (5.42^{-4})	8.19^{-1} (1.49^{-4})	9.42^{-1} (4.69^{-5})	9.91^{-1} (7.35^{-6})	9.99^{-1} (9.83^{-7})	1.00 (9.54^{-8})
15	4.38^{-3} (1.73^{-2})	6.62^{-1} (4.86^{-3})	9.09^{-1} (1.29^{-3})	9.72^{-1} (4.04^{-4})	9.96^{-1} (6.31^{-5})	9.99^{-1} (8.42^{-6})	1.00 (8.17^{-7})
16	3.47^{-2} (1.45^{-1})	7.28^{-1} (4.00^{-2})	9.28^{-1} (1.06^{-2})	9.77^{-1} (3.31^{-3})	9.97^{-1} (5.17^{-4})	1.00 (6.90^{-5})	1.00 (6.69^{-6})
17	1.32^{-1} (5.55^{-1})	7.61^{-1} (1.52^{-1})	9.37^{-1} (4.03^{-2})	9.80^{-1} (1.26^{-2})	9.97^{-1} (1.97^{-3})	1.00 (2.63^{-4})	1.00 (2.55^{-5})
18	1.83^{-1} (7.73^{-1})	7.76^{-1} (2.12^{-1})	9.41^{-1} (5.61^{-2})	9.81^{-1} (1.76^{-2})	9.97^{-1} (2.74^{-3})	1.00 (3.66^{-4})	1.00 (3.55^{-5})
$\to \infty$	1.92^{-1} (8.08^{-1})	7.78^{-1} (2.22^{-1})	9.41^{-1} (5.87^{-2})	9.82^{-1} (1.84^{-2})	9.97^{-1} (2.87^{-3})	1.00 (3.83^{-4})	1.00 (3.72^{-5})

The indices give the power of ten by which the coefficient values must be multiplied.

Table 1 (contd)

$T_e = 8 \times 10^3$ °K

p / lg n_e	2	3	4	5	7	10	15
$\rightarrow -\infty$	1.17^{-3} $(1.35^{-17} n_e)$	2.03^{-2} $(4.64^{-18} n_e)$	6.12^{-2} $(3.19^{-18} n_e)$	1.08^{-1} $(2.61^{-18} n_e)$	1.90^{-1} $(2.11^{-18} n_e)$	2.75^{-1} $(1.78^{-18} n_e)$	3.55^{-1} $(1.52^{-18} n_e)$
6	1.20^{-3} (1.35^{-11})	2.08^{-2} (4.64^{-12})	6.29^{-2} (3.18^{-12})	1.11^{-1} (2.60^{-12})	1.97^{-1} (2.08^{-12})	2.89^{-1} (1.72^{-12})	3.93^{-1} (1.39^{-12})
8	1.30^{-3} (1.35^{-9})	2.27^{-2} (4.64^{-10})	6.93^{-2} (3.17^{-10})	1.24^{-1} (2.57^{-10})	2.28^{-1} (2.01^{-10})	4.00^{-1} (1.46^{-10})	8.01^{-1} (4.66^{-11})
10	1.68^{-3} (1.35^{-7})	3.01^{-2} (4.62^{-8})	9.66^{-2} (3.09^{-8})	1.95^{-1} (2.38^{-8})	5.68^{-1} (1.15^{-8})	9.02^{-1} (2.50^{-9})	9.88^{-1} (2.95^{-10})
11	2.21^{-3} (1.35^{-6})	4.15^{-2} (4.58^{-7})	1.59^{-1} (2.91^{-7})	4.29^{-1} (1.73^{-7})	8.53^{-1} (4.15^{-8})	9.77^{-1} (6.44^{-9})	9.98^{-1} (6.67^{-10})
12	3.47^{-3} (1.35^{-5})	7.85^{-2} (4.46^{-6})	4.34^{-1} (2.06^{-6})	7.78^{-1} (7.46^{-7})	9.61^{-1} (1.24^{-7})	9.95^{-1} (1.69^{-8})	1.00 (1.66^{-9})
13	7.10^{-3} (1.35^{-4})	2.63^{-1} (3.81^{-5})	7.85^{-1} (9.56^{-6})	9.34^{-1} (2.80^{-6})	9.90^{-1} (4.11^{-7})	9.99^{-1} (5.33^{-8})	1.00 (5.11^{-9})
14	2.17^{-2} (1.33^{-3})	6.90^{-1} (2.26^{-4})	9.34^{-1} (4.56^{-5})	9.81^{-1} (1.26^{-5})	9.97^{-1} (1.78^{-6})	1.00 (2.28^{-7})	1.00 (2.17^{-8})
15	1.12^{-1} (1.20^{-2})	8.71^{-1} (1.50^{-3})	9.75^{-1} (2.89^{-4})	9.93^{-1} (7.91^{-5})	9.99^{-1} (1.11^{-5})	1.00 (1.41^{-6})	1.00 (1.34^{-7})
16	5.07^{-1} (5.92^{-2})	9.39^{-1} (7.07^{-3})	9.88^{-1} (1.36^{-3})	9.97^{-1} (3.71^{-4})	1.00 (5.20^{-5})	1.00 (6.61^{-6})	1.00 (6.27^{-7})
17	8.30^{-1} (9.79^{-2})	9.80^{-1} (1.16^{-2})	9.96^{-1} (2.23^{-3})	9.99^{-1} (6.09^{-4})	1.00 (8.55^{-5})	1.00 (1.09^{-5})	1.00 (1.03^{-6})
18	8.88^{-1} (1.05^{-1})	9.87^{-1} (1.24^{-2})	9.97^{-1} (2.39^{-3})	9.99^{-1} (6.52^{-4})	1.00 (9.14^{-5})	1.00 (1.16^{-5})	1.00 (1.10^{-6})
$\rightarrow \infty$	8.94^{-1} (1.06^{-1})	9.87^{-1} (1.25^{-2})	9.98^{-1} (2.41^{-3})	9.99^{-1} (6.57^{-4})	1.00 (9.21^{-5})	1.00 (1.17^{-5})	1.00 (1.11^{-6})

The indices give the power of ten by which the coefficient values must be multiplied.

Table 1 (contd)

$T_e = 1.6 \times 10^4$ °K

p / lg n_e	2	3	4	5	7	10	15
$\to -\infty$	2.29^{-2} ($1.20^{-17} n_e$)	9.63^{-2} ($4.09^{-18} n_e$)	1.73^{-1} ($2.78^{-18} n_e$)	2.36^{-1} ($2.24^{-18} n_e$)	3.25^{-1} ($1.78^{-18} n_e$)	4.05^{-1} ($1.48^{-18} n_e$)	4.69^{-1} ($1.26^{-18} n_e$)
6	2.31^{-2} (1.20^{-11})	9.72^{-2} (4.09^{-12})	1.75^{-1} (2.77^{-12})	2.39^{-1} (2.23^{-12})	3.31^{-1} (1.76^{-12})	4.15^{-1} (1.44^{-12})	4.95^{-1} (1.16^{-12})
8	2.40^{-2} (1.20^{-9})	1.02^{-1} (4.08^{-10})	1.84^{-1} (2.75^{-10})	2.54^{-1} (2.20^{-10})	3.60^{-1} (1.69^{-10})	5.01^{-1} (1.23^{-10})	8.20^{-1} (4.21^{-11})
10	2.71^{-2} (1.20^{-7})	1.17^{-1} (4.04^{-8})	2.20^{-1} (2.65^{-8})	3.27^{-1} (2.00^{-8})	6.41^{-1} (9.66^{-9})	9.16^{-1} (2.16^{-9})	9.90^{-1} (2.60^{-10})
11	3.12^{-2} (1.20^{-6})	1.40^{-1} (3.97^{-7})	2.95^{-1} (2.42^{-7})	5.42^{-1} (1.40^{-7})	8.82^{-1} (3.35^{-8})	9.81^{-1} (5.26^{-9})	9.98^{-1} (5.52^{-10})
12	3.98^{-2} (1.19^{-5})	2.06^{-1} (3.74^{-6})	5.73^{-1} (1.55^{-6})	8.38^{-1} (5.45^{-7})	9.72^{-1} (9.02^{-8})	9.96^{-1} (1.23^{-8})	1.00 (1.22^{-9})
13	6.10^{-2} (1.18^{-4})	4.66^{-1} (2.73^{-5})	8.61^{-1} (6.12^{-6})	9.59^{-1} (1.74^{-6})	9.94^{-1} (2.52^{-7})	9.99^{-1} (3.27^{-8})	1.00 (3.15^{-9})
14	1.30^{-1} (1.11^{-3})	8.29^{-1} (1.27^{-4})	9.66^{-1} (2.31^{-5})	9.91^{-1} (6.19^{-6})	9.99^{-1} (8.60^{-7})	1.00 (1.09^{-7})	1.00 (1.04^{-8})
15	4.42^{-1} (7.04^{-3})	9.45^{-1} (5.97^{-4})	9.90^{-1} (1.04^{-4})	9.97^{-1} (2.74^{-5})	9.99^{-1} (3.77^{-6})	1.00 (4.76^{-7})	1.00 (4.52^{-8})
16	8.66^{-1} (1.52^{-2})	9.88^{-1} (1.24^{-3})	9.98^{-1} (2.14^{-4})	1.00 (5.64^{-5})	1.00 (7.73^{-6})	1.00 (9.76^{-7})	1.00 (9.27^{-8})
17	9.69^{-1} (1.72^{-2})	9.97^{-1} (1.39^{-3})	1.00 (2.41^{-4})	1.00 (6.35^{-5})	1.00 (8.70^{-6})	1.00 (1.10^{-6})	1.00 (1.04^{-7})
18	9.81^{-1} (1.74^{-2})	9.99^{-1} (1.41^{-3})	1.00 (2.45^{-4})	1.00 (6.43^{-5})	1.00 (8.81^{-6})	1.00 (1.11^{-6})	1.00 (1.06^{-7})
$\to \infty$	9.83^{-1} (1.75^{-2})	9.99^{-1} (1.41^{-3})	1.00 (2.45^{-4})	1.00 (6.44^{-5})	1.00 (8.82^{-6})	1.00 (1.11^{-6})	1.00 (1.06^{-7})

The indices give the power of ten by which the coefficient values must be multiplied.

Table 1 (contd)

$T_e = 3.2 \times 10^4$ °K

p / lg n_e	2	3	4	5	7	10	15
$\to -\infty$	1.25^{-1} $(1.20^{-17} n_e)$	2.52^{-1} $(4.05^{-18} n_e)$	3.39^{-1} $(2.70^{-18} n_e)$	4.01^{-1} $(2.16^{-18} n_e)$	4.80^{-1} $(1.69^{-18} n_e)$	5.44^{-1} $(1.40^{-18} n_e)$	5.85^{-1} $(1.18^{-18} n_e)$
6	1.25^{-1} (1.20^{-11})	2.52^{-1} (4.04^{-12})	3.40^{-1} (2.69^{-12})	4.03^{-1} (2.14^{-12})	4.84^{-1} (1.67^{-12})	5.50^{-1} (1.35^{-12})	6.00^{-1} (1.10^{-12})
8	1.27^{-1} (1.20^{-9})	2.58^{-1} (4.02^{-10})	3.50^{-1} (2.66^{-10})	4.16^{-1} (2.11^{-10})	5.07^{-1} (1.60^{-10})	6.10^{-1} (1.18^{-10})	8.44^{-1} (4.38^{-11})
10	1.35^{-1} (1.20^{-7})	2.77^{-1} (3.96^{-8})	3.84^{-1} (2.55^{-8})	4.77^{-1} (1.91^{-8})	7.14^{-1} (9.49^{-9})	9.30^{-1} (2.22^{-9})	9.91^{-1} (2.76^{-10})
11	1.44^{-1} (1.19^{-6})	3.03^{-1} (3.87^{-7})	4.51^{-1} (2.31^{-7})	6.45^{-1} (1.33^{-7})	9.05^{-1} (3.29^{-8})	9.84^{-1} (5.30^{-9})	9.98^{-1} (5.67^{-10})
12	1.63^{-1} (1.19^{-5})	3.77^{-1} (3.55^{-6})	6.80^{-1} (1.42^{-6})	8.78^{-1} (5.02^{-7})	9.78^{-1} (8.48^{-8})	9.97^{-1} (1.18^{-8})	1.00 (1.18^{-9})
13	2.06^{-1} (1.15^{-4})	6.25^{-1} (2.34^{-5})	9.04^{-1} (5.15^{-6})	9.71^{-1} (1.47^{-6})	9.96^{-1} (2.17^{-7})	9.99^{-1} (2.85^{-8})	1.00 (2.77^{-9})
14	3.34^{-1} (9.87^{-4})	8.93^{-1} (9.64^{-5})	9.79^{-1} (1.74^{-5})	9.94^{-1} (4.67^{-6})	9.99^{-1} (6.56^{-7})	1.00 (8.42^{-8})	1.00 (8.10^{-9})
15	7.14^{-1} (4.21^{-3})	9.74^{-1} (3.15^{-4})	9.95^{-1} (5.45^{-5})	9.99^{-1} (1.44^{-5})	1.00 (2.00^{-6})	1.00 (2.55^{-7})	1.00 (2.44^{-8})
16	9.52^{-1} (6.25^{-3})	9.96^{-1} (4.53^{-4})	9.99^{-1} (7.78^{-5})	1.00 (2.05^{-5})	1.00 (2.84^{-6})	1.00 (3.62^{-7})	1.00 (3.47^{-8})
17	9.89^{-1} (6.57^{-3})	9.99^{-1} (4.74^{-4})	1.00 (8.15^{-5})	1.00 (2.15^{-5})	1.00 (2.97^{-6})	1.00 (3.79^{-7})	1.00 (3.63^{-8})
18	9.93^{-1} (6.60^{-3})	1.00 (4.76^{-4})	1.00 (8.19^{-5})	1.00 (2.16^{-5})	1.00 (2.99^{-6})	1.00 (3.81^{-7})	1.00 (3.65^{-8})
$\to \infty$	9.93^{-1} (6.61^{-3})	1.00 (4.76^{-4})	1.00 (8.19^{-5})	1.00 (2.16^{-5})	1.00 (2.99^{-6})	1.00 (3.81^{-7})	1.00 (3.65^{-8})

The indices give the power of ten by which the coefficient values must be multiplied.

Table 1 (contd)

$T_e = 6.4 \times 10^4$ °K

p / lg n_e	2	3	4	5	7	10	15
$\to -\infty$	3.50^{-1} $(1.29^{-17} n_e)$	4.72^{-1} $(4.29^{-18} n_e)$	5.45^{-1} $(2.83^{-18} n_e)$	5.90^{-1} $(2.24^{-18} n_e)$	6.46^{-1} $(1.75^{-18} n_e)$	6.88^{-1} $(1.44^{-18} n_e)$	7.02^{-1} $(1.21^{-18} n_e)$
6	3.50^{-1} (1.29^{-11})	4.73^{-1} (4.29^{-12})	5.45^{-1} (2.82^{-12})	5.91^{-1} (2.23^{-12})	6.47^{-1} (1.72^{-12})	6.90^{-1} (1.40^{-12})	7.08^{-1} (1.13^{-12})
8	3.53^{-1} (1.29^{-9})	4.78^{-1} (4.27^{-10})	5.53^{-1} (2.79^{-10})	6.01^{-1} (2.19^{-10})	6.64^{-1} (1.66^{-10})	7.29^{-1} (1.24^{-10})	8.76^{-1} (5.01^{-11})
10	3.62^{-1} (1.29^{-7})	4.94^{-1} (4.19^{-8})	5.78^{-1} (2.67^{-8})	6.42^{-1} (1.99^{-8})	7.95^{-1} (1.03^{-8})	9.46^{-1} (2.57^{-9})	9.93^{-1} (3.28^{-10})
11	3.73^{-1} (1.28^{-6})	5.15^{-1} (4.07^{-7})	6.24^{-1} (2.41^{-7})	7.51^{-1} (1.42^{-7})	9.30^{-1} (3.69^{-8})	9.88^{-1} (6.13^{-9})	9.99^{-1} (6.67^{-10})
12	3.94^{-1} (1.27^{-5})	5.73^{-1} (3.69^{-6})	7.80^{-1} (1.49^{-6})	9.13^{-1} (5.40^{-7})	9.84^{-1} (9.39^{-8})	9.98^{-1} (1.33^{-8})	1.00 (1.36^{-9})
13	4.41^{-1} (1.21^{-4})	7.54^{-1} (2.34^{-5})	9.36^{-1} (5.26^{-6})	9.80^{-1} (1.53^{-6})	9.97^{-1} (2.30^{-7})	1.00 (3.07^{-8})	1.00 (3.03^{-9})
14	5.72^{-1} (9.54^{-4})	9.34^{-1} (9.02^{-5})	9.87^{-1} (1.66^{-5})	9.96^{-1} (4.53^{-6})	9.99^{-1} (6.48^{-7})	1.00 (8.43^{-8})	1.00 (8.21^{-9})
15	8.57^{-1} (3.15^{-3})	9.87^{-1} (2.35^{-4})	9.98^{-1} (4.15^{-5})	9.99^{-1} (1.11^{-5})	1.00 (1.57^{-6})	1.00 (2.03^{-7})	1.00 (1.97^{-8})
16	9.78^{-1} (4.10^{-3})	9.98^{-1} (2.96^{-4})	1.00 (5.21^{-5})	1.00 (1.40^{-5})	1.00 (1.97^{-6})	1.00 (2.54^{-7})	1.00 (2.46^{-8})
17	9.94^{-1} (4.22^{-3})	1.00 (3.04^{-4})	1.00 (5.35^{-5})	1.00 (1.43^{-5})	1.00 (2.02^{-6})	1.00 (2.61^{-7})	1.00 (2.53^{-8})
18	9.96^{-1} (4.24^{-3})	1.00 (3.05^{-4})	1.00 (5.37^{-5})	1.00 (1.44^{-5})	1.00 (2.02^{-6})	1.00 (2.62^{-7})	1.00 (2.53^{-8})
$\to \infty$	9.96^{-1} (4.24^{-3})	1.00 (3.05^{-4})	1.00 (5.37^{-5})	1.00 (1.44^{-5})	1.00 (2.03^{-6})	1.00 (2.62^{-7})	1.00 (2.53^{-8})

The indices give the power of ten by which the coefficient values must be multiplied.

Table 1 (contd)

$T_e = 1.28 \times 10^5$ °K

p / lg n_e	2	3	4	5	7	10	15
$\to -\infty$	6.93^{-1} ($1.47^{-17}n_e$)	7.27^{-1} ($4.80^{-18}n_e$)	7.53^{-1} ($3.13^{-18}n_e$)	7.69^{-1} ($2.48^{-18}n_e$)	7.91^{-1} ($1.92^{-18}n_e$)	8.07^{-1} ($1.57^{-18}n_e$)	8.20^{-1} ($1.33^{-18}n_e$)
6	6.93^{-1} (1.47^{-11})	7.26^{-1} (4.79^{-12})	7.52^{-1} (3.12^{-12})	7.68^{-1} (2.46^{-12})	7.90^{-1} (1.89^{-12})	8.05^{-1} (1.53^{-12})	8.18^{-1} (1.25^{-12})
8	6.96^{-1} (1.47^{-9})	7.30^{-1} (4.77^{-10})	7.57^{-1} (3.09^{-10})	7.75^{-1} (2.42^{-10})	8.01^{-1} (1.83^{-10})	8.28^{-1} (1.38^{-10})	9.14^{-1} (6.11^{-11})
10	7.02^{-1} (1.46^{-7})	7.40^{-1} (4.68^{-8})	7.72^{-1} (2.96^{-8})	7.98^{-1} (2.22^{-8})	8.72^{-1} (1.21^{-8})	9.63^{-1} (3.20^{-9})	9.95^{-1} (4.24^{-10})
11	7.10^{-1} (1.45^{-6})	7.53^{-1} (4.54^{-7})	7.97^{-1} (2.69^{-7})	8.55^{-1} (1.63^{-7})	9.55^{-1} (4.51^{-8})	9.92^{-1} (7.74^{-9})	9.99^{-1} (8.59^{-10})
12	7.25^{-1} (1.43^{-5})	7.84^{-1} (4.11^{-6})	8.78^{-1} (1.71^{-6})	9.49^{-1} (6.43^{-7})	9.90^{-1} (1.15^{-7})	9.99^{-1} (1.67^{-8})	1.00 (1.72^{-9})
13	7.53^{-1} (1.36^{-4})	8.77^{-1} (2.62^{-5})	9.65^{-1} (6.09^{-6})	9.89^{-1} (1.81^{-6})	9.98^{-1} (2.78^{-7})	1.00 (3.76^{-8})	1.00 (3.75^{-9})
14	8.22^{-1} (1.02^{-3})	9.69^{-1} (9.82^{-5})	9.93^{-1} (1.86^{-5})	9.98^{-1} (5.15^{-6})	1.00 (7.49^{-7})	1.00 (9.89^{-8})	1.00 (9.72^{-9})
15	9.46^{-1} (2.99^{-3})	9.94^{-1} (2.29^{-4})	9.99^{-1} (4.17^{-5})	1.00 (1.14^{-5})	1.00 (1.63^{-6})	1.00 (2.14^{-7})	1.00 (2.10^{-8})
16	9.90^{-1} (3.71^{-3})	9.99^{-1} (2.77^{-4})	1.00 (5.01^{-5})	1.00 (1.36^{-5})	1.00 (1.95^{-6})	1.00 (2.56^{-7})	1.00 (2.50^{-8})
17	9.96^{-1} (3.80^{-3})	1.00 (2.83^{-4})	1.00 (5.11^{-5})	1.00 (1.39^{-5})	1.00 (1.99^{-6})	1.00 (2.61^{-7})	1.00 (2.55^{-8})
18	9.96^{-1} (3.81^{-3})	1.00 (2.83^{-4})	1.00 (5.12^{-5})	1.00 (1.39^{-5})	1.00 (2.00^{-6})	1.00 (2.61^{-7})	1.00 (2.56^{-8})
$\to \infty$	9.96^{-1} (3.81^{-3})	1.00 (2.83^{-4})	1.00 (5.12^{-5})	1.00 (1.39^{-5})	1.00 (2.00^{-6})	1.00 (2.61^{-7})	1.00 (2.56^{-8})

The indices give the power of ten by which the coefficient values must be multiplied.

Table 1 (contd)

$T_e = 2.56 \times 10^5$ °K

p / lg n_e	2	3	4	5	7	10	15
$\rightarrow -\infty$	1.09 $(1.74^{-17}n_e)$	1.00 $(5.61^{-18}n_e)$	9.76^{-1} $(3.64^{-18}n_e)$	9.64^{-1} $(2.87^{-18}n_e)$	9.55^{-1} $(2.22^{-18}n_e)$	9.46^{-1} $(1.82^{-18}n_e)$	9.38^{-1} $(1.53^{-18}n_e)$
6	1.09 (1.74^{-11})	9.99^{-1} (5.59^{-12})	9.74^{-1} (3.63^{-12})	9.61^{-1} (2.85^{-12})	9.51^{-1} (2.19^{-12})	9.40^{-1} (1.77^{-12})	9.29^{-1} (1.44^{-12})
8	1.09 (1.73^{-9})	1.00 (5.57^{-10})	9.77^{-1} (3.59^{-10})	9.65^{-1} (2.81^{-10})	9.56^{-1} (2.12^{-10})	9.49^{-1} (1.62^{-10})	9.66^{-1} (7.80^{-11})
10	1.09 (1.73^{-7})	1.00 (5.47^{-8})	9.81^{-1} (3.45^{-8})	9.70^{-1} (2.59^{-8})	9.71^{-1} (1.49^{-8})	9.89^{-1} (4.23^{-9})	9.98^{-1} (5.82^{-10})
11	1.10 (1.72^{-6})	1.01 (5.31^{-7})	9.84^{-1} (3.16^{-7})	9.79^{-1} (1.98^{-7})	9.91^{-1} (5.88^{-8})	9.98^{-1} (1.05^{-8})	1.00 (1.18^{-9})
12	1.10 (1.69^{-5})	1.01 (4.84^{-6})	9.92^{-1} (2.10^{-6})	9.94^{-1} (8.23^{-7})	9.98^{-1} (1.53^{-7})	1.00 (2.26^{-8})	1.00 (2.35^{-9})
13	1.09 (1.60^{-4})	1.01 (3.18^{-5})	1.00 (7.69^{-6})	9.99^{-1} (2.33^{-6})	1.00 (3.65^{-7})	1.00 (5.01^{-8})	1.00 (5.04^{-9})
14	1.07 (1.18^{-3})	1.01 (1.19^{-4})	1.00 (2.31^{-5})	1.00 (6.50^{-6})	1.00 (9.62^{-7})	1.00 (1.28^{-7})	1.00 (1.27^{-8})
15	1.02 (3.40^{-3})	1.00 (2.70^{-4})	1.00 (5.03^{-5})	1.00 (1.39^{-5})	1.00 (2.03^{-6})	1.00 (2.68^{-7})	1.00 (2.65^{-8})
16	9.98^{-1} (4.19^{-3})	1.00 (3.23^{-4})	1.00 (5.98^{-5})	1.00 (1.65^{-5})	1.00 (2.40^{-6})	1.00 (3.17^{-7})	1.00 (3.13^{-8})
17	9.96^{-1} (4.29^{-3})	1.00 (3.30^{-4})	1.00 (6.10^{-5})	1.00 (1.68^{-5})	1.00 (2.44^{-6})	1.00 (3.23^{-7})	1.00 (3.19^{-8})
18	9.96^{-1} (4.30^{-3})	1.00 (3.30^{-4})	1.00 (6.11^{-5})	1.00 (1.68^{-5})	1.00 (2.45^{-6})	1.00 (3.24^{-7})	1.00 (3.20^{-8})
$\rightarrow \infty$	9.96^{-1} (4.30^{-3})	1.00 (3.30^{-4})	1.00 (6.11^{-5})	1.00 (1.68^{-5})	1.00 (2.45^{-6})	1.00 (3.24^{-7})	1.00 (3.20^{-8})

The indices give the power of ten by which the coefficient values must be multiplied.

Table 2(a) Collisional-radiative coefficient S_{CR} ($cm^3\ sec^{-1}$)

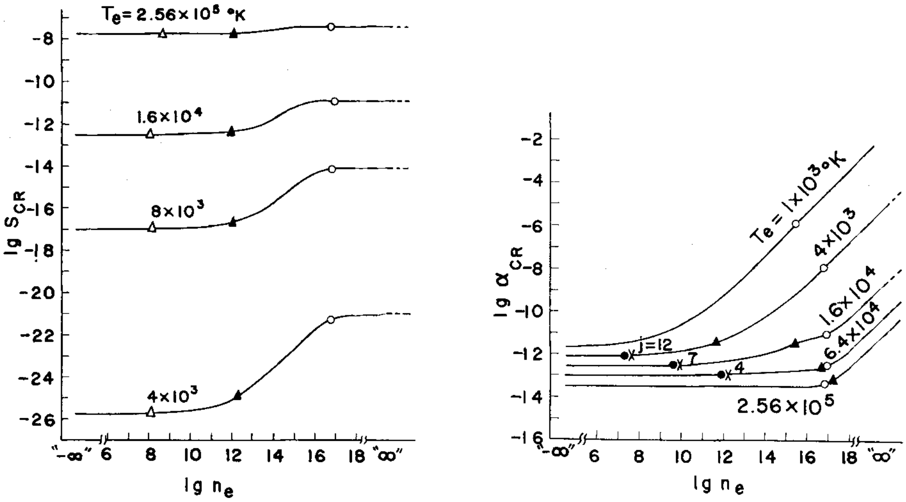
T_e (10^3 °K) / lg n_e	1	2	4	8	16	32	64	128	256
6	—	—	1.82^{-26}	1.02^{-17}	3.09^{-13}	7.00^{-11}	1.35^{-9}	7.23^{-9}	1.87^{-8}
8	—	—	1.97^{-26}	1.06^{-17}	3.15^{-13}	7.08^{-11}	1.36^{-9}	7.26^{-9}	1.87^{-8}
10	—	—	2.83^{-26}	1.23^{-17}	3.39^{-13}	7.36^{-11}	1.39^{-9}	7.37^{-9}	1.90^{-8}
11	—	—	4.40^{-26}	1.51^{-17}	3.74^{-13}	7.76^{-11}	1.44^{-9}	7.52^{-9}	1.93^{-8}
12	—	—	9.95^{-26}	2.26^{-17}	4.57^{-13}	8.66^{-11}	1.53^{-9}	7.86^{-9}	1.99^{-8}
13	—	—	3.61^{-25}	4.92^{-17}	7.06^{-13}	1.11^{-10}	1.79^{-9}	8.75^{-9}	2.17^{-8}
14	—	—	2.35^{-24}	1.78^{-16}	1.66^{-12}	1.97^{-10}	2.65^{-9}	1.16^{-8}	2.73^{-8}
15	—	—	2.00^{-23}	1.05^{-15}	6.28^{-12}	4.64^{-10}	4.57^{-9}	1.71^{-8}	3.75^{-8}
16	—	—	1.63^{-22}	4.89^{-15}	1.26^{-11}	6.31^{-10}	5.39^{-9}	1.90^{-8}	4.11^{-8}
17	—	—	6.23^{-22}	8.03^{-15}	1.41^{-11}	6.57^{-10}	5.50^{-9}	1.93^{-8}	4.15^{-8}
18	—	—	8.67^{-22}	8.58^{-15}	1.43^{-11}	6.60^{-10}	5.51^{-9}	1.93^{-8}	4.16^{-8}
$\to \infty$	—	—	9.07^{-22}	8.65^{-15}	1.43^{-11}	6.60^{-10}	5.51^{-9}	1.93^{-8}	4.16^{-8}

The indices give the power of ten by which the coefficient values must be multiplied.

Table 2(b) Collisional-radiative coefficient α_{CR} ($cm^3\ sec^{-1}$)

T_e (10^3 °K)	1	2	4	8	16	32	64	128	256
lg n_e									
6	2.57^{-12}	1.40^{-12}	8.20^{-13}	4.91^{-13}	2.95^{-13}	1.74^{-13}	9.98^{-14}	6.03^{-14}	3.15^{-14}
8	5.06^{-12}	1.97^{-12}	9.67^{-13}	5.31^{-13}	3.05^{-13}	1.76^{-13}	1.01^{-13}	6.05^{-14}	3.15^{-14}
10	2.79^{-11}	5.08^{-12}	1.58^{-12}	6.77^{-13}	3.42^{-13}	1.85^{-13}	1.03^{-13}	6.09^{-14}	3.15^{-14}
11	1.16^{-10}	1.23^{-11}	2.61^{-12}	8.79^{-13}	3.89^{-13}	1.96^{-13}	1.05^{-13}	6.13^{-14}	3.16^{-14}
12	7.06^{-10}	4.25^{-11}	5.56^{-12}	1.34^{-12}	4.83^{-13}	2.18^{-13}	1.10^{-13}	6.21^{-14}	3.16^{-14}
13	5.71^{-9}	2.10^{-10}	1.59^{-11}	2.53^{-12}	6.86^{-13}	2.59^{-13}	1.18^{-13}	6.35^{-14}	3.16^{-14}
14	5.35^{-8}	1.44^{-9}	6.48^{-11}	5.92^{-12}	1.12^{-12}	3.38^{-13}	1.33^{-13}	6.57^{-14}	3.14^{-14}
15	5.28^{-7}	1.20^{-8}	3.12^{-10}	2.03^{-11}	2.59^{-12}	5.25^{-13}	1.61^{-13}	6.92^{-14}	3.13^{-14}
16	5.27^{-6}	1.12^{-7}	2.46^{-9}	9.02^{-11}	5.00^{-12}	6.96^{-13}	1.87^{-13}	7.57^{-14}	3.34^{-14}
17	5.27^{-5}	1.11^{-6}	2.16^{-8}	2.99^{-10}	1.05^{-11}	1.31^{-12}	3.38^{-13}	1.30^{-13}	5.55^{-14}
18	5.27^{-4}	1.11^{-5}	2.03^{-7}	1.96^{-9}	6.11^{-11}	7.27^{-12}	1.83^{-12}	6.69^{-13}	2.77^{-13}
$\rightarrow \infty$	$5.27^{-22} n_e$	$1.11^{-23} n_e$	$2.00^{-25} n_e$	$1.84^{-27} n_e$	$5.61^{-29} n_e$	$6.62^{-30} n_e$	$1.66^{-30} n_e$	$6.00^{-31} n_e$	$2.46^{-31} n_e$

The indices give the power of ten by which the coefficient values must be multiplied.



(a) (b)

Fig. 2. (a) Collisional-radiative coefficient S_{CR} (cm^3 sec^{-1}) vs. n_e (cm^{-3}). Critical electron densities: ▲ and △; Eqs. (31), (20) and (17) with $q=4$ and 10, respectively. ○; Eq. (32). Limitting values: for lg $n_e \to$ "$-\infty$", ——; Eq. (36). For lg n_e $\to$ "∞", ——; the bottom row in Table 2(a), - - -; Eq. (38′).
(b) Collisional-radiative coefficient α_{CR} (cm^3 sec^{-1}) vs. n_e (cm^{-3}). Critical densities: ●; Eqs. (33), (12), (13) and (17), where j is indicated, ×; Eq. (18) with $p=2$, ○; Eqs. (35) and (32), ▲; Eq. (34′). Limitting values: for lg $n_e \to$ "$-\infty$", ——; Eq. (37). For lg $n_e \to$ "∞", ——; the bottom row in Table 2(b), - - -; Eqs. (39′) and (38′).

3. High- and Low-Density Limits

3.1. The population coefficients

Table 1 and Fig. 1(a) show that at a sufficiently small value of n_e the population coefficient $r_0(p)$ takes a constant, *i.e.*, the limitting value for lg $n_e \to$"$-\infty$", while with increase in n_e it increases or decreases to reach another constant, which tends to unity as p or T_e tends to infinity. Now the limitting values of $r_0(p)$ are denoted as $r_0{}^0(p)$ and $r_0{}^\infty(p)$ corresponding to the low- and high-density limits, respectively, and the critical electron densities at which $r_0(p)$ approximates to $r_0{}^0(p)$ or $r_0{}^\infty(p)$ are named $n_e{}^{00}(p)$ or $n_e{}^{0\infty}(p)$, respectively. In Fig. 1(a), for instance, $r_0(4)$ tends to $r_0{}^0(4)$ for $n_e \lesssim n_e{}^{00}(4) \approx 10^7$ cm^{-3} and to $r_0{}^\infty(4)$ for $n_e \gtrsim n_e{}^{0\infty}(4) \approx 10^{12}$ cm^{-3}. The similar nomenclatures also apply to $r_1(p)$. It is seen in Table 1 and Fig. 1(b) that $r_1(p)$ has, in general, theee parts*; for $n_e \lesssim n_e{}^{10}(p)$ the coefficient $r_1(p)$ has the low-density limit $r_1{}^0(p)$ which is not constant but proportional to n_e, for $n_e \gtrsim n_e{}^{1\infty}(p)$ it approaches the

* In ref. 18 these three parts are called the coronal, quasi-saturated and full-saturated parts in the order of increasing n_e.

high-density limit $r_1^{\infty}(p)$ and for $n_e^{10}(p) \lesssim n_e \lesssim n_e^{1\infty}(p)$ an intermediate part exists.

3.1.1. The critical electron densities—Equation (5) is rewritten with the substitution of Eq. (6) to give $r_0(p)$ and $r_1(p)$ in terms of $r_0(q)$'s and $r_1(q)$'s with $q \neq p$, respectively;

$$r_0(p) = \frac{1}{Z(p)} \frac{\alpha(p)n_e + \beta(p) + \sum_{q=2}^{p-1} Z(q)r_0(q)C(q,p)n_e + \sum_{q=p+1}^{s} Z(q)r_0(q)\{F(q,p)n_e + A(q,p)\}}{\{\sum_{q=1}^{p-1} F(p,q) + \sum_{q=p+1}^{s} C(p,q) + S(p)\}n_e + \sum_{q=1}^{p-1} A(p,q)}, \tag{10}$$

$$r_1(p) = \frac{1}{Z(p)} \frac{Z(1)C(1,p)n_e + \sum_{q=2}^{p-1} Z(q)r_1(q)C(q,p)n_e + \sum_{q=p+1}^{s} Z(q)r_1(q)\{F(q,p)n_e + A(q,p)\}}{\{\sum_{q=1}^{p-1} F(p,q) + \sum_{q=p+1}^{s} C(p,q) + S(p)\}n_e + \sum_{q=1}^{p-1} A(p,q)}. \tag{11}$$

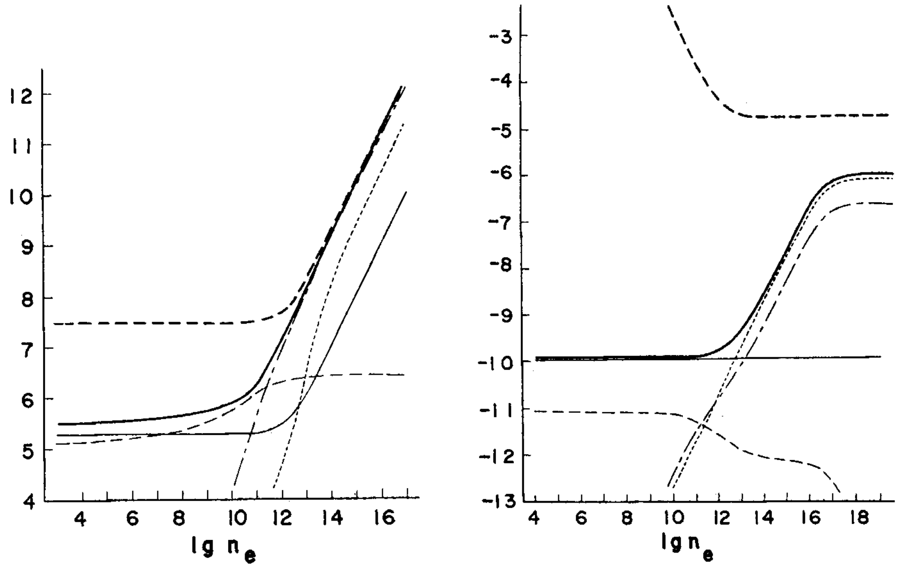
In these equations the contributions from various terms are examined by making use of the results of Table 1 for $r_0(q)$ and $r_1(q)$ with $2 \leqslant q \leqslant r$. An example is shown in Figs. 3(a) and (b) for $r_0(4)$ in Fig. 1(a) and $r_1(4)$ in Fig. 1(b), respectively. Figure 3(a) indicates that for $n_e \leqslant 10^7$ cm^{-3} both the numerator and denominator of Eq. (10) are independent of n_e. As n_e exceeds 10^7 cm^{-3} the contribution from the radiative cascade term gradually increases in the numerator, and at a certain value of n_e the amount of the increase, $\sum_{q>p} Z(q)\{r_0(q) - r_0^0(q)\}A(q,p)$, may be expressed roughly by $\sum_{q \geqslant j} Z(q)\{1 - r_0^0(q)\}A(q,p)$, where j is the lowest level having $r_0(q) = r_0^{\infty}(q) \approx 1$. It is assumed that when this increase is comparable to one tenth of $Z(p)r_0^0(p)\sum_{q<p} A(p,q)$, the coefficient $r_0(p)$ begins to deviate from $r_0^0(p)$. Thus one obtaines

$$n_e^{00}(p) = n_e^{0\infty}(j), \tag{12}$$

where j satisfies

$$\frac{1}{Z(p)} \sum_{q \geqslant j} Z(q)\{1 - r_0^0(q)\}A(q,p) \approx \frac{1}{10} r_0^0(p) \sum_{q<p} A(p,q). \tag{13}$$

With further increase in electron density $r_0(p)$ tends to $r_0^{\infty}(p)$. This is realized in Eq. (10) when the terms representing the collisional transitions dominate over the radiative ones both in the numerator and denominator (see Fig. 3(a)). When it is assumed that $r_0(q) = r_0^{\infty}(q) \approx 1$ for $q > p$ and $r_0(q) \ll 1$ for $q < p$, one can neglect



(a) (b)

Fig. 3. (a) Contributions from various terms in Eq. (10) against n_e for $r_0(4)$. $T_e = 4\times10^3$ °K. Ordinate shows following values in logarithmic scale. - - -; denominator, $\{\sum_{q=1}^{3} F(4,q)+\sum_{q>4} C(4,q)+S(4)\}n_e+\sum_{q=1}^{3} A(4,q)$. ——; numerator, ——; $\{\alpha(4)n_e+\beta(4)\}/Z(4)$, ········; $\sum_{q=2}^{3} Z(q)r_0(q)C(q,4)n_e/Z(4)$, — — — —; $\sum_{q>4} Z(q)r_0(q)F(q,4)n_e/Z(4)$, — —; $\sum_{q>4} Z(q)r_0(q)A(q,4)/Z(4)$.

(b) Contributions from terms in Eq. (11) for $r_1(4)$. $T_e = 4\times10^3$ °K. Ordinate shows following values in logarithmic scale. - - -; denominator, $\sum_{q=1}^{3} F(4,q)+\sum_{q>4} C(4,q)+S(4)+\sum_{q=1}^{3} A(4,q)/n_e$. ——; numerator, ——; $Z(1)C(1,4)/Z(4)$, ··········; $\sum_{q=2}^{3} Z(q)r_1(q)C(q,4)/Z(4)$, — — — —; $\sum_{q>4} Z(q)r_1(q)F(q,4)/Z(4)$, — —; $\sum_{q>4} Z(q)r_1(q)A(q,4)/Z(4)n_e$.

$\sum_{q<p} Z(q)r_0(q)C(q,p)$ in the numerator. Further, $\alpha(p)$ may be disregarded, and then the numerator gives a critical electron density;

$$\sum_{q>p} C(p,q)n_e \approx \Big\{\beta(p)+\sum_{q>p} Z(q)A(q,p)\Big\}/Z(p). \tag{14}$$

On the other hand the denominator gives

$$\Big\{\sum_{q<p} F(p,q)+\sum_{q>p} C(p,q)+S(p)\Big\}n_e \approx \sum_{q<p} A(p,q). \tag{15}$$

For large p or T_e the inequalities $\sum_{q<p} F(p,q) \ll \sum_{q>p} C(p,q)$ and $S(p) \ll \sum_{q>p} C(p,q)$ hold, and for large p the following asymptotic relations are allowed;[19]

$$\begin{cases} \beta(p)/Z(p) \simeq 2Kp^{-5} \ln p \\ \sum_{q>p} Z(q)A(q,p)/Z(p) \simeq Kp^{-5} \ln p \\ \sum_{q<p} A(p,q) \simeq 3Kp^{-5} \ln p, \end{cases} \tag{16}$$

where K is a constant. In this approximation, therefore, Eq. (14) is the same relation as Eq. (15), and the critical electron density is given by

$$n_e^{0\infty}(p) \approx \sum_{q<p} A(p,q) \Big/ \Big\{ \sum_{q<p} F(p,q) + \sum_{q>p} C(p,q) \Big\}. \tag{17}$$

As shown in Fig. 1(a) the critical electron densities $n_e^{0\infty}(p)$ defined by Eqs. (14) and (17) approach each other with increase in p.

The value of $n_e^{00}(p)$ is obtained from Eqs. (12), (13) and (17). The following numerical expression is found to be a good approximation for $n_e^{00}(p)$;

$$n_e^{00}(p) \approx 3\times 10^{16} p^{-17/2} \Phi(p, T_e) \text{ cm}^{-3}, \tag{18}$$

where

$$\Phi(p, T_e) = 10(kT_e/2\chi_H)^{-2.8 \lg p + 4.8}. \tag{19}$$

It is noted that the approximation (18) is valid as fas as $\Phi \lesssim 1$. In Fig. 1(a) there are also shown $n_e^{00}(p)$ given by Eqs. (12), (13) and (17) and the approximate value of Eq. (18).

Figure 3(b), which corresponds to Fig. 1(b), shows the contribution from each term in Eq. (11) for $r_1(4)$, where the values of the terms are divided by n_e both for the numerator and denominator. The denominator is the same as that of $r_0(4)$. The numerator is independent of n_e for $n_e \lesssim 10^{12}$ cm^{-3} and it increases with n_e for $10^{12} \lesssim n_e \lesssim 10^{17}$ cm^{-3}. This transition takes place because the direct excitation is overcome by the stepwise excitations from lower excited levels. It can be shown from a qualitative consideration that this transition occurs roughly at the same electron density as the critical one of Eq. (17) for the denominator;

$$n_e^{10}(p) = n_e^{0\infty}(p). \tag{20}$$

For $n_e \gtrsim 10^{13}$ cm^{-3} the sum $\sum_{q<p} Z(q) r_1(q) C(q,p)$ becomes dominant in the numerator. The saturation of this sum for $n_e \gtrsim 10^{17}$ cm^{-3} results in the saturation in $r_1(p)$. The former comes from the saturation of $r_1(2)$ at $n_e^{10}(2)$ (see Fig. 1(b)). Thus,

$$n_e^{1\infty}(p) = n_e^{10}(2). \tag{21}$$

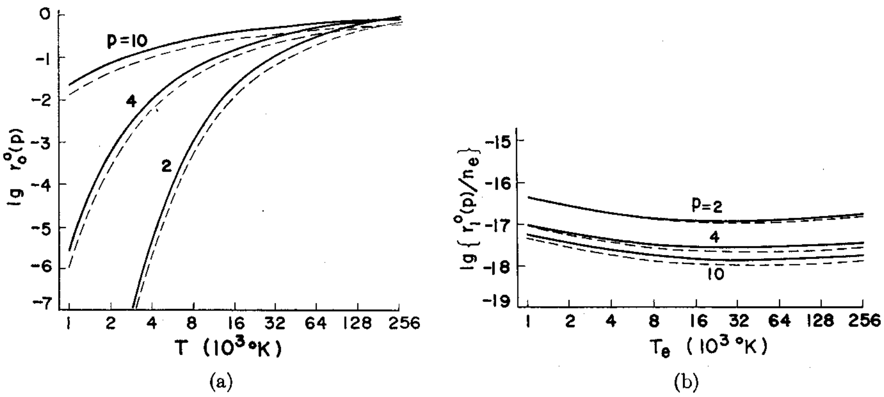


Fig. 4. (a) Limitting values $r_0{}^0(p)$ vs. $T_e(10^3\,°\mathrm{K})$. ——; the top row in Table 1. - - -; Eq. (22).
(b) Limitting values $r_1{}^0(p)$ divided by n_e. ——; the top row in Table 1. - - -; Eq. (23). See also Figs. 1(a) and (b).

3.1.2. The limitting values—Approximate expressions of the population coefficients are now derived for the low and high electron-density limits. Figure 3(a) suggests that for sufficiently low density only the term of radiative recombination into the level is retained in the numerator of Eq. (10);

$$r_0{}^0(p) \approx \beta(p)/Z(p)\sum_{q<p} A(p,q). \tag{22}$$

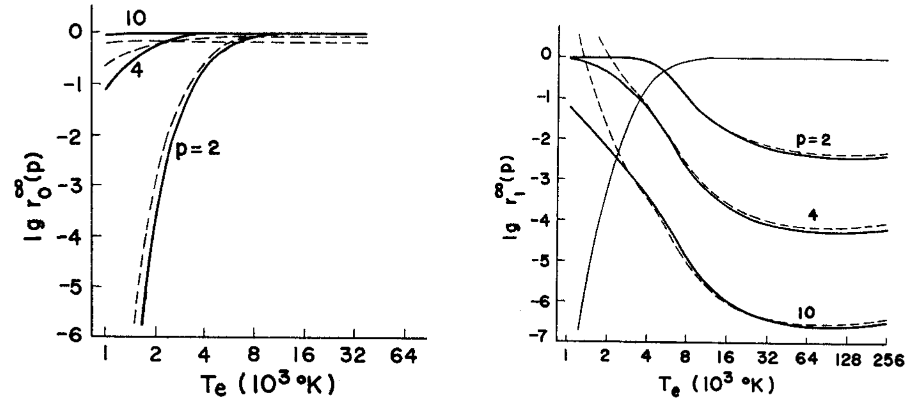
In Figs. 1(a) and 4(a) this approximation is compared with the top row, lg $n_e \to$ "$-\infty$", in Table 1.

As is easily seen from Fig. 3(b), equation (11) is readily transformed to the approximate form at the low electron-density limit;

$$r_1{}^0(p) \approx \left\{Z(1)/Z(p)\right\}\left\{C(1,p)/\sum_{q<p} A(p,q)\right\} n_e. \tag{23}$$

This result is shown in Figs. 1(b) and 4(b). In the approximations above the contribution of cascading from higher levels are neglected in Eqs. (10) and (11). The radiative capture-cascade model[17], which gives the top row in Table 1, includes these contributions in $r_0{}^0(p)$ and $r_1{}^0(p)$.

Figure 3(a) shows that in the limit of high electron density the dominant terms of Eq. (10) are the collisional populations from higher levels in the numerator and the collisional depopulations from the level in the denominator. This fact give the approximation;



(a) (b)

Fig. 5. (a) Limitting values $r_0^\infty(p)$ vs. $T_e(10^3\ ^\circ\text{K})$. ——; the bottom row in Table 1. - - -; Eq. (25).
(b) Limitting values $r_1^\infty(p)$. ——; the bottom row in Table 1. - - -; Eq. (28). ——; t given by Eq. (29). See also Figs. 1(a) and (b).

$$r_0^\infty(p) \approx \sum_{q>p} Z(q) r_0^\infty(q) F(q,p) / Z(p) \Big\{ \sum_{q<p} F(p,q) + \sum_{q>p} C(p,q) \Big\}. \tag{24}$$

If it is assumed that $r_0^\infty(q) \approx 1$ for $q>p$, this equation reduces to

$$r_0^\infty(p) \approx \sum_{q>p} C(p,q) / \Big\{ \sum_{q<p} F(p,q) + \sum_{q>p} C(p,q) \Big\}. \tag{25}$$

This approximation is shown in Figs. 1(a) and 5(a).

In the limit of high electron density Eq. (11) for $p=2$ leads to

$$r_1^\infty(2) \approx F(2,1) / \Big\{ F(2,1) + \sum_{q\geqslant 3} C(2,q) \Big\} \tag{26}$$

and for $p\geqslant 3$

$$r_1^\infty(p) \approx Z(p-1) r_1^\infty(p-1) C(p-1,p) \\ / Z(p) \Big\{ \sum_{q<p} F(p,q) + \sum_{q>p} C(p,q) \Big\}. \tag{27}$$

In general $C(p-1,p)$ is roughly equal to $\sum_{q\geqslant p} C(p-1,q)$, and for $T_e \geqslant 2\times 10^3\ ^\circ\text{K}$ the relation $\sum_{q<p} F(p,q) \ll \sum_{q>p} C(p,q)$ holds for $p\geqslant 3$. Therefore Eq. (27) approximates to

$$r_1^\infty(p) \approx Z(1) C(1,2) t / Z(p) \sum_{q>p} C(p,q), \tag{28}$$

where

$$t = 1/\left\{F(2,1)/\sum_{q\geqslant 3} C(2,q)+1\right\}. \tag{29}$$

This approximation is shown in Figs. 1(b) and 5(b). The deviation of the approximate values from the exact ones at low temperature (see Fig. 5(b)) is due to the neglect of $\sum_{q<p} F(p,q)$ for $p\geqslant 3$.

It is noted that Eqs. (5) and (6) gives the exact relation

$$r_0^{\infty}(p)+r_1^{\infty}(p)=1. \tag{30}$$

As shown in Figs. 5(a) and (b) equations (25) and (28) are approximately consistent with this relation.

3.2. The collisional-radiative coefficients

Tables 2(a) and (b) and Figs. 2(a) and (b) indicate that for sufficiently low electron density both S_{CR} and α_{CR} are independent of n_e, and that for sufficiently high density S_{CR} is independent of n_e while α_{CR} is proportional to it. These limitting values are named S_{CR}^0, α_{CR}^0, S_{CR}^{∞} and α_{CR}^{∞}, and the corresponding critical electron densities are written as n_e^{S0}, $n_e^{\alpha 0}$, $n_e^{S\infty}$ and $n_e^{\alpha\infty}$, respectively.

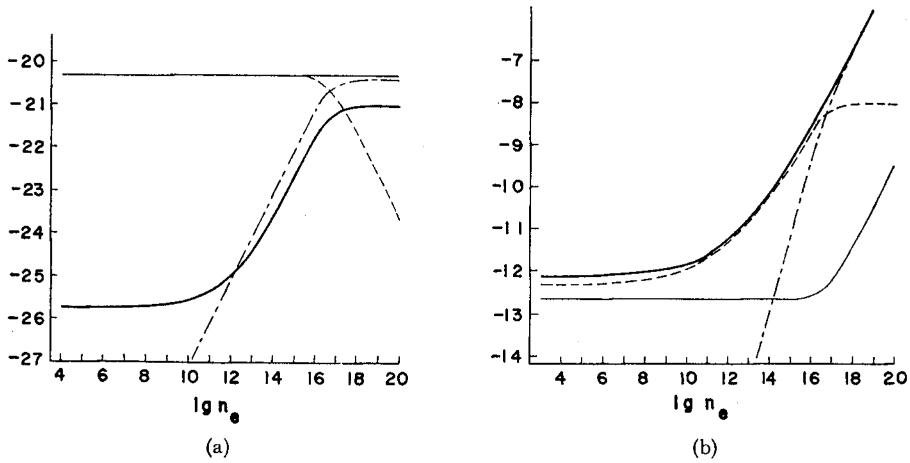


Fig. 6. (a) Contributions from various terms in Eq. (8) against n_e for S_{CR}. $T_e = 4\times 10^3$ °K. Ordinate shows following values in logarithmic scale. ——; S_{CR}, ——; $S(1)+\sum_{q\geqslant 2} C(1,q)$, — — — —; $\sum_{q\geqslant 2} Z(q)r_1(q)F(q,1)/Z(1)$, - - -; $\sum_{q\geqslant 2} Z(q)r_1(q)A(q,1)/Z(1)n_e$. (b) Contributions from terms in Eq. (9) for α_{CR}. $T_e=4\times 10^3$ °K. ——; α_{CR}, ——; $\alpha(1)n_e+\beta(1)$, — — — —; $\sum_{q\geqslant 2} Z(q)r_0(q)F(q,1)n_e$, - - -; $\sum_{q\geqslant 2} Z(q)r_0(q)A(q,1)$.

Equation (8) gives S_{CR} in terms of $r_1(q)$ with $2 \leqslant q \leqslant s$. Figure 6(a) shows an example of the contributions from various terms in Eq. (8) as a function of n_e for $T_e = 4 \times 10^3$ °K. As n_e increases from sufficiently low density (cf. Eq. (36)), the term of cascade transitions in the curly bracket in Eq. (8) reduces due to the decrease in $r_1(q)/n_e$ beginning at $n_e^{10}(q)$ for some large q (see Fig. 1(b)). This results in the increase in S_{CR} and, therfore, the critical electron density for the lower limit is given by

$$n_e^{S0} = n_e^{10}(q), \tag{31}$$

where q is taken as 4∼10. In Fig. 2(a) it is seen that the values of n_e^{S0} with $q=4$ and 10 indicate the transition zone from the low-density limit for S_{CR}.

The coefficient S_{CR} tends to S_{CR}^{∞} when n_e reaches such a high density that in Eq. (8) all the $r_1(q)$'s take constant values, say $r_1^{\infty}(q)$'s, and that the first term in the curly bracket becomes larger than the second one. The first condition is equivalent to $n_e \gtrsim n_e^{1\infty}(q) = n_e^{10}(2) = n_e^{0\infty}(2)$, and the second is realized when $F(q, 1)n_e \gtrsim A(q, 1)$ for $q \geqslant 2$, *i.e.*, when $F(2, 1)n_e \gtrsim A(2, 1)$ is satisfied. Here the inequality $A(2, 1)/F(2, 1) > A(2, 1)/\{F(2, 1) + \sum_{q\geqslant 3} C(2, q)\} \approx n_e^{0\infty}(2)$ holds. Therefore, the critical electron density for S_{CR}^{∞} is defined by

$$n_e^{S\infty} = A(2, 1)/F(2, 1). \tag{32}$$

This result is shown in Fig. 2(a).

Figure 6(b) indicates the component terms of α_{CR} at $T_e = 4 \times 10^3$ °K. At the limit of low electron density α_{CR} is expressed as the sum of the terms of direct recombination and radiative cascade in Eq. (9) (cf. Eq. (37)). With increase in n_e the cascade term increases due to the increase in $r_0(q)$, mianly that in $r_0(2)$ (see Fig. 1(a)). Therefore, the critical electron density for α_{CR}^0 is the same as that for $r_0^0(2)$ (see Fig. 2(b));

$$n_e^{\alpha 0} = n_e^{00}(2). \tag{33}$$

In order that α_{CR} has the limitting value α_{CR}^{∞} two conditions must be satisfied: the first is the same as the one imposed on $n_e^{S\infty}$, *i.e.*, $n_e \gtrsim A(2, 1)/F(2, 1)$, and the second is that in Eq. (9) the following relation must hold;

$$\sum_{q\geqslant 2} Z(q) r_0(q) F(q, 1) n_e \gtrsim \beta(1). \tag{34}$$

With the approximation of $r_0(q) \approx 1$ for $q \geqslant 2$ the left side is transformed to give

$$Z(1) \sum_{q\geqslant 2} C(1, q) n_e \gtrsim \beta(1). \tag{34'}$$

The values of n_e corresponding to Eq. (34′) are given in Fig. 2(b) and it is seen that the second condition is always satisfied when the first one is satisfied, except for very high temperature. Thus,

$$n_e^{\alpha\infty} = n_e^{S\infty}. \tag{35}$$

Next, the limitting values of S_{CR} and α_{CR} are considered. Using the method by Seaton[17] the following expressions are proven;

$$S_{CR}{}^0 = S(1), \tag{36}$$

$$\alpha_{CR}{}^0 = \sum_{q\geqslant 1} \beta(q). \tag{37}$$

At the limit of high electron density Eq. (8) becomes (see Fig. 6(a))

$$S_{CR}{}^\infty \simeq S(1) + \sum_{q>1} C(1, q)\{1 - r_1{}^\infty(q)\}. \tag{38}$$

In the summation the term of $q=2$ is the leading one, so that Eq. (38) may approximate to

$$S_{CR}{}^\infty \approx S(1) + \sum_{q>1} C(1, q)\left\{1 - r_1{}^\infty(2)\right\}. \tag{38'}$$

The recombination coefficient is written as (see Fig. 6(b))

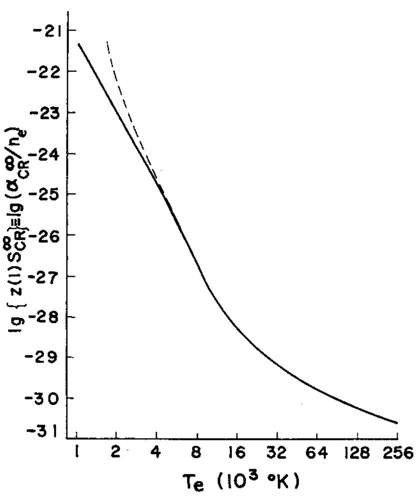


Fig. 7. Limitting value $Z(1)S_{CR}{}^\infty \equiv \alpha_{CR}{}^\infty / n_e$ (cm^6 sec^{-1}) vs. T_e (10^3 °K). ——; the bottom row in Table 2(b), - - -; Eq. (38′) or (39′). See also Figs. 2(a) and (b).

$$\alpha_{CR}^{\infty} \simeq \left\{\alpha(1)+\sum_{q>1} Z(q)r_0^{\infty}(q)F(q,\,1)\right\}n_e. \tag{39}$$

Substitution of Eq. (30) into (39) leads to

$$\alpha_{CR}^{\infty} = Z(1)S_{CR}^{\infty}n_e. \tag{39'}$$

The approximations (38′) and (39′) are shown in Fig. 7, where Eq. (26) is used for $r_1^{\infty}(2)$. Agreement with the exact calculation is satisfactory for $T_e \gtrsim 2\times 10^3$ °K.

4. Discussions

In the analysis of the preceding sections the plasma has been assumed to be optically thin. However, it is occasionally not the case for laboratory and astronomy plasmas, and radiation is absorbed partially or completely by them. In order to treat such plasmas an equation of radiative transfer must be incorporated with the rate equations for population density. As one of limitting cases the case B plasma is frequently considered, in which Lyman lines are completely absorbed and other radiations do not suffer any absorption. Rate equations for this case are readily obtained by setting $A(p,\,1)=0$ for $p\geqslant 2$ in Eq. (1), and the population and collisional-radiative coefficients are easily calculated for this case. As an example of the results Figs. 8(a)

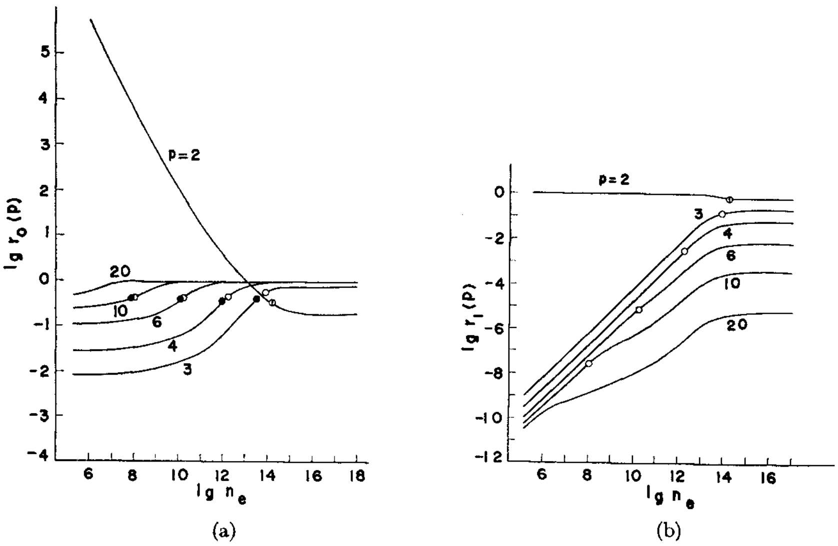
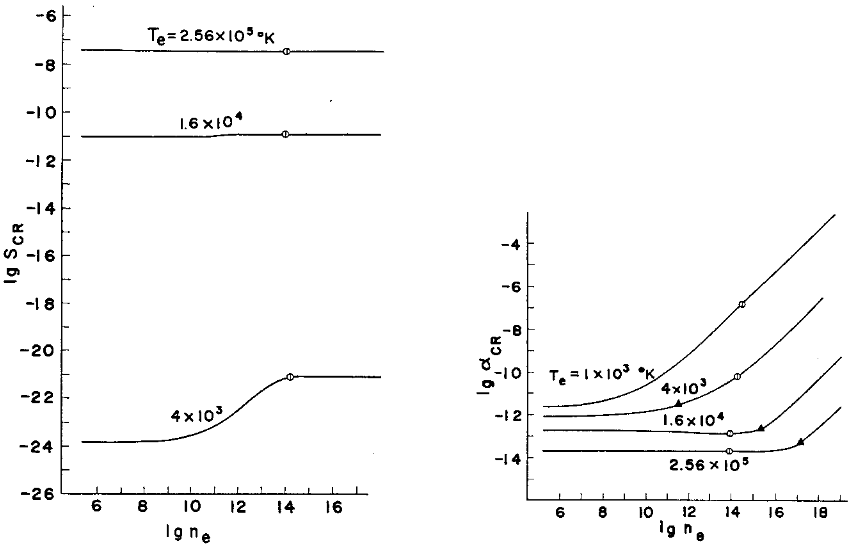


Fig. 8. (a) Population coefficients $r_0(p)$ in the case B for $T_e=4\times 10^3$ °K. Critical electron densities: ●; Eq. (40), ○; Eqs. (41) and (17), ⦶; Eq. (42).
(b) Population coefficient $r_1(p)$ for $T_e=4\times 10^3$ °K. Critical densities: ○; Eqs. (43), (41) and (17). ⦶; Eq. (44).



(a) (b)

Fig. 9. (a) Collisional-radiative coefficient S_{CR} in the case B. Critical density: ⏀; Eqs. (45) and (44).
(b) Coefficient α_{CR} in the case B. Critical densities: ⏀; $n_e^{\alpha\infty\prime} = A(3,2)/F(3,2)$, ▲; $n_e^{\alpha\infty\prime} = \beta(1)/Z(1)\sum_{q\geqslant 2} C(1,q)$.

and (b) show the population coefficients for $T_e = 4\times 10^3$ °K, and Figs. 9(a) and (b) give the collisional-radiative coefficients.

Here some of the critical electron densities are discussed. The level 2 must be treated separately from higher levels since $A(2, 1)=0$. From Eq. (17) the critical electron density for $r_0^\infty(p)$ $(p\geqslant 3)$ is given by

$$n_e^{0\infty}(p)' \approx \sum_{2\leqslant q<p} A(p,q) \Big/ \Big\{ \sum_{q<p} F(p,q) + \sum_{q>p} C(p,q) \Big\}, \tag{40}$$

where the prime on the critical density denotes the case B. The sum $\sum_{2\leqslant q<p} A(p, q)$ differs little from $\sum_{1\leqslant q<p} A(p, q)$, therefore Eq. (40) is replaced by Eq. (17);

$$n_e^{0\infty}(p)' \approx n_e^{0\infty}(p) \qquad \text{for } p \geqslant 3. \tag{41}$$

Figure 10 shows the contributions from various terms in Eq. (10) for $r_0(2)$. As in the case of $n_e^{S\infty}$, the density $n_e^{0\infty}(2)'$ is given by the two conditions. The first one is that the collisional terms become larger than the radiative cascade ones; this gives the density $A(q, 2)/F(q, 2)$ with $q\geqslant 3$ and the largest of which is $A(3, 2)/F(3, 2)$. The second condtion is that $r_0(q)=r_0^\infty(q)$ for $q\geqslant 3$, *i.e.*, $n_e \gtrsim n_e^{0\infty}(q)'$. The former condition apparently satisfies the latter. Therefore,

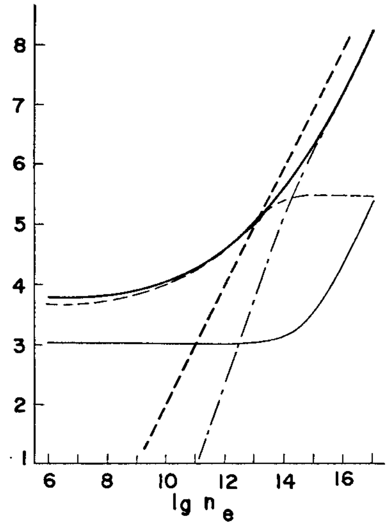


Fig. 10. Contributions from various terms in Eq. (10) for $r_0(2)$ in the case B. $T_e=4\times10^3$ °K. Ordinate shows following values in logarithmic scale. - - -; denominator, $\{F(2,1)+\sum_{q\geq3}C(2,q)+S(2)\}n_e$. ——; numerator, ——; $\{\alpha(2)n_e+\beta(2)\}/Z(2)$, — - — -; $\sum_{q\geq3}Z(q)r_0(q)F(q,2)n_e/Z(2)$, - - -; $\sum_{q\geq3}Z(q)r_0(q)A(q,2)/Z(2)$.

$$n_e^{0\infty}(2)'\approx A(3,2)/F(3,2) \tag{42}$$

Equations (40), (41) and (42) are indicated in Fig. 8(a).

Equation (20) also holds in the case B for $p\geqslant3$;

$$n_e^{10}(p)'=n_e^{0\infty}(p)' \quad \text{for } p\geqslant3. \tag{43}$$

In the same way as the derivation of $n_e^{0\infty}(2)'$ the critical density for $r_1^{\infty}(2)$ is given by

$$n_e^{1\infty}(2)'\approx A(3,2)/F(3,2). \tag{44}$$

Figure 8(b) shows these critical densities.

Noting $A(q,1)=0$ in Eq. (8) one obtains $n_e^{S\infty}$, from the condition of $r_1(q)=r_1^{\infty}(q)$ for $q\geqslant2$ (see Fig. 9(a));

$$n_e^{S\infty\prime}=n_e^{1\infty}(2)'. \tag{45}$$

When n_e becomes so high that in Eq. (9) the relation $r_0(q)=r_0^{\infty}(q)$ for $q\geqslant2$ is realized and that Eq. (34) is satisfied, α_{CR} tends to α_{CR}^{∞}. Figure 9(b) shows the densities corresponding to these two relations. Therefore, the larger one of the following gives the critical density;

$$n_e^{\alpha\infty\prime} = \begin{cases} A(3,2)/F(3,2) \\ \beta(1)/Z(1)\sum\limits_{q\geq 2} C(1,q). \end{cases} \tag{46}$$

In the case B plasma the limiting forms of $r_0^\infty(p)$, $r_1^\infty(p)$, S_{CR}^∞ and α_{CR}^∞ are the same as those for optically thin plasma. However, the ilmiting forms at low electron density, $r_0^0(p)$, $r_1^0(p)$, S_{CR}^0 and α_{CR}^0, are too complicated to be given by simple approximate expressions and they must be treated in detail for a given particular condition of a plasma.

In general, in order to discuss the plasma balance of ionization and recombination, rate coefficients due to the processes such as Penning ionization or wall recombination must be considered in addition to the collisional-radiative coefficients given in Table 2. The population coefficients in Table 1 then give enough information to determine a population density distribution among energy levels of atomic or ionic species in the plasma. Further, these coefficients will be very useful to give definite knowledge of local equilibrium in plasma such as LTE, coronal equilibrium and so forth. These problems will be discussed in separate papers.

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