Mean Particle Diameter in an Analysis of a Particulate Process

By

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Abstract

In the study of a particulate process, one of the most important subjects to consider is what mean particle diameter to employ. In this study, an experimental value is divided into two terms, one with some interaction between particles and the other without such interaction (the linear term). The mean particle diameter is defined only in terms of the latter, that is the linear term.

It is shown that the scattering in previously published data for the particulate process, is attributable to the fact that the mean diameter is not determined correctly. Further, if such a diameter as determined in this study is used, a satisfactory result with little scattering in the data is achieved.

In connection with the definition, the practical method for determining the mean particle diameter and suggestions as to its use are given, and the relation between the process variables observed when the distribution is log-normal, is also discussed.

1. Introduction

What has been said previously about the mean particle diameter is not definite enough, and this vagueness has caused much trouble. When studying the experimental data obtained for the same particulate process, for example, it is found that some authors employ the arithmetic mean diameter, others, the geometric mean diameter or the mass-surface mean diameter, and so on. Practically, the value of mean diameter varies so much depending on the definition employed, that in some extreme cases, even an assumed mean diameter may be used to bring the experimental data (measured values) into good agreement with the estimated values (theoretical values). If such is the case, one cannot tell whether the theoretical value is correct or not.

Another problem may arise as to what mean diameter to employ in quoting some of the results reported by other investigators. One may only apply such definitions as have been generally used, namely the arithmetic mean diameter, the geometric mean diameter, and so on.

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Another problem to be solved is how the theoretical and experimental results for mono-disperse particulate processes are to be applied to a poly-disperse particulate process.

A solution to such problems as pointed out above is to develop a definition for the mean diameter, which allows the value of the mean particle diameter to be estimated with high accuracy.

In an approach to this, C.E. Андреев and his collaborators[1] defined the mean particle diameter in general form. S. Miwa[2,3] has commented on their results.

In this study, the authors, from a consideration of these studies, define the mean particle diameter in terms of the linear estimate of the process variable, and show that it may be used to determine the mean diameter in any process. Alternative methods for determining the mean particle diameter are given in detail.

2. Definition of the Mean Particle Diameter

2.1. The experimental value of the process variable

To describe the state of the particles sampled, two parameters, "mean" and "variance" of the diameters are necessary, even if other conditions such as temperature, moisture or the shape of the particle are considered to be constant. The frequency distribution of the particle size is, therefore, a function of diameter D_p, mean m and variance s^2;

$$f = f(D_p, m, s^2) \tag{1}$$

Generally, the particles sampled vary in each experiment, the experimental value Y being a function of at least two variables, m and s^2. With interaction term γ, Y will be expressed in the form;

$$Y(m,s^2) = \int_0^\infty y(D_p)\ f(D_p, m, s^2)\, dD_p + \gamma(m, s^2) \tag{2}$$

Here $y(D_p)$ represents either a mono-disperse particulate process variable or a one-particle process variable. The first term of the right hand side, being a linear estimate, can be put in the form;

$$\bar{y}(m, s^2) = \int_0^\infty y(D_p)\ f(D_p, m, s^2)\, dD_p \tag{3}$$

Of course, the term $\int \cdot \ \cdot\ dD_p$ can be replaced with Σ when a discrete system is concerned.

2.2. Linear estimate $\bar{y}(m, s^2)$

Generally, the linear estimate $\bar{y}$ varies with the method of measurement such as weighing the particles or counting the particles and so on. Just for a simple

Table 1. Description of the six particles

Particle dia.	1	1	1	1	2	2
Process variable	y_1	y_1	y_1	y_1	y_2	y_2
Weight ratio	1	1	1	1	8	8

($y_1=15$, $y_2=30$)

example, take the case of six spherical particles with the same specific gravity. Four of the six have a diameter 1, and the remaining two have a diameter 2. Each particle fits the equation;

$$y=y(D_p) \text{ e.g. if the process follows Stokes' law } y=KD_p^2 \tag{4}$$

and has no interaction with any of the others (cf. Table 1). Then the experimental value on the count basis is;

$$Y^{(0)}=\frac{4}{4+2}y_1+\frac{2}{4+2}y_2=\frac{2}{3}y_1+\frac{1}{3}y_2 \text{ [mean on count basis]} \tag{5}$$

Here y_i stands for $y(D_{pi})$. On the other hand, the experimental value on the mass basis is;

$$Y^{(3)}=\frac{4}{4+(2\times 8)}y_1+\frac{2\times 8}{4+(2\times 8)}y_2=\frac{1}{5}y_1+\frac{4}{5}y_2 \text{ [mean on mass basis]} \tag{6}$$

If $y_1=15$ and $y_2=30$, it is clear that $Y^{(0)}=20$ and $Y^{(3)}=27$ from the last two Eqs. (5) and (6). So, in spite of the fact that the process is the same, the experimental values obtained by the different methods are not the same*). In this example, the experimental value can be estimated only by the linear term. So from this example if one wants to estimate the experimental values on any basis, it is found desirable to express the linear estimate in the form;

$$\bar{y}^{(a)}=\sum f^{(a)}y=\sum f^{(0)}D_p^a y/\sum f^{(0)}D_p^a \tag{7}$$

If "a" in the above equation is 0, the estimate is on the count basis, if "a" is 1, on the length basis, if "a" is 2, on the area basis, if "a" is 3, on the volume (or mass) basis. As discussed above, these values are different from each other. The method used, therefore, has to be stated definitly, and the experimental values have to be compared with the estimated values on the same basis.

2.3. Definition of the mean particle diameter

By use of the linear estimate, the mean particle diameter will be defined by the equation;

$$\bar{D}_p=y^{-1}(\bar{y}) \tag{8}$$

2.4. A method of applying the definition term by term

When the process variable y is given in the form of the summation of several

*) Note that the mean particle diameter of the six particles cannot be determined at this point. See also §4.3.

terms, it is convenient to apply definition (8) to each term. That is, if the process variable y is expressed by the equation;

$$y=\sum_{j=1}^{n} y_{(j)} \tag{9}$$

then the mean particle diameters are given by the equations;

$$\bar{D}_{p(j)}=y_{(j)}^{-1}(\bar{y}_{(j)}),\ j=1,\ 2,\ 3,\ldots,\ n \tag{10}$$

2.5 A graphical method

Now consider the case that the process variable y cannot be expressed in a simple equation of D_p, or cannot be expressed in any equation. In such a case, if the process variable can be expressed graphically or in a table, the mean particle diameter may be obtained directly by use of definition (8). See Fig. 1 to explain this method generally. In the figure, $g(D_p)$ is a function of D_p, e.g. Reynolds number, Nusselt number, Peclet number, Sherwood number, Inertia parameter, and so on, and here it is called the characteristic parameter of the particle diameter.

From Eq. (7), the linear estimate $\bar{y}$ is given by the equation;

$$\bar{y}=\sum_{i} f_i\, y(g(D_{pi})) \tag{11}$$

Having calculated $\bar{y}$, the characteristic parameter g of the mean particle diameter is obtained as shown by the arrows in Fig. 1. Then, from this value, the mean particle diameter can be determined by the following equation;

$$\bar{D}_p=g^{-1}(\bar{g}) \tag{12}$$

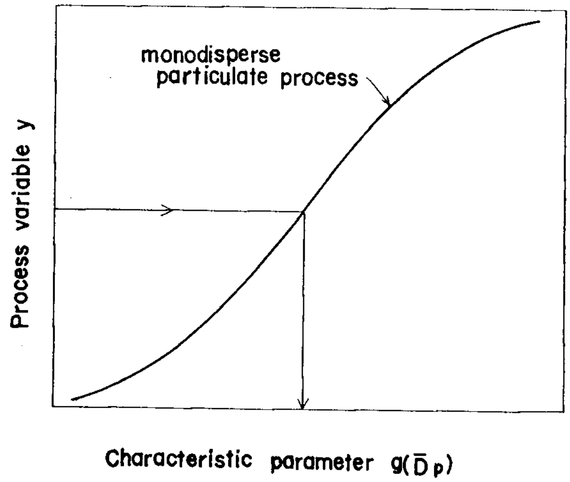


Fig. 1. Graphical method

3. Discussion

3.1 On the definition of mean particle diameter

The following discussion shows that definition (8) is adequate in scope and is of considerable importance. The discussion is based on the assumption that there is no interaction between particles. From Eq. (2), it follows that;

$$Y(m,s^2)=\int_0^\infty y(D_p)\,f(D_p,m,s^2)\,dD_p=\bar{y}(m,s^2) \tag{13}$$

From this equation it is found that the experimental value $Y(m, s^2)$ is a function of m and s^2. Hence, the experimental value $Y(m, s^2)$ may be expressed as a plane on the (m, s^2, Y)-space (cf. Fig. 2). This is interpreted as implying that if one point (m, s^2) is given, as a sampling of the particles, one experimental value $Y(m, s^2)$ will be given. If the mean particle diameter is given in accordance with definition (8), it is given by the equation;

$$\bar{D}_p=y^{-1}(\bar{y}(m, s^2))=y^{-1}(Y(m, s^2)) \tag{14}$$

So, $\bar{D}_p=$const. gives a curve in a (m, s^2)-space, and from the definition, the experimental values $Y(m, s^2)$ on this curve are constant, that is;

$$Y(m, s^2)=y(\bar{D}_p)=\text{const.} \tag{15}$$

From the above discussion, it may be concluded that the experimental values $Y(m, s^2)$ have a constant value $y(\bar{D}_p)$ on this mean particle diameter $\bar{D}_p$ (cf. Fig. 3). This will be applicable when the experimental values are studied using the mean particle diameter (cf. Fig. 4). On the other hand, any mean particle diameter that is defined other than by Eq. (8) has no such characteristic property as mentioned above. Such mean diameters will be discussed briefly here. Generally any one of these mean particle diameters may be formulated with a certain

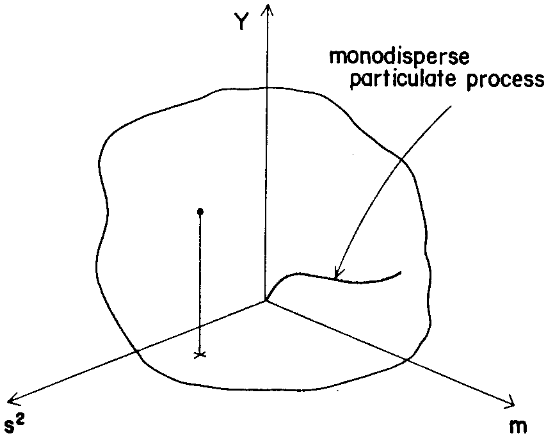


Fig. 2. General idea of experimental data in the (s^2, m, Y)-space

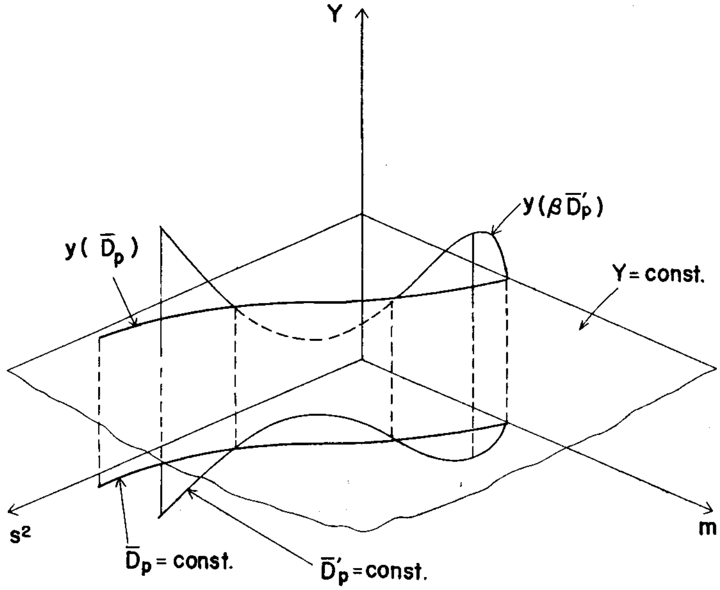


Fig. 3. Relation between the mean particle diameter and the experimental data

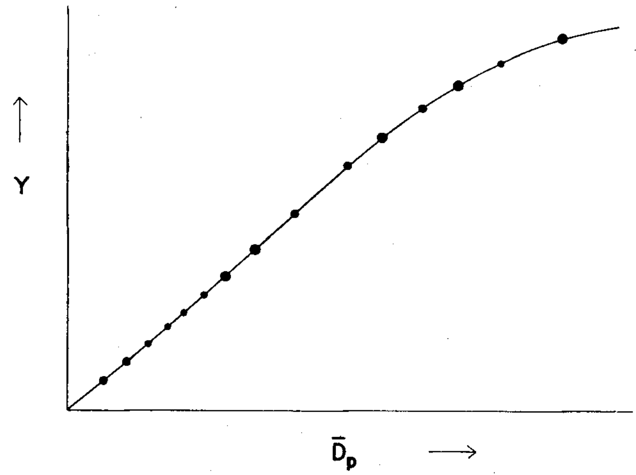


Fig. 4. Experimental data represented by use of the mean particle diameter based on definition (8)

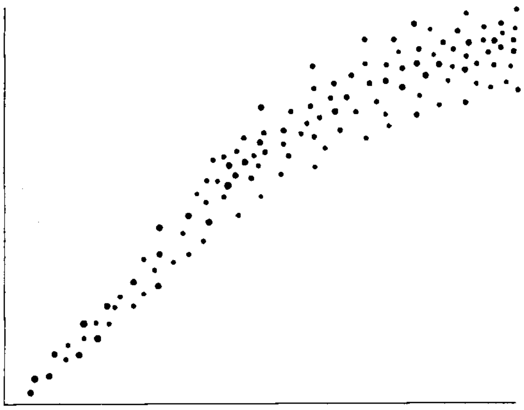
function $\beta(m, s^2)$ as follows;

$$\bar{D}_p' = \bar{D}_p/\beta(m, s^2) = y^{-1}(Y(m, s^2))/\beta(m, s^2) \tag{16}$$

Therefore,

$$Y(m, s^2) = y(\beta(m, s^2) \cdot \bar{D}_p') \tag{17}$$

The above equation means that $\bar{D}_p'$=const. cannot give a constant experimental value. In other words, the experimental values on the curve $\bar{D}_p'$=const. vary



$\bar{D}_p'$ ⟶

Fig. 5. Experimental data represented by use of other mean particle diameters than the correct one based on the proposed definition

with each (m, s^2). Fig. 3 shows the general idea. Note that if a wrong mean particle diameter is used, the values obtained, even from the most careful experiments, will turn out to scatter as shown in Fig. 5. Without any data as to (m, s^2), scarcely any satisfactory results may be obtained from the experiments. The following example will help to make the above discussion clear. A random sampling is carried out from log-normal distributed particles (cf. §3.3). Then an experiment is made on the process to conform to the equation;

$$y = KD_p^2 \tag{18}$$

The mean particle diameter to conform to definition (8) is;

$$\bar{D}_p = y^{-1}(\bar{y}) = \sqrt{\int_{-\infty}^{\infty} KD_p^2 f^{(0)}(\ln D_p, m, s^2)\, d\ln D_p / K} = \exp(m+s^2) \tag{19}$$

[for Q1 shown in Fig. 8, $\bar{D}_p = 57.4$ microns]

The experimental value $Y(m, s^2)$ is expressed as;

$$Y(m, s^2) = K\int_{-\infty}^{\infty} D_p^2 f^{(0)}(\ln D_p, m, s^2)\, d\ln D_p = K\bar{D}_p^2 \tag{20}$$

[for Q1, $Y/K = 3295$ microns²]

If $\bar{D}_p = \text{const.}$, the experimental values $Y(m, s^2)$ have a constant value $K\bar{D}_p^2$, independently of the value (m, s^2), and are correctly represented by Eq. (18). Such a process as discussed above is shown in Fig. 6. On the other hand, if some other mean particle diameter, for example, the length mean diameter;

$$\bar{D}_p' = \int_{-\infty}^{\infty} D_p f^{(0)}(\ln D_p, m, s^2)\, d\ln D_p = \exp(m+s^2/2) \tag{21}$$

[for Q1, $\bar{D}_p = 54$ microns]

is used, the relation between $\bar{D}_p'$ and $\bar{D}_p$ is as follows;

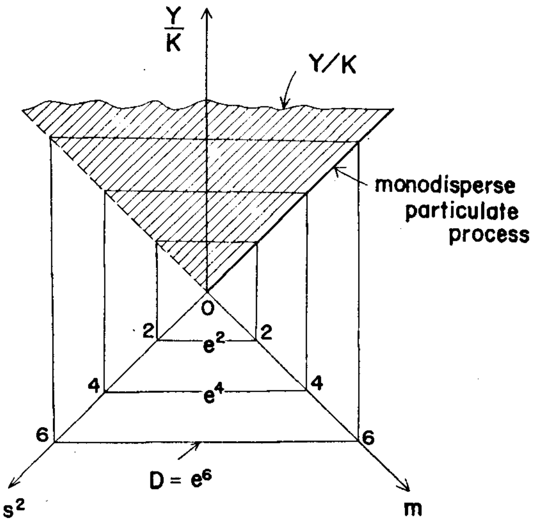


Fig. 6. The mean particle diameter—surface mean diameter—based on the definition and the experimental data in the $(s^2, m, Y/K)$-space

$$\bar{D}_p' = \exp(m+s^2)\exp(-s^2/2) = \bar{D}_p/\exp(s^2/2) \tag{22}$$

$\beta(m, s^2)$ used in the general discussion is $\exp(s^2/2)$. From Eq. (17) or Eqs. (20) and (22), the following equation is obtained.

$$Y(m, s^2) = y(\beta(m, s^2)\bar{D}_p') = K\{\beta(m, s^2)\bar{D}_p'\}^2 = K\cdot\exp(s^2)\bar{D}_p'^2$$

or,

$$Y(m, s^2)/K = \bar{D}'^2_p \exp(s^2) \tag{23}$$

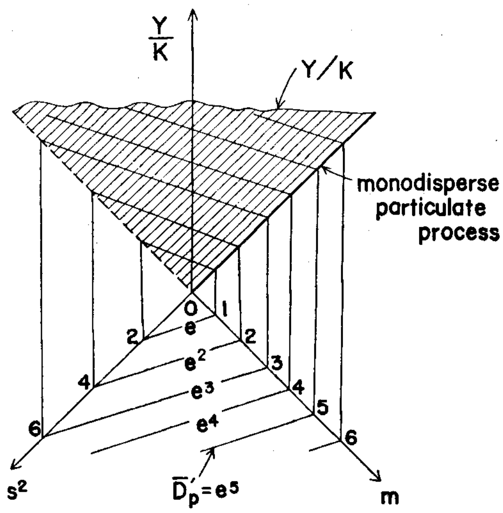


Fig. 7. The length mean particle diameter and the experimental data in the $(s^2, m, Y/K)$-space

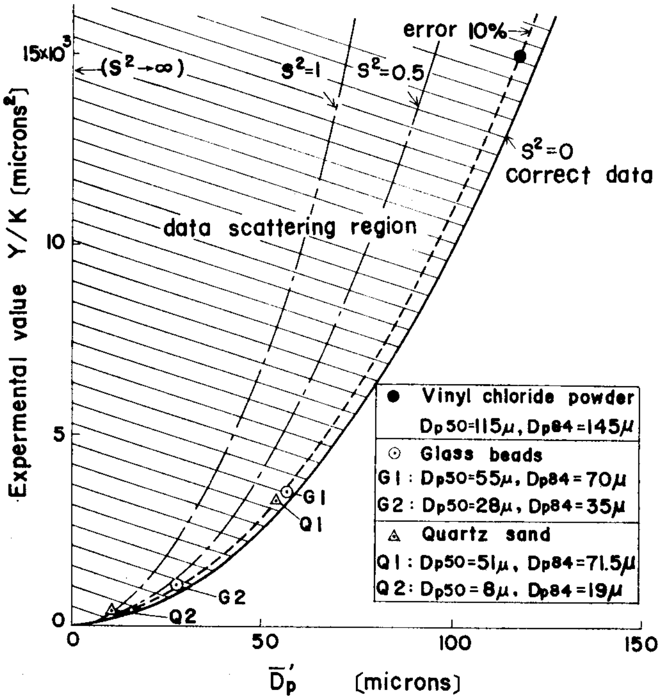


Fig. 8. Experimental data represented by use of other mean particle diameters than the correct one-e.g. : the length mean diameter

This is shown in Fig. 7. As shown in Fig. 8, the results represented by $\bar{D}_p'$ vary with the variance s^2. Therefore, even if the length-mean particle diameter $\bar{D}_p'$ is constant, the experimental values will scatter through the variance of powders. In general, this scatter has a bias. In this example, the experimental values are always larger than $K \cdot D_p^2$ when $\bar{D}_p'$ is used. In Fig. 8, corresponding experimental values for the kinds of powder often used in experiments are indicated by the designated symbols. These powders have fairly small variances. However, the results show about 10% error. When $\bar{D}_{p50}$ is used instead of $\bar{D}_p'$, the results become worse, and the errors are 20% or more.

It is most important in determining the mean particle diameters term by term (cf. §2.4) that the experimental value is compared with another only at the point where all the mean particle diameters coincide with one another. When the process variable is expressed in more than two terms, therefore, it is better to deal with the data using both the mean and the variance. Note that even in such a case, when the graphical method is introduced (cf. §2.5), it is adequate to study the data with the mean particle diameter or with its characteristic parameter $g(\bar{D}_p)$.

From the above discussion it is clear that to use a properly-defined mean particle diameter is very important, not only to study the various experimental values systematically but to attain satisfactory results with little scattering in the data.

3.2 Comments on the use of the mean diameter

The mean diameter being defined by the linear estimate, the process variable estimated, using the mean diameter, is a linear part of the variable. The non-linear part of it, therefore, has to be discussed separately from the linear part.

Especially in the case where the process variable depends both on the feed and on the product particles, careful consideration must be taken on the linear estimate. When the mean diameters defined term by term are used, the linear estimate is given by the equation;

$$\bar{y}=\sum_{i,j} f_{(F)i}\, f_{(P)j}\, y(D_{p(F)i},\ D_{p(P)j}) \tag{24}$$

where the suffixes F and P refer to the feed and the product, respectively. Therefore, the effects of the overlapping of the two frequency distributions are left out of consideration here. In the case where each particle of the product cannot be larger than it was before passing through the process, $\sum_{D_{p(F)}<D_{p(P)}} f_{(F)}\, f_{(P)}\, y(D_{p(F)},$ $D_{p(P)})$ must be subtracted from the estimate. Then Y is given by the equation;

$$Y=\bar{y}-\sum_{D_{p(F)}<D_{p(P)}} f_{(F)}\cdot f_{(P)}\cdot y(D_{p(F)},\ D_{p(P)})+\gamma \tag{25}$$

Conversely, when the term "larger than" is replaced with "smaller than", the summation should be carried out on $D_{p(F)}>D_{p(P)}$.

3.3 Other comments; Log-normal particle size distribution

It is convenient to make use of the log-normal particle size distribution;

$$f(\ln D_p, \mu, \sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{1}{2}\frac{(\ln D_p-\mu)^2}{\sigma^2}\right\} \tag{26}$$

where,

$$\mu=\int_{-\infty}^{\infty}\ln D_p f(\ln D_p, \mu, \sigma^2)\, d\ln D_p=\ln D_{p(50\%)} \tag{27}$$

$$\sigma^2=\int_{-\infty}^{\infty}(\ln D_p-\mu)^2 f(\ln D_p, \mu, \sigma^2)\, d\ln D_p \tag{28}$$

$$\sigma = \ln D_{p(84.1\%)}-\ln D_{p(50\%)} \tag{29}$$

Then, the relation between $\bar{y}^{(0)}$ and $\bar{y}^{(a)}$ is as follows;

$$\bar{y}^{(a)}(\mu, \sigma^2)=\int_{-\infty}^{\infty} y(D_p) D_p{}^a f^{(0)}(\ln D_p, \mu, \sigma^2)\, d\ln D_p \Big/ \int_{-\infty}^{\infty} D_p{}^a f^{(0)}(\ln D_p, \mu, \sigma^2)\, d\ln D_p$$

$$=\int_{-\infty}^{\infty} y(D_p) f^{(0)}(\ln D_p, \mu+a\sigma^2, \sigma^2)\, d\ln D_p$$

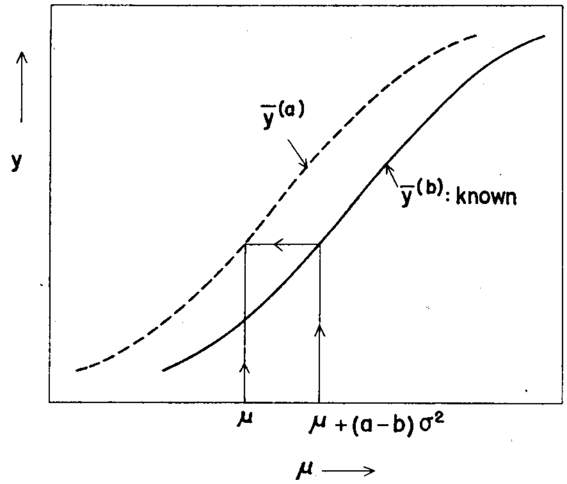


Fig. 9. The linear estimate of the process variable in case of the log-normal size distribution

$$=\bar{y}^{(0)}(\mu+a\sigma^2, \sigma^2) \tag{30}$$

That is, if the estimate is made on the count basis, the estimate on some other basis can be calculated with this equation. In more general form, Eq. (30) reads

$$\bar{y}^{(a)}(\mu, \sigma^2)=\bar{y}^{(b)}(\mu+(a-b)\sigma^2, \sigma^2) \tag{31}$$

Therefore if the estimate is made on any one of the bases, the estimate on another basis can be calculated with this equation. Figures, too, may be of use in the transformation (cf. Fig. 9). Let us take the process of terminal velocity for example. If the fact that the particles are of $D_p\,(50\%)=5$ [μ], $D_p\,(84.1\%)=8$ [μ], and $\rho_p=3$ [g/cm^3] is known, one may obtain $\mu=\ln D_{p(50\%)}=1.61$, and $\sigma=\ln D_{p(84.1\%)}-\ln D_{p(50\%)}=0.47$. Now, Stokes' law, $v_t=\{g(\rho_p-\rho_a)/18\mu_a\}D_p^2$ is applicable. This equation corresponds to the process variable y. Then on the count basis, one may have;

$$\bar{v}_t^{(0)}=\frac{g(\rho_p-\rho_a)}{18\mu_a}\int_{-\infty}^{\infty}D_p^2 f^{(0)}d\ln D_p=\frac{g(\rho_p-\rho_a)}{18\mu_a}\exp(2\mu+2\sigma^2) \tag{32}$$

On the other hand, applying Eq. (31) to (32), $\bar{v}_t^{(3)}$ (on the mass basis) will be

$$\bar{v}_t^{(3)}=\frac{g(\rho_p-\rho_a)}{18\mu_a}\exp\{2(\mu+3\sigma^2)+2\sigma^2\} \tag{33}$$

From $\mu_a=0.00018$ [g/cm sec] and $\rho_a=0.0012$ [g/cm^3], $\bar{v}_t^{(0)}$ and $\bar{v}_t^{(3)}$ will be 0.35 [cm/sec] and 1.33 [cm/sec] respectively. The difference between these two values is due to the fact that they are not on the same basis. For comparison, the calculated mean particle diameters are 6.24 [μ] on the count basis, and 12.1 [μ] on the mass basis. If $\bar{v}_t^{(0)}$ and $\bar{v}_t^{(3)}$ are calculated using these values, it is found that they coincide with the above mentioned values 0.35 and 1.33 respectively.

4. Example

4.1 $y=\sum_{j=1}^{n} K_j D_p^{a_j}$ (cf. §2.4)

One may have $y_{(j)}=K_j D_p^{a_j}$ by inspection, therefore

$$y_{(j)}^{-1}=(y_{(j)}/K_j)^{1/a_j}, \text{ and } \bar{y}_{(j)}=K_j \sum_i f_i D_{pi}^{a_j}$$

Then Eq. (10) reads

$$\bar{D}_{p(j)}=(K_j \sum_i f_i \cdot D_{pi}^{a_j}/K_j)^{1/a_j}=(\sum f D_p^{a_j})^{1/a_j},\ j=1,2,3,\ldots\ldots n \quad (34)$$

4.2 $\eta_I=1+D_p/d-1/(1+D_p/d)$ (cf. §2.4)

As the first term is a constant, it is omitted for now, then $y_{(1)}=D_p/d$ and $y_{(2)}=-1/(1+D_p/d)$, therefore

$$y_{(1)}^{-1}=dy_{(1)},\quad \bar{y}_{(1)}=\sum_i f_i D_{pi}/d,\quad y_{(2)}^{-1}=-d(1+1/y_{(2)}) \text{ and}$$

$$\bar{y}_{(2)}=-\sum\{f_i/(1+D_{pi}/d)\}. \text{ From Eq. (10), we have}$$

$$\bar{D}_{p(1)}=d\sum f_i D_{pi}/d=\sum f D_p,$$

$$\bar{D}_{p(2)}=-d\left\{1+\frac{-1}{\sum \frac{f_i}{1+\frac{D_{pi}}{d}}}\right\}=\frac{1}{\sum \frac{f}{d+D_p}}-d \quad (35)$$

Here, $\bar{D}_{p(1)}=\sum f D_p$ is the length mean diameter, but $\bar{D}_{p(2)}$ has never been taken into consideration before.

4.3 Example of six particles (cf. §2.5)

In this section the graphical method (cf. §2.5) will be explained in a less abstract way. Here the six particles (cf. §2.2) will be dealt with again. Now

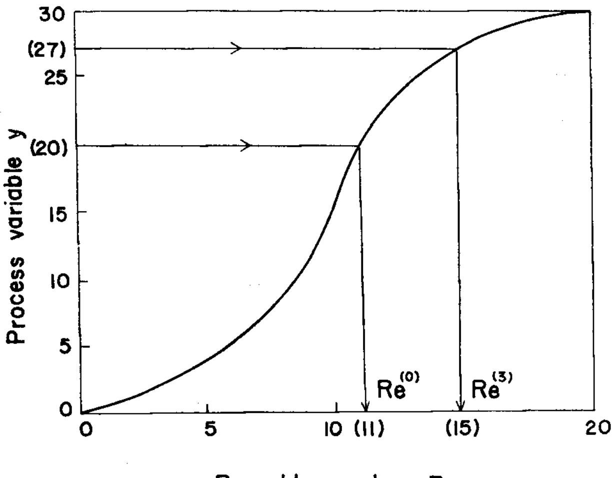


Fig. 10. Process variable in a six particle system

using $v\rho/\mu=10$, and $D_{p1}=1$ and $D_{p2}=2$, and one may have $R_{e1}=10$ and $R_{e2}=20$. On the assumption that the process variable y is obtained as in Fig. 10, $Re^{(0)}=11$ and $R_e^{(3)}=15$ are found from the figure. They were represented as g

Table 2. Mean particle diameters in the key processes

Process	Representative Equation	Mean Particle Dia.	Comment
Absorption	$\Delta p=\frac{150(1-\varepsilon)}{R_e}\frac{h(1-\varepsilon)G^2}{g_c\varepsilon^3 D_p\rho_a}+1.75\frac{h(1-\varepsilon)G^2}{g_c\varepsilon^3 D_p\rho_a}$	$\bar{D}_{p(1)}=\sqrt{1/(\sum f/D_p^2)}$ $\bar{D}_{p(2)}=1/(\sum f/D_p)$ $\bar{R}_e=\bar{D}_{p(1)}v\rho/\mu$	Pressure drop in packed beds S. Ergun. Chem. Eng. Progr., **48**, 89 (1952)
Adsorption	$J=\frac{k}{v}F\left(\frac{\mu}{\rho D_p}\right)^{2/3}$	$\left(\frac{1}{\sum \frac{f}{D_p^{2/3}}}\right)^{3/2}$	Fluid film coefficient C. Chou, et al., Chem. Eng. Progr., **49**, 141 (1953)
	$\mathrm{HETP}=2\lambda D_p+\frac{2\gamma D}{v}+\frac{8\mathrm{Hd}}{\pi^2(1+\mathrm{Hd})^2}\frac{d_f^2 v}{D_e}$	$\sum fD_p$	Effective height of theoretical plate van Deemter, et al., Chem. Eng. Sci., **5**, 271 (1956)
Agglomeration	$\sigma_Z=\frac{9(1-\varepsilon)}{8\pi D_p^2}kF$	$\frac{1}{(\sum f/D_p^2)^{1/2}}$	Adhesive strength of packed particles H. Rumpf, & E. Turba., Ber. Dtsch. Keram. Ges., **41**, 78 (1964)
	$\sigma=8(1-\varepsilon)T/\varepsilon D_p$	$1/(\sum f/D_p)$	
Drying	$p=2cT/D_p$	$1/(\sum f/D_p)$	Capillary suction pressure
	$\theta_{min}=0.23L/N^{0.9}\mathrm{SD}\pm 9.85\times LG/FD_p^{0.5}$ + counter-current − co-current	$\frac{1}{\left(\sum \frac{f}{\sqrt{D_p}}\right)^2}$	Minimum residence time of a rotary dryer S.J. Friedman & W.R. Marshall Jr., Chem. Eng. Progr., **45**, 482, 572 (1949)
Dust and mist collection	$U_t=n_eV/\{3\pi\mu D_pR\cdot\ln(R_2/R_1)\}$	$1/\sum f/D_p$	Electrical method ; moving velocity of a particle
Evaporation	$k_sa=0.148(k_sa)_0\ w/D_p$	$1/(\sum f/D_p)$	Capacity coefficient in crystallization
Fluidized beds	$N_u=0.004(D_pv\rho/\mu)^{1.5}$	$(\sum fD_p^{3/2})^{2/3}$	R. Toei, Kagakukikai Gijutsu (Japan), No. 15, 22 (1963)
	$\rho_{MB}=0.356\ \rho_p\ (\log D_p-1)$	$\log^{-1}(\sum f\log D_p)$	Maximum bed density
	$U_{mf}=\left(\frac{g\phi^2D_p^2}{200}\right)\times\left(\frac{\rho_s-\rho_f}{\mu_f}\right)/F(\varepsilon)_{\varepsilon=\varepsilon mf}$	$\sqrt{\sum fD_p^2}$	Minimum velocity for fluidization
Heat transfer	$k_e^\circ/k_g=\varepsilon\left[1+\frac{h_{rv}D_p}{k_g}\right]+\frac{3}{2}\frac{k_s}{k_g}\left[1-\frac{1}{1+\frac{2k_g}{3\phi k_s}+\frac{2h_{rs}D_p}{3k_s}}\right]$	$\bar{D}_{p(1)}=\sum fD_p$ $\bar{D}_{p(2)}=\frac{1}{\sum\frac{f}{A+D_p}}-A$ $A=\left(\frac{3}{2}k_s+\frac{k_g}{\phi}\right)/h_{rs}$	Effective thermal conductivities in packed beds S. Yagi & D. Kunii, A.I. Ch.E. Journal, **3**, 373 (1957) Kagaku Kōgaku, **18**, 576 (1954)
	$q_p=\frac{6(1-\varepsilon)}{D_p\phi_s}h_p(t-t_p)_{av.}$	$1/(\sum f/D_p)$	Heat transfer between packings and fluids

in §2.5. It is found that $R_e = D_p\ v\rho/\mu$, and then $g^{-1}(g) = \mu R_e/v\rho$. It follows that $\bar{D}_p^{(0)} = 11\ \mu/v\rho = 1.1$, and $\bar{D}_p^{(3)} = 15\ \mu/v\rho = 1.5$. These results are interpreted to mean that the estimates of this process are 20 on the count basis and 27 on the mass basis, and the mean diameters of the particles (six particles) are 1.1 on the count basis and 1.5 on the mass basis. Experimental values for this process may be plotted at the point of mean diameter 1.1 on the count basis and 1.5 on the mass basis, and they may be compared with the estimates on their respective bases. A comparison such as described above is the most systematic and proper way to deal with experimental and estimated values.

4.4 Application to other physical processes

The procedure discussed in this paper may be extended to other physical processes. The conclusions reached, when this is done, are listed in Table 2 and should have wide application. They were obtained through the methods described in §2.3 and §2.4. To use these mean diameters effectively, it must be kept in mind that they may be used to express the quantities in the left hand side of the equations in column 2, but not to express other quantities such as the square of them, a function of them, and so on.

5. Conclusion

The mean particle diameter is defined as $y^{-1}(\bar{y})$, where y^{-1} is an inverse function of the mono-disperse particulate process variable and $\bar{y}$ a linear estimate of the poly-disperse particulate process variable. When the mean particle diameters determined using the definition have the same values, the linear parts of the process variable also have the same values even if the parameters —e.g. mean and variance— are not the same. If the experimental data are studied by use of this mean particle diameter, therefore, no scattering in the data due to the wrong use of the mean particle diameter is observed. On the other hand, use of other mean particle diameters will lead to unsatisfactory results with scattering in the data. Thus, the various experimental data may be systematically studied only by use of the proposed definition. Furthermore, when the size distribution is log-normal in form, it is found that the linear estimate on the "a" basis with the mean μ is the same as that on the "b" basis with the mean $\mu + (a - b)\sigma^2$, where σ^2 is the variance. As is clear from this fact, the determination of the mean particle diameter is much simplified.

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