

The Effect of Stress Sequence on the Fatigue Strength under Program Loading

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Abstract

The annealed S35C carbon steel was tested under the many-fold multiple repeated stresses in four stress levels in rotating bending. The effect of the stress sequence on fatigue lives was investigated in association with the effect of the initial stress level and the magnitude of stress levels as a whole. The number of cycles in one program block, n_o , was chosen at 3600. The results show :

(1) The effects of the initial stress level and the stress sequence on the fatigue lives can not be seen, because the value of n_o in this experiment is very small in comparison with the total number of cycles to failure.

(2) When the maximum stress σ_1 is high and all the stresses are over stresses, the actual fatigue lives coincide nearly with the theoretical ones which are calculated by using the linear damage law. When σ_1 becomes low, the actual fatigue life N_f becomes smaller than the theoretical one N_{th} , and N_f reaches the smallest value at a certain stress level. When the all stresses except σ_1 are under stresses, N_f becomes equal to or larger than N_{th} .

1. Introduction

Various methods of calculations and experiments have been published in order to obtain the fatigue strengths or fatigue lives of metallic materials under service loads,¹⁾ but most of them have been conceived and can be applied for the special load histories. Material suffers fatigue damage by the stress repetition, but the progress of this fatigue damage is so complicated that it is impossible to obtain the fatigue lives or fatigue strengths under the service loads by calculation only. Therefore as the first step to obtain them, the random load tests and the program load tests are carried out.

In order to find out the applicable law for the general random loads it is necessary to compare the results under program load tests with those under random load tests and to search out the most suitable condition in conducting a program load test. For this purpose, we should clarify the effects of the factors in the program

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load tests on the fatigue properties. Those factors are as follows :

- (1) Stress count method²⁾ and frequency distribution of service load,³⁾
- (2) Magnitude of number of cycles in one program block, n_0 ,⁴⁾
- (3) Initial stress level,
- (4) Stress sequence in program load,
- (5) Number of stress steps in program load,⁵⁾
- (6) Stress level under program loading,

and so on. According to the results of past investigations, the most reliable fatigue lives were given by taking n_0 small, the number of stress steps larger than five or six, each stress level near the service load, and the stress sequence at random.^{1), 6)}

The object of this investigation is to obtain the effect of the stress sequence in a program load on the fatigue strength. Many investigators reported on this matter, but they recommended random sequence⁷⁾ or a Hi-Lo type sequence.⁸⁾ Or else, they insisted on no effect of load sequence,⁹⁾ or on the existing effect based on their results.¹⁰⁾ Those results may be right under their test conditions. The six factors mentioned above are independent ones, but are related with each other. Therefore in this paper the effect of load sequence on the fatigue strengths was considered in relation to the effects of initial stress level and the total stress levels in program load.

2. Specimen and Material

A rolled bar of 16 mm in diameter was tested, whose chemical composition is shown in Table I. The bar was annealed at 830°C for 30 minutes. The mechanical properties of the annealed bar is shown in Table II. The size and form of the specimen is shown in

Table I. Chemical compositions of the material (%).

C	Si	Mn	P	S
0.35	0.16	0.59	0.026	0.017

Table II. Mechanical properties of the material.

Upper yield point σ_{10} (kg/mm ²)	Lower yield point $\sigma_{0.2}$ (kg/mm ²)	Tensile strength σ_B (kg/mm ²)	Breaking strength on final area σ_T (kg/mm ²)	Elongation φ (%)	Reduction of area ψ (%)
32.1	30.5	55.4	100.4	38.8	56.4

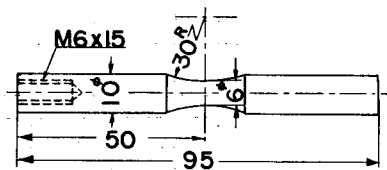


Fig. 1. Size and form of specimen.

Fig. 1, having the theoretical stress concentration factor (form factor) $\alpha = 1.03$.

3. Testing machine

The tests were conducted on a cantilever type rotating bending program fatigue

testing machine¹¹⁾. Cams which are set beforehand in programmed sequence operate limit switches and electric magnets, which load and unload weights on the specimen. The detail of the machine is reported in the reference (11). The rotating speed of the machine is 3600 rpm.

4. Test procedure

In this research rotating bending fatigue tests were carried out under many-fold multiple loads in four stress levels. Fig. 2 shows the stresses and the numbers of cycles in program loading in the case of Lo-Hi type sequence. As shown in Fig. 2 the lowest stress is named σ_1 and the highest σ_4 . The number of cycles in one program is given by n_i corresponding to σ_i . The total number of cycles in one program loading $n_0 \left(= \sum_{i=1}^4 n_i \right)$ is 3600, and each n_i is as follows: $n_1 = 1800 = n_0/2$, $n_2 = 900 = n_0/4$, and $n_3 = n_4 = 450 = n_0/8$. The twelve kinds of stress sequences tested are shown in Fig. 3. The initial stresses, i.e., the stress levels loaded at first on the specimen under program loading test, are chosen σ_1 and σ_4 . Each load sequence is named as shown in Fig. 3.

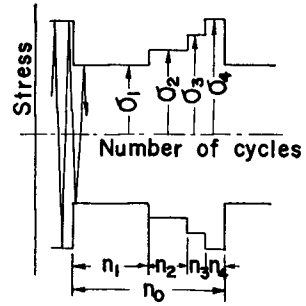


Fig. 2. Each stress level and its number of cycles in one program block in case of loading of Lo-Hi type.

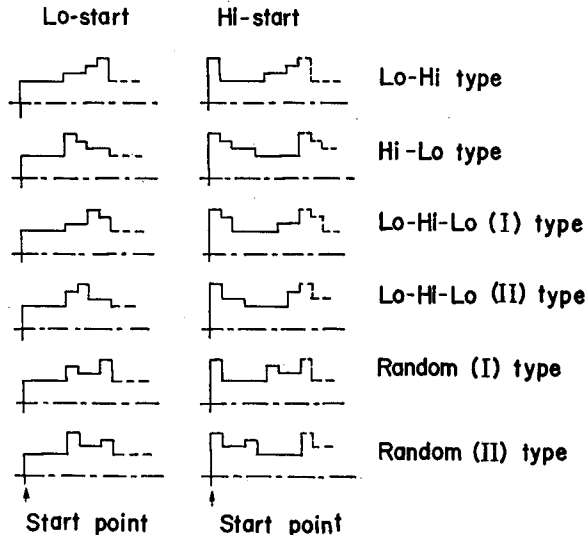


Fig. 3. Twelve kinds of stress sequences.

The magnitudes of the stress levels of total program loads in this research are listed in Table III. The stress difference between the adjacent two stress levels is

taken about 3 kg/mm². All the stresses in experiments (i) and (ii) are overstresses,

Table III. Values of stress level (kg/mm²):

Experiments	σ_1 $(\frac{n_1}{n_0} = \frac{1}{2})$	σ_2 $(\frac{n_2}{n_0} = \frac{1}{4})$	σ_3 $(\frac{n_3}{n_0} = \frac{1}{8})$	σ_4 $(\frac{n_4}{n_0} = \frac{1}{8})$
(i)	24.8	28.0	30.5	33.2
(ii)	22.5	26.1	28.6	32.0
(iii)	19.1	22.5	25.1	28.3
(iv)	16.4	19.6	22.8	25.3
(v)	14.5	17.6	20.8	24.0

and in experiments (iii), (iv) and (v) some of the stress levels are understresses. In Table III the stresses under the bold line are understresses of the material.

5. Test results and discussions

Fig. 4 shows the standard S-N curve of the material, and the endurance limit of the material was determined 22.5 kg/mm². The S-N curve drawn in Fig. 4 was

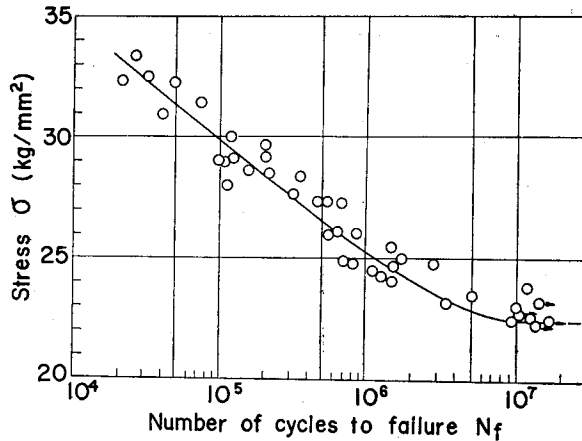


Fig. 4. The S-N curve of the material.

used as the standard of the discussion of program loading test results. The results of program fatigue tests under many-fold multiple loads in four stress levels are shown in Figs. 5 (a)-(d) by taking the fatigue life ratio in abscissa. The fatigue life ratio was calculated by the equation :

$$\frac{N_f}{N_{th}} = \left(\frac{N_f}{n_0}\right) \cdot \left(\sum_{i=1}^4 \frac{n_i}{N_i}\right),$$

where

N_f : the experimental number of cycles to failure under program loading,

N_{th} : the theoretical number of cycles to failure calculated by using the linear damage law and the standard S-N curve in Fig. 4,

N_i : the number of cycles to failure at stress level σ_i , and

n_0, n_i : the values shown in Fig. 2.

The vertical short bold lines in Fig. 5 show the algebraic mean value of fatigue life ratio for each condition.

5-1 The effect of initial stress

Comparing the results of Lo-start and Hi-start conditions in Fig. 5, both cases have nearly the same fatigue life ratios. The conditions were of the same stress level and of the same stress sequence. This means that there is no effect of initial stress level on the fatigue lives. In the tests the number of cycles in one program cycle

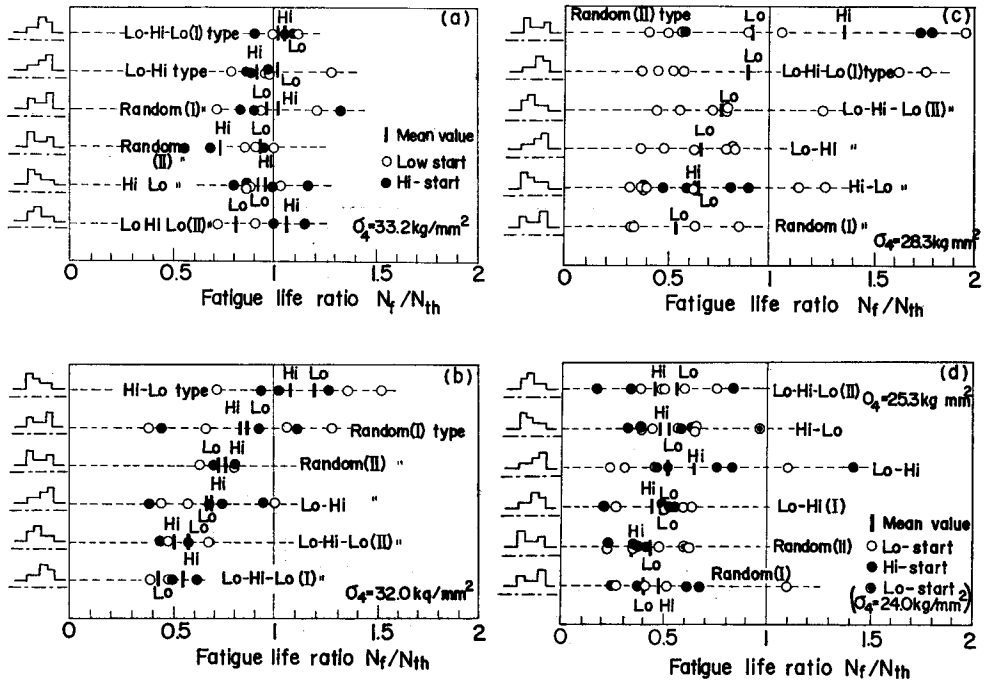


Fig. 5. Results of program loading tests.

n_0 , is 3600, so that the number of program cycles yields to a large one, from 19 to 11000. That is, n_0 is of small value in comparison with the total number of cycles to failure, which, we think, resulted from the fact that the initial stress level did not affect the fatigue life ratio. Therefore, when the number of cycles in one program cycle n_0 is large, the effect of the initial stress level may appear.¹²⁾

5-2 The effect of stress sequence

No special effect of stress sequence on fatigue lives can be seen from Fig. 5. Of course, in case of taking the algebraic mean value of fatigue life ratios, a little effect of stress sequence might be seen, but no significant difference among the algebraic means of fatigue life ratios could be said to exist from the statistical point of view (we mean *t*-calibration) with taking account of their scatters even for the case of Fig. 5 (b) where the difference among them was probably observed to be the most. That is, the effect of stress sequence does not appear like that of the initial stress. This is also thought to be due to the smallness of n_0 .

5-3 The effect of stress level under program loading

It was clarified that in the range of n_0 , chosen in this test series, the initial stress level and the stress sequence did not affect the fatigue lives. It is, however, observed from Figs. 5 (a)-(d) that the higher level of σ_4 gives the larger value of fatigue life ratio N_f/N_{th} . That is, in experiment (i) the fatigue life ratio N_f/N_{th} is nearly unity, and in experiment (iv) it takes the minimum value. But in experiment (v) where only σ_4 is overstress, N_f/N_{th} increases to unity or to larger than unity, this phenomenon coincides with the results under many-fold multiple loads in two⁽¹³⁾ and three stress levels.⁽¹⁴⁾ Fig. 6 shows the above phenomenon for Lo-Hi type, Lo-Hi-Lo

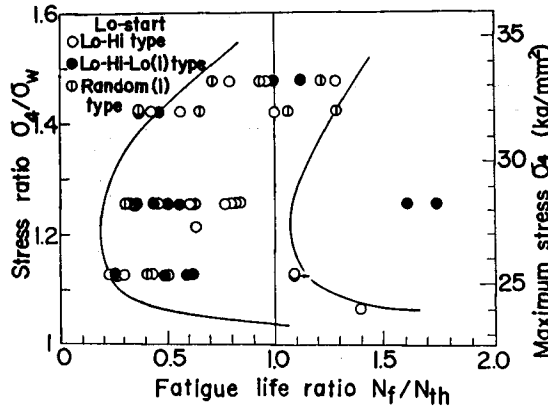


Fig. 6. Relation between the maximum stress and fatigue life ratio.

type and random (I) type of Lo-start, by taking the stress ratio σ_4/σ_w and σ_4 in ordinate axis, and fatigue life ratio in abscissa. The two curves show the range of scatter of the test results, which coincides with that of the original S-N curve in Fig. 4.

In many-fold multiple loads in two stress levels, when the lower stress level is equal to the endurance limit of the virgin material, the fatigue life ratio N_f/N_{th}

becomes the minimum value.¹³⁾ In this experiment also N_f/N_{th} becomes the minimum value when $\sigma_4 = 25.3 \text{ kg/mm}^2$, that is, when the value σ_3 is equal to the endurance limit of the virgin material.

It is thought to be a typical feature under the program loading tests that the fatigue life ratio varies greatly according to the stress level in program load, and that fatigue life ratio becomes a minimum value under certain conditions.

5-4 Fatigue lives in Lo-Hi and Hi-Lo types

As stated already there is no significant effect of stress sequence on the fatigue life. Therefore there will not appear any difference in fatigue lives between the Lo-Hi and Hi-Lo type sequences either. But in Fig. 5 the fatigue life ratio N_f/N_{th} of the Lo-Hi type is smaller than that of Hi-Lo type at high stress range, and at low stress level vice versa. In Fig. 7 the comparison of the fatigue curves for both cases are shown, and from this figure we can see the phenomenon stated above clearly.

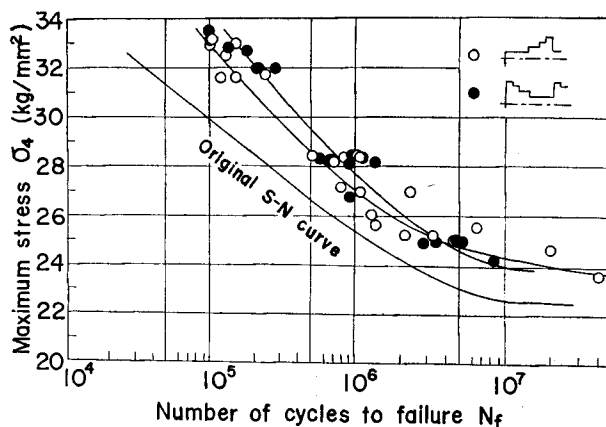


Fig. 7. Comparison of S-N curves in Lo-Hi and Hi-Lo types.

Considering, however, the scatter of the results, such an effect of the stress sequence on the fatigue life as shown in Fig. 7 will be negligible.

5-5 The stress level at fracture

Fig. 8 shows the relation between the stress level at the fracture of the specimen and the number of specimens fractured at that stress level, and the relation between the stress level at one step before the fracture and the number of specimens at that stress level. In the figure, the axis of the ordinate represents the ratio of the number of specimens, r . The number of specimens fractured at the stress level σ_1 is the largest, which is caused by that the number of cycles at σ_1 is larger than those at other stress levels. Fig. 9 shows the same relation as in Fig. 8 (a) by taking

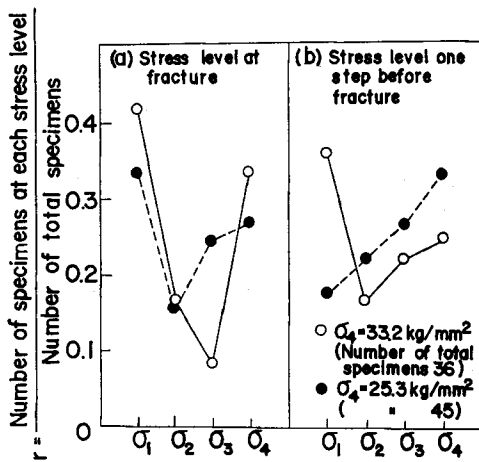


Fig. 8. The stress level at fracture in program loading tests.

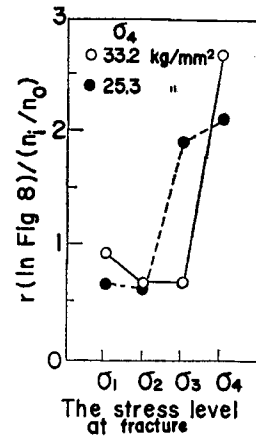


Fig. 9. The stress level at fracture when the comparison is made per unit cycling.

the ratio of r to cycle ratio n_i/n_0 in ordinate axis, and the figure reveals that the ratio becomes larger at higher stress level.

6. Conclusions

The effects of stress sequence, the magnitude of stress levels as a whole and the initial stress level in program loading test on the fatigue lives were investigated on S35C carbon steel under many-fold multiple repeated loads in four stress levels. The results obtained are as follows :

- (1) When the number of cycles in one program block, n_0 , is small as in this test, the effect of the initial stress level on the fatigue life does not appear.
- (2) On the effect of stress sequence the same thing can be said as stated in (1).
- (3) $\sigma_4/\sigma_w - N_f/N_{th}$ curves under many-fold multiple loads in four stress levels show the same behaviours as those under many-fold multiple loads in two or three stress levels.
- (4) A large number of specimens fractured at the stress level, σ_1 , the smallest stress level, but the high stress level is found to bring the larger number of fractured specimens if the comparison is made per unit cycling.

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