Service Load Counting and Fatigue Life Estimation by the Full Wave Count Method

By

Minoru KAWAMOTO,* Hiroshi ISHIKAWA* and Terutaka ONOE**

(Received September 30, 1972)

Abstract

Different methods have been already proposed in order to find load frequencies of service loads, and only peaks or ranges are given main attention in most of them. From the standpoint of fatigue, however, service loads should be analyzed in consideration not only of their peaks or ranges but also of their means.

There is proposed in this paper the full wave count method which analyzes a service load-time history into individual proper full wave with a corresponding mean value. There is also discussed how to carry out a program fatigue test according to the count result for the fatigue life estimation.

1. Introduction

Various count methods have been proposed up to the present in order to find load frequency distribution of service load[1], but their analytical results do not always coincide with one another. Consequently it is an important subject how to analyze service loads and then replace the count results into proper program loads. Each count method proposed in the past is thought to have its peculiar characteristics, and therefore has its applicable propriety according to circumstances. When only stress peaks or ranges are given attention, the fatigue lives are apt to be estimated shorter or longer, respectively, which is considered in part due to the lack of thought about mean stresses. In investigating the effects of service loads on the fatigue properties, not only stress peaks or ranges but also mean stresses have thus to be considered[2]. So the authors propose in this paper the full wave count method and also discuss how to carry out program fatigue tests for life estimation according to the load frequencies obtained from the original service loads.

* Department of Mechanical Engineering
** Department of Mechanical Engineering (Present Address : Osaka Gas Company, Ltd.)

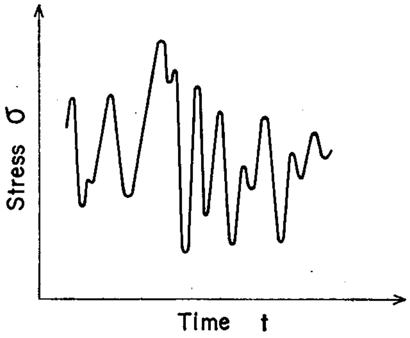

Fig. 1. An example of service load-time history.

In this report the service load means a quite irregularly varying load as shown in Fig. 1. In the case of varying loads with some regularity as shown in Fig. 2, they had better be treated in other way as the occasion may demand.

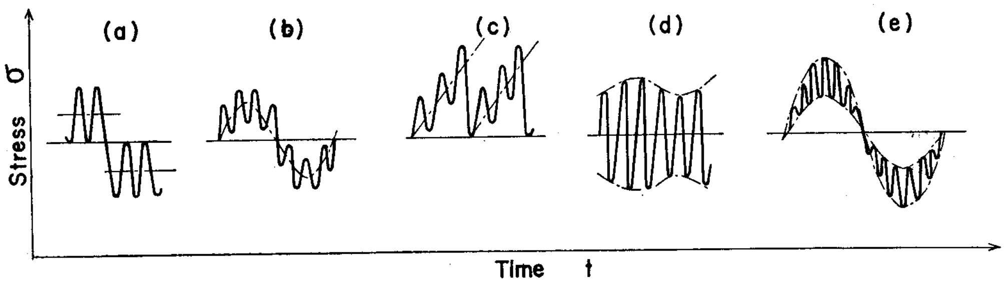

Fig. 2. Examples of varying loads with some regularity.

2. Load count methods proposed in the past

As there have been some previous general remarks[1),3),4)], the details are entrusted to them. Here follow the classification and simple discussion according to the authors.

2.1 Classification of load count methods in the past

(1) Such methods as take only stress peaks into account

(a) The peak count method[3),5)].

(b) The mean crossing maximum peak count method[5)].
The P_{30}- or P_{100}- method[6)] is involved in this item.

(c) The all peaks count method[2),5)].

(d) The peak-pair count method[7)].

(2) Such a method as takes only stress ranges into account
The range count method[3)].

(3) Such methods as take both peaks and ranges into account
  (a) The range-mean count method[5].
  (b) The half wave method[8].
(4) Methods for counting peaks or ranges
  (a) The level crossings count method[5].
  (b) The range-pair count method[4),9].
  (c) The fatigue-meter count method[5].
(5) Others
  (a) The hereditary method[10].
  (b) The Main-Kloss' method[11].
  (c) The R. M. S. (or Fralich's) method[12].

2.2 Comparison and discussion of load count methods already proposed

No methods in the past can be said sufficient from the viewpoint of reappearance of the original wave. Therefore, it may be advisable to use a suitable one according to one's discretion. Here follow a brief comparison and discussion.

In the peak or all peaks count method, no information can be acquired on the difference between adjacent two peaks. The service load wave usually has dispersion not only of stress peak but also of individual mean stress, so that it is impossible in this case to restore the original service load wave by using the count result[13].
If the program test will be carried out on the basis of the load frequency distribution acquired by this count method, a shorter fatigue life may be given.

In the range count method, the longer fatigue life will be estimated because each absolute level of load is left out of consideration. The more irregularity the original wave has, the stronger trend like this may appear, which means an unsafe estimation of the fatigue life. This is also shown by Kowalewski[14]. Further the analytical result by the range or range-mean count method is often affected by a small variation in stress, the omission of which will increase the number of counts of larger

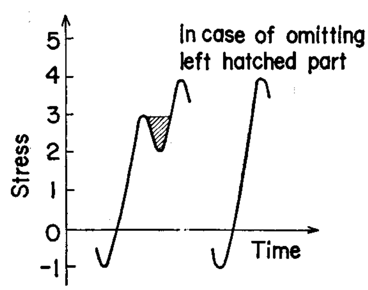

Fig. 3. Difference of count results between the omission and non-omission of a small variation in stress in the range count method.

Table 1. Analytical result by the range count method.

	Stress range
Without omission of small variation in stress	+4 -1 +2
With omission	+5

stress range, and will decrease the total number of counts of waves as shown in Fig. 3 and in Table 1.

Consequently, the count result in the case of omitting such small variations in stress cannot be acquired only by subtracting that of only small variations from that with no omission, which implies that ground swells of waves are left out of consideration. From the standpoint of fatigue, the load count should be made both without oversight of such ground swells of waves, and with omitting small variations in stress as invalid amplitudes. However, a case has to be considered where even small variations in stress may act as strengthening amplitudes, especially in the case of many superposed small waves[15].

As for the range-pair count method, when the count is made of a certain range-pair larger than r_p, ranges smaller than r_p will be neglected. If the count result of range-pairs smaller than r_p is subtracted from that of all range-pairs, that of range-pairs larger than r_p can be given. Small amplitudes are thus able to be omitted at will in the range-pair count method, where this method is superior to the range count method. But in this method the individual mean stress is not taken into account. Hence, two wave components with different mean stresses will give the same count result as shown in Fig. 4.

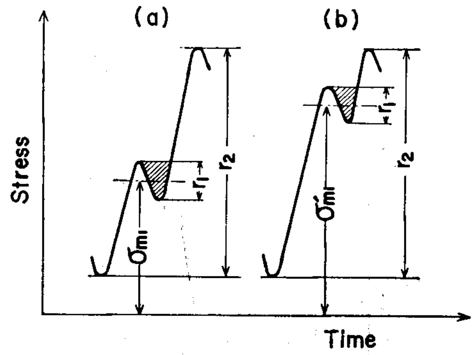

Fig. 4. Two stress waves with the same stress amplitude but with different mean stress.

3. The full wave count method

3.1 Fundamental idea of the full wave count method

Let's suppose a stress-time history as shown in Fig. 5 (a). Peaks are named in order as σ_1, σ_2, σ_3 and σ_4, respectively, where the magnitude is set $\sigma_4 < \sigma_2 < \sigma_1 < \sigma_3$.

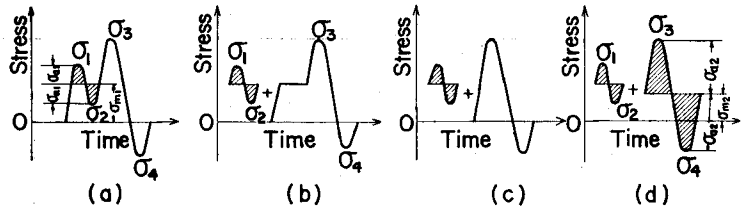

Fig. 5. Fundamental idea of the full wave count method. A stress-time history in Fig. (a) can be analyzed into two full waves with mean stresses in Fig. (d).

The minimum stress range between the adjacent two peaks is first detected and taken out. In this case, the hatched part in the figure is separated as a full wave, both with a mean stress of $\sigma_{m1} = (\sigma_1 + \sigma_2)/2$ and with a stress amplitude of $\sigma_{a1} = (\sigma_1 - \sigma_2)/2$. Namely, the original wave is analyzed into two waves in Fig. 5 (b). Then the right wave in Fig. 5 (b) is assumed to be equivalent to that in Fig. 5 (c), which is also considered equivalent to the right hatched wave in Fig. 5 (d) both with a mean of $\sigma_{m2} = (\sigma_3 + \sigma_4)/2$ and with an amplitude of $\sigma_{a2} = (\sigma_3 - \sigma_4)/2$. The original wave in Fig. 5 (a) is thus analyzed into two full waves in Fig. 5 (d). Although the right wave in Fig. 5 (b) has the corresponding horizontal part of stress due to separating the left small full wave, it may be regarded as that in Fig. 5 (c), since this replacement is thought to have little influence. This has been actually confirmed by the experiments concerned with the effect of pause of loading on the fatigue strength[16].

The full wave count method is called, as mentioned above, the method to find out a three-dimensional load frequency distribution of a service load with respect to mean, amplitude and number of occurrences.

3.2 Procedure for analysis of service load by the full wave count method

The concrete analytical procedure of a service load is as follows. Fig. 6 (a) shows an example of a service load-time history. Each peak is named in order as shown in the figure. The magnitude is made, for simplicity, an integer times as large as a divisional width, $\Delta\sigma$. In the first stage, the smallest disparity of stress between adjacent two peaks is detected and then separated as a full wave. In the figure, the smallest disparity of stress, $1 \cdot \Delta\sigma$, can be found between the following adjacent two peaks ; namely, (1 and 2), (11 and 12) and (20 and 21). When wave

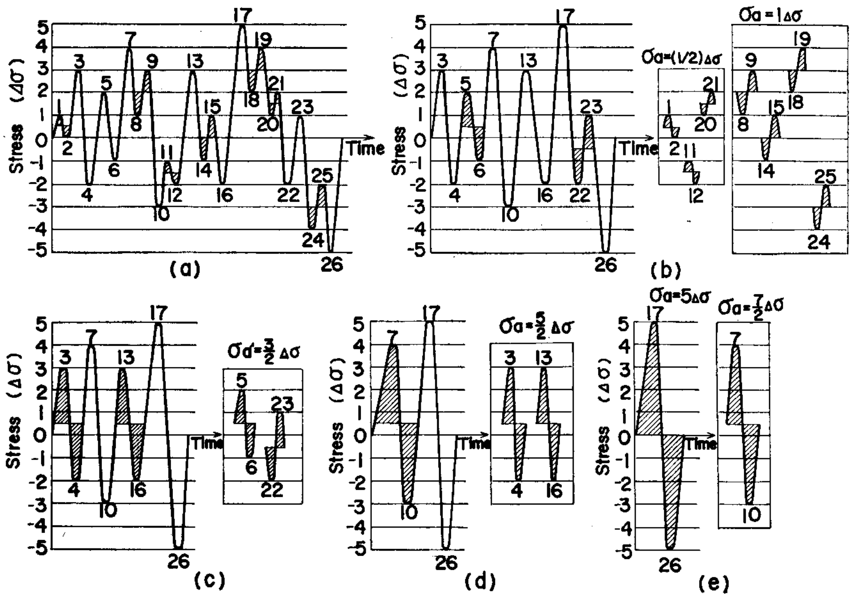

Fig. 6. The concrete analytical procedure by the full wave count method.

components of this stress range are separated from the original wave as full waves indicated in the hatched part in Fig. 6 (a), the adjacent two peaks of (1 and 2), of (11 and 12) and of (20 and 21) will be thought to make full waves both with an amplitude of $\Delta\sigma/2$ and with means of $\Delta\sigma/2$, $(-3/2)\cdot\Delta\sigma$ and $(3/2)\cdot\Delta\sigma$, respectively. Full waves with a stress range of $2\cdot\Delta\sigma$ are then separated in a similar way, and the remaining stress waves are shown in Fig. 6 (b). In other words, the original wave in Fig. 6 (a) is analyzed into separated full waves and corresponding residual waves as shown in Fig. 6 (b).

Quite similarly, full waves with a stress range of $3\cdot\Delta\sigma$ are designated in the hatched part in Fig. 6 (b), and the remaining waves, after excluding these full waves, are shown in Fig. 6 (c). Full waves with a stress range of $5\cdot\Delta\sigma$ and $7\cdot\Delta\sigma$ are taken out as indicated in Figs. 6 (d) and (e), respectively. Thus the original wave is discomposed into full waves with means. Table 2 gives the result of the frequency distribution of the separated full wave. A programing of loads is then made according to it, which will be referred to later.

There can be observed two kinds of separated full waves. One is the full wave being loaded from the positive side, and the other from the negative side. In this report, however, no difference between them is taken into account, which means no consideration about Bowshinger's effect.

It is evident from the above-stated analytical procedure that there arises no

Table 2. Count result by the full wave count method.

Stress amplitude [Δσ]	Mean stress [Δσ] -3	-5/2	-2	-3/2	-1	-1/2	0	1/2	1	3/2	2	5/2	3	7/2
1/2				1				1		1				
1	1						1				1		1	
3/2						1		1						
2														
5/2								2						
3														
7/2								1						
4														
9/2														
5							1							
11/2														

diversity of a count result in the full wave count method, since full waves are taken out in order one by one from the smallest amplitude to the largest one. Further such drawback is excluded in this method as mean stresses are not considered in the range-pair count method (see Fig. 4.). Finally this method is thought to be rational for the counting of the service load.

4. Programing procedures for fatigue life estimation on the basis of the count results of service loads

Analytical results by the full wave count method will be, for example as shown

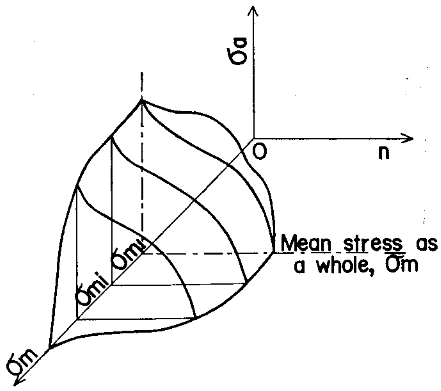

Fig. 7. An example of the three-dimensional load frequency distribution acquired by the full wave count method.

in Fig. 7, a three-dimensional frequency distribution with respect to mean, amplitude and number of occurrences. The figure is described for simplicity only in the case of positive mean stresses. Two-dimensional frequency curves are indicated in Fig. 8 as parameters of mean stresses in Fig. 7.

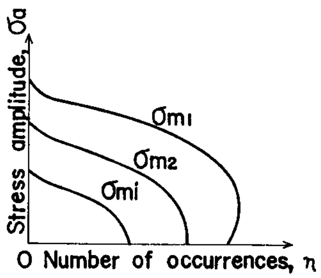

Fig. 8. Two-dimensional representation of load frequency distribution as a parameter of each level of mean stress, σ_{mi}.

Two procedures can be considered in carrying out a program fatigue test according to those load frequency distributions. One is the method where mean stresses are varied as they are, and the other where each wave with a mean is replaced into the corresponding equivalent, but completely reversed. The former will be considered more desirable since the latter will require some assumptions. It may be, however, very difficult to vary simultaneously both means and amplitudes in a program test, so they will be carried out in practice according to such frequency curves as shown in Fig. 8 by considering proper mean levels σ_{mi} $(i=1, 2, 3, \cdots)$. With consideration about the idea of the full wave count method, the positive and negative cummulative

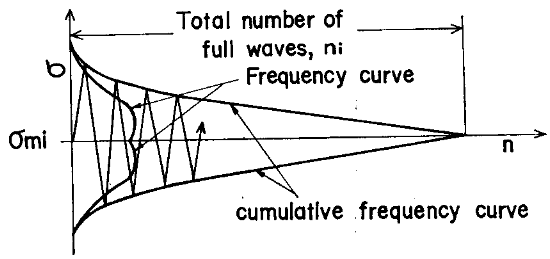

Fig. 9. Programing procedure for full waves with the same mean stress of σ_{mi}.

frequency curves will be said to be completely symmetrical for each mean level as shown in Fig. 9. It will be also said that this corresponds to the case where the full waves with the same mean value are arranged orderly from the maximum amplitude to the smallest one.

Fig. 10 gives an example where each mean level goes downwards, and practical program tests have only to be carried out according to the way in the figure. The example in the figure shows a case of both means and amplitudes decreasing

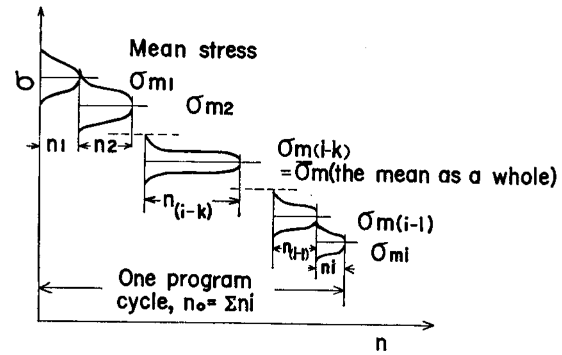

Fig. 10. An example of practical procedure for program test. (In the case of gradually decreasing mean stress level)

gradually, but there may be many cases about the loading sequence. Fig. 10 corresponds to the case of beginning the loading from the maximum level of mean, but it may be better said to begin the loading from an intermediate level of mean in a

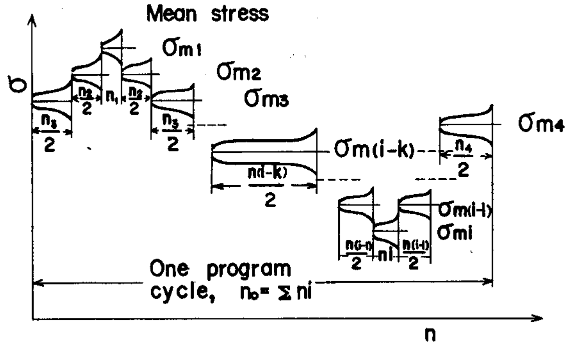

Fig. 11. An example of practical procedure for program test. (In the case of the order of mean stress level being Lo↗Hi↘Lo.)

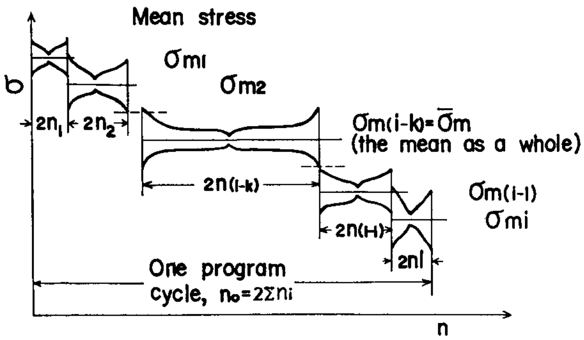

Fig. 12. An example of practical procedure for program test.

practical test. Problems on initial mean stress are associated with those on initial stress. When the initial mean stress is considerably high, there will appear the effect of cyclic prestressing, and when fairly low, the coaxing effect will appear. In order to exclude these effects it will be useful to start loading from an intermediate level of mean. For example, one way is to choose a nearly equal level of mean to the value of mean as a whole of the original service wave. Furthermore, various cases may by considered about the formation of program cycles. Figs. 11 and 12 give examples of programing according to analytical results by the full wave count method. In all events, it is necessary that moderate programing methods are decided on the basis of Fig. 10 by paying deliberate attention to loading sequences, number of cycles per a program period, namely, block size and so forth.

In all cases mentioned above, the fall of stress arises when a mean level is

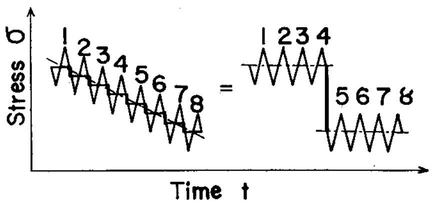

Fig. 13. Explanatory figure for step-like approximation of mean stress.

made to vary into another one, which will be thought due to a step-like approximation of mean stress, as shown, for example, in Fig. 13 with bold lines. The accumulation of the bold lines in the original wave will correspound to such a fall of stress.

Further it will not always be easy in a practical program test to vary stress amplitude smoothly as stated until now. Consequently the replacement into a staircase load had better be applied as indicated in Fig. 14.

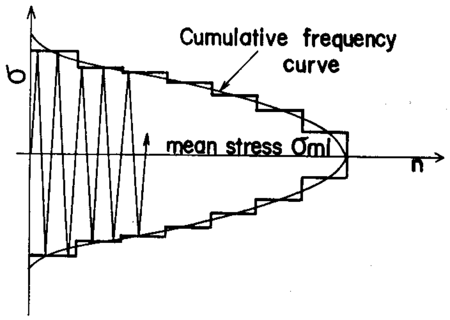

Fig. 14. Replacement into staircase load.

The above-stated cases correspond to those where amplitudes vary at a certain level of mean. However, the cases may also be naturally considered where means vary for a same stage of amplitude on account of a three-dimensional load frequency by the full wave count method, an example of which is shown in Fig. 15.

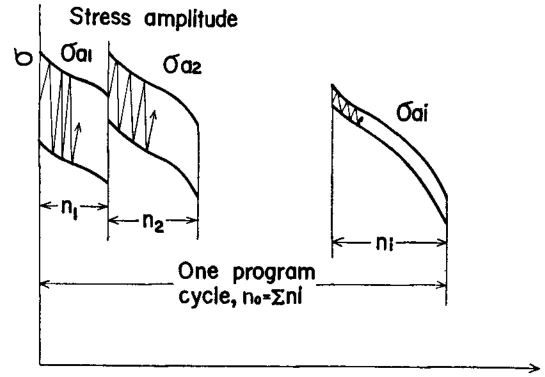

Fig. 15. An example of practical procedure for program test. (In the case of a parameter of stress amplitude.)

5. Comparison of the full wave count method with others

Count results by the full wave count method can be compared with those of others by replacing each full wave with an individual mean into an equivalent completely reversed wave under some assumptions. It is good in this replacement to apply a method such as modified Goodman's law or Gerber's diagram over the stress range above the fatigue limit. Thus, by applying proper modified fatigue limit diagrams, the three-dimensional frequency distribution is then converted into a two-dimensional one with respect to equivalent reversed stresses and number of occurrences ; and the effect of each mean stress will come to be appreciated.

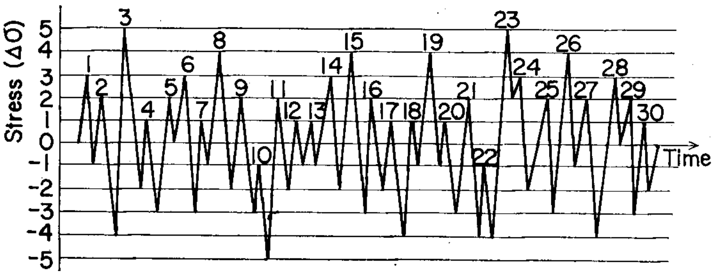

Fig. 16. An example of service load-time history. In the wave the mean stress as a whole equals zero, and the value of N_0/N_p is nearly 0.93.)

Now consider the service load wave shown in Fig. 16. The number both of the positive peaks and of the negative ones was made thirty respectively. For simplicity, each peak value was made an integer times as large as the divisional width. The

Table 3. Count result by the all peaks count method.

Stress peak [Δσ]	Number of occurrences	Cumulative number of occurrences
5	1.5	1.5
4	4.5	6
3	6	12
2	8	20
1	7.5	27.5
0	1	28.5
-1	1	29.5
-2	0.5	30

Table 4. Count result by the range count method.

Stress amplitude [Δσ]	Number of occurrences	Cumulative number of occurrences
5	-	-
4.5	1	1
4	-	1
3.5	3	4
3	2.5	6.5
2.5	7	13.5
2	5.5	19
1.5	5.5	24.5
1	5	29.5
0.5	0.5	30

Table 5. Count result by the range-pair count method.

Stress amplitude [Δσ]	Number of occurrences	Cumulative number of occurrences
5	1	1
4.5	2	3
4	1	4
3.5	3	7
3	3	10
2.5	2	12
2	3	15
1.5	6	21
1	8	29
0.5	1	30

Table 6. Frequency of the equivalent reversed stress acquired from the count result by the full wave count method.

Equivalent reversed stress σ_{eq} [Δσ]	Number of occurrences	Cumulative number of occurrences
5	1	1
4.8	2	3
4	1	4
3.7	3	7
3.5	1	8
3	2	10
2.5	3	13
2	3	16
1.5	5	21
1	8	29
0.5	1	30

algebraic mean of all positive and negative peaks becomes nearly zero. Tables 3, 4 and 5 indicate the analytical results by the all peaks count method, by the range count method and by the range-pair count method, respectively. The results, both by the all peaks count method and by the range count method, are shown there by the average number of occurrences between each positive and negative peak or range with the same absolute value. This is because frequencies both of positive and of negative peaks or ranges are not always symmetrical above and below the mean as a whole on account of a little distortion of the original wave. For this reason, there are some cases where the number of occurrences becomes one half. Table 6 indicates the frequency distribution of the equivalent reversed stress σ_{eq} acquired by applying modified Goodman's diagrams in Fig. 18 described from the supposed S-N diagram in Fig. 17 to the count result by the full wave count method.

Fig. 19 gives the cumulative frequency curves described according to the data in

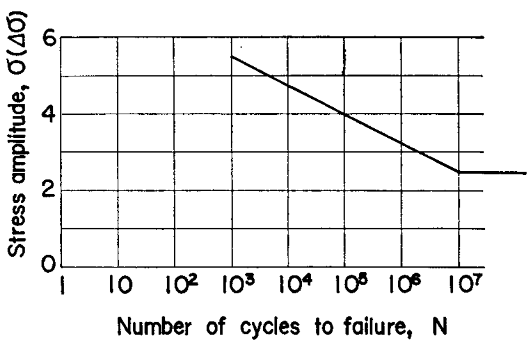

Fig. 17. Supposed *S-N* diagram. (The strengths are assumed as follows : $\sigma_B = 6 \cdot \Delta\sigma$, $\sigma_w = 2.5 \cdot \Delta\sigma$)

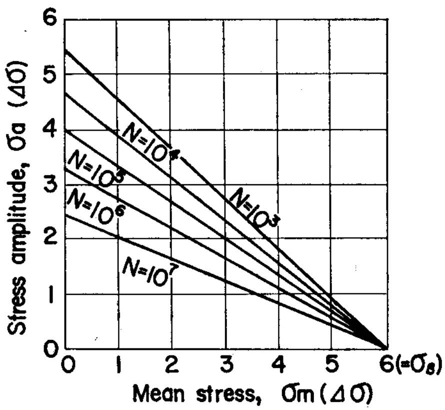

Fig. 18. Modified Goodman diagrams.

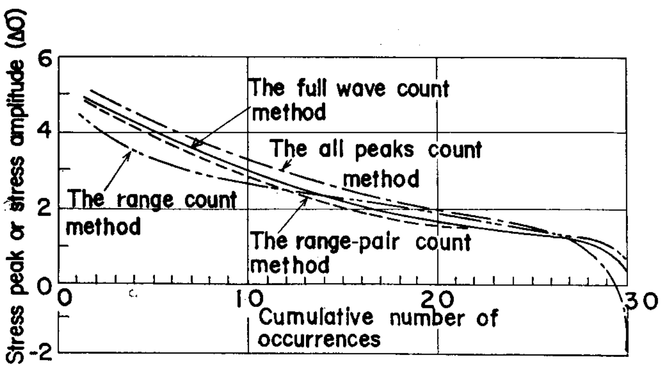

Fig. 19. Difference among cumulative frequency curves according to various count methods. (In the case of a service load wave with $N_0/N_p \cong 0.93$.)

Tables 3~6. As shown clearly in the figure, the all peakes count method gives the most severe estimation of fatigue life, while the range count method gives the mildest estimation. The range-pair count method is found to estimate fatigue life longer than the full wave count method because it does not consider any individual mean stress. In the service load wave in Fig. 16 the value of N_0/N_p which represents the extent of the irregularity of the wave is almost equal to 0.93, and so this wave is regarded as a fairly reversed one. Here N_p represents the total number of positive peaks, and N_0 does the number of occurrences where the wave intersects the zero level with the positive gradiant ($d\sigma/dt > 0$). The cumulative frequency curve by the full wave count method lies between those by the all peaks and range count method, which will imply that the full wave count method is suitable for counting of service load.

The difference between the full wave and range-pair count method is not perceived very clearly in this example, which is due to the little irregularity of the wave. If the wave has a smaller value of N_0/N_p, that is, if the wave becomes more irregular, the difference between the two will appear more clearly, because the effects of mean stresses come out distinctly. Moreover as observed in Fig. 18, the cumulative frequency curve by the full wave count method is closer to that by the all peaks count method than that by the range-pair count method. This means that the full wave count method is superior to the range-pair count method if the consideration is taken of Kowalewski's report[14] that fatigue lives evaluated from the analytical results by the peak count method did not change so much, irrespective of the extent of irregularity of the wave and were about one half of those obtained under actual random fatigue tests.

6. Conclusions

In order to obtain fatigue strengths or lives under service loads, it is said desirable to perform suitable program fatigue tests according to analyzed load frequency curves. For this purpose, it is very important how to find out the load frequency distributions of service loads. Various count methods already proposed have their peculiar characterisitcs, but they cannot be regarded as sufficient because of their lack of consideration about mean stresses.

The authors proposed, therefore, the full wave count method which analyzed a service load wave into an individual proper full wave with a corresponding mean and amplitude. The new method proved to be very practical for the counting of service load.

The authors also discussed how to carry out a program fatigue test for life estimation according to the load frequency distribution found out by the full wave count method. More precise evaluation may be made of fatigue strength or life under the service load by this programing method.

References

1) M. Kawamoto and T. Shibata : Jour. Japan Soc. Mat. Sci., **15**-156 (1966), 575.
2) K. Yoshiwara : Trans. Soc. Auto. Engrs., Japan, **8** (1961), 79.
3) M. Seki : Jour. Japan Soc. Mat. Sci., **12**-113 (1963), 79.
4) J. Schijve : Fatigue of Aircraft Structures, (1961), 115, Pergamon Press.
5) H. A. Leybold and E. C. Naumann : Proc. ASTM, **63** (1963), 717.
6) H. Nakamura and M. Tanaka : Trans. Japan Soc. Mech. Engrs., **32**-244 (1966), 1755.
7) T. Yamada and S. Kitagawa : Jour. Japan Soc. Mat. Sci., **13** (1964), 619.
8) W. L. Starkey and S. M. Marco : Trans. ASME, **79**-6 (1957), 1329.
9) M. Kikukawa, et al. : Jour. Japan Soc. Mech. Engrs., **70**-585 (1967), 1495.
10) M. Kawamoto and K. Koibuchi : Trans. Japan Soc. Mech. Engrs., **30**-212 (1964), 475.
11) F. L. Main and D. C. Kloss : Jour. SAE, **71**-9 (1963), 86.
12) R. E. Fralich : NACA Memo., 412-519 (1964), 619.
13) K. Yoshiwara : Jour. Soc. Mat. Sci., Japan, **8**-72 (1959), 76.
14) J. Kowalewski : Full Scale Fatigue Testing of Aircraft Structures, (1961), 60, Pergamon Press.
15) M. Kawamoto, et al. : Trans. Japan Soc. Mech. Engrs., **17**-61 (1951), 143.
16) M. Kawamoto, et al. : Preprint of Trans. Japan Soc. Mech. Engrs. (Kansai Branches), (1969), 10.