On Galloping Oscillation of Square Prism in Two Dimensional Uniform Flow

By

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Abstract

In the connection of frequently used structural members of long-spanned bridges, the aeroelastic characteristics of a square prism are investigated, based on the quasi-steady theory. The experimental results of the wind tunnel tests are compared with the theoretical characteristics, using the theory given by G. V. Parkinson. In this paper, some considerations on the critical wind velocity for galloping phenomenon of square prism are presented. Also, the qualitataive method to determine the induced amplitude is discussed.

Introduction

Recent trends of high-rise and long spanned structures in civil engineering fields now more than ever require accurate knowledge about natural external loads such as seismic and wind forces, because of the large size and the long natural period of structures. There are noted numerous investigations on aerodynamic stability problems on suspension bridges since the accident of the Tacoma Narrows Bridge in 1940. However one should note that the difficulties in an analysis of this kind are due to the facts that most of the structures are of an unstream-like crosssectional form, and the air stream considered is eventually anisotropically turbulent. In this report, one of the fundamental characteristics associated with a square prism in uniform flow is treated in order to clarify the so-called galloping type instability.

As already investigated, two types of aerodynamic characteristics are mentioned for a square prism as dynamic responses in a plane normal to the mean flow, namely aeolian and galloping oscillations, respectively. The aeolian oscillation is generally a kind of forced oscillation, caused by the periodical vortex in the back flow of the body. On the other hand, the galloping oscillation is considered a kind of self-con-

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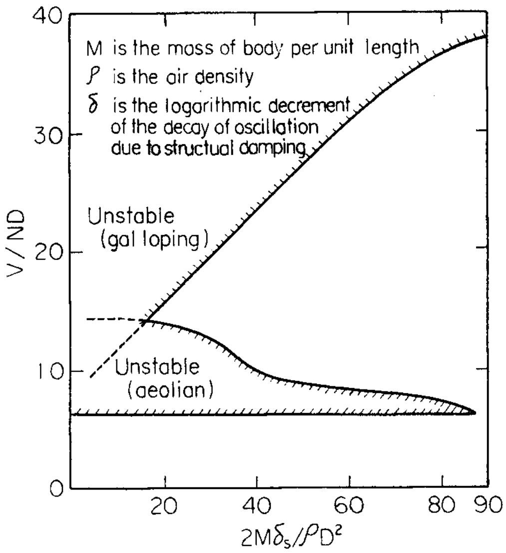

Fig. 1. Aerodynamic characteristics of square prism (by C. Scruton).

trolled oscillation. These unstable phenomena are known to occur when the velocity of the wind reaches a certain level, which stability diagram is exemplified by Fig. 1[1] as reported by C. Scruton. This figure shows the relation of the aeolian and galloping critical wind velocities with the mass and damping effects of the structures. The non-dimensional aeolian critical velocity is independent of the mass and damping of structures, while the galloping velocity increases almost linearly with the mass and damping. However, Scruton's result show only qualitative characteristics. The quantative studies of these phenomena were reported by G. V. Parkinson[2] and M. Novak,[3] based on the quasi-steady theory to explain the galloping phenomenon particularly.

In addition to the characteristics mentioned above, in the low wind velocity, the characteristics of increasing the damping of structures together with the wind velocity are pointed out very recently by several researchers, K. Washizu,[4] M. Ito[5] and N. Shiraishi.[6] Furthermore, in the frequency characteristics, the rock-in state as a kind of aeolian oscillation is discussed by N. Shiraishi, M. Matsumoto, Y. Morimitsu,[7] *et al.*

In this paper, the galloping phenomena of the square prism in a two dimensional uniform flow are discussed by using the quasi-steady theory mentioned above, and then they are compared with the results reported by G. V. Parkinson.

Aerostatical Forces

Generally, the aerostatical forces acting on variable sections are related with the angle of attack of the wind. We call the forces in the wind direction the drag

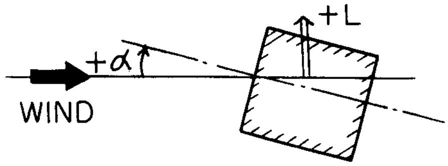

Fig. 2. Definition of positive angle of attack.

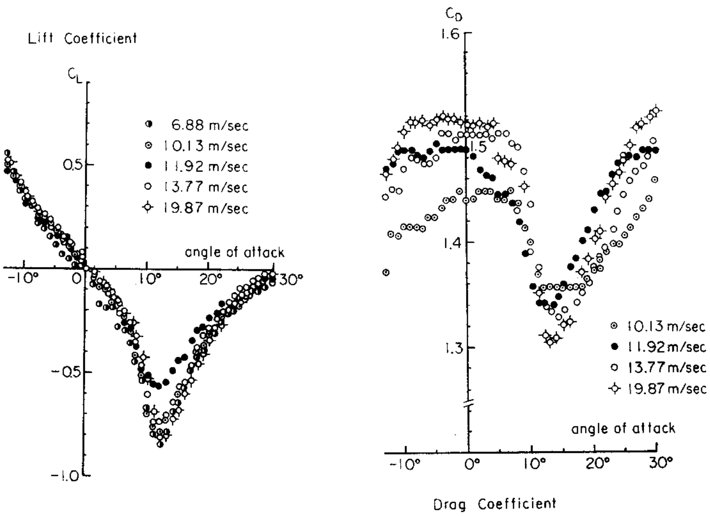

Fig. 3. Characteristics of lift and drag coefficients v.s. angle of attack (experimental results).

force or the lift force respectively. Besides, we define the angle of attack as positive when the leading side of the section rises up as shown in Fig. 2.

Defining the aerostatical forces and the angle of attack as mentioned above, we can show the experimental results of the lift force and the drag force acting on a square prism (vs the angle of attack) in Fig. 3. In our experiments, a sectional model with the dimensions of the cross section 20 cm × 20 cm and span length 93 cm is used, and the wind tunnel used here is located at the Department of Civil Engineering in Kyoto University. These results give us much information as follows;

(i) When the angle of attack is positive, the lift forces of the lift coefficient are negative.

(ii) The particular changes of both the lift and drag coefficient are recognized at the angle of attack 13°. These characteristics are thought to be caused by the reattachment of the separated flow at the leading corner of the square section.

(iii) The drag coefficient is about 1.5 at the angle of attack 0°, which is rather small in comparison with the one reported before, 2.0. The reasons for this difference are thought to be based on the condition of the surface of the model, the scale of the section and the Reynolds number in the experiments etc.

Formulation of Aerodynamical Forces

In the quasi-steady theory, the aerodynamical forces acting on an oscillating square prism in a flow are related with the aerostatical forces by the relative angle of attack as follows;

$$\alpha = \arctan(\dot{x}/U) \tag{1}$$

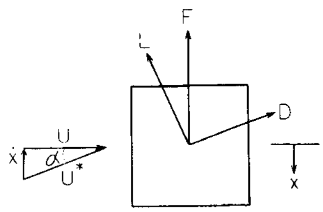

Fig. 4. Direction of aerostatic forces and Induced angle of attack.

Then, the aerodynamical forces inducing the oscillation in the direction normal to the flow (F in Fig. 4), are thought to be the sum of the vector components of the lift and the drag force reduced by the relative velocity, U^*. On the other hand, this aerodynamical force F is represented as follows;

$$F = \frac{1}{2}\rho U^2 C_F A \tag{2}$$

in which the symbols, ρ, U, A and C_F represent the air density, the mean velocity of flow, the revealed area of the section and the coefficient of the aerodynamical force, respectively. Hence, we can get the following equation by considering the relative angle of attack given by eq. (1) and using the lift and the drag coefficients, C_L and C_D.

$$C_F = -(C_L + D_D \tan\alpha)\sec\alpha \tag{3}$$

Hereupon, use is made of the following;

(i) the relative angle of attack is expanded in a series of $(\dot{x}/U)$,

(ii) the higher order terms are neglected

and

(iii) the characteristics of the lift and the drag coefficient (vs the angle of attck) are considered.

We can express the aerodynamical force coefficient C_F in eq. (3) as an odd function up to seventh order of $(\dot{x}/U)$ such as;

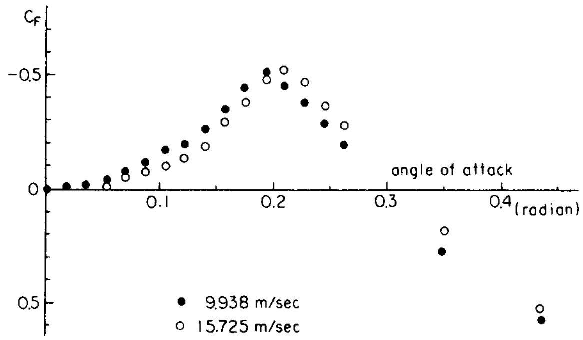

Fig. 5. Characteristics of C_F coefficient v.s. angle of attack (experimental results).

$$C_F = k_1\left(\frac{\dot{x}}{U}\right) - k_3\left(\frac{\dot{x}}{U}\right)^3 + k_5\left(\frac{\dot{x}}{U}\right)^5 - k_7\left(\frac{\dot{x}}{U}\right)^7 \tag{4}$$

On the other hand, we show our experimental results of the aerostatical coefficient C_F (vs the angle of attack) in Fig. 5, and the curves simulated by the following equations;

$$C_F = 0.5\left(\frac{\dot{x}}{U}\right) + 52.67\left(\frac{\dot{x}}{U}\right)^3 + 1760.18\left(\frac{\dot{x}}{U}\right)^5 - 44171.9\left(\frac{\dot{x}}{U}\right)^7 \tag{5}$$

$$C_F = 2.69\left(\frac{\dot{x}}{U}\right) - 168\left(\frac{\dot{x}}{U}\right)^3 + 6270\left(\frac{\dot{x}}{U}\right)^5 - 59900\left(\frac{\dot{x}}{U}\right)^7 \tag{6}$$

in Fig. 6. In this figure, the solid line shows the curve calculated by eq. (5) obtained by our experimental results shown in Fig. 5. The dotted line shows the curve calculated by eq. (6) as reported by G. V. Parkinson. When we compare C_F gained by our experiments with the C_F reported by G. V. Parkinson, a rather large difference is recognized. This difference is thought to be caused by the several differences of the experimental conditions like as follows:

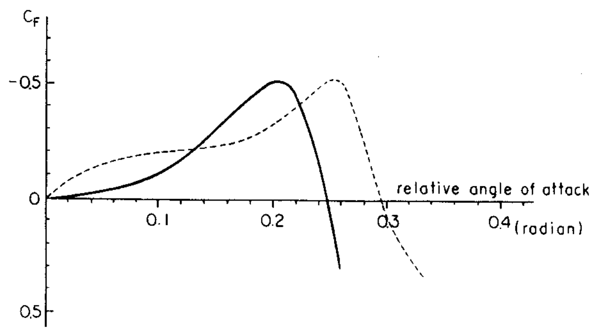

Fig. 6. Simulated curves of C_F coefficient (Solid line is obtained by authors and dotted line is reported by G. V. Parkinson.).

(i) as regards Reynolds' number, about 200,000 in our experiment, on the other hand, about 66,000 in G. V. Parkinson's,

(ii) as regards the dimension of the square section used in the experiments, 20 cm × 20 cm in ours, on the other hand 2 inch × 2 inch in G. V. Parkinson's, and

(iii) as for other factors, the differences of the dimension of the wind tunnel used, the effects of the section effects due to the model in the wind tunnel etc.

Response Based on Quasi-steady Theory

When the aerodynamical forces acting on the oscillating square prism are given by following equation;

$$F_{dyn} = -U^2.A.\left\{k_1\left(\frac{\dot{x}}{U}\right)-k_3\left(\frac{\dot{x}}{U}\right)^3+k_5\left(\frac{\dot{x}}{U}\right)^5-k_7\left(\frac{\dot{x}}{U}\right)^7\right\} \tag{7}$$

the system of the motion is nonlinear. The analysis of is nonlinear system has been already performed by G. V. Parkinson or M. Novak. The detailed process of the analysis is refered to references (2) or (3). In this paper we show only the brief process in the following.

The equation of the motion of the square prism in the normal to the wind direction is expressed by using eq. (7);

$$m\ddot{x}+c\dot{x}+kx = F_{dyn} \tag{8}$$

in which the symbols, m, c, k and F_{dyn} represent the mass of the square prism per unit length, the damping constant, the stiffness of the spring and the aerodynamical force per unit length, respectively. By transforming eq. (8), we gain the following equation;

$$\ddot{x}+2\zeta_0\omega_0\dot{x}+\omega_0^2x = \frac{\rho b}{m}U^2\left\{k_1\left(\frac{\dot{x}}{U}\right)-k_3\left(\frac{\dot{x}}{U}\right)^3+k_5\left(\frac{\dot{x}}{U}\right)^5-k_7\left(\frac{\dot{x}}{U}\right)^7\right\} \tag{9}$$

in which, $\omega_0^2 = k/m$ and $\zeta_0 = \dfrac{C}{2m\omega_0}$

Hereupon, we introduce two nondimensional parameters like as follows:

$V = U/2b\omega_0$; reduced velocity

and

$\varepsilon = \dfrac{m}{2\pi\rho b^2}$; mass parameter

into eq. (9). Thus, we can obtain the following the nonlinear equation.

$$\ddot{x}+2\left(\zeta_0\omega_0-\frac{\omega_0 V}{2\pi\varepsilon}k_1\right)\dot{x}+\omega_0^2 x$$
$$=\frac{2b\omega_0^2 V^2}{\pi\varepsilon}\left\{-k_3\left(\frac{\dot{x}}{V}\right)^3+k_5\left(\frac{\dot{x}}{V}\right)^5-k_7\left(\frac{\dot{x}}{V}\right)^7\right\} \tag{10}$$

In eq. (10), the coefficient of the first derivative of the deflectional response with respect to time(t) expresses the damping of the oscillating system at the reduced velocity, V. When this coefficient is zero, it is the boundary between the stable and the unstable conditions. Therefore, from this boundary condition, we can obtain the following critical reduced velocity.

$$V_0=\frac{2\pi\zeta_0\varepsilon}{k_1}=\frac{2\pi\bar{\varepsilon}}{k_1} \tag{11}$$

in which, $\bar{\varepsilon}=\frac{m\zeta_0}{2\pi\rho b^2}$; mass and damping parameter

The critical equation shown in eq. (11) coincides with den Hartog's condition. In the next, by using the Krylov-Bogoliubov method in the analysis of the nonlinear equation shown in eq. (10), we can obtain the following high order algebraic function (equation) in regard to the nondimensional amplitude symbolized by $(x_0/2b)$.

$$\frac{36}{64}\frac{k_7}{V^6}\left(\frac{x_0}{2b}\right)^6-\frac{5}{8}\frac{k_5}{V^4}\left(\frac{x_0}{2b}\right)^4+\frac{3}{4}\frac{k_3}{V^2}\left(\frac{x_0}{2b}\right)^2+\frac{2\pi\bar{\varepsilon}}{V}-k_1=0 \tag{12}$$

Hereupon, we introduce the following new variables defined as follows;

$$r=\frac{16V^4}{35k_7}\left(k_3-\frac{20}{63}\frac{k_5^2}{k_7}\right),\qquad \text{and}$$
$$s=\frac{32V^6}{35k_7}\left(\frac{10k_3k_5}{35k_7}-\frac{2\pi\bar{\varepsilon}}{V}+k_1+\frac{80k_5^3}{k_7^2}\right)$$

Then, the characteristics of the solution, $(x/2b)^2$, obtained from eq. (12) can be classified by the sign of (r^3+s^2) as following;

$r^3+s^2>0$; one real solution and two complex solutions
$r^3+s^2=0$; three real solutions (two of them are equal at least.)
$r^3+s^2>0$; three real solutions.

Furthermore, we can obtain the following two types of the critical reduced velocities which give the boundary of the hysterisis of the nonlinear response, respectively.

$$\mathrm{V}_1=\frac{2\pi\bar{\varepsilon}}{M+N} \quad (13), \qquad \text{and} \qquad V_2=\frac{2\pi\bar{\varepsilon}}{M-N} \quad (14)$$

in which, the symbols M and N represent the coefficient; defined by the coefficients k_i only, and they are given by the following equations, respectively.

$$M=k_1+\frac{10k_5}{35k_7}\left(\frac{40k_5^2}{189k_7}-k_3\right)$$

$$N=\left\{\frac{4}{35k_7}\left(\frac{20k_5^2}{63k_7}-k_3\right)\right\}^{3/2}$$

Therefore, the responses are shown as follows;

(i) the nondimensional amplitude (x_0/b) out of the hysterisis,

$$\left(\frac{x_0}{b}\right)_0=2\sqrt{2}\,V\left[\left(\frac{4}{35k_7}\right)^{1/3}\left\{\left(M-\frac{2\pi\bar{\varepsilon}}{V}+\sqrt{\left(M-\frac{2\pi\bar{\varepsilon}}{V}\right)^2-N^2}\right)^{1/3}+\left(M-\frac{2\pi\bar{\varepsilon}}{V}-\sqrt{\left(M-\frac{2\pi\bar{\varepsilon}}{V}\right)^2-N^2}\right)^{1/3}\right\}+\frac{4k_5}{21k_7}\right]^{1/2} \tag{15}$$

(ii) the nondime nsional amplitude (x_0/b) in the hysterisis,

$$\left(\frac{x_0}{b}\right)_1=4\sqrt{2}\left[\frac{k_5}{21k_7}+\cos\left(\frac{1}{3}\arccos\frac{M-\frac{2\pi\bar{\varepsilon}}{V}}{N}\right)\times\left\{\frac{1}{35k_7}\left(\frac{20k_5^2}{63k_7}-k_3\right)\right\}^{1/2}\right]^{1/2} \tag{16}$$

$$\left(\frac{x_0}{b}\right)_{2,3}=4\sqrt{2}\left[\frac{k_5}{21k_7}+\cos\left(\frac{1}{3}\arccos\frac{M-\frac{2\pi\bar{\varepsilon}}{V}}{N}\pm60^\circ\right)\times\left\{\frac{1}{35k_7}\left(\frac{20k_5^2}{63k_7}-k_3\right)\right\}^{1/2}\right]^{1/2} \tag{17}$$

As short discussions we can know the matters described as follows;

(i) the critical reduced velocities obtained from eqs. (11), (13) and (14), namely V_0, V_1 and V_2 have the relation of the linearity with the nondimensional mass and damping parameter symbolized by $\bar{\varepsilon}$,

(ii) the amplitudes of the response shown in eqs. (15), (16) and (17) are given as the functions of the reduced velocity, the mass and the damping of the structures.

Numerical Caluculation

Concerning the characteristics of the response of the square prism reduced by the quasi-steady theory, the critical reduced velocity and the nondimensional amplitude depend upon the aerodynamic coefficients k_i in the right hand side of eq. (4) and the mass and the damping parameter $\bar{\varepsilon}$ of the structures. As described before, in these coefficients k_i rather large differences containing the change of the sign are recognized between the results as reported by G. V. Parkinson and by us. Therefore, the critical reduced velocity and the nondimensional amplitude reduced by using two kinds of the results k_i are thought to show different values of the same parameter $\bar{\varepsilon}$. In this section, for convenience we will call the computed results reduced by G. V. Parkinson's k_i and ours, case 1 and case. 2 respectively. Then, the numerical calculations were accomplished by using the electrical computor (FACOM 230-60) at Kyoto University.

The aerodynamical coefficients used in the calculations are rewritten as follows;

case 1. $k_1=2.69$, $k_3=168$, $k_5=6270$, $k_7=59900$

case 2. $k_1=0.50$, $k_3=-52.67$, $k_5=1760.18$, $k_7=44171.9$

In the calculation, we change the parameter $\bar{\varepsilon}$ from 0.1 up to 0.7

(i) Critical Reduced Velocity

Two types of the critical reduced velocity, namely those which give the starting of the oscillation and the boundary of the hysterisis of the response, are symbolized by the signs V_0, V_1 respectively, and are computed from eqs. (11), (13) and (14). We show the results of the critical reduced velocities corresponding to the values of the parameter $\bar{\varepsilon}$ in the following table.

Table 1 critical reduced velocity of galloping

mass and damping parameter	case 1			case 2		
	critical reduced velocity			critical reduced velocity		
$\bar{\varepsilon}$	V_0	V_1	V_2	V_0	V_1	V_2
0.1	0.234	0.289	0.429	1.257	0.272	2.790
0.2	0.467	0.578	0.857	2.513	0.544	5.579
0.3	0.701	0.867	1.286	3.770	0.815	8.369
0.4	0.934	1.156	1.715	5.027	1.087	11.158
0.5	1.168	1.445	2.143	6.283	1.359	13.948
0.6	1.411	1.734	2.572	7.540	1.631	16.738
0.7	1.635	2.023	3.001	8.796	1.902	19.527

As shown the upper table, the following relation in the three critical reduced velocities, V_0, V_1 and V_2 reduced from coefficients k_i obtained by G. V. Parkinson is substantiated.

$$V_0 < V_1 < V_2$$

Moreover, as regards the relation between the amplitude of the response and the reduced velocity, when the reduced velocity drops to V_0 reaches V_0, the response starts and then the amplitude grows with the increase of the reduced velocity up to V_2 continuously. However, when the reduced velocity is V_2, the amplitude of the response grows up suddenly and discretely, and then grows up with the increase of the reduced velocity. On the contrary, the amplitude of the response becomes continuously smaller with the decrease of the reduced velocity down to V_1. At the critical reduced velocity V_1, after the sudden change of the amplitude, the response becomes smaller and vanishes completely at the critical reduced velocity V_0.

On the other hand, when we use the coefficients, k_i gained by our experiments, the relation is substantiated as follows;

$$V_0 < V_1 < V_2$$

In this case, the response shows the following characteristics. Namely, at the critical reduced velocity V_0, the amplitude of the response starts suddenly and its magnitude is rather large. It then grows continuously with the increase of the reduced velocity. On the contrary, the response becomes smaller with the decrease of the reduced velocity down to the critical one, V_1. At this one V_1, the response suddenly vanishes to zero.

As described above, as regards the characteristics of the response, rather large differences are recognized. Yet, as regards the critical reduced velocity which predominantes the stability of the structures, we can list up V_0 for case 1 and V_1 for case 2. Therefore, by the comparison of the critical reduced velocity V_0 for case 1 with V_1 for case 2, the latter is slightly larger than the former at the same values of the mass and damping parameter of the structures.

However, as for the structures with a small mass and damping parameter, the predominant critical reduced velocities V_0 for case 1 or V_1 for case 2 are smaller than the aeolian critical reduced velocity V (nearly 1.5) or equal to the one. In such a case, as reported by C. Scruton, actually the response does not show the characteristics as described above, when the reduced velocity reaches to the aeolian one, the first response starts by the vortex excitation and then the response veers to the galloping oscillation with the increase of the reduced velocity.

(ii) Characteristics of Response

The nondimensional amplitudes of the structures with a square section to each reduced velocity at case 1 or case 2 are calculated from eqs. (15), (16) and (17). These results of case 1 or case 2 are shown in Fig. 7 or Fig. 8 respectively by using the parameter symbolized by $\bar{\varepsilon}$ (mass and damping parameter of the structures). From these results, the following characteristics of the response are pointed out in common with case 1 and case 2. Namely, the amplitudes in the region over the critical reduced velocity V_1 grow up linearly with the increase of the reduced velocity. Furthermore, in the large amplitude the difference owing to the mass and damping parameter $\bar{\varepsilon}$

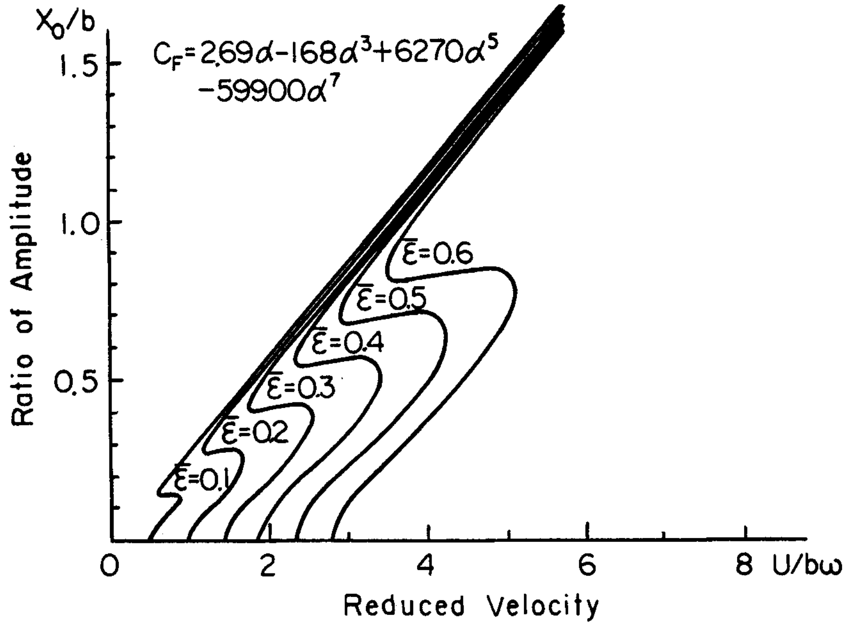

Fig. 7. Aerodynamic response curve calculated by use of C_F coefficient (by G. V. Parkinson).

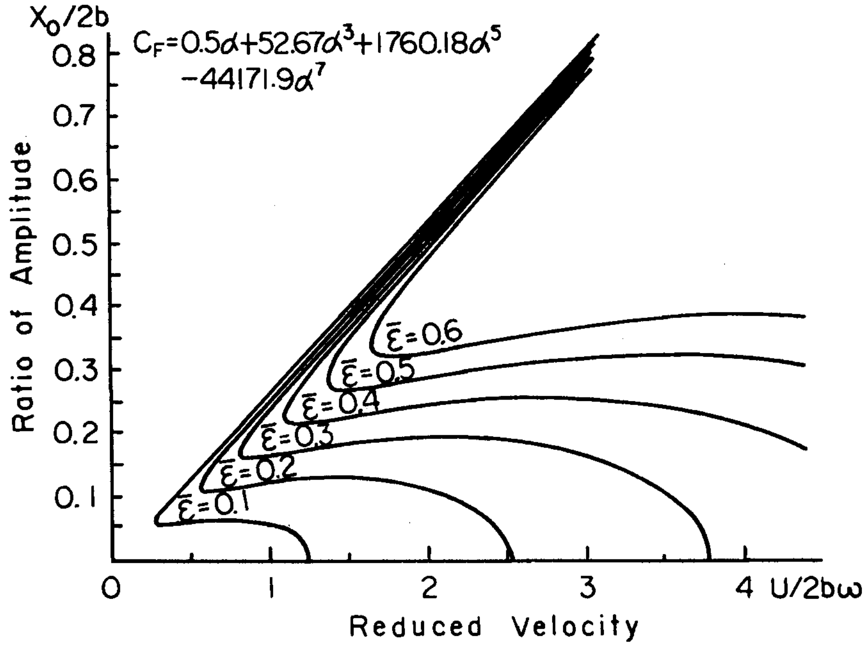

Fig. 8. Aerodynamic response curve calculated by use of C_F coefficient obtained by authors.

is very small. However, in the small amplitude the characteristics of the amplitude containing a zero starting critical velocity are influenced conspicuously by the mass and damping parameter $\bar{\varepsilon}$.

In comparison with the characteristics of the response reduced by the aerostatical coefficients k_i reported by G. V. Parkinson (case 1) and the ones obtained by our exparinents (case 2), at the region over the critical reduced velocity V_1, the amplitude in case 1 is a little larger than in case 2 to the same reduced velocity and the mass and damping parameter $\bar{\varepsilon}$. Besides, as regards the amplitude at the lowest reduced velocity in the hysterisis — namely, the amplitude at the critical reduced velocity V_1 — to each the mass and damping parameter $\bar{\varepsilon}$, the one in case 1 shows about 1.2 times value in case 2.

From the above several results, in spite of the rather large difference of the aerostatical coefficients k_i obtained by G. V. Parkinson's reports and the reports of our experiments as shown in eqs. (5) and (6), in the critical reduced velocity governing the stability of the aerodynamics of the structures with a square section the critical reduced velocity symbolized by V_0 in case 1 is nearly equal to the one symbolized by V_1 in case 2. Moreover, in the region over the critical reduced velocity V_1 the great difference of the amplitude between case 1 and case 2 is not recognized.

Conclusion

As mentioned before, using the quasi-steady theory to explain the characteristics of the aerodynamics of the square prism-galloping oscillation, the coefficients of the aerostatical force are considered essential factors. However, as regards the experimental results of the aerostatical force acting on a square section are concerned, there were some differences between the findings of G. V. Parkinson and our own. So in this paper, treating differences in the dynamical response according to the differences of the aerostatic coefficients, we consequently obtained the following results;

(i) The equations of the oscillating system are nonlinear because of the nonlinearity of the aerostatical forces. Then, at a low wind velocity, note the difference in the forms of the hysterisis between G. V. Parkinson's and our reports.

(ii) In the small amplitude of the response, the differences of the response due to the coefficients of the aerostatical force are remarkable.

(iii) In the region over a certain level of the amplitude, one hardly recognizes the above discrepancy.

(iv) As for the critical wind velocity, the differences between the results obtained by G. V. Parkinson and ours are small.

In future works, we should like to study these aerostatical forces of the square prisms with some kinds of the scales to consider the aerodynamical characteristics of the square section by combining the aerolian oscillation to the galloping oscillation in detail.

Finally, the authors would like to acknowledge the valuable assistance and the sustaining efforts of Messers Toru Saito, Shigeo Takei and Shigeru Komae for the wind tunnel experiments.

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