A Planning Model for Industrial Development by Using Integer Programming

By

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Abstract

We propose one static and two dynamic industrial developmental planning models for choosing the strategic areas and for determining the associated scale of development over a multiperiod planning horizon. The criterion of those models is to minimize the present worth of the total cost required for a series of developmental programs, which must satisfy the given demand for development in the regions as a whole with respect to each planning period. We assume that the total cost function can be well specified as a step function of the scale. This way of treatment can be justified by taking account of the data availability and the existence of thresholds. Emphasis of this study is put on the formulation of three various models in the form of O-1 Integer Programming, referring to their methods for solution and their sensitivity analysis.

In most cases where we can break down the project into several parts and fulfill them in a series of stages, some additional cost accrues as compared with their joint fulfillment. The second model applies to the case where this additional cost, which we call "Set-up cost", is so small as to be negligible. The third model is devised for cases where the set-up cost stemming from the atagewise fulfillment will not be negligible. Thus the third model is the most general model among the proposed three models in this paper.

1. Introduction

For purposes of industrial developmental planning, we propose one static and two dynamic models for choosing the strategic areas and for determining the associated scale of development over a multiperiod planning horizon.

The criterion built into these models is to minimize the present value of the total cost required for a series of developmental programs. These programs must also satisfy the given demand for development in the regions as a whole with respect to each planning period.

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As for the total cost which is introduced as our objective function, it is highly dependent on the scale of development. Keeping this fact in mind, we assume that the total cost function can be well specified as a step function of the scale. This way of treatment might be further justified by taking account of the data availability and the existence of thresholds.

We explicitly formulate three varied models in the form of 0–1 Integer Programming.

The first can be called a static model, because time is not considered. It should be noted, however, that even this model works well so long as the total demand for the development is independent of the time.

In contrast, the second and the third models are called dynamic ones in which the time span is dealt with. The difference between them can be stated as follows:

In most cases when we can break down a project into several parts and carry them out at different times, some "additional cost" accrues as compared with the joint fulfillment of the project. Yet, there exist some cases, where this "additional cost" is so small as to be negligible. The second model applies to these cases.

The third model is devised for cases in which the "additional cost" stemming from the stage-wise fulfillment will not be negligible. In this model, the objective function is quadratic. In other words, the third model is the most general model among the proposed three models in the following sense:

(1) In the second model, we do not take into account the "additional costs" which are considered in the third model,

(2) In the first static model, we do not deal with the time span which is considered in the second and the third model.

The solution of these three models is, then, illustrated with the aid of the Branch-and-Bound method and/or Dynamic Programming, which we modify to simplify the computation procedures. When there are competing methods for solution, we have compared one with another from the viewpoint of the facility of the computation and of the sensitivity analysis.

The inputs in these models, such as the coefficients of our objective functions and total demand for the development, in most cases, have a certain amount of uncertainty. Therefore it is important to know the range of variation of these inputs within which the obtained optimal solution does not change. In this context, we have also investigated the feasibility of the models for such sensitivity analyses.

2. Characterization of Cost Function

For the purpose of simplicity, let us suppose a single candidate site for industrial development.

When it comes to allocating a certain amount of land for plants or factories in this area, we must construct new infrastructures, such as ports, railways, highways, etc. At the same time we must carry out well-organized zoning, in order to satisfy a certain level of quality, judging from social, economical and ecological criteria. Therefore, a project for industrial development is defined a a reasonable land-use plan with respect to the predetermined scale of development, e.g., the land area designated for the installation of the plants.

Once the infrastructure is completed, it can be very hard to reform it later on. In order to eliminate this difficulity, the projects under study must be presented as being mutually exclusive. In other words, if the infrastucture of a certain scale for the project can by no means be compatible with that of the broader one, those two projects are mutually exclusive.

As a matter of fact, even to make one project requires much time and labor, since it must meet the multiple purposes regarding the land-use, transportation, water resources, etc. Therefore, practical restrictions force us to make only a few projects associated with the different scales of development. This signifies that we have only a few points as data on the function which relates the cost to the scale of development. We then have to estimate the unknown costs sitting somewhere between two given projects.

For an illustration, let us suppose that we know only the three points A, B and C

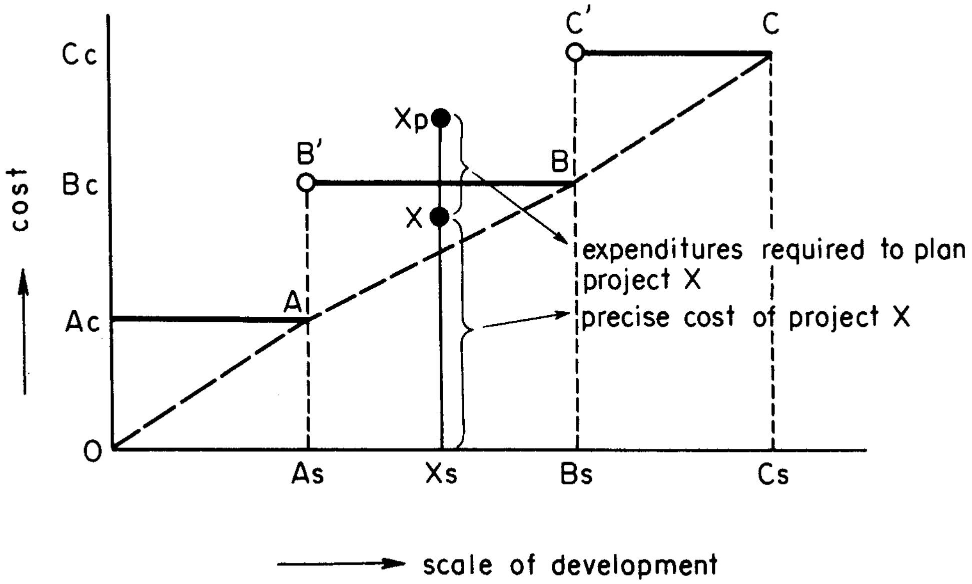

Fig. 1. Estimation of Cost Function.

in Fig. 1. The problem is, then, how to estimate the cost when the scale takes on some value between OA_s, A_sB_s and/or B_sC_s.

There are at least two ways of solving this problem. The first one is to link the given points with continuous straight lines segments which are shown as dotted lines in Fig. 1. The second one is to assume that the cost is invariant in each internal; hence, it takes a form of a step function shown as the solid lines in Fig. 1.

The latter is the development which we follow in this paper. The reasons for this are as follows:

(1) Points A, B and C correspond to completely different plans of land-use. Hence, a project associated with point A, which we call project A, is only appropriate for a scale of development within the range of 0 through A_s. In other words, over-expansion of the scale (-if we regard point A_s as the critical one, then it constitutes the upper limit for project A) causes a rapid increase in the cost owing to the traffic congestion, lack of water, air pollution, etc. In this sense, project A cannot be used whenever the scale of development exceeds the point A_s in Fig. 1.[1,2,3]

Similarly the project associated with B, which we call project B, can be used unless the scale exceeds point B_s.

(2) When the scale happens to be somewhere between A_s and B_s, there might be a more appropriate project for that scale. This project has many facilities in it which have some minimum units, and hence cannot be broken down thereafter. This means that once the scale given by a project happens to exceed this minimum unit level, then the same project will raise the cost to the next larger meaningful unit. Now that these sorts of projects are discrete in terms of their capacity, we can conceptually assume the existence of thresholds somewhere between A and B.[1,2,3]

(3) Furthermore, we have the following strong reason for justification.

Consider a project which corresponds to the scale taking on some value between A_s and B_s, which we call project X. Then the cost for that project might be somewhere between A_c and B_c. Obviously, there is no guarantee that it always lies on the line AB.

Suppose that the precise cost for project X is estimated from the outset, which automatically requires new expenditures for time spent and labor used to make the project. Therefore, we can regard the total cost of Project X as the sum of the expenditures required to plan project X plus the estimated precise cost for project X. This total cost of X may be greater than, or equal to, or less than B_c.

If we adopt the viewpoint of the project maker, he would surely calculate the precise cost so long as he knows that the total cost of X is less than B_c.

In reality, however, he has not done that yet. This indicates that the total cost of X is greater than B_c, whose project B can be used for the scale of X. In short, this is the most crucial factor which prevents him from estimating the precise cost for project X.

This suggests to us the efficiency of making use of B_c as a substitute for the cost of project X.

Thus if we can assume a step function as the cost function, any arbitary point on a segment parallel to the scale's axis (say C'C in Fig. 1.) which shows the same level of cost as C_sC, has the scale of development less than or at most equal to C_s. Therefore we can conclude that project C is more efficient than any other project associated with any scale taking on some value within B_s to C_s. It suggests to us that we need to consider Point A, B and C alone as the efficient project in the situation under study.

Finally, let us pay attention to one particular project. If it can be broken down into some feasible parts, then we can carry out the project in stages. In this case, our cost function also takes the form of a step function according to each small part. This is because any project always has some "minimum unit" which can by no means be broken down into smaller parts. This minimum unit takes a fundamental role in our analysis.

3. The Static Model

3.1. Assumption

Suppose we take an economy, say a state, which intends to fulfill the industrial development based on the given economic plan as a whole. It can be divided into N regions as their suitable candidates for industrial development. In each region i, we also assume that there are M_i development projects, mutually exclusive of the varied scales of fulfillment.

Our criterion for optimality is to minimize the total cost required to accomplish an aim given by the economic planning agency.

Under the assumption stated above, our model is entitled to choose the most strategic regions for industrial development as well as to determine the scale of operations.

3.2. Notations

$(i.j)$: indicates the (j)th project in the (i)th region.

D: required level of development for the region as a whole, as an aim given by the economic agency.

A_{ij}: the scale of the (j)th project in the (i)th region.

C_{ij}: the cost of (i, j)

$X_{ij}=1$: signifies the fulfillment of (i, j)
0: otherwise

3.3 Formulation

Our model can be, then, formulated as follows;

$$\min Z = \sum_{i=1}^{N} \sum_{j=1}^{M_i} C_{ij} X_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{M_i} X_{ij} \leqq 1 \quad (i=1, \cdots, N) \tag{1}$$

$$\sum_{i=1}^{N} \sum_{j=1}^{M_i} A_{ij} X_{ij} \geqq D$$

$$X_{ij} = 0 \text{ or } 1 \quad (i=1, \cdots, N;\ j=1, \cdots, M_i)$$

3.4. Methods for solution

There are two algorithms available:

a. Dynamic Programming[5,6]
b. Branch-and-Bound[7,8,9]

The sensitivity analysis may be worked out with respect to the following factors;

a. C_{ij}, A_{ij}, D
b. N, M_i

DP is convenient for a sensitivity analysis with respect to N, M_i and D.[4] Brach-and-bound is convenient for a sensitivity analysis with respect to C_{ij}.[4] However, the sensitivity analysis of A_{ij} is a remaining problem.

Therefore, it is desirable to solve the static model by both DP and Branch-and-Bound.

4. The Multistage Model without Set-up Cost

4.1. Assumption

The required level for the developmental plan must be given in advance with respect to each planning period.

The developmental projects in each region are mutually exclusive as before. Furthermore, each of them (i,j) can be broken down into several parts (let it be K_{ij}

in general) so that we can carry out the initial projects in successive stages. To be more specific, each part can be labeled by a number in such a way that the part of number 1 can be carried out without any restriction. However, the following parts,

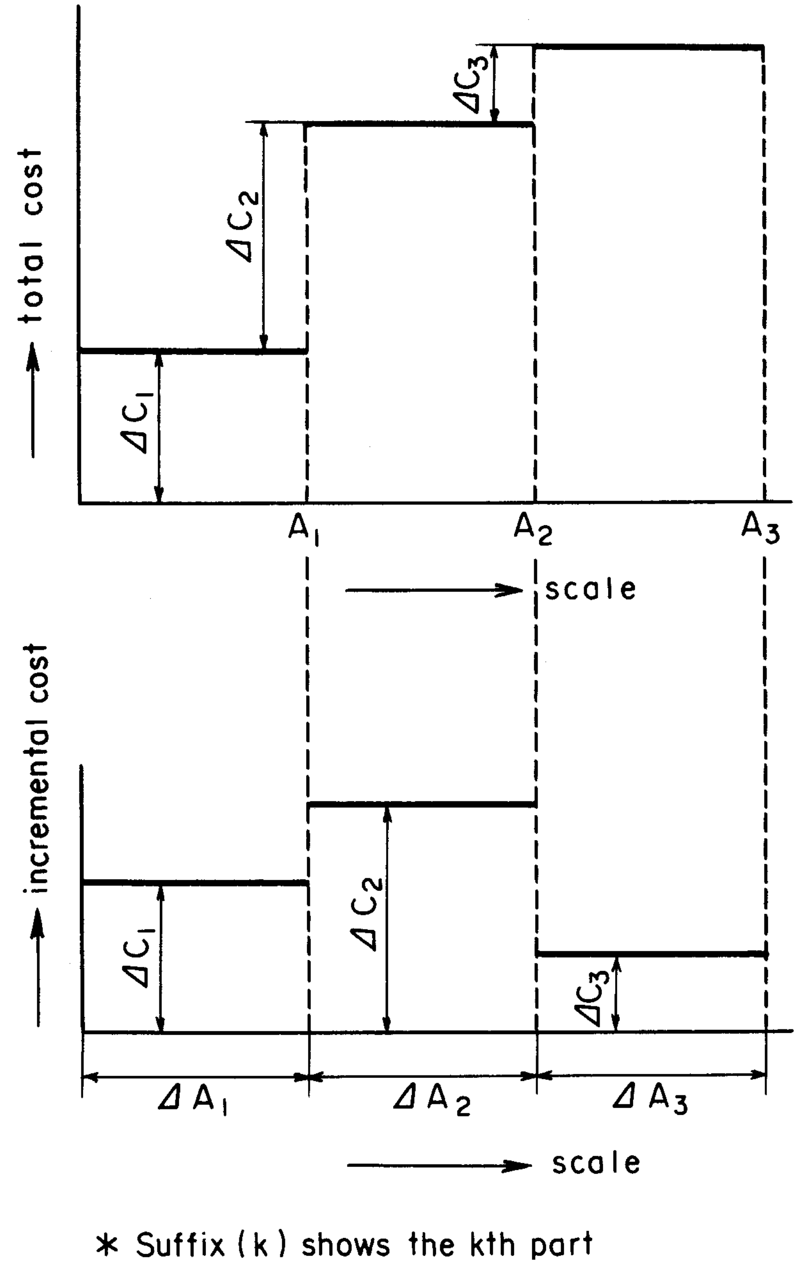

Fig. 2. Cost Function without Set-up Cost.

say the (k)th part can only be carried out provided that the parts, whose labels are less than k, have already been carried out before, or at the time when the (k)th part is going to be done.

In most cases where we break down the initial project into several parts and then carry out them separately over time, some "additional costs" accrue as compared with the joint fulfillment of the initial project. However, in this case, we assume that such an additional cost does not occur no matter how we break down the initial project and carry them out disjointly.

Thus the cost function for this type of stage development can be represented by a single step function. (see Fig. 2)

Hence, our problem is to determine when the various parts of the separate projects should be carried out in their respective regions in order to minimizer the present value of the total cost.

4.2. Notation

(i, j, k): indicates the (k)th part with respect to the (j)th project in the (i)th region

D_t: required level of development for the regions *as a whole* at period t, as an aim given by the economic agency. $(t=1, \cdots, T)$

ΔA_{ijk}: the net (or incremental) scale associated with the (k)th part with respect to the (j)th project in the (i)th region. $(i=1, \cdots, N;$ $j=1, \cdots, M_i;\ k=1, \cdots, K_{ij})$

ΔC_{ijk}: the net (or incremental) cost associated with (i, j, k)

ΔC_{ijkt}: the present value of ΔC_{ijk} consumed at the period t.

Mathematicaly it can be expressed as follows;

$$\Delta C_{ijkt} = \frac{\Delta C_{ijk}}{(1+r)^{n(t-1)}}$$

where r: social discount rate.

n: number of years per unit period.

$X_{ijkt} = 1$: signifies the fulfillment of (i, j, k) at period t

0: otherwise

4.3 Formulation

$$\min Z = \sum_{i=1}^{N} \sum_{j=1}^{M_i} \sum_{k=1}^{K_{ij}} \sum_{t=1}^{T} \Delta C_{ijkt} X_{ijkt} \tag{2·a}$$

s.t.
$$\sum_{i=1}^{N}\sum_{j=1}^{M_i}\sum_{k=1}^{K_{ij}}\sum_{\tau=1}^{t}\Delta A_{ijk}X_{ijk\tau}\geqq D_t \qquad (t=1,\cdots,T) \tag{2·b}$$

$$\sum_{t=1}^{T}X_{ijkt}\leqq 1 \qquad (i=1,\cdots,N,\ j=1,\cdots,M_i,\ k=1,\cdots,K_{ij}) \tag{2·c}$$

$$\sum_{\tau=1}^{t}X_{ij(k-1)\tau}\geqq X_{ijkt} \qquad \begin{pmatrix} i=1,\cdots,N,\ j=1,\cdots,M_i \\ k=2,\cdots,K_{ij},\ t=1,\cdots,T \end{pmatrix} \tag{2·d}$$

$$\sum_{j=1}^{M_i}\sum_{t=1}^{T}X_{ij1t}\leqq 1 \qquad (i=1,\cdots,N) \tag{2·e}$$

$$X_{ijkt}=0 \ \text{ or } \ 1 \qquad (i=1,\cdots,N,\ j=1,\cdots,M_i,\ K=1,\cdots,K_{ij},\ t=1,\cdots,T)$$

The constraints may be interpreted as follows:

(2·b) shows the total scales (or amounts which have been carried out before, or at each period t) must not be less than the required level for the industrial developmental plan at period t.

(2·c) shows that once (i, j, k) is carried out at period t, then it can not be allowed to occur at any different period. In short, this reflects the assumption of the "exclusiveness" concerning the period.

(2·d) shows that for each project j in each region i, any part k, but for k=1, can only be carried out provided that the foregoing parts (i.e. the 1,2,..., and (k-1)th parts) have already been completed or are in progress at the same time as the kth parts. In short, this reflects the other assumption of "preoccupied rule" about the execution ordering.

(2·e) shows that at most one project can be carried out for each region. This reflects the assumption of "exclusiveness" concerning projects.

4.4. Methods for Solution

As far as the author's knowledge is concerned, the Branch-and-Bound method is available for solution.

The formulation and its algorithms using Dynamic Programming are remaining problems.

A sensitivity analysis may be worked out with respect to the following factors;

(a) ΔC_{ijk}, ΔA_{ijk}, D_t
(b) N, M_i, K_{ij}, T
(c) r

Using the Branch-and-Bound procedure, the author developed an algorithm for the sensitivity analysis with prespect to ΔC_{ijk}.[10] However, the Sensitivity analysis for the other factors remains to be solved.

5. The Multistage Model with Set-up Cost

5.1. Assumption

This model is devised to deal with case in which an additional cost accrues from the stage-wise fulfillment of the initial project, as compared with its joint fulfillment.

Generally speaking, the grand total cost summed over the stagewise fulfillment of a certain project j in region i is much higher than that of joint fulfillment of the associated project. It might be attributed to the existence of a "set-up cost".

In order to elucidate this point, let us consider the following case:

We have a choice of whether a certain project j in region i should be fulfilled at one time or in stages over different periods. For simplicity we assume only two parts and two distinct periods.

If we carry out the given project (i,j) at one time, then the total cost required for that project can be written as C, which is conceptually equal to $\Delta C_1 + \Delta C_2$, where ΔC_1 and ΔC_2 indicate the cost required for the fulfillment of the first and the second part, respectively.

On the other hand, if we break down the same (i, j) into two parts, and fulfill them separately during two distinct periods, then the total cost may differ and hence it can be expressed as $\Delta C_1 + (\Delta C_2 + \Delta \tilde{C}_2)$. Note that the latter is larger by $\Delta \tilde{C}_2$, which we define an "additional cost" for (i,j,2)

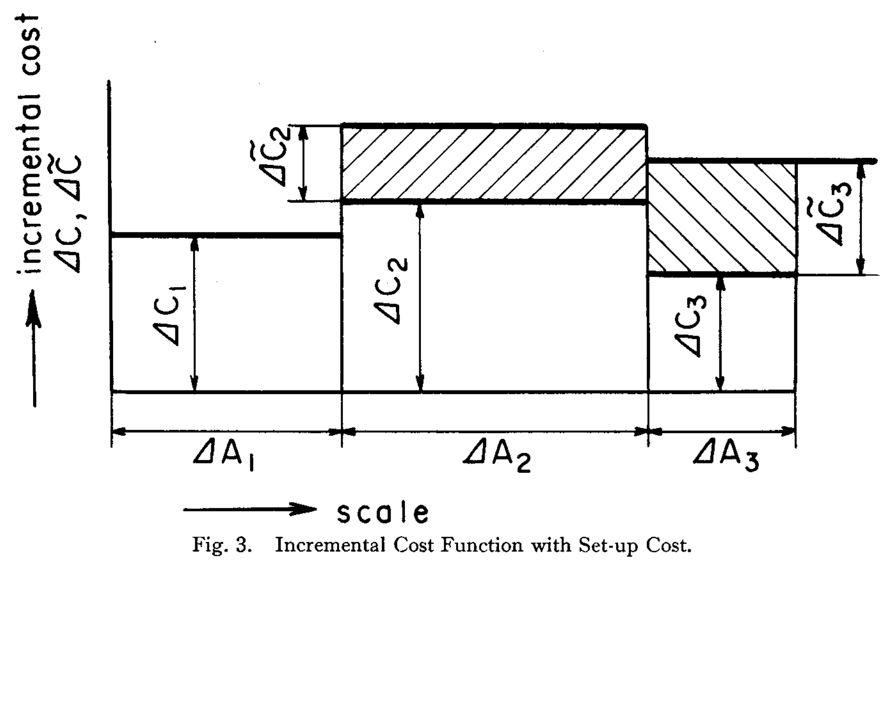

Fig. 3. Incremental Cost Function with Set-up Cost.

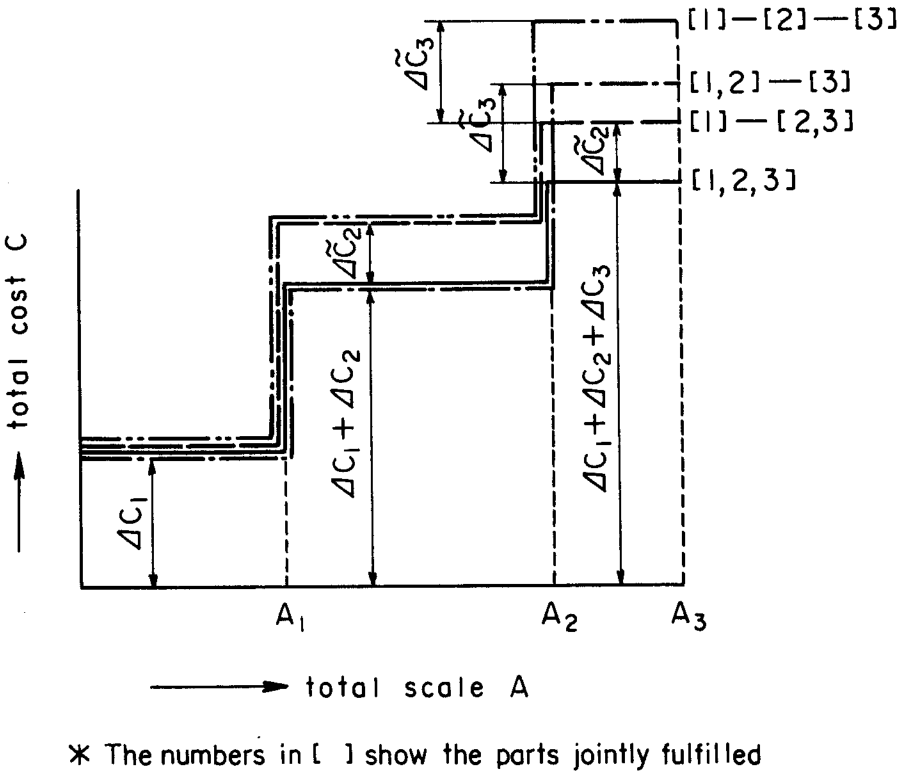

Fig. 4. Total Cost Function with Set-up Cost.

Diagrammatically, the above situations with 3 parts are represented in Figure 3.

Figure 4 shows how the sequence of fulfillment affects the grand total cost under the same scale of a certain developmental project.

Thus the cost function for the general multistage development cannot be representes by a single curve but by a family of curves differentiated by the sequence of joint and disjoint fulfillment of the project parts.

5.2. Notation

ΔC_{ijk}: the required cost associated with the joint fulfillment of (i. j, k),

$\Delta \tilde{C}_{ijk}$: the additional cost of the separate fulfillment of (i, j, k), where $k \neq 1$.

In comparison with the case of the joint fulfillment, the partial cost for the disjoint fulfillment can be expressed as $\Delta C_{ijk}+\Delta \tilde{C}_{ijk}$.

ΔC_{ijkt}: the present value of ΔC_{ijk} consumed at period t,

$\Delta\tilde{C}_{ijkt}$: the present value of $\Delta\tilde{C}_{ijk}$ consumed at period t, where $k\neq 1$ and $t\neq 1$.

5.3. Formulation

The constraints of this model do not differ from the previous ones (2·b)～(2·e). The difference emerges only in the objective function.

It can be specified as follows:

$$Z=\sum_{i=1}^{N}\sum_{j=1}^{M_i}\sum_{k=1}^{K_{ij}}\Delta C_{ijk1}X_{ijk1}+\sum_{i=1}^{N}\sum_{j=1}^{M_i}\sum_{t=2}^{T}\Delta C_{ij1t}X_{ij1t}$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M_i}\sum_{k=2}^{K_{ij}}\sum_{t=2}^{T}\Big[\Delta C_{ijkt}+\Delta\tilde{C}_{ijkt}\Big(\sum_{\tau=1}^{t-1}X_{ij(k-1)\tau}\Big)\Big]X_{ijkt} \tag{3}$$

The interpretation of our new objective function is as follows:

(1) *First term*; if (i, j, k) is carried out at the first period (that is, $X_{ijk1}=1$), then (i,j,1),...(i,j,k) must be jointly carried out according to the assumption shown by the constraint (2·d). Therefore the coefficient of X_{ijk1} should be ΔC_{ijk1} instead of $\Delta c_{ijk1}+\Delta\tilde{C}_{ijk1}$ as shown by the first term in eq.(3).

(2) *Second term*; for the first part of any project in any region, no matter whether it is carried out jointly or disjointly, its cost does not change as shown in Fig. 4. The second term of eq.(3) shows this fact.

(3) *Third term*; suppose $X_{ijkt}=1$ ($k\neq 1$, $t\neq 1$). By virtue of the constraint (2·d),

$$\sum_{\tau=1}^{t}X_{ij(k-1)\tau}\geqq X_{ijkt}=1 \tag{4}$$

Furthermore, with the aid of (2,c)

$$\sum_{\tau=1}^{t}X_{ij(k-1)\tau}+\sum_{\tau'=t+1}^{T}X_{ij(k-1)\tau'}\leqq 1$$

$$\therefore\ \sum_{\tau=1}^{t}X_{ij(k-1)\tau}\leqq 1 \tag{5}$$

From (4) and (5), we obtain the following relation:

$$\sum_{\tau=1}^{t}X_{ij(k-1)\tau}=\sum_{\tau=1}^{t-1}X_{ij(k-1)\tau}+X_{ij(k-1)t}=1 \tag{6}$$

Keeping eq.(6) in mind, let us examine the following two possibilities separately:

a) $$\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau}=1 \quad \text{and} \quad X_{ij(k-1)t}=0$$

With $X_{ijkt}=1$, this case implies that the (k-1) part has been carried out before the period t in which (i,j,k) is going to be carried out.

Therefore this case implies the separate fulfillment of (i,j,k) at the period t. In this case, the coefficient of X_{ijkt} in the objective function is

$$\Delta C_{ijkt}+\Delta \tilde{C}_{ijkt}\left(\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau}\right)=\Delta C_{ijkt}+\Delta \tilde{C}_{ijkt} \tag{7}$$

as it should be.

b) $$\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau}=0 \quad \text{and} \quad X_{ij(k-1)t}=1$$

In this case, the fact that $X_{ij(k-1)t}=1$ and $X_{ijkt}=1$ jointly hold, implies that the (k)th stage is carried out together with the (k−1)th stage at the same period t. In this case, the coefficient of X_{ijkt} in the objective function is

$$\Delta C_{ijkt}+\Delta \tilde{C}_{ijkt}\left(\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau}\right)=\Delta C_{ijkt} \tag{8}$$

as it should be.

The above examinations exhaustively cover all cases of interest.

5.4. Method for Solution

One of methods for solving this model is the linearlization of 0-1 Integer Quadratic Programming into 0-1 Integer Linear Programming.[11)]

Concentrate our attention on the product terms

$$\left(\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau}\right)X_{ijkt}$$

in eq.(3). We have already known from eq.(6) that

$$\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau}=0 \quad \text{or} \quad 1 \tag{9}$$

To deal with the product term $(\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau})X_{ijkt}$, we need only replace it with new variable Y_{ijkt} and add the three contraints:

$$\left.\begin{array}{ll} \text{(i)} & \sum_{\tau=1}^{t-1} X_{ij(k-1)\tau} + X_{ijkt} - Y_{ijkt} \leqq 1 \\ \text{(ii)} & -\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau} - X_{ijkt} + 2Y_{ijkt} \leqq 0 \\ \text{(iii)} & Y_{ijkt} = 0 \quad \text{or} \quad 1 \end{array}\right\} \tag{10}$$

where

$$\mathrm{i}=1, \cdots, \mathrm{N}, \quad \mathrm{j}=1, \cdots, M_i, \quad \mathrm{k}=2, \cdots, K_{ij}, \quad \mathrm{t}=2, \cdots, \mathrm{T}$$

To assess whether the constraints in eq.(10) will cause Y_{ijkt} to have the values of $(\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau})X_{ijkt}$ would have, given specified values for $\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau}$ and X_{ijkt}, let us examine the possible cases.

a) When $\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau} = X_{ijkt} = 0$, constraint (i) will not constrain Y_{ijkt}, but constraint (ii) will cause $Y_{ijkt}=0$ as it should be.

b) When $\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau}=0$, $X_{ijkt}=1$ or $\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau}=1$, $X_{ijkt}=0$, constraint (ii) will not constrain Y_{ijkt}, but (i) will cause $Y_{ijkt}=0$ as it should be.

c) When $\sum_{\tau=1}^{t-1} X_{ij(k-1)\tau} = X_{ijkt} = 1$, constraint (i) becomes

$$1+1-Y_{ijkt} \leqq 1 \quad \text{or} \quad Y_{ijkt} \geqq 1 \tag{11}$$

and constraint 2 becomes

$$-1-1+2Y_{ijkt} \leqq 0 \quad \text{or} \quad Y_{ijkt} \leqq 1 \tag{12}$$

Together (11) and (12) assure that $Y_{ijkt}=1$ as it should be.

Thus after the linearlization, we can use the Branch-and-Bound to solve the transformed 0–1 linear Integer Programming problem.

6. Conclusion

In this paper, we have proposed a static and two dynamic models for choosing the strategic areas and for determining the associated scale of industrial development over a multiperiod planning horizon. These three models were formulated by using 0–1 IP.

The main conclusions derived from the analysis in this paper are as follows:

(1) A project for industrial development can be defined as a reasonable land-use plan

with respect to the predetermined scale of development.

(2) It is reasonable to assume that the projects associated with the different scales are mutually exclusive.

(3) Practical restrictions force us to make only a few projects associated with the different scales of development due to data availability.

(4) The static model can be easily formulated by what is called the Knapsack problem. As for the method for solution, both Dynamic Programming and Branch-and-Bound are convenient for a sensitivity analysis.

(5) When we encounter cases where some project can be broken down into several parts so that we can carry them out in successive stages, the cost function in the model can be represented by a step function.

(6) The case without "additional costs" accrued from the step-wise development has a single step cost function. On the contrary, the case with them has a family of step cost functions. Therefore, the latter can be represented by a quadratic form of cost function. Nonetheless, the constraints for the latter have exactly the same form as the former.

(7) Among the necessary constraints for the dynamic models, even what is called "preoccupied rule" can be formulated by using the characteristics of binary variables.

(8) By converting the quadratic form into a linear form of cost function, we can use Branch-and-Bound as the method for solution for both dynamic models.

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