Parameters of Mounting and Foundation Affecting the Structural Dynamics of Machine Tools

By

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Summary

The effect of mount and foundation on the structural dynamics of machine tool was studied by theoretical analysis along with experimental methods.

The frequencies and mode shapes of the resonance modes to which the mount and foundation have decisive effect are analyzed by a theoretical model, a scale model test, and tests on several actual machines.

Based on the harmonic response test of a concrete foundation, the empirical values of the principal parameters of the mount and foundation influential to the machine dynamics are identified. They include the stiffness of the concrete foundation surface on which the machine is mounted, and the ground modulus of the soil supporting the foundation.

1. Introduction

A series of study has been undertaken on the foundation under the machine and the method used to mount the machine on the foundation, with respect to their effects on the dynamic behavior of the machine tools.

To be able to obtain a good surface finish by cutting or grinding operations, it is critically important that the machine has little vibration when operated at an idling condition1). This necessitates that the vibrations caused by the idling operation do not resonate the machine tool structure. It often happens, however, that the structural resonances of machine tools at relatively low natural frequencies, (generally less than 100 Hz), are found at close vicinities of the disturbance frequencies caused by rotational elements of the machine. Hence it causes diffculties and inaccuracy when exacting work is to be done. At the resonance modes of such low orders, the resonance frequencies are influenced by the design of the foundation and the mounting method. This is because the motion in the system principally

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Note: Numbers in brackets designate references at the end of the text.

consists of the translation or the rotation of the machine as a whole. The purpose of this study is to understand the mechanism of such low order resonance modes, and to identify as well as to quantify important parameters which affect the resonance frequencies.

2. Methods for the study

The study has been made on a particular type of the concrete foundation which is often used as the bases for high precision machine tools and instruments. As seen in Fig. 1, the bottom of the concrete foundation is set on the compacted soil. The five iron rails are embedded at the top of the concrete foundation. The rails are not for direct mounting of the machine base, but for use when needed to position the square heads of the screw studs which are used to press down the machine base, and hence the mounting blocks towards the top surface of the concrete foundation. In setting the test machines on the foundation, the mounting blocks are located on the concrete portion of the top surface of the foundation whenever possible. Estimated total weight of the concrete foundation is 12.3t, 11.5t for the concrete and 0.8t for the iron rail and the I-beams.

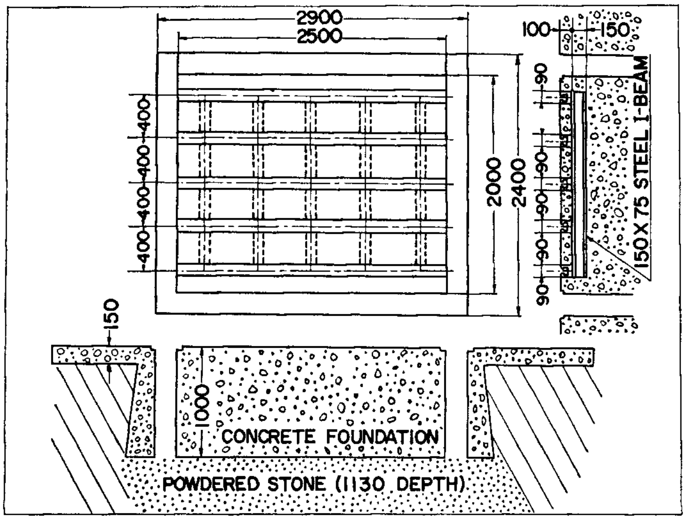


Fig. 1. Illustration of the concrete foundation used for the study.

Theoretical analysis and experimental study have been undertaken as outlined in the following:

i) *Analysis of low order resonance modes*

  i-1) *Theoretical analysis* Theoretical analysis has been made on simplified models of a machine, a foundation, and a combined set of the two.

  i-2) *Analysis by scale models* The theory is examined by dynamic tests of simple models made analogous to the machine and the foundation.

ii) *Measurement of low order modes of real machine*

Experimental data with actual machines are taken from dynamic test records of several machine tools which have been tested for general dynamic evaluation on the concrete foundation.

iii) *Excitation test of the foundation*

Dynamic tests of the foundation itself have also been made to study its mode shapes and the resonance frequency. Also special excitation tests have been performed to quantify important parameters inherent to the example foundation.

iv) *Evaluation of the flexible mounting*

A particular diamond lathe has been selected which, by standard specification, is mounted on flexible rubber pads. Excitation tests are performed to identify low order resonances of this machine standing on such a specified mounting, and also on a conventional mounting using iron blocks instead of the rubber pads.

3. Analysis of low order resonance modes

3.1 Theoretical analysis

For the resonance modes of the lower orders, it is first assumed that the motion of the machine and the foundation can be approximated by that of rigid bodies. Major flexibility in the system is taken by the soil under the foundation and the mounting mechanism used between the machine and the foundation. The oscillations of the system are readily understood, starting from the motion of the machine and the foundation, when they are separated and standing alone on the mounting mechanism and the earth soil respectively. This problem of a rigid body supported flexibly at its bottom is theoretically analyzed, and four principal modes are shown to exist as illustrated in Fig. 2. Since the last two of the four modes [modes (1) and (2)] in Fig. 2 are able to occur in two horizontal directions (x- and y-directions) respectively, a total of six resonance modes are able to occur in a rigid body supported at its bottom. They are:

mode Z,	translation in vertical direction
mode rZ,	rotation around vertical axis

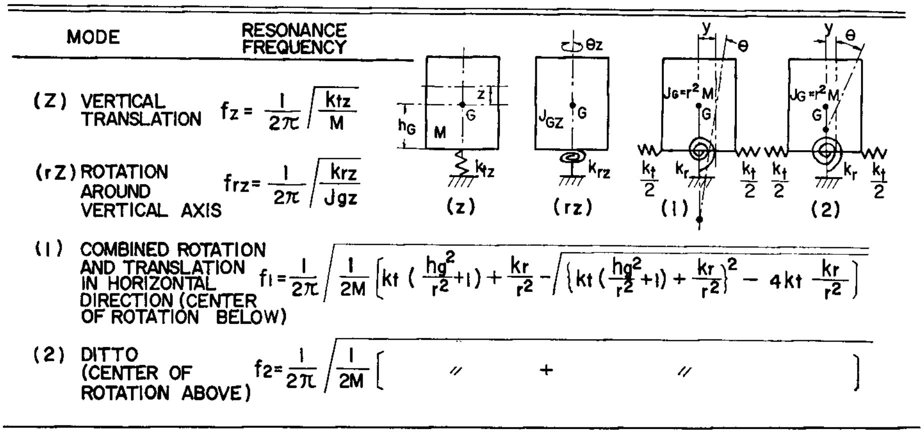
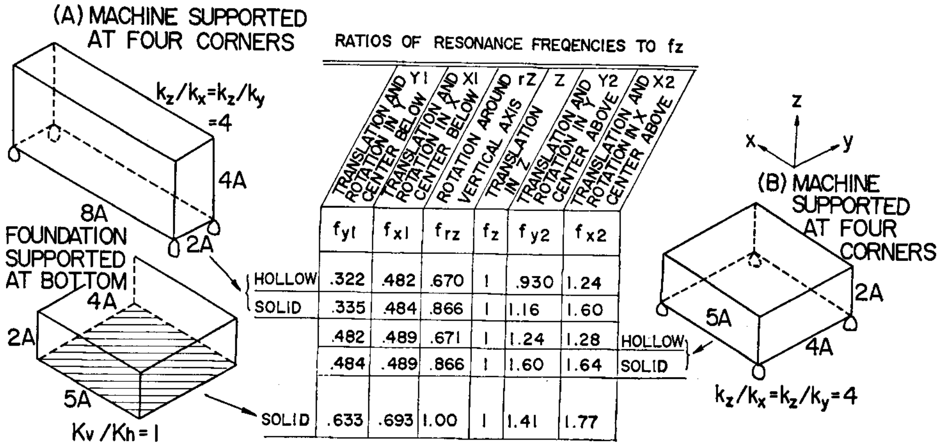


Fig. 2. Principal theory of low order modes and resonance frequencies of a rigid stucture that has horizontal symmetry.
M: Mass of structure, G: Center of gravity,
J_{GZ}: Moment of inertia of the structure around the vertical axis
J_G: Moment of inertia of the structure around the axis normal to figure.

mode $X1$,	combined translation and rotation in x-direction (direction of the longer side) with instantaneous center of rotation below the body
mode $X2$,	combined rotation and translation in x-direction with instantaneous center of rotation above the bottom of the body.
mode $Y1$,	combined translation and rotation in y-direction (direction of the shorter side), with instantaneous center of rotation below the body
mode $Y2$,	combined rotation and translation in y-direction with instantaneous center of rotation above the bottom of the body

In applying the above principle of the rigid body motion to the machine tools, it accounts for the fact that they are usually designed to be supported at several points under the bottom surface. As an example, equations of the resonance frequencies are listed in Appendix 1 for the case when a machine with a horizontal symmetry is supported at the four corners of its bottom. For a concrete foundation supported uniformly at its bottom surface, as is the case in this study, the equations are listed in Appendix 2. Theoretical resonance frequencies are computed for several examples, and the result is shown in Fig. 3. (A) illustrates a rather long and narrow machine having a 2–4–1 height-length-width proportion supported at the four corners. The resonance frequencies are shown in their ratios to that of the vertical translation mode (mode Z), both when the machine is a hollow structure and when it is a solid structure. Equations in the Appendix 1 are used for the



RATIOS OF RESONANCE FREQEENCIES TO fz

	TRANSLATION AND ROTATION IN Y CENTER BELOW Y1	TRANSLATION AND ROTATION IN X CENTER BELOW X1	ROTATION AROUND VERTICAL AXIS rZ	TRANSLATION IN Z Z	TRANSLATION AND ROTATION IN Y CENTER ABOVE Y2	TRANSLATION AND ROTATION IN X CENTER ABOVE X2	
	f_{y1}	f_{x1}	f_{rz}	f_z	f_{y2}	f_{x2}	
HOLLOW	.322	.482	.670	1	.930	1.24	
SOLID	.335	.484	.866	1	1.16	1.60	
	.482	.489	.671	1	1.24	1.28	HOLLOW
	.484	.489	.866	1	1.60	1.64	SOLID
SOLID	.633	.693	1.00	1	1.41	1.77	

Fig. 3. Examples of theoretical resonance frequencies of rigid structures.

computations, in which the vertical stiffness k_z of the four springs, representing the mounting mechanisms, is assumed to be four times greater than the horizontal stiffness. (B) shows the result of similar computations for a flat machine having a 2–5–4 height-length-width proportion. (C) shows a solid foundation analogous to the one used for this study, computed by equations of Appendix 2. In this computation, equality is assumed for the vertical and horizontal stiffnesses of the supporting ground (ground moduli K_v and K_h). It is understood as a common feature in those examples that mode *Y*1, (combined translation and rotation in the direction of the shorter side with the instantaneous center of rotation below the body), occurs at the lowest frequency among the six modes. On the other hand, mode *X*2 (the same in the direction of the longer side with the instantaneous center of rotation above the bottom of the body) occurs at the highest frequency.

Knowing the dynamic behavior of the machine and the foundation when each of them are standing alone as described above, analysis is made for the case when they are combined. In the actual system, the resilient ground supports the foundation, on top of which mount mechanisms are set to support the machine. In this case, the pair of the resonance modes possible for each body alone is coupled in the vertical (z), rotational (rz) and horizontal (x and y) directions respectively, and is transformed into new resonance modes which occur at different frequencies. To take an example in the vertical (z) direction, the resonance mode of the foundation alone $F(Z)$ and that of the machine alone $M(Z)$ as shown in the upper part of Fig. 4 are coupled and produce two resonance modes $Z1$ and $Z2$ as illustrated in the lower part of the figure. Mode $Z1$ occurs at a resonance frequency f_{Z_1} which is lower than either of the two resonance frequencies f_{ZF} and f_{ZM} possible for the foundation and

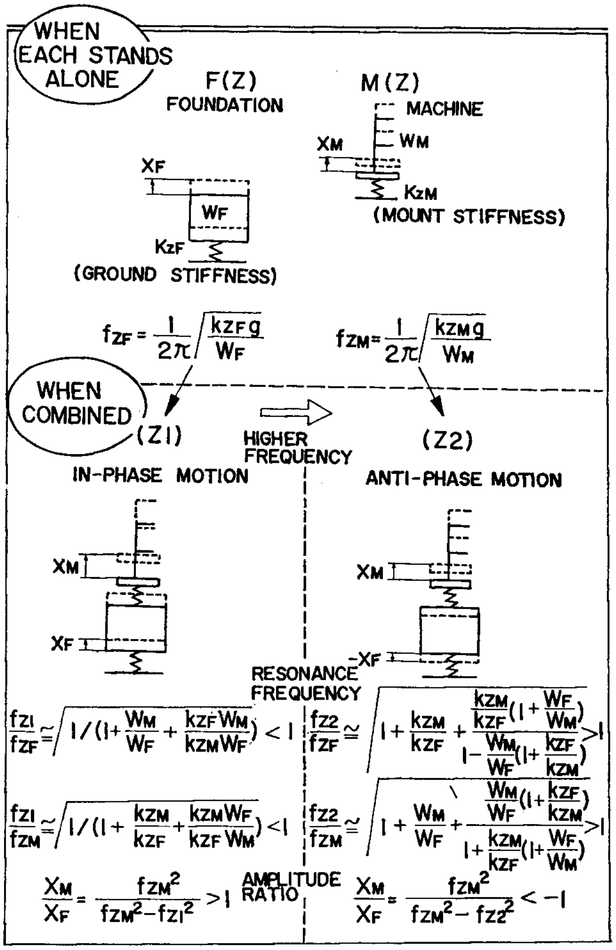


Fig. 4. Resonance modes and frequencies of a combined set of a machine and a structure compared to those when the machine and the foundation are standing alone. (Vertical translation mode).

the machine before coupling. The motion of the machine is greater than that of the foundation and they occur in-phase to each other in mode Z1. Mode Z2 occurs at a resonance frequency f_{Z_2} which is higher than either of f_{ZF} and f_{ZM}, the motion of the machine being greater than that of the foundation, but in this case they occur in opposite directions. Theoretically, similar couplings of the resonance modes will occur in the rotational motion around the vertical axis, and the horizontal motions respectively.

3.2 Analysis by scale models

Since the ground under the foundation has the inherent damping of a substantial amount, the oscillation of the real foundation is not measurable at some certain modes. Therefore, it is difficult to experimentally identify all of the combined resonance modes using an actual machine tool-foundation system. For this reason, experiments have been made on scale models which represent a rigid machine and a rigid foundation, in combination with two springs which represent the resilient ground and the mount mechanism. Two wood blocks are prepared as the rigid bodies. The brass plates are glued on all surfaces of the wood blocks in order to facilitate measurement of the oscillation by use of a proximity pickup. The springs of the mount mechanism are represented by four small pieces of foam rubber glued under the bottom of the machine model. The resilience of the ground is also represented by a mattress of foam rubber. A small piece of magnet is glued on the surface of the model, which is excited without contact by an alternate magnetic field produced by supplying alternate electric current of the excitation frequency to a conductive coil located in the vicinity. The experiments proceed first by exciting several points of the model in *x*-, *y*-, or *z*-directions for the measurement of the response curves, then measuring the mode shapes at the resonance frequencies.

In the upper row of Fig. 5, six resonance modes are illustrated as they are observed on the machine model, standing alone and supported at the four corners

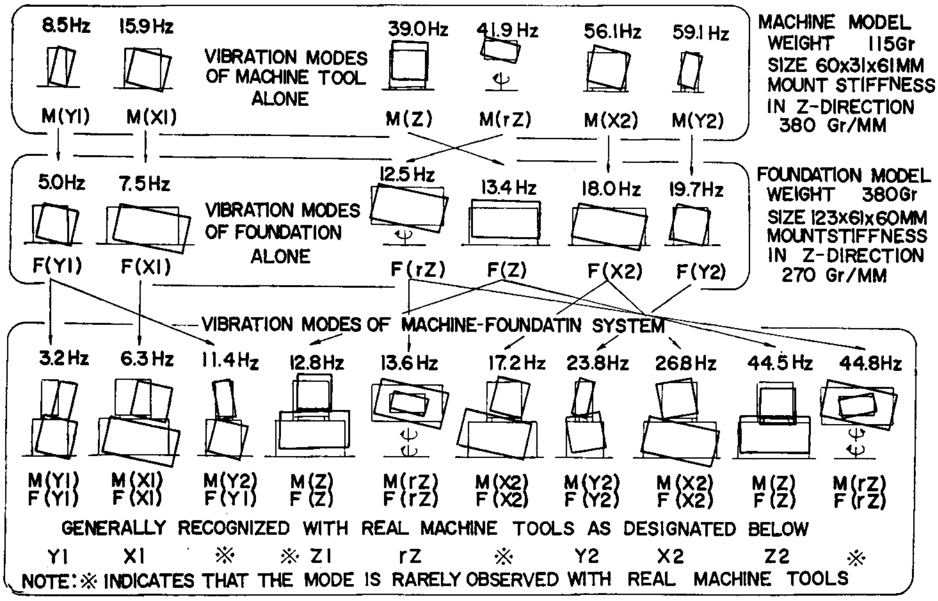


Fig. 5. Mode shapes and resonance frequencies obtained by excitation tests of scale models of the machine tool alone standing on mounting mechanisms (top row), the foundation alone supported by flexible ground (middle row), and their combination (bottom row).

by small pieces of foam rubber. The middle row of the figure illustrate the six resonance modes measured on the foundation model standing alone on a foam rubber mattress. The six mode shapes are all displayed by the models in both cases as expected from the theory. In the lower row of Fig. 5, ten resonance modes are illustrated which are observed when the two bodies are combined. Looking at the oscillations in z-direction, it is seen that the modes $M(Z)$ of the machine alone and $F(Z)$ of the foundation alone are coupled to produce two different resonance modes as expected from the theory. A similar coupling is seen in the vibration around the vertical axis. In the horizontal y-direction, a total of four modes before coupling [$M(Y1)$, $M(Y2)$ of the machine alone, and $F(Y1)$, $F(Y2)$ of the foundation alone] are found to produce a total of three resonance modes after the coupling. Similarly, three modes appear in the horizontal x-direction. From the scale model experiment, it is known as a consequence that the combined rigid body motion of the machine and the foundation occurs at ten possible resonance modes, which comprise two in z-, two in rz-, three in y- and three in x-directions.

4. Low order resonance of real machines

Several machine tools have been tested for general dynamic performance on the foundation. Since some of those tests were performed without paying particular attention to the resonance modes of lower orders, the data does not necessarily cover all possible resonance modes at low orders, but describes those modes which are readily measurable.

All of the low order resonance modes observed on real machine-foundation system have been found to correspond to some of the modes depicted in the lower row of Fig. 5; and they are listed in Table 1. In this table, the observed modes are arranged under the headings $Z1$, $Y1$, $X1$, $Y2$, rZ, $Z2$, and $X2$, which correspond to the mode shapes illustrated in the lower row of Fig. 5, as measured in the scale model experiments. Among the ten modes confirmed by the scale model experiments, three are not found on any of the seven machines tested. Mode $Z1$ (vertical translation of the machine in-phase to the foundation) is observed only once, indicating that this mode rarely occurs in the actual system. Six other modes can be often observed. Among those six, mode $Y1$ generally appears at the lowest frequency. Mode $Z2$ (vertical translation of the machine in anti-phase with the foundation) is most often found at the second from the top, the top being mode $X2$, whose frequency is above 100 Hz in all of the cases illustrated.

The above observations justify the primary assumption that the outline of resonance modes at low orders are understood as the motions of rigid bodies. In a

Table 1. Resonance frequencies of several machine tools mounted on the concrete foundation as taken from dynamic test records.

Recorded cases	Weight Length × Width (mm × mm)	Number of mounts	Resonance frequency (Hz) of the modes as designated in Fig. 5						
			[Z1]	[Y1]	[X1]	[Y2]	[rZ]	[Z2]	[X2]
A Internal grinder	2.5t 1410×1100	6	—	17.0 (.16)	—	31.7* (.31)	56.6* (.55)	103* (1)	151* (1.5)
B Centerless grinder	5.67t 1790×1100	6	—	—	29.8 (.31)	—	—	97.2* (1)	—
C Internal grinder	1.6t 1450×565	4	12 (.14)	26* (.30)	34* (.39)	—	73* (.84)	87* (1)	—
D Universal grinder	3.0t 2700×930	10	—	25.3 (.29)	29.5 (.34)	95.7 (1.1)	59.1 (.69)	85.8* (1)	107* (1.2)
E Diamond lathe	1.4t 1100×690	6	—	22.7 (.25)	30.2 (.33)	28.0 (.31)	41.0 (.45)	89.6 (1)	109 (1.2)
F Small milling machine	0.6t 700×500	4	—	—	25.4 (.24)	—	58.5 (.55)	107* (1)	—
G Hobbing machine	6.9t 2000×1000	8	—	18.2	22.7	—	53.3*	—	—

Number in ( ) designates ratio of the resonance frequency to that of the Z2 mode.
* designate that local deformations are observed within the machine.

strict view, however, the inertial force should be considered which occurs throughout the whole machine due to the oscillation, which necessarily causes elastic deformation in the machine. This influences the mode shape and the resonance frequencies to a little extent. However, a more significant effect is that the elastic deformation causes vibratory relative displacement between the tool and the work, even though it is a small amount. Since the inertial force is proportionate to the square of the resonance frequency, the effect is more significant at the higher frequencies. This is seen in Table 1 where * is marked to show the resonance modes which are confirmed by the mode shape measurment to be accompanied by local elastic deformations. Even at lower resonance frequencies, elastic deformation of a slight amount occurs which is insensible to vibration meters, but is harmful to the finish quality. When any of those low order modes are resonated by either of the vibration sources within the machine, intermittent cutting, or vibration transmitted from the outer floor, the geometrical texture of the finished surface is deteriorated by the vibration.

5. Excitation test of the concrete foundation

5.1 Measurement of the resonance modes

As devised by Bandyopadhyay and Taylor[2), an electro-dynamic exciter suspended by air springs on the foundation itself provides means to excite the foundation surface through a force pickup, as illustrated by the upper left picture of Fig. 6. The same principle is applicable to excite the top of the foundation in a

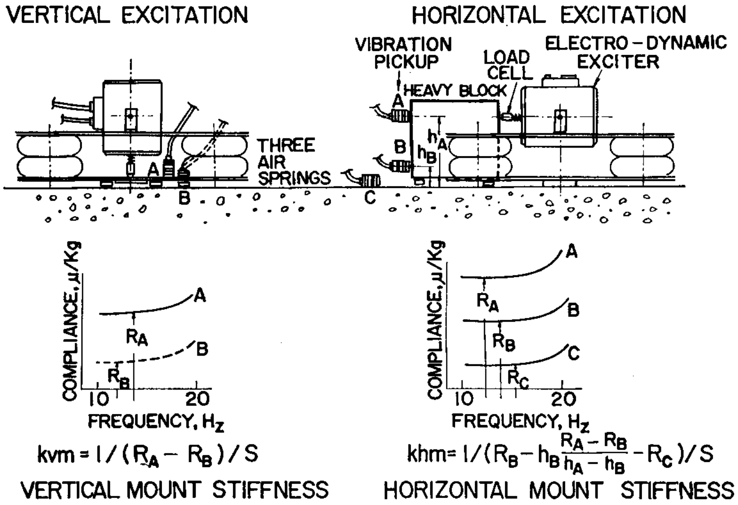


Fig. 6. Procedures to excite foundation and measure the contact stiffnesses due to local deformation of the concrete surface. S designates total area of contact.

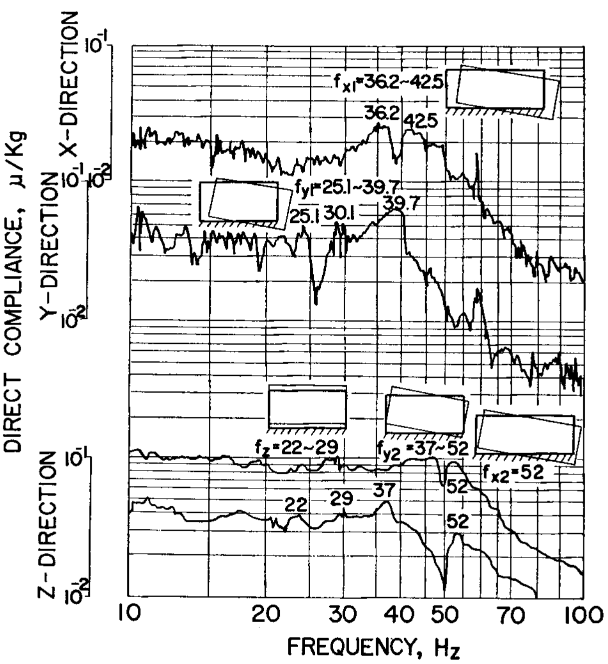


Fig. 7. Harmonic responses and mode shapes of the concrete foundation alone.

horizontal direction as seen in the upper right picture. In this case, a heavy block of steel is tightly fastened to the foundation. Using these methods, dynamics are measured as shown in Fig. 7, and the resonance frequencies of the concrete foundation have been identified as follows.

Modes	Range of resonance frequency	Mean frequency
F(Z)	22∼29Hz	26Hz
F(Y1)	25∼40Hz	32Hz
F(X1)	36∼42Hz	39Hz
F(Y2)	37∼52Hz	45Hz
F(X2)	52Hz	52Hz
F(rZ)	(Not observed in the test)	

From those frequency values, computations have been made, based on the theoretical equations of Appendix 2, and estimates of the stiffness (ground moduli of stiffness K_v and K_h) of the soil supporting the foundation are obtained. The mean frequency of mode $F(Z)$ is put into the equation of f_Z in Appendix 2, and the estimate of the ground modulus in the vertical direction K_v is obtained. Also, that in the horizontal direction K_h is estimated by taking the difference of the squares of the equations for f_{x1} and, f_{x2}, or f_{y1} and f_{y2} respectively, and applying the corresponding frequency values.

The estimated values are:

vertical direction	$K_v=6.4$ Kg/cm^3
horizontal direction	$K_h=7.5$ Kg/cm^3

Compared to the values listed in a reference[3)] (see table below), the estimated value of K_v seems reasonable.

Ground modulus K_v listed in reference (3)		
soft clay	2.0	kg/cm^3
top clay	3∼5	"
loam	3∼5	"
fine sand layer	5∼6	"
sand layer	8∼10	"
gravel layer	11∼13	"

5.2 Stiffness of mount mechanism

Considering the vertical translation of the machine-foundation system, experimental values of the resonance frequency f_{Z2} for several machines, and the vertical ground mudulus K_v are already given. By solving a theoretical equation, the vertical stiffness of the mount mechanism k_Z can be estimated as follows. Namely, f_{Z2} is a solution for ω in the following eigenvalue equation.

$$\omega^4-(\omega_{11}^2+\omega_{12}^2+\omega_{21}^2)\omega^2+\omega_{11}^2\omega_{21}^2 = 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(a)}$$

In the present problem, the following two terms contain the unknown k_Z:

$$\omega_{21}^2 = \frac{gk_Z}{W_M}, \quad \omega_{12}^2 = \frac{gk_Z}{W_F} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(b)}$$

and others are already known as:

$$\omega_{11}^2 = \frac{gk_{zF}}{W_F} \quad \omega^2 = (2\pi f_{z2})^2 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(c)}$$

In the above, notations designate the following parameters;

W_M: weight of machine, W_F: weight of foundation (=12.3t)

k_{ZF}: spring constant of the ground supporting the foundation in vertical direction (=bottom area $\times K_v$=200 cm $\times$ 250 cm $\times$ 6.4 kg/cm^3=32 Kg/μ)

Equations (a) (b) and (c) are reduced to the following formula:

$$k_Z = \frac{1}{g}\left\{\frac{1}{(2\pi f_{Z2})^2 W_M}+\frac{1}{(2\pi f_{Z2})^2 W_F - gk_{ZF}}\right\} \cdots\cdots\cdots\cdots\cdots\text{(d)}$$

Dividing k_Z as obtained by equation (d) by the number of supporting points for the machine tested, the vertical stiffness of a set of the mount mechanism is estimated. Plotted by double circles in Fig. 8 are k_{vm} values which are the k_Z values thus obtained, divided by the number of supporting points, and further divided by the apparent contact area of a mounting block on the concrete surface. As seen in Fig. 8, the double circles indicate that the mount stiffness is strongly dependent on the contact pressure. At this stage, it is inferred that this occurs due to the increased real contact area of the concrete surface at higher pressure, and therefore, the mount stiffness is principally connected to the localized deformation of the asperities of the concrete surface which are in contact with the mounting block.

In order to evaluate this inference, an excitation test has been planned for measurement of the vertical flexibility only of the surface of the concrete foundation under various contact pressures. Flexibility is measured in this case, not including that of the ground soil but including only that due to the localized deformation of the concrete surface asperities which are in real contact with the loading plate. The test procedures are shown in Fig. 6. As seen in the left picture, small steel pieces are located between the excited plate and the concrete surface. By exciting those pieces at a 10 to 20 Hz frequency range where the foundation does not resonate, the

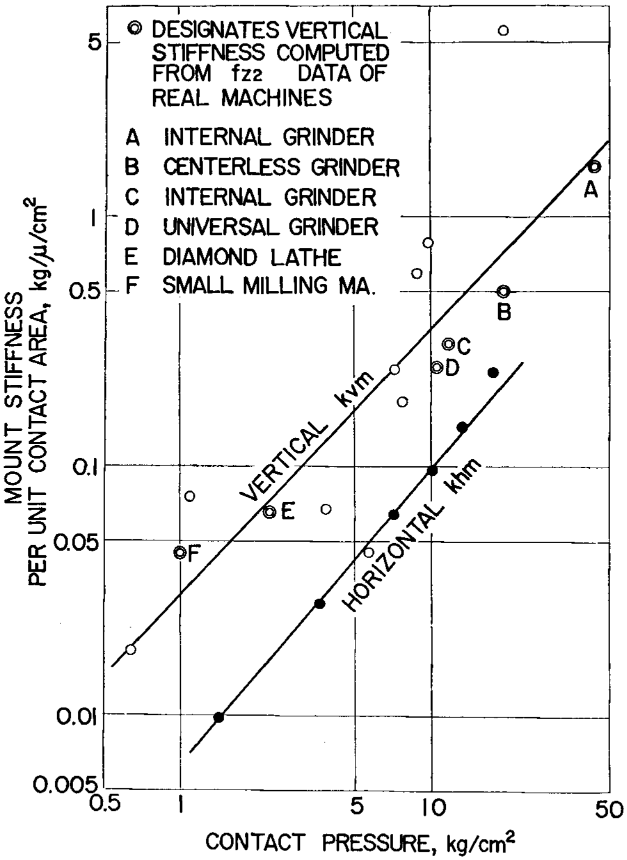


Fig. 8. Experimental values of the mounting stiffnesses k_{vm} (vertical direction) and k_{hm} (horizontal direction) of the concrete foundation surface per unit contact area, versus various contact pressures.

compliances R_A and R_B are measured above a small steel piece, and above the concrete surface remote from the piece. The difference between R_A and R_B represents the flexibility of the concrete surface, whose reciprocal divided by the contact area gives the vertical contact stiffness per unit contact area k_{vm}. Tests are performed at various contact pressures by varying the number and size of the small steel pieces and also by putting weights on the exciter. The same principle can be applied when measuring the horizontal stiffness associated with the deformation of the concrete surface, as seen in the right picture of Fig. 6. In this case, however, the tilting oscillation of the heavy block should be considered. Therefore, the oscillation is measured at two different heights h_A and h_B; and the horizontal oscillation of the block at its bottom is calculated.

As shown by the blank circles in Fig. 8, the measured stiffness is found to depend on the contact pressure. The data is plotted around the equivalent mounting stiffness computed from the experimental resonance frequencies of mode $Z2$. Therefore, it is confirmed that the major part of the flexibility of the mounting mechanisms which are used between the machine and the foundation is attributed to the localized deformation of the asperities of the concrete surface under contact load. The horizontal stiffness of the surface asperities of the concrete under the contact load is found to be smaller than the vertical stiffness, about one forth on the average, as seen in the lower portion of Fig. 8.

6. Example of flexible mounting

When a machine tool is mounted on a concrete foundation or floor in a conventional manner, the resonance frequencies of lower orders are essentially determined by the mounting stiffness of the concrete surface as well as by the weight and configuration of the machine. The resultant frequencies are often likely to be at close proximities to some of the disturbance frequencies caused by rotational elements contained in the machine. Deliberate use of flexible mounting mechanisms, such as rubber pads, are expected to shift the whole set of the rigid motion frequencies far lower than the disturbing frequencies.

A precision lathe for turning by a diamond tool is equipped with eight rubber pads as its standard mounting mechanism. Fig. 9 shows the lower order resonance

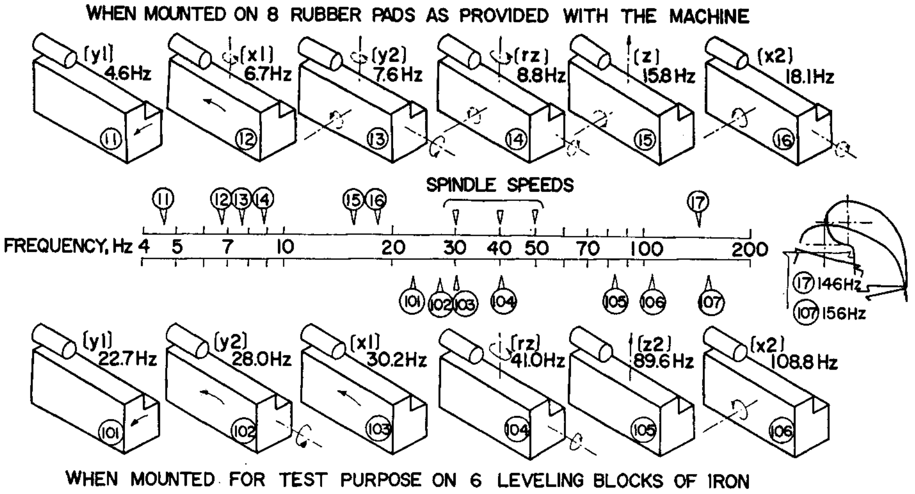


Fig. 9. Effect of flexible mounting mechanism illustrated by exitation tests of a diamond lathe having flexble mountings as standard specification (above), and the same machine mounted conventionally on the concrete foundation (bottom).

modes of this diamond lathe as tested at its standard mounting, and at a conventional-type mounting using six iron blocks. When mounted on the rubber pads, the six low order resonances occur at 4.6 to 18.1 Hz, above which no resonance occurs up to 146 Hz where a vibratory elastic deformation appears in the machine. The fundamental vibration due to the spindle rotation occurs at 30, 40, or 50 Hz which by no chance resonates the machine. When mounted on the iron blocks, the six low order modes occur between 22.7 and 108.8 Hz and some of them contain possibility of resonance with the spindle rotation.

The above example illustrates that the flexible mounting is used effectively in providing a superior performance of finish accuracy for high precision machine tools.

7. Conclusions

Theoretical analysis and experiments have been undertaken toward the study of the mounting and the foundation of the machine tools with respect to their effects on the dynamic performance of the machine. The following conclusions have been drawn from the study.

(1) Foundation and mounting mechanisms affect resonance modes of relatively low orders, for which the outline of the oscillation is readily understood by considering the rigid body motions of the machine and the foundation. In this simplified model, the resilient ground is assumed to support the foundation, further on top of which resilient mechanisms are assumed to mount the machine.

(2) When machine tools of small to medium sizes are mounted on the concrete foundation used in this study, ten resonance modes are able to occur theoretically by the combined motion of the machine and the foundation. Among these, only the following six modes are readily observed in actual cases.

mode $X2$	combined rotation and translation of the machine in the longitudinal direction with the instantaneous center of rotation above the bottom of the machine.
mode $Z2$	vertical translation of the machine in anti-phase to the foundation.
mode rZ	rotation of the machine around the vertical axis.
mode $Y2$	combined rotation and translation of the machine in the lateral direction with the instantaneous center of rotation above the bottom of the machine
mode $X1$	combined rotation and translation of the machine in the longitudinal direction with the instantaneous center of rotation below the machine
mode $Y1$	same as above in the lateral direction

Resonance frequencies of those modes depend on the shape, size, and weight of the machine and the foundation, the stiffness of the mount mechanism and the stiffness of the ground. In most cases, mode $X2$ occurs at highest frequency among those modes, followed by mode $Z2$. Resonance of mode $Y1$ occurs at the lowest frequency.

(3) Although those fundamental oscillations at relatively low orders of mode are principally approximated by the rigid body motion of the machine, elastic deformation of the machine structure, thus a relative displacement between the tool and work points, occurs along with the oscillation due to intertial forces distributed throughout the structure. This is particularly true when the resonance occurs at a high frequency. Therefore, when any of those low order resonances is excited by either of the vibration sources within the machine, intermittent cutting, or vibration transmitted from outside, it affects the quality of the surface finished by the machine tool.

(4) In the conventional method of mounting, the localized deformations which occur at the asperities of the concrete surface in contact with the iron mounting blocks determine the stifiness of the mount mechanism. The resultant stiffness depends on the contact pressure between the iron block and the concrete surface, and the stiffness per unit contact area increases with the contact pressure.

(5) Flexible mount mechanisms such as rubber pads can be used to control those low order resonance modes to occur at extremely low frequencies. This principle is also applicable for improving the machining performance of the machine tools used for intermittent cutting or a high quality surface finish.

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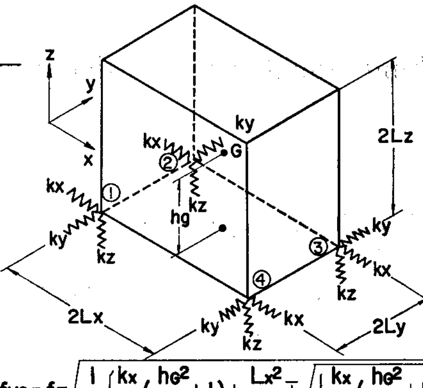
MODE RESONANCE FREQUENCY

(Z) VERTICAL TRANSLATION

$$f_z = \frac{1}{2\pi}\sqrt{\frac{4k_z}{M}}$$

(rZ) ROTATION AROUND VERTICAL AXIS

$$f_{rz} = f_z\sqrt{\frac{L_x^2 k_y - L_y^2 k_x}{r_z^2 k_z}}$$



$k_{rx} = 4L_y^2 k_z$

$k_{ry} = 4L_x^2 k_z$

$k_{rz} = 4(L_x^2 k_y + L_y^2 k_x)$

$k_{tx} = 4k_x$

$k_{ty} = 4k_y$

$k_{tz} = 4k_z$

$r_x^2 = J_{Gx}/M$

$r_y^2 = J_{Gy}/M$

$r_z^2 = J_{Gz}/M$

(X1) COMBINED ROTATION
(X2) AND TRANSLAION IN x-DIRECTION

$$f_{X1},\ f_{X2} = f_z\sqrt{\frac{1}{2}\left\{\frac{k_x}{k_z}\left(\frac{h_G^2}{r_y^2}+1\right)+\frac{L_x^2}{r_y^2}\mp\sqrt{\left\{\frac{k_x}{k_z}\left(\frac{h_G^2}{r_y^2}+1\right)+\frac{L_x^2}{r_y^2}\right\}^2-\frac{4L_x^2}{r_y^2}\frac{k_x}{k_z}}\right\}}$$

(Y1) COMBNED ROTAION AND
(Y2) TRANSLATION IN y-DIRECTION

$$f_{Y1},\ f_{Y2} = f_z\sqrt{\frac{1}{2}\left\{\frac{k_y}{k_z}\left(\frac{h_G^2}{r_x^2}-1\right)+\frac{L_y^2}{r_x^2}\mp\sqrt{\left\{\frac{k_y}{k_z}\left(\frac{h_G^2}{r_x^2}+1\right)+\frac{L_y^2}{r_x^2}\right\}^2-\frac{4L_y^2}{r_x^2}\frac{k_y}{k_z}}\right\}}$$

Appendix 1. Theoretical resonance frequencies of a symmetrical rigid structure when mounted at four corners.
J_{Gx}, J_{Gy}, J_{Gz}: moments of inertia aound *x*-, *y*-, and *z*-axes.

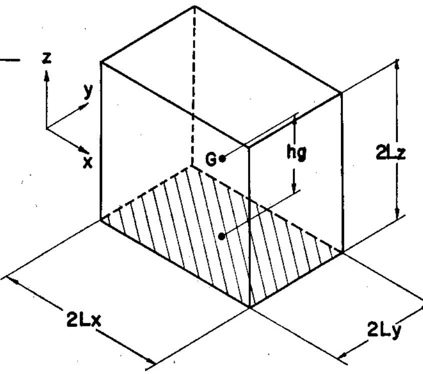
MODE RESONANCE FREQUENCY

(Z) VERTICAL TRANSLATION

$$f_z = \frac{1}{2\pi}\sqrt{\frac{4L_xL_yK_v}{M}}$$

(rZ) ROTATION AROUND VERTICAL AXIS

$$f_{rz} = f_z\sqrt{\frac{(L_x^2+L_y^2)}{3r_z^2}\frac{K_h}{K_v}}$$



$k_{rx} = \frac{4L_xL_y^3}{3}K_v$

$k_{ry} = \frac{4L_x^3L_y}{3}K_v$

$k_{rz} = \frac{4L_xL_y(L_x^2+L_y^2)}{3}K_h$

$k_{tx} = 4L_xL_yK_h$

$k_{ty} = 4L_xL_yK_h$

$k_{tz} = 4L_xL_yK_v$

$r_x^2 = J_{GX}/M$

$r_y^2 = J_{Gy}/M$

$r_z^2 = J_{GZ}/M$

(X1) COMBINED ROTATION
(X2) AND TRANSLATION IN x-DIRECTION

$$f_{x1}, f_{x2} = f_z\sqrt{\frac{1}{2}\left[\frac{K_h}{K_v}\left(\frac{h_G^2}{r_y^2}+1\right)+\frac{L_x^2}{3r_y^2}\mp\sqrt{\left\{\frac{K_h}{K_v}\left(\frac{h_G^2}{r_y^2}+1\right)+\frac{L_x^2}{3r_y^2}\right\}^2-\frac{4L_x^2}{3r_y^2}\frac{K_h}{K_v}}\right]}$$

(Y1) COMBINED ROTAION
(Y2) AND TRANSLATION IN y-DIRECTION

$$f_{y1}, f_{y2} = f_z\sqrt{\frac{1}{2}\left[\frac{K_h}{K_v}\left(\frac{h_G^2}{r_x^2}+1\right)+\frac{L_y^2}{3r_x^2}\mp\sqrt{\left\{\frac{K_h}{K_v}\left(\frac{h_G^2}{r_x^2}+1\right)+\frac{L_y^2}{3r_x^2}\right\}^2-\frac{4L_y^2}{3r_x^2}\frac{K_h}{K_v}}\right]}$$

Appendix 2. Theoretical resonance frequencies of a symmetrical rigid stucture when supported at the bottom.
J_{Gx}, J_{Gy}, J_{Gz}: moments of inertia around *x*-, *y*-, and *z*-axes.
K_h and K_v: vertical and horizontal ground moduli.