A System Analysis for the Planning of Facilities in Inter-basin Water Distribution Problems

By

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Synopsis

With a high increase in water demands in urbanized areas, there has been a growing concern about inter-basin water distribution problems. The primary objective of this study is to present a mathematical model which may serve as an available tool of system analysis for this kind of problem. In the proposed model are incorporated three functions, namely a choice of alternatives in dam planning, a choice of alternatives in channel planning and the integrating process of the above-mentioned two functions. To dissect the incorporated functions in terms of mathematics, the decomposition principle is employed in solving the model. The applicability and validity of the model are evidenced by a case study conducted for the water distribution problem in Hyogo Prefecture.

1. Introduction

The rapid increase in water demand has caused a serious scarcity of water in many parts of the country and the pressure on the supply of usable fresh water is becoming more difficult. Moreover, the scarcity of water might impede further growth of the region. In order to find a solution for this serious problem, various methods of water resource development planning including a large-scale water distribution, have become a major concern of the responsible administrations.

Although water is scarce in those parts of the country which are located in the watersheds of main rivers, it is also true that water would be available if the development could be conducted in such a large-scale that would permit water to be transported from one watershed to others.

Here, it should also be noted that if the development is conducted in a wide area, there will be some regions where the water supply will not yet be serious because of a low demand for water. This fact spurs the possibility of conveying water from low

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demand regions to high demand regions.

In the conventional way of water resource planning, an efficient combination of physical facilities has not yet been carefully examined. Necessary facilities to meet a demand have been constructed separately in each limited area.

When water resource development will be made in a large-scale, efficient water distribution would not be expected from the present conventional way of planning. This means that the planning agencies concerned are required to change their traditional policies. They must pursue a more efficient planning process from the standpoint of the entire broad region in order to reach an acceptable adjusted planning.

2. Inter-basin Water Distribution Planning

Inter-basin water distribution planning involves various problems. It requires a series of interdisciplinary researches from various angles. It should be examined to make a planning of related facilities such as reservoirs and water channels and studies from the standpoint of flood controls, hydrology and hydraulics are also needed. It also needs ecological, geological and geographycal examinations. It needs economists to check the problem from an economical point of view, and also executive officials to set up a framework of the planning and administratively reexamine the feasibility of the planning.

In spite of the various aspects of the problems to be checked, this study is mainly concerned with a problem of water distribution planning, and presents a model for the planning of related facilities in inter-basin water distribution planning.

We first observe the existence of system levels in water distribution systems. Water is carried from upstream reservoirs to downstream reservoirs through rivers and channels. Let us call this "the system of first level." In the system that follows, water is transported from the downstream reservoirs to further downstream reservoirs or to municipalities. In the system concerned with terminal distribution, water is conveyed to the point of use from a service reservoir in the municipality. It must be noticed here that the system of each level has two main components. One is the facilities that impound water which can be thought of as reservoirs, the other is the facilities that convey the stored water to the reservoirs of a lower level. In a broader sense, these can be regarded as channels. The way of constructing channels, as widely defined above, is directly concerned with the interests of the region located along the routes of the channels. This means that local agencies and community associations concerned with agricultural water are very interested in constructing the channels. On the other hand, dams are planned, constructed and managed by the Ministry of Construction, the Ministry of Agriculture and those related authorities. This is partly because the construction of dams is so expensive that local agencies and small associations cannot afford the construction of dams. Another reason is that the planning of dams must be made in the frame of national regional development plannings.

In this respect there are two different competitive interests in the planning of dams and channels. Therefore, the allocation of needed funds among those two groups of different interests becomes a very important problem.

For this reason, an efficient way of adjusting this kind of conflict becomes an important problem in making water distribution planning.

In this context, this paper is concerned with the model analysis of an integrating process in the planning of inter-basin water distribution. The model assumes that facilities are limited to dams and channels.

3. Model Presentation for Inter-basin Water Distribution Planning

(1) The Structure of The Model

The model to be presented here has the following functions.

- 1. Function to select the alternatives of sites and sizes of dams to be constructed.
- 2. Function to select the alternatives of routes and sizes of channels connecting dams and demand regions to be constructed.
- 3. Function to find the comprehensively adjusted alternative from the modified alternatives through 1. and 2.

Here, the comprehensively adjusted alternative is checked with the following criterion. The criterion to be set in this model is the total construction cost of dams and channels, and the alternative is regarded as comprehensively adjusted if it takes the minimum total cost and satisfies the constraint that the total capacity of dams to be constructed exceed the total demand.

(2) Model Formulation

In the formulation of the model, the following notations are used.

- D_j : Quantity of water demand in the demand region $j(j=1, 2, \dots, n)$
- D: Total quantity of demand in the entire region $(=\sum_{j=1}^{n} D_j)$
- A_i : Maximal capacity of dam $i(i=1, 2, \dots, m)$
- y_i : Index representing the construction size of dam $i(i=1, 2, \dots, m)$, meaning the size of dam i
- x_{ij} : Quantity of water conveyed from dam *i* to the demand place *j*

In the model two kinds of variables are introduced. One kind represents the quantity of water conveyed from the dam i to the demand region j. The other is y_i which represents the construction size of dam i.

These two kinds of variables must satisfy the constraint that the total capacity of constructed dams exceeds the total quantity of demands. These variables are also required to satisfy the constraint for each demand region that the total quantity of water conveyed to the demand region from all the dams to be constructed is more than required in the demand region. The other constraint is that the maximal capacity of each dam exceeds the quantity transported from it to all the demand regions.

These constraints are formulated by inequalities as follows,

$$\begin{bmatrix}
\mathbf{m} \\
\sum_{i=1}^{m} x_{ij} \ge D_j, \\
x_{ij} \ge 0 \quad (i=1, 2, \cdots, m; j=1, 2, \cdots, n).
\end{bmatrix}$$
(1)
$$\begin{bmatrix}
\mathbf{m} \\
\sum_{i=1}^{m} A_i y_i \ge D, \\
0 \le y_i \le 1 \quad (i=1, 2, \cdots, m).
\end{bmatrix}$$
(2)

Equations (1) and (2) are considered to represent feasibility constraints for the alternatives of channel planning and dam planning respectively. However, those alternatives satisfying equation (1) will not necessarily satisfy equation (2) and vice versa. Moreover, even if such an alternative should be obtained that satisfies both equation (1) and (2), it may not nescessarily satisfy the total feasibility constraint that the total quantity of water transported from each dam to all the demand regions cannot exceed the maximal capacity of each dam. This is written as

$$-\sum_{j=1}^{n} x_{ij} + A_i y_i \ge 0. \tag{3}$$

If such an alternative that satisfies the above three constraints should be obtained, it must be checked with the criterion that is formulated in the form of the objective function of variables x_{ij} and y_i . This is expressed as:

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij} + \sum_{i=1}^{m} R_i y_i \longrightarrow \min.$$
(4)

where

 C_{ij} : the unit cost for the construction of the channel linking the dam *i* with the demand region *j*,

 R_i : the cost for the construction of the dam *i* to the maximal capacity.

The mathematical model as formulated by Equations (1) to (4) is a large-scale linear programming, which can be solved by the use of simplex algorithm or other techniques developed for specified linear programmings.

However, when the number of constraints is extremely large, the solution cannot be efficiently obtained by such techniques. Moreover, it must be noted here that the study is not merely concerned with the solution of the above-formulated model, but also intends to obtain a mathematical interpretation of the integration mechanism existing

in the process of inter-basin water distribution planning.

In the discussions that follow, the decomposition principle initially developed by G. B. Dantzing will be applied to the model. It will be explained how the mathematical algorithm based on this principle can explain the integration process in the inter-basin water distribution planning.

(3) Integrating Process of Water Distribution Planning

First, we define any set of solutions that satisfy equation (1) as "an alternative of channel planning." Similarly, any set of solutions that satisfy equation (2) will be referred to as "an alternative of dam planning." In the following discussions where there is no danger of ambiguity, the terms "an alternative of channel planning" and "an alternative of dam planning" will be simply referred to as "channel-alternative" and "dam-alternative" respectively.

Here, let us assume that channel alternatives are selected by a certain agency concerned with channel planning (which will be simply referred to as a channel agency.) Similarly, we assume that dam alternatives are selected by a certain agency concerned with dam planning (which will be simply referred to as a dam agency.) We also assume that there is no direct exchange of information between the two agencies to find any comprehensively adjusted alternative of inter-basin water distribution planning.

The process to reach a comprehensively adjusted alternative, which will be called "the integrating process" is assumed to be menaged by a third agency (a comprehensive agency.)

Under these assumptions the integrating process of water distribution planning will be explained as follows.

(a) Channel-alternatives and dam-alternatives are independently selected by a channelagency and a dam-agency, respectively. They are committed to a third agency.

(b) Then, it examines the new alternatives; and tries to adjust the new one to those which have already been committed.

(c) If it proves to be unsatisfactory from the viewpoint of the given criterion, a third agency requests the other two agencies to propose another new alternative by using the information provided by the agency reporting how the preceding alternatives are unsatisfactory from the viewpoint of the given criterion.

(d) Then, the process returns to process (a) and a series of processes from (a) to (c) will be repeated until the adjusted alternative satisfies the given criterion of the third agency.

If it proves to be reasonable to adjust the alternatives by assigning a proper weight to each alternative, namely the adjusted alternative of the third agency, it is expressed as Mamoru HARUNA and Norio OKADA

$$\sum_{k} \lambda^{(k)} \xi_i^{(k)} + \sum_{i} \mu^{(l)} \eta_i^{(l)} = s_i \tag{5}$$

where

$$\xi_i = \sum_{j=1}^n x_{ij}, \quad \eta_i = -A_i y_i. \tag{6}$$

The right-hand supercripts (k, l), (k), (l), represent the kinds of alternatives. $\lambda^{(k)}$, $\mu^{(l)}$ stand for the assigned weights to the alternatives k and l. These weights must satisfy the following equations.

$$\sum_{k} \lambda^{(k)} = 1$$

$$\sum_{l} \mu^{(l)} = 1.$$
(7)

The criterion of the third agency is also formulated as:

$$Z^{(k,l)} = \sum_{k} \lambda^{(k)} Z_1^{(k)} + \sum_{l} \mu^{(l)} Z_2^{(l)} \longrightarrow \text{min.}$$
(8)

where

$$Z_{1}^{(k)} = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}, \qquad Z_{2}^{(l)} = \sum_{i=1}^{m} R_{i} y_{i}.$$
(9)

 $Z_1^{(k)}$ represents the total cost for the construction of the channels. Similarly, $Z_2^{(l)}$ represents the total cost for the construction of the dams. As a matter of fact, if the constraints are convex set, theorems of linear algebra assure that any feasible solution for the constraints may be equal to a certain combination of basic feasible solutions. This means the validity of the above assumption, that those adjusted alternatives of the third agency which are feasible solutions of Equations (1) to (3), can be expressed by a linear combination of certain alternatives.

Then, our interest is to find the assigned weights in process (c), and how to proceed to process (a) or how to terminate all the processes.

In the following discussions, we will explain these processes by the application of the decomposion algorithms.

(4) Decomposition Algorithm

Under the assumption that the set of the feasible solutions for Equation (1) is bounded convexly, any feasible solution consisting of the set for Equation (1) can be represented by a convex combination of an appropriate number of feasible solutions in this set. This is also true with feasible solutions in the set for Equation (2). Let $\{x_{ij}^{(k)}\}$ and $\{y_i^{(l)}\}$ represent any appropriate number of feasible solutions in the set for Equation (1) and that for Equation (2) respectively. Then, any feasible solutions $\{x_{ij}\}$ and $\{y_i\}$, are replaced by k kinds of $\{x_{ij}^{(k)}\}$ and l kinds of $\{y_i^{(l)}\}$ in the following forms:

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$$x_{ij} = \sum_{k=1}^{K} \lambda^{(k)} x_{ij}^{(k)} \qquad (i=1, 2, \cdots, m; j=1, 2, \cdots, n)$$
(10)

$$\sum_{k=1}^{K} \lambda^{(k)} = 1 \tag{11}$$

$$y_i = \sum_{i=1}^{L} \mu^{(l)} y_i^{(l)} \qquad (i = 1, 2, ..., m)$$
(12)

$$\sum_{i=1}^{L} \mu^{(l)} = 1.$$
 (13)

Equations (3), (4) are expressed by the linear combination of k kinds of $\{x_{ij}^{(l)}\}$ and l kinds of $\{y_i^{(l)}\}$ in the form:

$$Z = \sum_{k=1}^{K} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}^{(k)} \right) \lambda^{(k)} + \sum_{i=1}^{L} \left(\sum_{l=1}^{m} R_{i} y_{i}^{(l)} \right) \mu^{(l)} \longrightarrow \text{min.}$$
(14)

$$\sum_{k=1}^{K} (\sum_{j=1}^{n} x_{ij}^{(k)}) \lambda^{(k)} + \sum_{l=1}^{L} (-A_{ij} y_{l}) \mu^{(l)} = 0, \qquad (15)$$

when we set

$$Z_{1}^{(k)} = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}^{(k)}$$
(16)

$$Z_2^{(l)} = \sum_{i=1}^{m} R_{i} y_i^{(l)}$$
(17)

$$\xi_i^{(k)} = \sum_{i=1}^n x_{ij}^{(k)} \qquad (i=1, 2, \cdots, m)$$
(18)

$$\eta_i^{(l)} = -A_i y_i \qquad (i=1, 2, \cdots, m).$$
 (19)

Equations (3) and (4) are rewritten as follows:

$$\left(Z = \sum_{k=1}^{K} Z_1^{(k)} \lambda^{(k)} + \sum_{l=1}^{L} Z_2^{(l)} \mu^{(l)} \longrightarrow \min. \right)$$
(20)

$$[\mathbf{I}] \begin{cases} \sum_{k=1}^{K} \xi_{i}^{(k)} \lambda^{(k)} + \sum_{l=1}^{L} \eta_{i}^{(l)} \mu^{(l)} = 0 \quad (i=1, 2, \cdots, m) \end{cases}$$
(21)

$$\sum_{k=1}^{K} \lambda^{(k)} = 1$$
(11)

$$\left(\sum_{l=1}^{L} \mu^{(l)} = 1.$$
(13)

Problem [1] which is called the "Master Problem" represents the condition that a combination of those alternatives, which satisfies Equation (1) or (2), constitutes a globally optimal feasible solution. Since Problem [1] is expressed in the form of linear programming, the optimal condition that the basic feasible solution should satisfy

is written as:

$$\overline{Z}_1^{(k)} = Z_1^{(k)} - \Pi \nu_1^{(k)} \ge 0 \text{ for all } k$$
 (22)

$$\bar{Z}_{2}^{(l)} = Z_{2}^{(l)} - \boldsymbol{\Pi} \, \boldsymbol{\nu}_{2}^{(l)} \ge 0 \text{ for all } l, \tag{23}$$

where

$$v_1^{(k)} = {}^t [\xi_1^{(k)} \xi_2^{(k)} \dots \xi_m^{(k)} \ 1 \ 0]$$
$$v_2^{(l)} = {}^t [\eta_1^{(l)} \eta_2^{(l)} \dots \eta_m^{(l)} \ 0 \ 1]$$

and

$$\boldsymbol{\Pi} = [\Pi_1, \Pi_2, \cdots, \Pi_i, \cdots, \Pi_{m+2}]$$

$$\Pi_i \qquad (i=1, 2, \cdots, i_{0-1}, i_{0+1}, \cdots, m+2)$$
(24)

are simplex multipliers calculated for the basic matrix. Here, i_0 is a certain number representing the number of that simplex miltiplier which should be excluded from the set of Π_i for the following reason. Since (m+2) number of Equations (11), (13) and (21) in Problem [1] cannot constitute any linearly independent combination, the rank of a basic matrix for Problem [1] is (m+1); and the number of the simplex multipliers is therefore (m+1) on the assumption that from Equation (21) an arbitrary number of constraints is excluded. After a certain series of exchanges of indices, the basic matrix for Problem [1] is written as:

$$\begin{array}{c} \widehat{\mathbf{x}} \\ & & \\$$

To check the optimal condition represented by Equations (22) and (23), the minimum values of \overline{Z}_1 and \overline{Z}_2 are calculated and the non-negativity of the lower one of each minimum is examined.

First we rewrite $\overline{Z}_1^{(k)}$ and $\overline{Z}_2^{(l)}$ as follows.

$$\bar{Z}_{1}^{(k)} = \sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij} x_{ij}^{(k)} - \Pi_{m+1}$$
(26)

$$\bar{Z}_{2}^{(l)} = \sum_{i=1}^{m} g_{i} y_{i}^{(l)} - \Pi_{m+2}$$
(27)

where

$$f_{ij} = C_{ij} - \Pi_i \tag{28}$$

$$g_i = R_i + \Pi_i A_i. \tag{29}$$

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To obtain the minimum values of $\overline{Z}_1^{(k)}$ and $\overline{Z}_2^{(l)}$ we may omit thier constant terms for the moment, to modify the conditions, as follows:

$$\begin{cases} F^{(k)} = \sum_{i=1}^{m} f_{ij} x_{ij}^{(k)} \longrightarrow \min. \end{cases}$$
(30)

$$[II] \begin{cases} \sum_{i=1}^{m} x_{ij}^{(k)} = D_j \end{cases}$$
(31)

$$x_{ij}^{(k)} \ge 0$$
(32)

$$\begin{pmatrix}
G^{(l)} = \sum_{i=1}^{m} g_i y_i^{(l)} \longrightarrow \min.
\end{cases}$$
(33)

$$[III] \begin{cases} m \\ \sum_{i=1}^{m} \mathcal{A}_{i} \gamma_{i}^{(l)} = D \end{cases}$$
(34)

$$0 \le y_i^{(l)} \le 1. \tag{35}$$

Then, we obtain the optimum values of $F^{(k)}$ and $G^{(l)}$, denoted by $F^{(k^*)}$ and $G^{(l^*)}$ respectively, and calculate the values of the simplex multipliers for the optimal basis.

After obtaining the minimum values of $F^{(k)}$ and $G^{(l)}$ denoted by $F^{(k^*)}$ and $G^{(l^*)}$ respectively which are the optimal solutions for Problem [II] and Problem [III], and subtracting the values of $F^{(k^*)}$ and $G^{(l^*)}$ by the values of Π_{m+1} , Π_{m+2} respectively to get the values of $\overline{Z}_1^{(k)}$ and $\overline{Z}_2^{(l)}$, we calculate the value of H defined as:

$$H = \min(\bar{Z}_1^{(k)}, \bar{Z}_2^{(l)}). \tag{36}$$

Then, the global optimal condition expressed by inequalities (22) and (23) in the preceding discussions can be rewritten as:

$$H \ge 0. \tag{37}$$

As is clear in the above discussions, since Problem [II] and Problem [III] are used to constitute the new Master Problem [I], they are called 'Sub Programs' with respect to the "Master Program."

The global optimal condition expressed by (37) may not generally be satisfied by the optimal solution obtained by Problem [I], and therefore if

 $H < 0 \tag{38}$

holds for $F^{(k^*)}$ and $G^{(l^*)}$, then we add to the former Master Program a set of optimal feasible solution $\{x_{ij}^{(k)}\}$ or $\{y_i^{(l)}\}$, simply symbolized by k^* or l^* as an added column vector of the matrix constituting the Master Program. Since (38) holds, the added column k^* or l^* (which we rewrite as (k+1) or (l+1) for the convenience of notation) is to be put into the new basis of the improved Master Program. Also, a certain column representing an alternative is chosen through pivotal operations to be excluded from the former basis. For the new Master Program which gives a new optimal solution, we calculate the new values of the simplex multipliers as well as those of $Z_1^{(k)}$, $Z_2^{(l)}$

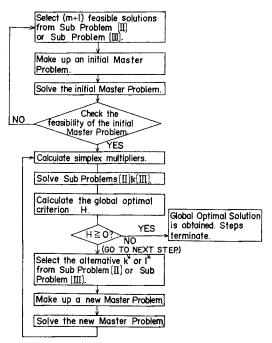


Fig. 1. Solution Algorithm of the Model by use of Decomposition Principle.

and H. Then, we check the sign of H to examine whether the global optimal condition expressed by (37) is attained. If (37) holds, the improved Master Program has given the global optimal solution which is a linear combination of those alternatives obtained from a series of the preceding Sub Programs. Otherwise we again add to the Master Program a column of another set of new feasible solution chosen from the previous Sub Program to constitute a new Master Program. After that, quite similar procedures continue until the global optimal condition of (37) holds.

There may be many ways to construct an initial Master Program. We take such a method that will automatically satisfy the feasibility of the Master Program. First, we select an arbitrary global feasible solution and divide it into the part of Sub Problem [II] and that of Sub Problem [III]. Then, we assign unities to their weights (say $\lambda^{(1)}$, $\mu^{(1)}$) and obtain the other (m-1) alternatives from Sub Problem [II] and [III], with all the assigned weights fixed to zero.

Thus, we obtain an initial feasible solution for the initial Master Program. Fig.-1 shows the processes to attain the global optimal solution applied to by the aid of the Decomposition Principle developed by G. B. Dantzig.

(5) Mathematical Interpretation of the Integrating Process

As discussed before, the model consists of three parts which function as follows:

(a) The part that represents the alternatives of the dam construction planning with the sizes and sites of dams to be constructed.

(b) The part that represents the alternatives of the channel construction planning with the sizes and routes of channels to be constructed.

(c) The part that represents the way to adjust those alternatives obtained from (a) and (b) with the assigned weights of the alternatives.

Then, in virtue of the Decomposition Principle, we can obtain a mathematical interpretation of the process to attain the integrated alternatives.

In other words, Sub Problem [II], defined by Equations (30), (31) and (32), and Sub Problem [II], defined by Equations (33), (34), and (35), may be considered to correspond to the parts of the above-mentioned (b) and (a) respectively. The alternatives thus obtained are reasonably weighted by the Master Program [I], and this process corresponds to the part of (c). Unless the global optimal condition is satisfied, another possible alternative is chosen from Sub Problem [II] or Sub Problem [III] to compose a new global feasible solution.

In this respect, the values of $\lambda^{(k)}$ and $\mu^{(l)}$ may be thought of as indices that quantitatively illustrate the adjustment processes.

Therefore the proposed model, if applied to by the Decomposition Principle, can present some basic information for the analysis of the integrating process of dam and channel plannings in the inter-basin water distribution planning.

4. Case Study

-The Inter-basin Water Resource Development in Hyogo Prefecture-

(1) The Water Use Pattern of Hyogo Prefecture

Hyogo Prefecture is geographycally characterized by two coastal areas, one along the Japan Sea, the other along both the Osaka Bay and the Seto Inland Sea. Furthermore, in the latter area, there are not only several dominant industrial zones such as the Hanshin Industrial District and the Harumi Industrial District playing an important role in Japan's industrial activities, but also Kobe Harbor, one of the largest harbors in Japan.

As a result, throughout these areas including their hinterland regions, industrial activities and economical activities are extremely active. There has been a remarkable number of industries for the processing and distribution of goods located in those coastal districts of Hanshin and Eastern Harima.

Moreover, those areas have recently been developed as a residential district for the families of those people working in Osaka, Kobe and other cities in the Hanshin District. This is because the tremendous increase in the price of land has made it impossible to

have their homes near their offices, and also because the increased convenience of traffic services and transportation facilities have greatly improved access to their working places. In these circumstances, the demand for water is rapidly increasing in these districts; and there is a high possibility of a serious shortage of a future water supply in many parts of the district. Especially, in the Hanshin District consisting of Kobe, Nishinomiya, Amagasaki etc. the available water supply is supposed to become physically impossible, consequently causing crucial water problems at the latest by 1985. To cope with these difficult problems, there is an increasing concern for an inter-basin water resource development which aims at water transfers to those high-demand regions from the regions affluent in suppliable water.

This sort of water transfer, although it has may difficulties to overcome, has much likelihood in the sense that in the northern part of the prefecture located along the Japan Sea, and in the basin of the Maruyama River, there will be far less demand for water in the future. Also, much fresh water can be developed even if a demand increase may result from a development of industries and residential regions.

Furthermore, as regards to the climatic characteristics of the region, in winter it is blessed with much water owing to abundant snowfalls. In the same season, however, the region facing the Seto Inland Sea has very little rain. In order to balance the gaps existing both in the supply and demand of water and in rainfall characteristics, it seems reasonable to consider inter-basin water transfers for the districts of Hanshin and Harima from the Maruyama River in northern Hyogo. On this matter, the Agency of Hyogo Prefecture has presented future development concepts of water resources on the basis of a regional development program for each part of the prefecture.

In the discussions that follow, we first present as a case study,, a basic frame of the water distribution planning of Hyogo Prefecture by specifically limiting the problem to the planning of dam and channel constructions.

For the framework of the planning, we first technologically selected those candidate sites for dam construction and channels. Then, we examined the maximal limits to the construction of each dam, roughly predicted the demands for water by a given period of time, and considered other problems by focussing our attention on the basins of the Maruyama, Kakogawa, Ichikawa, Chigusa, Yumesaki and Ibo Rivers. In this respect, we proposed two possible channels diverted from the Maruyama River down to the Kakogawa and Ichikawa Rivers. Also, we proposed some channels interconnecting these rivers (excluding the Maruyama River) which run southward, parallel with one another.

(2) The Predicted Water Demands in Each Part of the Region

To obtain information necessary for forecasting the water demand, we used the data presented by the Water Resource Section of the Planning Division of Hyogo

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Derestere			1965		1975				1980				1985			
Region		A	В	T ₁	A	в	T ₂	C ₂	A	В	T ₃	С3	Α	B T4 1602 3124 478 816 2080 3940 1717 2091 1196 1555 231 284		C4
	(1)	619	670	1289	1071	1059	2130	841	1302	1383	2685	1396	1522	1602	3124	183
Hanshin	(2)	61	117	178	198	281	474	296	266	392	658	480	338	478	T4 02 3124 78 816 80 3940 17 2091 96 1555 31 284 44 3930 84 1115 28 188	63
	(T ₁)	680	787	1467	1264	1340	2604	1137	1568	1775	3343	1876	1861	2080		247
	(3)	99	338	437	243	1040	1283	846	311	1499	1810	1373	374	1717	2091	165
Harima	(4)	120	591	711	218	899	1117	406	287	1141	1428	717	359	1196	1555	84
riarima	(5)	17	62	79	46	129	175	96	51	189	240	161	53	231	T4 2 3124 2 3124 2 3124 2 3124 3 816 0 3940 7 2091 16 1555 1 284 4 3930 4 115 28 188 9 195	20
	(T ₂)	236	991	1227	507	2068	2575	1348	649	2829	3478	2251	786	3144		270
Tamba	1	8	16	24	23	47	70	46	28	67	95	71	31	84	115	9
River Maruyam	IA.	30	9	39	60	56	116	77	60	82	142	103	60	128	188	14
Awaji		14	19	33	43	46	83	55	358	62	120	87	76	119	195	16
SUM		968	1822	2790	1897	3556	5453	2363	2363	4815	7178	4388	2813	5555	8368	551

Table 1. Predicted Water Demands.

 $(1000 \text{ m}^3/\text{yr.})$

where A : Municipal Water Demand, B: Industrial Water Demand

 $T_i: \text{Total}=A+B, C_i=T_i-T_1$ (i=1,2,3,4) (i=1,2,3,4)

(1): Coastal District, (2): Hinterland District

(3): Toban (Basin of River Kakogawa)

(4): Chuban (Basins of River Ichikawa, Ibo & Yumesaki)

(5): Seiban (Basin of Chigusa)

 (T_1) = Total Demands of (1) and (2)

Total

 (T_2) = Total Demands of 3(3), (4) and (5)

Table 2.	New Demands III V	(m ³ /sec)
Watersheds	New Demands	N.B.
R. Kakogawa	19.2)
R. Ichikawa	6.3	
R. Yumesaki	1.1	$31.4 \text{ (m}^{3}\text{/sec})$
R. Ibo	2.4	
R. Chigusa	2.4	J
R. Maruyama	1.7	

33.1

Table 2. New Demands in Watersheds

Prefectural Office as the predicted water demands for each part of the prefecture as shown in Table 1. Table 2 shows the assigned quantities of the predicted demands to the demand region in each watershed by use of the traditional ratios of water demands which owe their source to the neighboring rivers. (For this, we used the data of the traditional ratios of municipal water supply). In other words, the increased demand for water in Eastern Harima is assigned to the Kakogawa River, the demand in Western Harima is assigned to the Chigusa River; and that in the Tajima District is assigned to the Maruyama River. As for the demand in Middle Harima, some of it is assigned to the

Ibo River, some to the Yumesaki River, and the rest to the Ichikawa River by use of recent practical information of their shares.

(3) Alternatives of Routes and Construction of Channels

Channels can be laid under ground along roads and streets, but we did not choose this system because it would require pipes of more than 2.5 m in diameter, provided that the flow velocity with pipes is 2.0 m^3 /sec and the design flow 10 m^3 /sec. Therefore, the construction of pipes would cause many difficulties in those urban areas of highly integrated activities. Moreover, channels to be constructed are required to convey water from one river to another. For this purpose, channels are needed to have some 'distribution-reservoir-like facilities' which might be, appropriately located in hilly areas. In view of above considerations, we take a system of tunnels in hilly areas, and open channels in other areas. Then, for the assessment of the unit costs for channel construction, we used such basic data as shown in Table-3 and Table-4.

(4) Estimated Construction Costs of Dams

Construction costs of dams are estimated for those watersheds of the Chigusa, Ichikawa, Kakogawa and Maruyama Rivers. As a result, the assumed linearity between the costs for developing fresh water through reservoir retention (we regard these costs as the construction costs of dams) and its scales proved to hold good. Fig. 2

Flows	$Q=10 \mathrm{m^{3/sec}}$	$Q = 20 \mathrm{m^3/sec}$	Q=30 m ³ /sec	$Q = 40 \mathrm{m^3/sec}$
Section Area (m ²)	5.0	10.0	15.0	20.0
Diameter (m)	2.50	3.60	4.40	5.10
Thickness (m)	0.30	0.45	0.55	0.65
Escavation (m ³)	7.83	16.00	23.97	32, 32
Concrete (m ³)	2.83	6.00	8.97	12.32
Main Construction Costs (yen)	67450	140000	209500	284800
Preparatory Costs (yen)	6745	14000	20955	28480
The Others (yen)	33725	70000	104775	142400
Total Costs (yen)	107920	224000	335280	455680

Table 3. Associated Costs for Channel Construction (Tunnels) per m.

Table 4. Associated Costs for Channel Construction (Bridges) per m.

Flows Items	$Q=10\mathrm{m}^3/\mathrm{sec}$	$Q=20\mathrm{m}^3/\mathrm{sec}$	Q=30 m ³ /sec	$Q = 40 \mathrm{m}^3/\mathrm{sec}$
Diameter (mm)	2500	3600	4400	5100
<i>t</i> (mm)	12	17	20	23
W (kg/m)	743.4	1516	2180	2906
Construc7ion Costs (yen)	594400	1212800	1744000	2324800

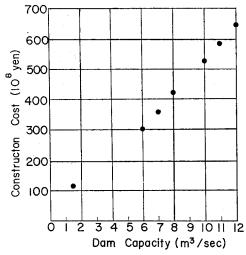


Fig. 2. Relation between Dam Capacity and Construction Cost (Case of River Maruyama).

Dams to be Constructed in the Watersheds of:	Unit Construction Costs (10 ⁸ yen/m ³ ·sec)	Maximal Capacity of Dams (m ³ /sec)	Costs for the Maximal Capacity (10 ⁸ yen)		
River Chigusa	38	6.0	228		
River Ibo	37	7.0	259		
River Yumesaki	31	2.5	77.5		
River Ichikawa	65	6.0	390		
River Kakogawa	65	10.0	650		
River Maruyama	54	10.3	556.2		

Table 5 Associated Costs for Dam Construction.

shows an example of those estimated costs. The unit costs for the construction of dams are listed in Table 5.

(5) Some Modification on the Model

Fig. 3 illustrates a simplified expression of the inter-basin water distribution system, in which frame the model is formulated.

Here we assumed that the construction costs for existing rivers are zero. Therefore, we can exclude those routes corresponding to them. At the same time we set up the following two assumptions. One is that all of the water demands in the downstream areas of the Maruyama River depend on this river. This means that we can exclude the route (5-13) from the system by subtracting that demand from the capacity of dam 5 so as to obtain the same result. The other assumption is that those routes (6-12)and (11-12) require the same cost, which means there is no need to include both of them for optimal calculations. Therefore, the demand in 12 is previously added to that in

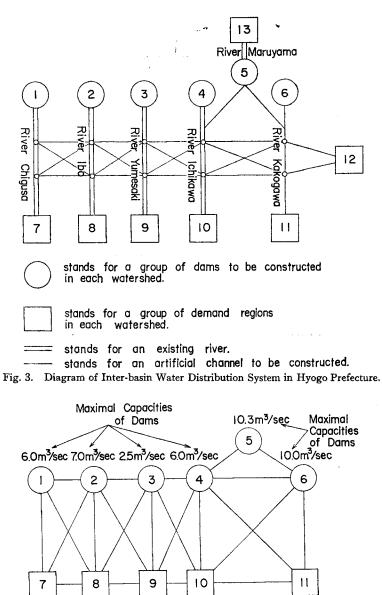


Fig. 4. Modified Diagram of the Inter-basin Water Distribution System in Hyogo Prefecture.

Demands

19.2m³/sec

2.4m3/sec 2.4m3/sec 1.1m3/sec 6.3m3/sec

11 to obtain the modified amount of demand in 11. On these assumptions, we simplify the original system as illustrated in Fig. 4, in which frame calculations by the model are performed. Moreover, as to the formulation of the model, there is need for some

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modification owing to the network structure of the system with which the model deals. Herewith, let m represent the maximal number of dam candidates, p represent the number of relay points, and n represent the number of water demand regions. Then, we assume that each of them is a supply point and a demand regions at the same time, regardless of the fact that each actually belongs to supply points, relay points, or demand regions.

We also assume that the supply amount of dam candidate *i* is equal to $A_i y_i$, those of demand regions and relay points equal to zero, the demand amount of demand regions *j* and equal to D_j those of dam candidates and relay points equal to zero. Here the notation of suffixes, *i*, *j* is used in the form:

$$i, j = \begin{cases} 1, 2, \dots, m: & \text{for dam candidates} \\ m+1, \dots, m+p: & \text{for relay points} \\ m+p+1, \dots, m+p+n: & \text{for demand places.} \end{cases}$$

On these assumptions, the model is formulated in a modified form such that:

Objective function

$$Z = \sum_{i=1}^{m+n+p} \sum_{j=1}^{m+n+p} C_{ij} x_{ij} + \sum_{i=1}^{m} R_i y_i \longrightarrow \min.$$
(39)

subject to

$$\sum_{i=1}^{m+p+n} x_{ij} - A_i y_i = 0 \qquad (i=1, 2, \cdots, m)$$
(40)

$$\sum_{j=1}^{m+p+n} x_{ij} = 0 \qquad (i = m+1, \cdots, m+p+n)$$
(41)

$$\sum_{i=1}^{m+p+n} x_{ij} = 0 \qquad (j=1, 2, \cdots, m, m+1, \cdots, m+p)$$
(42)

$$\sum_{i=1}^{m+p+n} x_{ij} = D_j \qquad (j = m+p+1, \cdots, m+p+n).$$
(43)

As explained before, any feasible solution is replaced by a linear combination of certain feasible solutions if the set of feasible solution is convex,

$$x_{ij} = \sum_{k=1}^{K} \lambda^{(k)} x_{ij}^{(k)} \tag{44}$$

$$\sum_{k=1}^{K} \lambda^{(k)} = 1 \tag{45}$$

$$y_i = \sum_{l=1}^{L} \mu^{(l)} y_i^{(l)}$$
(46)

$$\sum_{l=1}^{L} \mu^{(l)} = 1$$
 (47)

By substituting (44) and (46) into (39), we get

$$Z = \sum_{k=1}^{K} Z_1'^{(k)} \lambda^{(k)} + \sum_{l=1}^{L} Z_2^{(l)} \mu^{(l)} \longrightarrow \text{min.}$$

$$\tag{48}$$

where

$$Z_{1'(k)} = \sum_{i=1}^{m+n+p} \sum_{j=1}^{m+n+p} C_{ij} x_{ij}^{(k)}$$
(49)

$$Z_2^{(l)} = \sum_{i=1}^{m} R_{i\mathcal{Y}i}^{(l)}$$
(17)

By virtue of (44) and (46), Equation (40) is re-expressed in the form:

$$\sum_{k=1}^{K} \xi_{i}^{\prime(k)} \lambda^{(k)} + \sum_{l=1}^{L} \eta_{i}^{(l)} \mu^{(l)} = 0$$

$$(i=1, 2, \cdots, m)$$
(50)

where $\xi_{i}^{\prime(k)}$ represents the total amount of water distributed from dam candidate *i* to its neighboring demand regions and channels leading from it to the neighboring relay points.

5. Discussions on the Results

Analyses were performed for the following two cases: Case 1.

We will not include water distribution to the Kobe Area in the system. This case is characterized by a remarkable surplus amount over that needed. (The maximal amount of water development is estimated as 41.8 m^3 /sec., while the total sum of water demand as 31.4 m³/sec.)

Case 2.

We assume 5 m^3 /sec. of water distribution to the Kobe Area. (This means in calculations to add such an amount to 19.2 m³/sec of water demand in the downstream area of the Kako River. Then its amount is changed to 24.2 m³/sec.) This case is

	Regions (Case 1).			Regions (Case 2).	
	Demand Places (in the Watersheds of:)	Demand Quantity (m³/sec)		Demand Places (in the Watersheds of:)	Demand Quantity (m ³ /sec)
7	River Chigusa	2.4	7	River Chigusa	2.4
8	River Ibo	2.4	8	River Ibo	2.4
9	River Yumesaki	1.1	9	River Yumesaki	1.1
10	River Ichikawa	6.3	10	River Ichikawa	6.3
11	River Kakogawa	19.2	11	River Kakogawa	19.2
12	Kobe Area	0.0	12	Kobe Area	5.0

Table 6. Demand Quantity in Demand

Table 7.	Demand Quantity in Demand
	Regions (Case 2).

characterized by a smaller surplus amount over that demanded in the entire region. (The total sum of water demand is estimated as 36.4 m^3 /sec for the 41.8 m^3 /sec of water development limit.)

The estimated demands in analyzing these two cases are listed in Table 6 and Table 7.

(1) Calculation Results

The initial feasible solutions of the Master Program for Case 1 and Case 2 are illustrated in Fig. 5 and Fig. 6, respectively. Their global optimal solutions are shown in Fig. 7 and Fig. 8. They were obtained by a linear combination of those solutions (alternatives) weighted by the values of $\lambda^{(k)}$ and $\mu^{(l)}$ as shown in Table 8 and Table 9. Fig. 9 shows six alternatives of channel planning among those seven feasible solutions (alternatives) consisting of the global optimal solution of Case II. These alternatives are not necessarily global feasible solutions by themselves; and have been obtained in a series of steps that compose the solution process of the model applied by the decomposition principle. However, if they were linearly combined with those weights obtained by the Master Program, there would be an improved global feasible solution in each step. The criterion to check the global optimality of an improved global feasible solution proved to change in such a pattern as illustrated in Fig. 10 and Fig. 11.

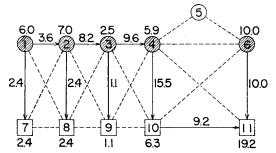


Fig. 5. Feasible Solution for the Initial Master Problem (Case 1).

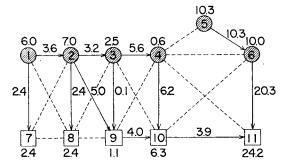


Fig. 6. Feasible Solution for the Initial Master Problem (Case 2).

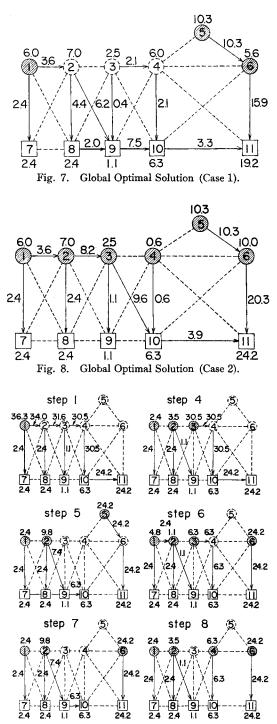


Fig. 9. Six Alternatives of Cannel Project among those composing the Global Optimal Solution (Case 2).

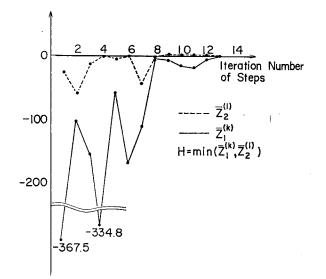


Fig. 10. Changing Pattern of H and $\overline{Z}_1^{(k)}$, $\overline{Z}_2^{(l)}$ (Case 1).

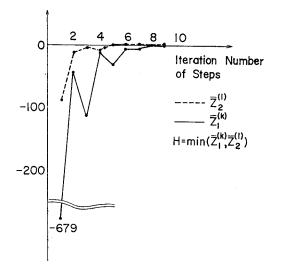


Fig. 11. Changing Pattern of H and $\overline{Z}_1^{(k)}$, $\overline{Z}_2^{(l)}$ (Case 2).

The process of the improvement of each global feasible solution will be explicitly understood by the changing pattern of objective function Z, as illustrated in Fig. 12 and Fig. 13. These figures clearly explain the fact that in earlier stages Z changes drastically, but in the latter stages the change is remarkably small, actually nothing in the final stages. Moreover, according to the results shown in Fig. 12 and Fig. 13, the earlier stages correspond to the process of improving those alternatives of dam construction

	no. of k, l	Weight	which is introduced in step:	step 1	step 2	step 3	step 4	step 5	step 6	step 7	step 8	step 9	step 10	step 11	step 12	step 13
2	1	μ ⁽¹⁾	1	1.0	1.0	1.0	1.0	0.761	0.517	0.141	0.131	*	*	*	*	*
(dam)	2	μ ⁽²⁾	1	0.0	0.0	0.0	0.0	0.239	0.483	0.859	0.869	1.0	1.0	1.0	1.0	1.0
	1	λ(1)	1	1.0	1.0	1.0	1.0	*	*	*	*	*	*	*	*	*
	2	λ(2)	1	0.0	0.0	0.0	*	*	*	*	*	*	*	*	*	*
	3	λ(3)	1	0.0	0.0	0.0	0.0	0.743	0.508	0.012	*	*	*	*	*	*
	4	$\lambda^{(4)}$	1	0.0	*	*	*	*	*	*	*	*	*	*	*	*
	5	λ ⁽⁵⁾	1	0.0	0.0	*	*	*	*	*	*	*	*	*	*	*
	6	λ(6)	2		0.0	0.0	0.0	0.104	0.142	0.212	0.213	0.202	0.172	0.172	0.172	0.172
	7	λ(7)	3			0.0	0.0	0.076	0.145	0.318	0.320	0.199	0.140	0.075	0.075	*
	8	λ(8)	4				0.0	0.042	0.032	*	*	*	*	*	*	*
k (channel)	9	λ(9)	5					0,030	*	*	*	*	*	*	*	*
(10	λ(10)	6						0.171	0.337	0.341	0.367	0.338	0.274	0.274	0.274
	11	λ(11)	7							0.121	0.124	0.0	0.0	0.0	*	*
	12	λ ⁽¹²⁾	8								0.0	0.092	*	*	*	*
	13	λ(13)	9									0.138	0. 193	*	*	*
	14	λ ⁽¹⁴⁾	10										0.152	0.217	0.217	0.217
	15	λ(15)	11									Ì		0.263	0.263	0.263
	16	λ(16)	12												0.0	0.0
	17	λ(17)	13													0.075

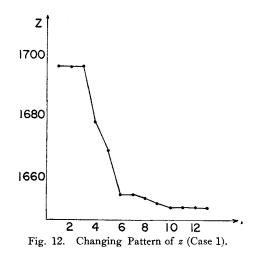
Table 8. Changing Pattern of Weight Variables $\lambda^{(k)}$, $\mu^{(l)}$ with respect to Steps (Case 1).

* indicates that the weight variable is sweeped out of the basis of Master Problem.

Blank indicates that the weight variable is not yet introduced into the basis of Master Problem.

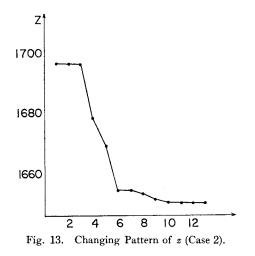
	no. of k, l	Weight	which is introduced in step:	step 1	step 2	step 3	step 4	step 5	step 6	step 7	step 8	step 9
/ (dam)	1	μ ⁽¹⁾	1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	1	λ(1)	1	0.0	*	*	*	*	*	*	*	*
	2	λ(2)	1	1.0	1.0	0.100	0.100	0.100	0.100	0.100	*	*
	3	λ(3)	1	0.0	0.0	0.065	0.071	0.091	*	*	*	*
	4	$\lambda^{(4)}$	1	0.0	0.0	0.0	*	*	*	*	*	*
	5	λ ⁽⁵⁾	1	0.0	0.0	*	*	*	*	*	*	*
	6	λ ⁽⁶⁾	1	0.0	0.0	0.383	0.383	0.359	0.048	*	*	*
k	7	λ ⁽⁷⁾	2		0.0	0,057	0.056	0.058	0.089	0.090	0.090	0.101
(channel)	8	λ ⁽⁸⁾	3		ļ	0.421	0.394	0.394	0.346	0.328	0.328	*
	9	λ(9)	4				0.0	*	*	*	*	*
	10	λ(10)	5					0.0	0.058	0.056	0.056	0.062
i	11	λ(11)	6						0.340	0.359	0.359	0.426
	12	λ ⁽¹²⁾	7							0.066	0.066	0.097
	13	λ ⁽¹³⁾	8								0.100	0.221
	14	λ ⁽¹⁴⁾	9									0, 095

Table 9. Changing Pattern of Weight Variables $\lambda^{(k)}$, $\mu^{(l)}$ with respect to Steps (Case 2).



planning with a needed modification of alternatives of channel construction planning. However, the latter stages represent a process to improve alternatives of channel construction planning after the optimal alternative, that will eventually compose the global optimal solution, is already obtained in the preceding stages.

This fact proves the validity of the existing planning process which attaches much importance to the planning of dams, and afterwards treats of the planning channels.



6. Conclusions

The water supply problem will undoubtedly see a severe shortage of available water. To solve this kind of problem, it is reasonable to develop fresh water in a broad scope. Especially in those regions undergoing an unbalanced use of water, such as a tremendous increase in water demand by one part of the region, and a much lesser demand of water with oppulent fresh water to develop in another part of the region, it is reasonable to take the means to develop water in an extensive area of the region, including an interbasin water distribution planning. As far as the financial aspect is concerned, it proves to be reasonable to make a plan of dams and channels in a broad scope.

In that case, a question arises whether the planning of dams should preceed that of channels, as traditionally peformed in our conventional plannings. This is based on the fear that this kind of large-scale water development might become highly expensive, and that the traditional way of planning may not lead to a reasonably integrated plan, even from the viewpoint of total cost.

In this respect, the following three functions are incorporated into the model: (a) The function that corresponds to selecting alternatives of both dam sites and scales in the planning of dams.

(b) The function that corresponds to selecting alternatives of the routes and scales of channels connecting each demand regions with each dam in the planning of channels.(c) The function that corresponds to the integrating process where those alternatives obtained by (a) and (b) are mutually adjusted.

We have already explain that our model possesses the three functions, and that the integrating process which corresponds to the 3rd function may be mathematically interpreted in terms of the algorithm based on the decomposition principle.

A System Analysis for the Planning of Facilities in Inter-basin Water Distribution Problems

The results of the presented case studies applied to the interbasin water distribution planning in Hyogo Prefecture ensures the validity of the traditional way of planning, which means the planning of channels before that of dams, and never systematically treats both plannings in the same framework. However, there is much likelihood that costs for constructing channels on a large scale will be more expensive in other places and in future years, owing to technical, geographical and other physical reasons as well as social and economical reasons. Another point that should be checked is to what extent the amounts of water demands may differ from those taken in the model. In such cases, it is obviously understood that such traditional planning will not lead to a reasonable plan. This possibility is suggested by the results of the sensitivity analysis conducted for larger amounts of water demand. The results proved to be different from those obtained before.

(1) Cost Estimations

As explained above, the estimated costs ensure the validity of linear relation between the construction of dams or channels and their scales. However, this assumption will not necessarily hold true with more precisely estimated costs, or those calculated in a different manner. This is obvious merely by the fact that more than half of the construction costs of dams is consumed by the costs for land purchase and attendant costs such as repair work of roads, building etc.

This means that in a more rigorous sense of the word the relation between construction costs and their scales may be nonlinear. However, the assumption of linearity as set up in the model is considered to be reasonably sufficient in the light of the required preciseness in the model. The model will merely serve as available information for considering future patterns of water distribution in a rough framework.

In this connection there is another problem, namely the evaluation criterion which we took as the total amount of construction costs in our study. The total cost may be taken as a reasonable criterion if our main concern is with the financial aspects of the planning. The measurability of costs is also a significant qualification for an evaluation criterion.

However, it is also true that there will be a limit to the use of funds. Studies are needed to deal with the trade-off which is inevitably attendant to these kinds of multipurpose problems.

(2) Water Demand Estimations

In our case studies, demand regions are set in the downstream area of each watershed under the assumption that demand regions located there will owe their water supply to the nearest river. This also assumes that the distribution of pipes conveying water to each user will be laid down by following the existing distribution system. However, the estimation will differ according to the way of allotting the water demands to each watershed. In that case, calculation with different parameter values will result in another outcome. In addition, there is a problem of error attendant with the forecast of water demand.

One of the available approaches to solve this kind of problem is to improve the preciseness of forecast by (1) analyzing dominant factors which have much effect on the increase of water demands, and by (2) carefully considering their possible future trends. It should also be noted that any improved preciseness of forecast in each factor will not necessarily lead to a more precise estimate of water demand forecast.

Another possible approach is to apply a sensitivity technique, which will present other solutions for given changes in parameters. Our main concern in using the technique is to know whether, or how the original optimal solution may change for such changes.

(3) Other Problems Precluded in The Model

The model precludes an examination of various problems associated with water distribution problems. They may be listed as follows.

(a) There are different kinds of system levels at which water distribution planning may be made. For example, the model does not consider municipal water supply problems.

(b) The model does not consider the different technical problems such as hydrological, hydraulic and potamological problems.

(c) The model also excludes ecological examinations such as the impacts on the environment of the constructed inter-basin channels etc.

(d) In this study, social and economical problems such as the impacts of the projects on the regional activities are not taken into consideration.

In spite of the various problems unsolved by the model, our study has shown that the model may present important information for the inter-basin water distribution planning in years to come. We are ready to improve the model in an effort to solve the above-mentioned problems.

Aknowledgments

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