A Mathematical Model for the Capacity Expansion Problem of Inter-regional Water Supply Facilities

By

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Synopsis

This paper concerns the capacity expansion problem of inter-regional water supply facilities, and presents a mathematical model to analyze the problem. The model postulates such an inter-regional water supply system that is managed by a body which is independent of municipalites; and provides water to municipalities by conveying purified water to distribution reserviors in the municipalities through conduits laid between them. The formulated model, which belongs to a nonlinear programming, is solved by both an enumeration method and dynamic programming. A case study was conducted by applying the model to the capacity expansion problem in the region consisting of Takasago, Kakogawa and Akashi Cities in Hyogo Prefecture The calculated results show that such a management system as is presumed in the model, is economically effective mainly because joint expansion for the increased supply in several municipalities will save associated costs, provided that unnecessarily excessive expansion is avoided.

1. Introduction

In Japan, a popular method of expanding water supply capacities is that each municipality manages to build water supply facilities, completely independent from other municipalities, to secure water which is demanded within the municipality. However, such a method of expansion is not considered reasonable for the following reasons.

(1) Unexpected high rates of demand increase are forcing each municipality into a situation where water demands exceed the forecast quantity.

(2) In highly urbanized areas, most available water sources have been developed. Some have become unsuitable for water intake mainly because the aggravated water quality resulting from the integrated urban activities.

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(3) Increased amounts of water demands are burdening each municipality with extremely heavy expenses.
(4) The urban growth is extending beyond city limits. This means that capacity expansion of water supply facilities within each municapality would not be necessarily reasonable.
(5) Especially for this reason, many regional development plannings are carried out in a broad scope, and land use plans are drawn up. Moreover, a water distribution project is required to be made in such a large-scale that would permit water to be conveyed from one river to another through artificial channels. This tendency means the necessity of providing a water supply project in a broader frame, where the traditional management system of water facilities need not necessarily followed; and the possibility of an inter-regional water supply system is taken into account.

For those reasons, the study mainly deals with the capacity expansion problem of an inter-regional water supply system, which is assumed to be managed by a third sector completely independent of the concerned municipalities.

The capacity expansion problem involves a variety of matters, and should be discussed from various angles. Amongst those many matters, our main focus is placed on the analysis of location and time of expansion of water supply facilities. Hereafter, water supply facilities are limited to two kinds of facilities, namely filtration plants and distributing conduits. These facilities are assumed to be constructed between located filtration plants and those municipalities requiring water to be transported from other municipalities.

In the analysis of this problem those two points are important.
(1) A larger type of expansion will save costs chiefly due to scale merits. This suggests that separate expansions by municipalities would not be effective from an economical point of view. Therefore, such expansion projects should be unified in order to carry out a joint expansion in a scale as large as possible.
(2) An unecessarily large scale of expansions would not be considered effective even from economical viewpoints, because such expansions would lead to an "'idle' state, which means that part of the capacity of a facility is not used.
(3) A more integrated scale of expansion will require longer and larger distributing conduits, which would consequently require higher costs associated with this method.

In view of the above points, we will present a mathematical model for the analysis of the problem.

2. Assumptions of the Model

(1) In each stage of the project, the sum of all the capacities of the filtering plants exceeds, or at least equates, the total amount of water demands throughout the entire

region.

(2) Distribution Conduits conveying water from filtering plants to those cities needing water are constructed at the outset of the project. This is done in a scale equal to the maximal amounts of water through the conduits over the project period.

(3) The total amount of water which undergoes purification is already secured by certain means which will not be specified here.

(4) The system treats those processes from water purification to water distribution up to service reservoirs, excluding a water transmission process for conveying water to the individual user of water.

3. Model Formulation

First notations used here are given as follows.

k, l are indices representing cities.

M is the number of those cities included in the system.

i, j are indices representing stages of the project period.

T is the project period.

τ is the time length of each stage, with each length being equal.

Thus $T = n\tau_d$.

x_i^k represents the scale of the extended filtering plant in city k in stage i.

y_i^{kl} represents the amount of water transported from city $k(k \in A_i)$ to city $l(l \in \bar{A}_i)$ in stage i.

A_i denotes the set of those cities with a surplus of suppliable water.

$\bar{A}_i$ denotes the set of those cities in need of suppliable water.

$D^k(i)$ represents the water demand in city k in stage i. This is a function with respect to i.

$O^k(i)$ represents the construction cost of the extended filtering plant in city k in stage i. This is a function of x_i^l.

$O(w_i^k)$ represents the management cost of the extended filtering plant in city k in stage i. This is a function of w_i^k which is defined as:

$$w_i^k = \begin{cases} D^k(i) + \sum_l y_i^{kl} & \text{for } k \in A_i,\ l \in \bar{A}_i, \\ D^k(i) - \sum_l y_i^{kl} & \text{for } k \in \bar{A}_i,\ l \in A_i. \end{cases} \tag{1}$$

$K(\hat{y}^{kl})$ represents the construction cost of the water main connecting the service reservoir of city k with that of city l. This is a function of $\hat{y}^{kl}$ which is defined as:

$$\hat{y}^{kl} = \max_i \{y_1^{kl}, y_2^{kl}, \cdots, y_i^{kl}, \cdots, y_n^{lk}\}. \tag{2}$$

$P(y_i^{kl})$ represents the management cost of the distribution conduit connecting the service reservoir of city k with that of city l in stage i. For convenience of notation $D^k(i)$ is also expressed as D_i^k, if there is no danger of ambiguity.

According as the notations are defined above, the constraints of the model are formulated as follows:

$$\sum_{i=1}^{j} x_i^k \geqq D_j^k + \sum_l y_j^{kl} \qquad (k \in A_i,\ i \in \bar{A}_i),\ (j=1, 2, \cdots, n) \tag{3}$$

$$D_j > \sum_{i=1}^{j} x_i^k \geqq D_j^k - \sum_l y_j^{lk} \qquad (k \in A_i,\ l \in \bar{A}_i),\ (j=1, 2, \cdots, n) \tag{4}$$

$$\sum_{i=1}^{j} \sum_{k=1}^{M} x_i^k \geqq \sum_{k=1}^{M} D_j^k \qquad (j=1, 2, \cdots, n). \tag{5}$$

Before going into the formulation of the objective function, its underlying criterion for an evaluation of the system will be presented.

The evaluation criterion taken by the model is that such a pattern of expansion is optimal, if it can estimate the minimal total amount of repayments plus management costs over the project period. The reasons are as follows.

(1) Since most of the capital needed for facility expansion projects are due to public bounds, the major concern of the public organization is the amount of periodical repayments rather than the total cost of the project.

(2) The amount of repayment within a certain period is regarded as the extent of financial pressure on the organization. This means that if the facility is overly capacitated, the amount may be considered as a dominant criterion for the estimation of 'idle costs'.

(3) In terms of cost accounting of the facility as fixed property, depreciation accounts over the project period correspond to the concept of the sinking fund for redemption. Moreover, since the price of water supplied to each user is calculated on the basis of cost price, total depreciation accounts over the period may be a greater concern to each user as well as to the public organization.

(4) In this context, total repayments over the period should be calculated as total depreciation accounts—by use of the following coefficient $g(r)$:

$$g(r) = \frac{r(1+r)^m}{(1+r)^m - 1},$$

where r and m represent the annual interest rate of the loan capital and the amortization period respectively. This is called the 'Coefficient of the Sinking Fund Depreciation', which is multiplied by the total cost to give annual repayments.

Then, the sum of repayments over the project period for the filtration plants constructed in stage i is expressed as

$$S(x_i^k, i) = (n-i+1) \cdot \tau_d \cdot \frac{r(1+r)^{30}}{(1+r)^{30} - 1} \cdot C(x_i^k). \tag{6}$$

As stated above, water mains connecting city k with city l are constructed at the outset of the project period. Then the sum of repayments over the project period for the constructed distribution conduits is expressed as

$$R(\hat{y}^{kl})=n\tau_d\cdot\frac{r(1+r)^{30}}{(1+r)^{30}-1}\cdot K(\hat{y}^{kl}). \tag{7}$$

Through the above discussions, the objective function framed into the model is written as follows:

$$Z=\sum_{i=1}^{n}\sum_{k=1}^{M}S(x_i^k, i)+\sum_{k}\sum_{l}R(\hat{y}^{kl})+\sum_{i=1}^{n}\sum_{k=1}^{M}O(w_i^k)+\sum_{i=1}^{n}\sum_{k=1}^{M}\sum_{l=1}^{M}P(y_i^{kl}) \longrightarrow \text{min.} \tag{8}$$

Thus, we have formulated our model which is expressed by equations (1) to (8) where unknown variables are x_i^k, y_i^{kl}. ($i=1, 2, \cdots, n$; $k, l=1, 2, \cdots, m$) The model belongs to a nonlinear programming which has already been approached through various methods according to the feature of formulated programs. We observe a large number of unknown variables as well as constraints, which might involve complications and also a tremendous number of calculations. In this view, we propose two efficient methods, namely an enumeration method and dynamic programming in order to solve the model.

4. Algorithm for Solving the Model

We first observe that the minimum and maximum of the scale of the filtration plant may reasonably be set by practically considering the technical and economical problems associated with the construction and operation of the filtration plant.

Let $\varDelta$ denote the minimum unit of the scale of the filtration plants, and $x_{\max}$ the maximum of the scale. The values of x_i^k, y_i^{kl} are continuous and can take any set of values that satisfy the constraints. However, we assume that x_i^k can merely take the following discrete values as

$$x_i^k=0, \varDelta, 2\varDelta, \cdots, R\varDelta\leqq x_{\max}.$$

For the given x_i^k the range of the values of y_i^{kl} is correspondently determined, since both x_i^k and y_i^{kl} are mutually correlated to satisfy the constraints. Thus, for the given $\varDelta$ and $x_{\max}$, both the minimum and maximum of the scale of y_i^{kl} can be determined. Let δ denote the minimum unit of the scale of y_i^{kl} and $y_{\max}$ the maximal unit of the scale. Then we may set the values of y_i^{kl} as:

$$y_i^{kl}=0, \delta, 2\delta, \cdots, r\delta\leqq y_{\max}.$$

For x_i^k and y_i^{kl} as thus determined, the number of combinations of the values of

x_i^k can be calculated as $(R\Delta)^{nM}$. This means that with an increase in the number of cities M and terms n, the number of combinations of the values of x_i^k exponentially increases.

The same is true of the combinations of the values of y_i^{kl}. In this respect, the enumeration method is applicable only to a small number of cities and terms.

5. Enumeration Method

After setting the minimum unit of plants and their maximum scale, we can select all the possible combinations (patterns) of sclaes of plants with respect to project stages. They are expressed in terms of a matrix Ξ as:

$$\Xi = \begin{bmatrix} \xi_{11} & \xi_{12} & \cdots & \xi_{1n} \\ \vdots & \vdots & & \vdots \\ \xi_{p1} & \xi_{p2} & \cdots & \xi_{pn} \\ \vdots & \vdots & & \vdots \\ \xi_{p1} & \xi_{p2} & \cdots & \xi_{pn} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\xi}_1 \\ \vdots \\ \boldsymbol{\xi}_p \\ \vdots \\ \boldsymbol{\xi}_p \end{bmatrix} \tag{9}$$

, where ξ_{pi} $(i=1, \cdots, n; p=1, \cdots, p)$ represents the expansion scale in pattern p at stage i; and $\boldsymbol{\xi}_p=(\xi_{p1} \cdots \xi_{pi} \cdots \xi_{pn})$ is vector representing the expansion scales of pattern p.

Let P_l represent the pattern of city l; and we then get

$$x_i^l = \xi_{p_l,i}. \tag{10}$$

Any one of the possible combinations of the pattern with respect to each city is expressed by the matrix X_q as:

$$X_q = \begin{bmatrix} \xi_{P_11} & \xi_{P_12} & \cdots & \xi_{P_1n} \\ \vdots & \vdots & & \vdots \\ \xi_{P_l1} & \xi_{P_l2} & \cdots & \xi_{P_ln} \\ \vdots & \vdots & & \vdots \\ \xi_{P_L1} & \xi_{P_L2} & \cdots & \xi_{P_Ln} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\xi}_{P_1} \\ \vdots \\ \boldsymbol{\xi}_{P_2} \\ \vdots \\ \boldsymbol{\xi}_{P_L} \end{bmatrix} \tag{11}$$

, where q represents an alternative of possible combinations of all the patterns P_1, P_2, $\cdots$, P_L with respect to each city. The total number of all the possible combinations is expressed as:

$$Q=(P)^L.$$

For given the ξ_{P_li}, the values of y_i^{kil} are obtained and referred to as $\eta_{i,q}^{kl}$, which satisfies the following constraints:

$$\sum_{i=1}^{i} \xi_{P_ki} - \sum_{l} \eta_{i,q}^{kl} = D_i^k \qquad \text{for} \quad k \in A_i \tag{12}$$

$$\sum_{i=1}^{i} \xi_{P_li} + \sum_{k} \eta_{i,q}^{kl} = D_i^l \qquad \text{for} \quad l \in \bar{A}_i \tag{13}$$

$$\sum_{l=1}^{L} \sum_{i=1}^{i} \xi_{P_li} = \sum_{l=1}^{L} D_i^l \qquad (i=1, 2, \cdots, n). \tag{14}$$

If some patterns of expansions do not satisfy the above constraints, they are excluded from feasible patterns.

Then for each set X_q, whose elements are $\eta_{i,q}^{kl}$, the value of the objective function is calculated and referred to as Z_q. Likewise, S_i^l, $R_{i,q}^{kl}$, $O_{i,q}^l$, $P_{i,q}^{kl}$ are referred to as $S_{i,q}^l$, $R_{i,q}^{kl}$, $O_{i,q}^l$, $P_{i,q}^{kl}$.

$$Z_q=\sum_{l=1}^{L}\sum_{i=1}^{n} S_{i,q}^l+\sum_{k}\sum_{l} R_{,q}^{kl}+\sum_{l=1}^{L}\sum_{i=1}^{n} O_{i,q}^l+\sum_{k}\sum_{l}\sum_{i=1}^{n} P_{i,q}^{kl}. \tag{15}$$

Then, the approximate optimal solution for the enumaration problem corresponds to the set $X_q{}^0$ for Z^0 as obtained by comparing the values of Z_q ($q=1, 2, \cdots, Q$).

$$Z^0=\min_{q^0}(Z_1, Z_2, \cdots, Z_q, \cdots, Z_Q).$$

6. Dynamic Programming Approach

As stated before, the enumeration method can be practically applied to deal with those cases where the number of cities and terms is small, at most 3 or 4. Otherwise, a method using the dynamic principle can be efficiently applied to solve the problem.

First, we rewrite the objective function Z as:

$$Z=\sum_{i=1}^{n}\left\{\sum_{k=1}^{M} S(x_i^k,i)+\sum_{k}\sum_{l} R(y_i^{kl})+\sum_{k=1}^{M} O(y_i^k)+\sum_{k}\sum_{l} P(y_i^{kl})\right\} \tag{16}$$

where

$$R(y_i^{kl})=\begin{cases} O & \text{for } i=1, 2, \cdots, n-1 \\ R & \text{for } i=n \end{cases} \tag{17}$$

$$y_i^k=(y_i^{k1}, \cdots, y_i^{kl}, \cdots, y_i^{kn}).$$

If we set

$$Z_j(\boldsymbol{x}_j, Y_j)=\min\sum_{i=1}^{j}\left\{\sum_{k=1}^{M} S(x_i^k, i)+\sum_{k}\sum_{l} R(y_i^{kl})+\sum_{k=1}^{M} O(y_i^k)+\sum_{k}\sum_{l} P(y_i^{kl})\right\} \tag{18}$$

and

$$\boldsymbol{x}_j=(x_j^1 \cdots x_j^M)$$

$$Y_j=(y_j^1 \cdots y_j^k \cdots y_j^M)$$

, then by applying the dynamic principle to the problem we get the following recursive relation as:

$$Z_j(\boldsymbol{x}_j, Y_j)=\min\left[\left\{\sum_{k=1}^{M} S(x_j^k,j)+\sum_{k}\sum_{l} R(y_j^{kl})+\sum_{k=1}^{M} O(y_j^k)+\sum_{k}\sum_{l} P(y_j^{kl})\right\}\right.$$
$$\left.+Z_{j-1}(\boldsymbol{x}_{j-1}, Y_{j-1})\right], \tag{19}$$

where

$$Z_1(\boldsymbol{x}_1, Y_1)=\sum_{k=1}^{M} S(x_1^k, 1)+\sum_{k}\sum_{l} R(y_1^{kl})+\sum_{k=1}^{M} O(y_1^k)+\sum_{k}\sum_{l} P(y_1^{kl}). \tag{20}$$

The outline of the process of the solution algorithm may be stated as follows. First we begin with the case for $j=1$ and set

$$x_1^k=0, \Delta, \cdots, R\Delta \leqq x_{\max} \qquad (k=1, \cdots, M),$$

and

$$y_1^{kl}=0, \delta, \cdots, r\delta \leqq y_{\max} \qquad (k=1, \cdots, M).$$

Then for each value of x_1^k (x_1^l) and y_1^{kl} we check whether city $k(l)$ belongs to the set A_1 or $\bar{A}_1$ and whether the values of x_1^k (x_1^l) and y_1^{kl} satisfy the constraints. Next, for those values of x_1^k and y_1^{kl} which are found to be feasible, we calculate $Z_1(\boldsymbol{x}_1, Y_1)$. Likewise for $i=2$, we calculate $Z_1(\boldsymbol{x}_2, Y_2)$ by checking the values of x_2^k and y_2^{kl} and using the values of $Z_1(\boldsymbol{x}_1, Y_1)$ as determined in the previous process. By continuing the same procedure we finally obtain the value of $Z_n(\boldsymbol{x}_n, Y_n)$ for $j=n$. Here we find $\hat{Z}_n(\hat{\boldsymbol{x}}_n, \hat{Y}_n)$ such that

$$\hat{Z}_n(\hat{\boldsymbol{x}}_n, \hat{Y}_n)=\min\{Z_n(\boldsymbol{x}_n, Y_n)\}. \qquad (21)$$

It seems to be reasonable to conclude that $\hat{\boldsymbol{x}}_n$ and $\hat{Y}_n$ are the optimal solutions set for the problem and $\hat{Z}_n(\hat{\boldsymbol{x}}_n, \hat{Y}_n)$ gives the value of the objective function for it.

7. Case Study

(1) Region

We applied the mathematical model as formulated above to the capacity expansion planning of filtration plants in the region comprising the cities of Kakogawa and Takasago through which the Kakogawa River runs into the Seto Inland Sea. The main reasons for taking this region are as follows:

a) Both cities bear some close resemblances in geometry, land use pattern and population. For example, most parts of the two areas are open fields. They both have residential and commercial areas in their central parts. In their southern parts along the Seto Inland Sea, there are industrial areas where heavy industries demanding much water are dominant.

b) Both cities collect water from the Kamo River.

c) In one section, an inter-regional water supply system is already in operation between the two cities.

d) For those reasons, it might be quite significant to consider an inter-regional water supply system in the region which includes the two cities.

(2) Predicted Water Demand

In predicting the water demand for the coming years in the region concerned, we assumed

$$q_t = at + b$$

$$P_t = \frac{K}{1 + e^{\alpha - \beta x}}$$

$$Q_t = q_t P_t,$$

where

q_t: maximum quantity to be supplied per capita in year t.

P_t: population in year t.

Q_t: total quantity to be supplied in year t.

$a, b; \alpha, \beta_t$: parameters determined by use of regression analysis to the past data on P_t and q_t.

Predicted Q_t thus estimated is illustrated in Fig. 1. This shows that Q_t may be approximately regarded as a linear function of t.

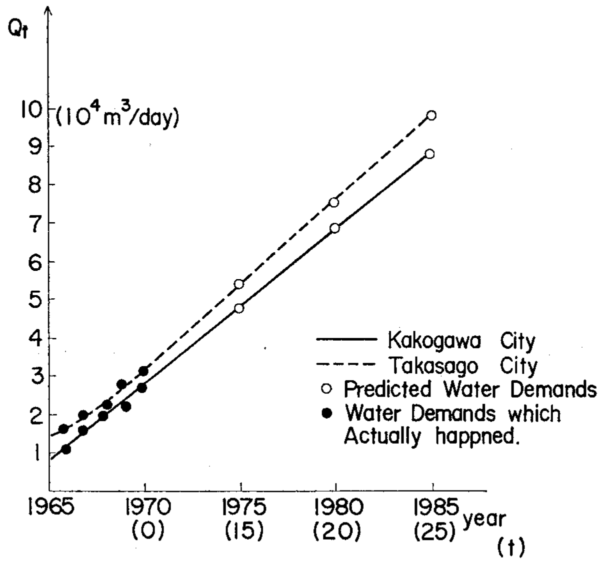

Fig. 1. Predicted Water Demands.

(3) Functions and Parameters

Functions and values of their parameters are listed in Table 1.

(4) Results of the Case Study

With the values and forms of parameters and functions as shown in Table 1, we considered the following cases.

a) Case 1 and Case 2

We consider Case 1 as a fundamental example. The annual interest rate r is set at 0.065 for Case 1, and 0.070 for Case 2. Q_t, which is the total quantity of water demand at time t is assumed to follow the predicted linear equation with respect to

Table 1. Input Data of Case Studies

SYMBOL & NOTATION	STANDS FOR	ARE ASSUMED TO TAKE THE FOLLOWING VALUES OR FORMS
T	entire time length (project period)	15 yrs.
τ_d	length of each period (constant for each)	5 yrs. for Cases 1 to 3. 3 yrs for Case4.
n	number of periods	3 for Cases 1 to 3. 5 for Case 4.
r	annual rate of interest	0.065 (rate for the local bond) 0.070 (rate for the national treasury)
$C(x_i^k)$	Construction cost for the filtration plant in city k in period i (function of x_i^k)	$\{1.40\times10^7\times(x_i^k)^{-0.75}+1000\}\cdot x_i^k$
$O(w_i^k)$	Operation cost for the filtration plant in city k in period i (function of w_i^k)	$1.4\,w_i^k$
$K(\bar{y}^{kl})$	Construction cost for the distribution conduits between cities k and l. (function of $\bar{y}^{kl}$)	see Table 4
$P(y_i^{kl})$	Operation cost for the distribution conduits between cities k and l. (function of y_i^{kl})	$1.5\,y_i^{kl}$

PATTERNS OF ANNUAL DEMAND INCREASE (Case 1, Case 2, Case 3)

DEMAND PATTERN (PATTERN OF ΔQ_t)	Kakogawa City (10^4 m^3/day/yr.)	Takasago City (10^4 m^3/day/yr.)
A	0.41	0.45
B	0.33	0.36
C	0.50	0.54

PATTERNS OF DEMAND INCREASE (Case 4)

NO. OF CITY \ PERIOD	PERIOD 1	PERIOD 2	PERIOD 3	PERIOD 4	PERIOD 5
1	1.0 (10^4 m^3/day)	1.0	2.0	3.0	5.0
2	1.0	2.0	3.0	4.0	5.0
3	2.0	4.0	5.0	6.0	7.0

time, as mentioned before. Therefore, the annual demand increase ΔQ_t, which is equal to $Q_t - Q_{t-1}$, is assumed to take 0.41×10^4m^3/day/yr. and 0.45×10^4m^3/day/yr. for Kakogawa City and Takasago City, respectively.

Other parameters are set to take the valuesas listed in Table 1.

The results of calculations using both the enumeration method and dynamic programming are shown in Table 2 and Fig. 2. They show that:

1) It is better to expand a filtration plant on a large scale in one of the cities rather

Table 2 Calculation Results

(r=0.065)

DEMAND PATTERN	ALTERNATIVE	Capacity Expansion of City Kakogawa (10^4 m^3/day)			Capacity Expansion of City Takasago (10^4 m^3/day)			Management Cost for Dsitribution Conduits (over the project period)	Sum of Repayments for the Construction Costs for Filtration Plants (over the project period)	Sum of Repayments for the Construction Costs for Distribution Conduits (over the project period)	Management Cost for Filtration Plants (over the project period)	Total Associated Costs
		PERIOD 1	PERIOD 2	PERIOD 3	PERIOD 1	PERIOD 2	PERIOD 3					
DEMAND PATTERN A	1	0.00	4.50	4.50	4.50	4.50	9.00	4, 912	150, 223	2, 006	497, 416	654, 557
	2	4.50	4.50	4.50	0.00	4.50	9.00	5, 111	150, 223	2, 006	497, 416	654, 758
	3	0.00	4.50	9.00	4.50	4.50	4.50	5, 846	150, 223	20, 06	497, 416	655, 492
	4	4.50	4.50	6.75	4.50	4.50	6.75	0	180, 582	0	497, 416	677, 998
DEMAND PATTERN B	1	0.00	3.60	3.60	3.60	3.60	7.20	3, 938	134, 685	1, 192	397, 933	530, 069
	2	3.60	3.60	3.60	0.00	3.60	7.20	4, 097	134, 685	1, 192	397, 933	530, 229
	3	0.00	3.60	7.20	3.60	3.60	3.60	4, 644	134, 685	1, 192	397, 933	530, 820
	4	3.60	3.60	5.40	3.60	3.60	5.40	0	153, 206	0	397, 933	551, 139
DEMAND PATTERN C	1	0.00	5.40	5.40	5.40	5.40	10.80	5, 836	173, 083	2, 112	596, 899	777, 825
	2	5.40	5.40	5.40	0.00	5.40	10.80	6, 075	173, 083	2, 112	596, 899	778, 064
	3	0.00	5.40	10.80	5.40	5.40	5.40	6, 946	173, 083	2, 112	596, 899	778, 936
	4	5.40	5.40	8.10	5.40	5.40	8.10	0	207, 500	0	596, 899	804, 399

←(Million Yen)→

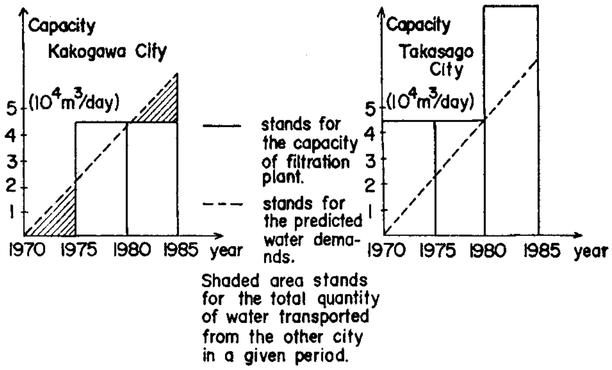

Fig. 2. the Optimal Expansion Pattern of Filtration Plants in the Region Comprising Kakogawa City and Takasago City.

than build plants in both of the cities.

2) Although the expansion of a plant on a large scale in either of the cities is more advantageous than separate expansions of plants in each city, the scale should not exceed the total demand occured over the period in the entire region.

3) This does not necessarily mean an advantage of constructing plants in every period in a fixed city. It is also necessary to consider the increased costs of the distribution conduits which convey water from one city to other cities. The model presents the optimal pattern of plant expansions over the entire period.

4) An increased rate of interest by a certain percentage will increase each construction cost equally by the same percentage. Consequently, it will lead to the increase of that percentage in the total amount of construction costs. This means that the increase in the annual interest rate r will not change the optimal pattern of expansion if, and only if, it will not change the optimal pattern of distribution conduits.

Quite similarly, if the interest rate r decreases, the total amount of construction costs will decrease. However, the optimal pattern of plants and other associated facilities will not change if, and only if, the optimal pattern of distribution conduits remains the same.

The results of those cases where the interest rate r increases are shown in Table 3. They show that because of a relatively higher order of construction costs, the optimal pattern of plants and other associated facilities will take the same pattern as the original value of r.

b) Case 3

Since the prediction of water demand growth can hardly be free from uncertainty factors and therefore, possibilities that the water demands will not really grow as predicted, it would be reasonable to consider the case where the growth pattern of water demands shifts from the predicted growth pattern. For this purpose, we set forth two

Table 3 Calculation Results

($r=0.070$)

DEMAND PATTERN	ALTERNATIVE	Capacity Expansion of City Kakogawa (10^4 m^3/day)			Capacity Expansion ot City Takasago (10^4 m^3/day)			Management Cost for Distribution Conduits (over the project period)	Sum of Repayments for the Construction Costs for Filtration Plants (over the project period)	Sum of Repayments for the Construction Costs for Distribution Conduits (over the project period)	Management Cost for Filtration Plants (over the project period)	Total Associated Costs
		PERIOD 1	PERIOD 2	PERIOD 3	PERIOD 1	PERIOD 2	PERIOD 3					
DEMAND PATTERN A	1	0.00	4.50	4.50	4.50	4.50	9.00	4,912	158,088	2,112	497,416	662,528
	2	4.50	4.50	4.50	0.00	4.50	9.00	5,111	158,088	2,112	497,416	662,728
	3	0.00	4.50	9.00	4.50	4.50	4.50	5,846	158,088	2,112	497,416	663,463
	4	4.50	4.50	6.75	4.50	4.50	6.75	0	190,036	0	497,416	687,452
DEMAND PATTERN B	1	0.00	3.60	3.60	3.60	3.60	7.20	3,938	133,654	1,255	397,933	536,781
	2	3.60	3.60	3.60	0.00	3.60	7.20	4,097	133,654	1,255	397,933	536,941
	3	0.00	3.60	7.20	3.60	3.60	3.60	4,644	133,654	1,255	397,933	537,532
	4	3.60	3.60	5.40	3.60	3.60	5.40	0	161,226	0	397,933	559,159
DEMAND PATTERN C	1	0.00	5.40	5.40	5.40	5.40	10.80	5,836	182,144	2,112	596,899	786,992
	2	5.40	5.40	5.40	0.00	5.40	10.80	6,075	182,144	2,112	596,899	787,232
	3	0.00	5.40	10.80	5.40	5.40	5.40	6,946	182,144	2,112	596,899	788,134
	4	5.40	5.40	8.10	5.40	5.40	8.10	0	218,362	0	596,899	815,261

← (Million Yen) →

growth patterns of water demands other than the original pattern (of Case 1.) The results are shown in Table 2 and Table 3. They show that the optimal pattern of expansion will not change even for the increased water demand of each city by 20%. The main reason is that the marginal cost of plant construction slightly changes with the increase in the scale of plants in question.

c) Case 4

In Cases 1 to 3, we dealt with the region comprising Kakogawa City and Takasago City, and considered the optimal pattern of the expansion of plants and other associated facilities. Here, we consider the expansion problem of another region, comprising three unspecified cities in 3 periods, and then re-examine the general results obtained from the previous cases. In this case, the approach of the dynamic principle becomes more useful than the enumeration method because of the increase in the number of cities and periods.

Fig. 3 shows the results, whereby it is clear that even if the number of cities and periods increases, the general results obtained in Cases 1 to 4 will have the same tendency.

8. Conclusions

Since the marginal cost of a filtration plant decreases with the increase in scale, separate constructions in each city will not be effective if our interest is to minimize the total cost.

The optimal scale of the expansion in each period should be no more than the total demand growth in the concerned municipalities.

The municipality which should receive preference over others in the expansion of concerned facilities is the one which exceeds others in the rate of demand growth.

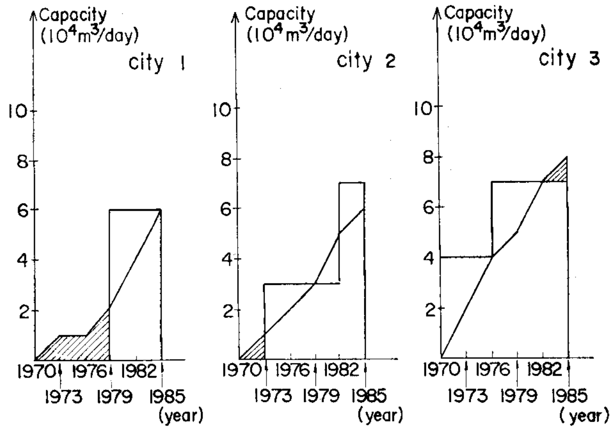

Fig. 3. the Optimal Expansion Pattern of Filtration Plants in the Region Comprising unspecified 3 cities.

Table 4 Estimated Costs for the Construction of Distribution Conduits (by pumping)
(per m)

*Optimal Diameter (mm)	Distribution Capacity of Conduits (m^3/day)	Materials Costs for Distribution Conduits (m^3/day)	Total Construction Costs for Distribution Conduits (yen)
200	2590	631	1011
300	5000	952	2118
400	8640	1376	2616
500	18000	1868	3341
600	30000	2464	4248
700	43200	3263	5442
1000	86400	6000	8973

* There may be many kinds of combinations of diameter and capacity of conduits with attached pumping facilities. But the optimal diameter from economical viewpoints can be selected among them.

In practice, within the scale as considered here, it might be reasonable to examine first the optimal pattern of plant expansion. Next comes the examination of the costs needed to distribute water from one municipality to others. Then, by taking into account both costs, the optimal pattern of the expansion of plants and related distribution facilities can be decided.

It was found that the above results hold true even if the interest rate, costs and demands should change their values.

It is not true that a pattern of construction most advantageous to an entire region will be just as favorable to particular sections within the region.

The model excludes other important facilities such as distribution reservoirs, distributing pipes, service pipes etc. Actually, the costs of the excluded facilities may sometimes amount to as much as those considered in the model. In such cases, the excluded costs will not be negligible, and those models which can include such costs should be developed.

The estimated value of costs associated with the facilities in each city are assumed to be independent of the location places, and set to be uniquely equal. However, in many cases this will not hold true, and an estimation of the costs should be carefully examined so that local differences in values can be reasonably reflected.

If the numbers of municipalities and the entire time length amount to more than 5, the necessary calculations will become so great that practical applications of the model without any fujther modifications would be difficult, even with the implementation of the most modern high-speed electronic computers. In these cases, one way to solve the problem is to decompose the model, and to find some efficient algorithms to combine the results.

As mentioned in the introduction, the problem of capacity expansion involves a variety of matters, some of which are precluded from the study. One precluded important technical problem is the possibility of conveying water from one municipality to another in a certain period of time; and then the reverse, from another place back to the municipality in another period, always using the same conduits connecting two regions. It has already been examined by technical experts in this field, and felt to be faintly possible. However, it would require some preparatory operations before switching the direction of distribution.

With those problems yet to be examined in further detail, the model proved to be of practical help for studying the complicated problem chiefly from a financial point of view.

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